Exotic phases of Finite Temperature $SU(N)$ gauge theories with massive fermions: F, Adj, A/S

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Lattice 2008
Motivation: To understand the effects of massive fermions on the phase diagram of $SU(N)$ gauge theories for fermions in various representations.

- Phase diagram of a simple deformed Yang-Mills theory formulated on the lattice
- Phase diagram of a more complicated deformed Yang-Mills theory (compare with QCD(Adj))
- Phase diagram of $SU(N)$ gauge theories with massive fermions from the one-loop effective potential
  - Fermion representations: Fundamental (F), Antisymmetric (A), Symmetric (S), and Adjoint (Adj)
  - Boundary conditions: periodic (PBC), antiperiodic (ABC)
  - $N_c = 2$ through 9.
  - various $N_f$
- One-loop contribution to $\langle \bar{\psi} \psi \rangle_R$ ($R = F, A, S, Adj$)
Lattice model

Last year we analyzed a simple deformed Yang-Mills theory on the lattice (Myers and Ogilvie 2008):

\[ S_{\text{lat, def}} = S_W + \sum_x V_{\text{lat, def}}[P(x)] \]

\[ V_{\text{lat, def}}[P(x)] \equiv H_A \text{Tr} A P(x) = H_A \left( |\text{tr} P(x)|^2 - 1 \right) \]

- Simulations in \( SU(3) \) and \( SU(4) \) revealed two interesting new phases.
- The simulations also showed that confined phase could be accessed perturbatively in \( SU(3) \).

\[ \langle P(x) \rangle \text{ in } SU(3), \quad \beta = 6.5, \quad H_A = -0.055 \]

\[ \langle P(x) \rangle \text{ in } SU(4), \quad \beta = 11, \quad H_A = -0.12 \]
Deformed Yang-Mills theory

To keep the confined phase accessible for $N > 3$ additional terms were required in the deformation potential (Ogilvie et al 2007, Unsal and Yaffe 2008):

$$V_{\text{def}}(P) = \frac{1}{\beta} \sum_{n=1}^{\lfloor N/2 \rfloor} a_n |\text{tr}(P^n)|^2 = \frac{1}{\beta} \sum_{n=1}^{\lfloor N/2 \rfloor} a_n \sum_{i,j=1}^{N} \cos[n(v_i - v_j)]$$

where $\lfloor N/2 \rfloor$ is the integer part of $N/2$.

Including the boson contribution from pure Yang-Mills theory

$$V_{\text{model}}(P) = \frac{1}{\beta^4} \left[ \frac{1}{24\pi^2} \sum_{i,j=1}^{N} [v_i - v_j]^2 (2\pi - [v_i - v_j])^2 - \frac{\pi^2}{45} (N^2 - 1) \right]$$

$$+ \frac{1}{\beta} \sum_{n=1}^{\lfloor N/2 \rfloor} a_n \sum_{i,j=1}^{N} \cos[n(v_i - v_j)]$$

We minimize this potential to determine the phase diagram for a range of values of the $a_n$. 
One-loop effective potential

The one-loop effective potential for $N_f$ Majorana fermions ($N_{f, Dirac} = \frac{1}{2} N_f$) of mass $m$ in a background Polyakov loop $P = \text{diag}\{e^{i v_1}, e^{i v_2}, ..., e^{i v_N}\}$ gauge field is (Meisinger and Ogilvie 2001):

$$V_{\text{eff}}(P, m) \equiv -\frac{1}{\beta V_3} \ln Z(P, m)$$

$$= \frac{1}{\beta V_3} \left[ -N_f \ln \det (-D_R^2(P) + m^2) + \ln \det (-D_{adj}^2(P)) \right]$$

$$= \frac{m^2 N_f}{\pi^2 \beta^2} \sum_{n=1}^{\infty} \frac{( \pm 1)^n}{n^2} \text{Re} [\text{Tr}_R (P^n)] K_2 (n \beta m)$$

$$+ \frac{1}{\beta^4} \left[ \frac{1}{24 \pi^2} \sum_{i,j=1}^{N} [v_i - v_j]^2 (2\pi - [v_i - v_j])^2 - \frac{\pi^2}{45} (N^2 - 1) \right]$$

where we have $(\pm 1)^n$ for periodic boundary conditions (PBC) and $(-1)^n$ for antiperiodic boundary conditions (ABC) applied to fermions.

