Computation of the string tension in three dimensional Yang-Mills theory using large N reduction

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Outline

- Quick result
- Introduction
- Details
- Conclusion
Quick result

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- $5^3$ lattice
- $N = 47$
- $b = \frac{1}{g^2 N} = 0.6$ to 0.8
- Wilson loops 1x1 to 7x7
- $\sqrt{\sigma b} = 0.1964 \pm 0.0009$ (continuum extrapolation)
Introduction

- Large N
- Large N reduction
- Phase structure
- Project description
Large $N$

- Expansion parameters \( \alpha(Q^2) \) or \( 1/N \)
- \( N \to \infty \) simplifications
- Planar graphs
- Factorization
- Non-interacting mesons
- OZI rule
- \( 1/3 \approx 1/\infty \)
Large N reduction

- Reduction to a one point $1^d$ lattice (Eguchi-Kawai)
- $Z^d_N$ center symmetry
- But broken at weak coupling
Work-arounds

- Quenched E-K
- But Bringoltz and Sharpe
- Twisted E-K
- But Teper and Vairinhos
- Continuum or partial reduction
  - i.e. reduction to finite physical size
  - \( l > 1/T_c \)
Center symmetry breaking at physical scale

\[ Z_N^{d} \rightarrow Z_N^{d-1} \rightarrow Z_N^{d-2} \rightarrow \ldots \]

4D 2-loop $\beta$–function for $L_e(b)$

Tadpole Improved

Kiskis, Narayanan, and Neuberger
Phase structure

3 dimensions

Figure 8: Summary of large $N$ QCD in $d = 3$ on $L^3$ lattice

Narayanan and Neuberger
Project

Context

- Karabali, Kim, and Nair
  \[ \sqrt{\sigma b} = \frac{1}{\sqrt{8\pi}} \approx 0.1995 \]

- Bringoltz and Teper
  - Large lattices
  - \( N \) up to 8
  - Polyakov loops
  \[ \sqrt{\sigma b} = 0.1975 \pm 0.0002 - 0.0005 \]
This work

- $5^3$ lattice

- $N = 47$

- $b = 0.6$ to $0.8$

- Smear space-like links with staples in the same time slice

- Wilson loops $1x1$ to $7x7$

- Fit to get quark-antiquark potential and string tension
Details

Wilson gauge field action with bare coupling $g$

$b = \frac{1}{g^2N}$ Tadpole improved to $b_I = e(b)b$ with $e(b)$ the average plaquette

Space-like and time-like separations $K, T$ in lattice units.

Physical units $k = K/b_I$ and $t = T/b_I$
\[ U' = P_{SU(N)}[(1 - f)U + \frac{f}{2}S_+ + \frac{f}{2}S_-] \]

Iterate \( n \) times

\[ \tau = fn \]

\[ f = 0.1 \quad n = 25 \quad \tau = 2.5 \]
Compute all Wilson loops 1x1 to 7x7

Fit to \( W(k, t) = e^{-a - m(k)t} \)
Fit $m(k)$ to

$$m(k) = \sigma b_I^2 k + c_0 b_I + \frac{c_1}{k}$$
Extrapolate: \( b_I \rightarrow \infty \quad \sqrt{\sigma b_I} \rightarrow 0.1964 \pm 0.0009 \)
Are N and L large enough?

\[ m(k) = \begin{cases} 
0.0342(11)k + 0.140(3) - 0.101(2)/k & N=47 \\
0.0341(14)k + 0.140(3) - 0.100(3)/k & N=41 \\
0.0339(14)k + 0.139(4) - 0.099(3)/k & N=37 \\
0.0330(17)k + 0.143(4) - 0.102(3)/k & N=31 \\
0.0323(22)k + 0.141(6) - 0.097(4)/k & N=23 
\end{cases} \]
$m(k) = 0.0316(23)k + 0.108(5) - 0.117(6)/k$

$5^3$

$
\begin{align*}
\text{Solid line:} & \quad 0.0316(23)k + 0.108(5) - 0.117(6)/k \\
\text{Dashed line:} & \quad 0.0296(15)k + 0.129(4) - 0.160(4)/k
\end{align*}$
Are the results sensitive to smearing?

\[ \tau = 1.25 \quad 0.0354(12)k + 0.134(3) - 0.093(3)/k \]

\[ \tau = 2.5 \quad 0.0342(11)k + 0.140(3) - 0.101(2)/k \]
Do Creutz ratios work well?

\[ \chi(k,k) \]

\[ b, I \]
Conclusion

- $5^3$ lattice
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- Wilson loops 1x1 to 7x7
- $\sqrt{\sigma b} = 0.1964 \pm 0.0009$ (continuum extrapolation)