The QCD transition with 2+1 dynamical flavors

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1. Discrepancy in the transition temperature
2. $T = 0$ simulations at the physical point
3. $N_t = 12$ simulations at finite temperature
Problem

hotQCD finite T results are very different from ours
(hotQCD data points: 0710.1655, 0711.0661, 0804.4148, RBRC workshop 04.08)
(our old data points: hep-lat/0609068)

chiral susceptibility, rescaled (quark masses are different)

$$\chi_{\bar{\psi}\psi} = m_l^2 \frac{\partial^2}{\partial m_l^2} (f(T) - f(T = 0))$$

chiral condensate

$$\Delta_{l,s} = \frac{\langle \bar{l}l \rangle - m_l/m_s \langle s\bar{s} \rangle}{\langle \bar{l}l \rangle_{T=0} - m_l/m_s \langle s\bar{s} \rangle_{T=0}}$$
Possible resolutions

hep-lat/0609068: \( N_t = 4, 6 \) of 'p4fat3' are too coarse, no controlled continuum limit, status 2008: fine \( N_t = 8 \) somewhat better but still large discrepancy

our simulations:

• scale set by \( f_K \), non-Goldstone pions distort chiral extrapolation or continuum limit
• naive staggered dispersion relation has large artefacts

hotQCD:

• nonphysical quark masses \( \rightarrow \sim 5 \text{ MeV} \) Soeldner's talk
• scale set by \( r_0^{\text{HPQCD, UKQCD}} = 0.469(7) \text{ fm} \)

\( r_0^{\text{ETM}} = 0.444(4) \text{ fm}, r_0^{\text{QCD SF}} = 0.467(6) \text{ fm}, r_0^{\text{PACS-CS}} = 0.492(6)(+7) \text{ fm} \)

both:

• universality problem of staggered discretization
• bug in computer code
• ...

maybe a bit of all

systematic errors are simply underestimated
Improving our previous results

1. improving $T = 0$ simulations
   previously: $m_\pi > 240\text{MeV} + \text{chiral extrapolations}$
   now: $m = m^{\text{phys}}$, no need for chiral extrapolations
   $\Rightarrow$ more precise scale/renormalization

2. improving $T > 0$ simulations
   previously: $N_t = 4, 6, 8, 10$ at the physical point
   now: $N_t = 12$ at the physical point
   $\Rightarrow$ more control over lattice artefacts
Simulation setup: finite $T$

nVidia GeForce 8800 Ultra
768 MB video memory
103.7 GB/sec bandwidth
two cards per machine

multishift inverter on $12 \cdot 36^3$ fits to the video memory and runs with 32 Gflop
gauge force on the video card: 15 Gflop

only single precision arithmetics, HMC-force is not needed more precisely, for HMC-energy mixed precision inverters ($\epsilon = 10^{-8}$)

50 dual GPU PC’s in Wuppertal $\rightarrow$ 3 Tflops $\sim$ 1 BGP rack
cluster computing: ideal for finite $T$ with many parameter sets
Simulation setup: zero T

zero T lattices are too large for a single video card → BG/P supercomputer in Juelich
Simulation setup: zero T

simulations directly at the physical point
choose lattice sizes, so that finite volume corrections are below 0.5% for $f_\pi, m_\pi, f_K, m_K$ (cont. formula of Colangelo, Durr, Haefeli ’05)

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$N_t^{\text{crit}}$</th>
<th>lattice</th>
<th>#traj</th>
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<tbody>
<tr>
<td>3.45</td>
<td>$\sim 4$</td>
<td>$24^3 \times 32$</td>
<td>1500</td>
</tr>
<tr>
<td>3.55</td>
<td>$\sim 6$</td>
<td>$24^3 \times 32$</td>
<td>3000</td>
</tr>
<tr>
<td>3.67</td>
<td>$\sim 8$</td>
<td>$32^3 \times 48$</td>
<td>1500</td>
</tr>
<tr>
<td>3.75</td>
<td>$\sim 10$</td>
<td>$40^3 \times 48$</td>
<td>1500</td>
</tr>
<tr>
<td>3.85</td>
<td>$\sim 13$</td>
<td>$48^3 \times 64$</td>
<td>1500</td>
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</tbody>
</table>
chiral extrapolations (not staggered $\chi$PT !) work amazingly well for all analyzed spacings the extrapolation error for $f_\pi, m_\pi, f_K, m_K$ is $\leq 1\%$

hep-lat/0609068: "2% is the accuracy of our LCP."
extend spectrum analysis to $\Omega$
red bands are the experimental values with uncertainties
$K^*$ decays in the physical point, width is also given (pink) smaller spacings and $r_0$ are currently under analysis
Strange quark number susceptibility

Finite T results

preliminary results, 300-500 trajectories in each point

good agreement with old \( N_t = 10 \) data

hep-lat/0609068: "For the transition temperature in the continuum limit one gets: \( T_c(\chi_s) = 175(2)(4) \) MeV"
Finite $T$ results

renormalized chiral susceptibility

\[
\chi_{\bar{\psi}\psi}
\]

nice agreement with old $N_t = 8, 10$ data

hep-lat/0609068: "the transition temperature based on the chiral susceptibility reads $T_c(\chi_{\bar{\psi}\psi}) = 151(3)(3)$ MeV"
Summary

• improving determination of thermodynamical observables by
  1. zero T simulations with physical quark masses
  2. finite T simulations with $N_t = 12$

• chiral extrapolations were correct on the 1% level

• preliminary results for chiral susceptibility and strange suscep-
tibility on $N_t = 12$ are in good agreement with our old data