Chiral Condensate:

$$\langle \bar{\psi} \psi \rangle_{1-\text{loop}}(m) = -\lim_{V_4 \to \infty} \frac{1}{V_4 N_f} \frac{\partial}{\partial m} \ln Z(m) = \frac{1}{N_f} \frac{\partial}{\partial m} V_{\text{eff}}(P, m)$$
Possible phases of QCD for PBC and ABC

- **ABC**
  - confined phase
    \[ \mathbf{v} = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\} \]
  - deconfined phase
    \[ \mathbf{v} = \{0, 0, 0\}, \{\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{2\pi}{3}\}, \{\frac{4\pi}{3}, \frac{4\pi}{3}, \frac{4\pi}{3}\} \]

- **PBC**
  - confined phase
  - deconfined phase
  - $\mathcal{C}$-breaking phase ($P$ is not invariant under $P \rightarrow P^*$.)
    \[ \mathbf{v} = \left\{ \frac{2\pi}{3}, \frac{2\pi}{3}, \frac{2\pi}{3} \right\} \]

Note: In QCD(F) with PBC on fermions, $\mathcal{C}$-symmetry is only broken for $N$ odd. For $N$ even, $Tr FP$ is magnetized along the negative real axis ($\mathbf{v} = \{\pi, \pi, \ldots\}$).
$V_{\text{EFF}}$ and $\langle \bar{\psi} \psi \rangle$ in perturbative QCD

- We calculate $V_{\text{eff}}$ for fermions in the fundamental representation to which ABC are applied.

\begin{align*}
V_{\text{FUND}} & = \frac{V_{\text{FUND}}}{\beta^4} \\
\langle \bar{\psi} \psi \rangle & = \frac{\langle \bar{\psi} \psi \rangle}{\beta^3} \\
V_{F,-}, \ Nc = 3, \ Nf = 2 \ (1 \ \text{Dirac flavour}) & \quad \text{only the deconfined phase is accessible in the perturbative limit.}
\end{align*}

- The fermion contribution to $V_{\text{eff}}$ vanishes as $m\beta \to \infty$.
- The inflection point in $V_{\text{EFF}}$ at $m\beta \approx 1.4$ implies a large one-loop contribution to $\langle \bar{\psi} \psi \rangle$. 

\begin{align*}
\langle \bar{\psi} \psi \rangle_{F,1\text{-loop}} \text{ for } N_c = 3
\end{align*}
Phases of adjoint QCD: $N_c = 3, N_c = 4$, PBC, $N_f > 1$

$N_c = 3$

confined: $\mathbf{v} = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

$U(1) \times SU(2)$: $\mathbf{v} = \{0, \pi, \pi\}$

deconfined: $\mathbf{v} = \{0, 0, 0\}$

$N_c = 4$

confined: $\mathbf{v} = \{0, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}\}$

$SU(2)$ conf: $\mathbf{v} = \{-\frac{\pi}{2}, \frac{\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}\}$

deconfined: $\mathbf{v} = \{0, 0, 0, 0\}$
Phases of adjoint QCD: $N_c = 5$, $N_c = 6$, PBC, $N_f > 1$

$N_c = 5$

confined: $v = \{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}\}$

inconst: $v = \{0, -\phi, -\phi, \phi, \phi\}$

$SU(2) \times SU(3)$ dec: $v = \{\pi, \pi, 0, 0, 0\}$

deconfined: $v = \{0, 0, 0, 0, 0\}$

$N_c = 6$

confined: $v = \{\frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}\}$

$SU(3)$ conf: $v = \{0, \frac{2\pi}{3}, -\frac{2\pi}{3}, 0, \frac{2\pi}{3}, -\frac{2\pi}{3}\}$

$SU(2)$ conf: $v = \{-\frac{\pi}{2}, \frac{\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}\}$

deconfined: $v = \{0, 0, 0, 0, 0, 0\}$
$SU(3)$ Adjoint QCD (PBC) $N_f = 2$

- The data points (black dots) were found by minimizing $V_{\text{eff}}$ with respect to the Polyakov loop eigenvalues $\nu_i$.
- The confined phase is accessible perturbatively for $m\beta \leq 1.6$.
- There is a dramatic jump in $\langle \bar{\psi} \psi \rangle$ corresponding to the deconfinement transition.
- The model has the same phases as QCD(Adj)
$SU(4)$ Adjoint QCD (PBC) $N_f = 2$

- The confined phase is accessible perturbatively for $m\beta \leq 1.0$.

- The model has the same phases as QCD(Adj), and more, but the additional phases can be circumnavigated.

$V_{ADJ}$, $N_c = 4$, $N_f = 2$

$\langle \lambda \lambda \rangle_{ADJ}$

$V_{model}$, deformed YM, $N_c = 4$
$SU(5)$ Adjoint QCD (PBC) $N_f = 2$

- The confined phase is accessible perturbatively for $m\beta \leq 0.8$.
- A moving phase is found between the confined and $SU(2) \times SU(3)$-dec phases.

