Stochastic All-to-All Propagators for Baryon Correlators

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Motivation

LHPC Spectrum effort:

- Goal: extract a large number of low-lying excited hadron states
- Requires a large variational basis of operators
- Multi-particle and non-zero momentum operators are required to identify multi-hadron states in the spectrum
Estimate all elements of the quark propagator, $M^{-1}_{(\alpha a|\beta b)}(x, t|x_0, t_0)$:

- Generate $N_r$ random ($Z_4$) sources: $\eta^{(r)}_{\alpha a}(x, t)$
- Solve

$$M_{(\alpha a|\beta b)}(x, t|x', t') \phi^{(r)}_{\beta b}(x', t') = \eta^{(r)}_{\alpha a}(x, t)$$

for the $\phi^{(r)}_{\alpha a}(x, t)$
- The quark propagator is given by

$$M^{-1}_{(\alpha a|\beta b)}(x, t|x_0, t_0) = E[\phi_{\alpha a}(x, t) \eta^*_{\beta b}(x_0, t_0)]$$
The Dilution Method [1]

- Instead of adding more noise sources, **Dilute** a single noise source:

\[
\eta_{\alpha a}^{(r)}(x, t) = \sum_{d=1}^{N_d} P_{(\alpha a|\beta b)}^{[d]}(x, t|x', t') \eta_{\beta b}^{(r)}(x', t')
\]

\[
= \sum_{d=1}^{N_d} \eta_{\alpha a}^{(r)[d]}(x, t)
\]

- Solve

\[
M_{(\alpha a|\beta b)}(x, t|x', t') \phi_{\beta b}^{(r)[d]}(x', t') = \eta_{\alpha a}^{(r)[d]}(x, t)
\]

- Examples of dilution schemes:
  - Time
  - Time + spin + color
  - Time + spatial even-odd
Given the $\phi^{(r)}_{\alpha a}(x, t)$ and $\eta^{(r)}_{\alpha a}(x, t)$,

- Form source and sink baryon operators:

$$
\Gamma^{(r)[dA dB dC]}_{\ell}(t) = c^{(\ell)}_{\alpha \beta \gamma; ijk} \sum_x \epsilon_{abc} \phi^{(r)[dA]}_{\alpha ai}(x, t) \phi^{(r)[dB]}_{\beta bj}(x, t) \times \\
\phi^{(r)[dC]}_{\gamma ck}(x, t)
$$

$$
\Omega^{(r)[dA dB dC]}_{\ell}(t) = c^{(\ell)}_{\alpha \beta \gamma; ijk} \sum_x \epsilon_{abc} \eta^{(r)[dA]}_{\alpha ai}(x, t) \eta^{(r)[dB]}_{\beta bj}(x, t) \times \\
\eta^{(r)[dC]}_{\gamma ck}(x, t)
$$

- Combine them to form two-point functions
Advantages of the Dilution Method

- Expected to approach exact all-to-all \textit{faster than} $1/\sqrt{N_d}$ as $N_d \rightarrow N_t \times N_s \times N_c \times V$

- \textit{Complete factorization} of source and sink in correlation functions
  - Great for a large variational basis
  - Use the same operators to make multi-hadrons

- All elements of quark propagator are calculated
  - Non-zero momentum projections require spatial sum at source
  -Disconnected Diagrams
Dilution Scheme Tests

- 100 quenched gauge configurations with: \( L_s = 12, \)
  \( L_t = 48, \) \( a_s \approx 0.1 \) fm, \( \beta = 6.1, \) \( m_\pi \approx 700 \) MeV

- Choose a few relevant observables for comparison of dilution schemes, point-to-all

**Question**: Assuming time dilution, is it better to add more noise sources or more dilution projectors?
Examined Single-Site, Singly-Displaced, and Triply-Displaced baryon operators

- Single-site
- Singly-displaced
- Triply-displaced

Diagonal correlators evaluated at several time separations is the measure of choice
Results

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Single-Site, \( t = 5 \)

- \( \text{time} \)
- \( \text{time} + \text{space}_\text{eo} \)
- \( \text{time} + \text{color} \)
- \( \text{time} + \text{spin} \)
- \( \text{time} + \text{color} + \text{space}_\text{eo} \)
- \( \text{time} + \text{spin} + \text{space}_\text{eo} \)
- \( \text{time} + \text{spin} + \text{color} \)
- \( \text{time} + \text{spin} + \text{color} + \text{space}_\text{eo} \)
- \( \text{pt-to-all} \)
- \( \text{noise limit} \)

Relative Error vs. \( 1/\sqrt{N_{\text{inv}}} \)
Results

Singly-Displaced, $t = 5$

Relative Error vs $1/\sqrt{N_{inv}}$

- time
- time + space eo
- time + color
- time + spin
- time + color + space eo
- time + spin + space eo
- time + spin + color
- time + spin + color + space eo
- pt-to-all
- noise limit

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Results

\[ \frac{1}{\sqrt{N_{\text{inv}}}} \]

Relative Error

\[ 0 \quad 0.02 \quad 0.04 \quad 0.06 \quad 0.08 \quad 0.1 \]

\[ 0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \]

\[ \text{time} \]

\[ \text{time} + \text{space}_{\text{eo}} \]

\[ \text{time} + \text{color} \]

\[ \text{time} + \text{spin} \]

\[ \text{time} + \text{color} + \text{space}_{\text{eo}} \]

\[ \text{time} + \text{spin} + \text{space}_{\text{eo}} \]

\[ \text{time} + \text{spin} + \text{color} \]

\[ \text{time} + \text{spin} + \text{color} + \text{space}_{\text{eo}} \]

\[ \text{pt-to-all} \]

\[ \text{noise limit} \]

Triply-Displaced, \( t = 5 \)
Results - Effective Masses

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Conclusions

- Adding more dilution projectors beats adding more noise sources, up to a point.
- Time + spin + color dilution is roughly equivalent to point-to-all method.
- Time + spin + color + spatial-even-odd dilution is consistent with the gauge noise.
- Currently working on an alternative method that gives exact all-to-all for less. Stay Tuned!
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