The model includes the phases of QCD(Adj).

The (non-$C$-breaking) moving phase of the model is the same as that of QCD(Adj).
Accessibility of the confined phase as $N \to \infty$, or as $N_f$ is increased

- As $N \to \infty$ the maximum $m\beta$ for which the confined phase is accessible, $(m\beta)_{crit}$, decreases.
- However, as $N_f$ increases, $(m\beta)_{crit}$ increases (we must have $N_f \leq 5$ Majorana flavours to preserve asymptotic freedom).

Range of $m\beta$ for which the confined phase is accessible in QCD(Adj) with $N_f = 2$

$V_{eff}$ for $N_c = 6$, $N_f = 2$

$V_{eff}$ for $N_c = 6$, $N_f = 3$
Orientifold Planar Equivalence

The story:

- **Armoni, Shifman, and Veneziano (2003 - 2004)** prove non-perturbatively the equivalence of the bosonic sectors of QCD(Adj) with $N_f$ Majorana fermions and QCD(AS/S) with $N_f$ Dirac fermions, in the planar limit.

- **Unsal and Yaffe (2006)** show that on $S^1 \times \mathbb{R}^3$ $\mathcal{C}$-symmetry is broken in QCD(A/S) when PBC are applied to fermions.

- **DeGrand and Hoffman (2007), Lucini et al (2007)** showed using lattice simulations that the $\mathcal{C}$-breaking is lifted as $S^1$ is decompactified.

- **Lucini et al. (2008)** non-perturbatively prove orientifold equivalence in the quenched approximation (in the absence of $\mathcal{C}$-breaking) using lattice simulations and calculate the quark condensate in QCD(A/S/Adj).

We compare (to 1-loop order) the phase diagrams of $QCD(A)$, $QCD(S)$ with $N_f = 2$ (1 Dirac flavour), to $QCD(Adj)$ with $N_f = 1$ (Majorana flavour), for massive fermions with PBC.
$SU(6)$ QCD(A) (left), (S) (middle), and (Adj) (right) for PBC on fermions.

$V_{A,+}, N_c = 6, N_f = 2$

$V_{S,+}, N_c = 6, N_f = 2$

$V_{Adj,+}, N_c = 6, N_f = 1$

$\langle \bar{\psi} \psi \rangle_A$ for $N_c = 6$

$\langle \bar{\psi} \psi \rangle_S$ for $N_c = 6$

$\langle \lambda \lambda \rangle_{Adj}$ for $N_c = 6$
The $\mathcal{C}$ breaking phase of QCD(AS/S)

- The $\mathcal{C}$-breaking phase is favoured in the case where PBC are applied to fermions in the $A$ and $S$ representations (When ABC are used the deconfined phase is favoured).
- For example, when $N_c = 6$ the $\mathcal{C}$-breaking phase has the Polyakov loop eigenvalues

$$\mathbf{v} = \left\{ \frac{2\pi}{6}, \frac{2\pi}{6}, \frac{2\pi}{6}, \frac{2\pi}{6}, \frac{2\pi}{6}, \frac{2\pi}{6} \right\}$$

- $P$ is clearly not invariant under $P \rightarrow P^*$. 
Conclusions

One-loop PT:

- In QCD(Adj) for $N_f \geq 2$ there are several exotic phases occurring between the confined and deconfined phases.
- In QCD(Adj) for $N_f \geq 2$, as $N$ increases, $(m\beta)_{\text{crit}}$, below which the confined phase is accessible, decreases.
- In QCD(Adj) for $N_f \geq 2$, as $N_f$ is increased, the confined phase is accessible for a larger $(m\beta)_{\text{crit}}$.
- In QCD(A/S) with PBC for fermions the $\mathcal{C}$-breaking phase is favoured for all $m\beta$.
- For all representations there is a clear one-loop contribution to $\langle \bar{\psi} \psi \rangle$ for small $m\beta$.
  - In QCD(AS), QCD(S), QCD(F) there is an inflection point in $V_{\text{eff}}$ at which $\langle \bar{\psi} \psi \rangle \neq 0$, in the deconfined phase.
  - In QCD(Adj) for $N_f \geq 2$ (with PBC on fermions) the chiral condensate peaks at the transition to the deconfined phase.
- In QCD(Adj) for $N_f = 1$ the deconfined phase is favoured for all $m\beta$.

The deformed Yang-Mills theory finds all the phases of QCD(Adj), and the $a_n$ can be slowly varied to go through the phases in the same order.