Vector meson electromagnetic form factors

M. Gürtler  P. Hägler
QCDSF collaboration

Lattice 2008, College of William & Mary
Form factors

- Internal structure of hadrons $\rightarrow$ (generalised) form factors
- Low energy quantities $\rightarrow$ lattice
- Nucleon, pion, $\rho$ meson
- Heavy pions, $\rho$ meson stable
- Representative for spin 1 particle
**Introduction**

**Results**

**Conclusion/Outlook**

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**Form factors**

**interaction hadron - e.m. current**

\[
\langle p', s' | J^\alpha | p, s \rangle = \left( 2 \sqrt{E_\rho(p')} E_\rho(p) \right)^{-1} \epsilon_{\tau}^*(p', s') J^{\tau \alpha \sigma}(p', p) \epsilon_{\sigma}(p, s)
\]

**for spin one particle parametrised by three form factors**

\[
J^{\tau \alpha \sigma}(p', p) = -G_1(Q^2) g^{\tau \sigma}(p^\alpha + p'^\alpha) \\
- G_2(Q^2) (g^{\alpha \sigma} q^\tau - g^{\alpha \tau} q^\sigma) \\
+ G_3(Q^2) \left( q^\sigma q^\tau \frac{p^\alpha + p'^\alpha}{2m^2_\rho} \right)
\]

momentum transfer \( Q^2 = -q^2 = -(p' - p)^2 \)

polarisation vectors \( \epsilon \)

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Form factors

Sachs form factors

\[ G_C(Q^2) = G_1(Q^2) + \frac{2}{3}\eta G_Q(Q^2) \]
\[ G_M(Q^2) = G_2(Q^2) \]
\[ G_Q(Q^2) = G_1(Q^2) - G_2(Q^2) + (1 + \eta) G_3(Q^2) \]

\[ \eta = \frac{Q^2}{4m^2} \]

Interesting static quantities

- charge radius \( \langle r^2 \rangle = -6 \frac{\partial G_C}{\partial (Q^2)} \bigg|_{Q^2=0} \)
- magnetic moment \( \mu_M = \frac{e}{2m_p} G_M(0) \), aka g factor
- quadrupole moment \( \mu_Q = \frac{e}{m_p^2} G_Q(0) \)
What to expect

- Samsonov et al: $\mu_M = 1.8(3)$ (QCD sum rules in ext. fields) JHEP 0312:061, 2003
- Bhagwat et al: $\langle r^2 \rangle = 0.54$ fm$^2$, $\mu_M = 2.01$, $\mu_Q = -0.41$ fm$^2$ (Dyson-Schwinger eqs.) Phys.Rev.C77:025203, 2008
- Aliev et al: $G_M/G_C > 2$ (light cone sum rules; don’t work at small $Q^2$) Phys.Rev.D70:094007, 2004
- Choi et al: $\mu_M = 1.92$, $\mu_Q = -0.43$ fm$^2$ (Light front quark model) Phys.Rev.D70:053015, 2004
- Hedditch et al: $\langle r^2 \rangle \sim 0.6$ fm$^2$, $\mu_M \sim 2.3$, $\mu_Q \sim -0.005$ fm$^2$ (quenched lattice simulation, standard 3pt technique) Phys.Rev.D75:094504, 2007
Lattice method

compute three point functions involving $\langle p', s' | J^\alpha | p, s \rangle$
(and two point functions)

system of equations $R^{\alpha}_{\mu\nu}(p, p') = \sum_i c_i G_i$ for each $Q^2$

solve numerically ($\chi^2$ minimisation) $\rightsquigarrow G_i(Q^2)$
Lattice matrix elements can be extracted from three point functions

\[ G_{\mu\nu}^{\alpha}(t, \tau, \vec{p}', \vec{p}) = \sum_{\vec{x}, \xi} e^{-i\vec{p}'(\vec{x}-\xi)} e^{-i\vec{p}\xi} \left\langle \Omega \left| \chi_\mu(x) J_\alpha^{\alpha}(\xi) \chi_\nu^\dagger(0) \right| \Omega \right\rangle \]

choice of \( t \) critical
we will also need the two point functions

\[ G_{\mu\nu}(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\vec{x}} \left\langle \Omega \left| \chi_\mu(x) \chi_\nu^\dagger(0) \right| \Omega \right\rangle \]
transfer matrix formalism; $0 << \tau << t$

\[
\lim_{t \to \infty} G_{\mu\nu}(t, \vec{p}) = -\frac{e^{-E_{\rho}(\vec{p}) t}}{2E_{\rho}(\vec{p})} \lambda_{\rho}(\vec{p}) \bar{\lambda}_{\rho}(\vec{p}) \left( g_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{m_{\rho}^2} \right)
\]

\[
\lim_{\tau \to \infty} G_{\mu\nu}^{\alpha}(t, \tau, \vec{p}', \vec{p}) = \frac{e^{-E_{\rho}(\vec{p}') (t-\tau)} e^{-E_{\rho}(\vec{p}) \tau}}{4 E_{\rho}(\vec{p}') E_{\rho}(\vec{p})} \lambda_{\rho}(\vec{p}') \bar{\lambda}_{\rho}(\vec{p}) \times \left( g_{\mu\tau} - \frac{p_{\mu}' p_{\tau}'}{m_{\rho}^2} \right) J^{\tau\alpha\sigma} \left( g_{\sigma\nu} - \frac{p_{\sigma}p_{\nu}}{m_{\rho}^2} \right)
\]

$\bar{\lambda}$-overlap of interpolating operator $\chi_{\mu}^{\dagger} = \bar{d} \gamma_{\mu} u$ with $\rho$

\[
\langle \Omega | \chi_{\mu}(0) | \rho(\vec{p}, s) \rangle = \sqrt{2 E_{\rho}(\vec{p})}^{-1} \lambda_{\rho}(\vec{p}) \epsilon_{\mu}(p, s)
\]

sum over polarisations using transversality condition $\sum_s \epsilon_{\mu}(p, s) \epsilon_{\nu}^*(p, s) = -g_{\mu\nu} + p_{\mu}p_{\nu}/m_{\rho}^2$
Ratios

\[
R_{\mu\nu}^{(\alpha)}(\tau, \vec{p}', \vec{p}) = \frac{G_{\mu\nu}^{(\alpha)}(t, \tau, \vec{p}', \vec{p})}{G_{\mu\mu}(t, \vec{p}')} \sqrt{\frac{G_{\nu\nu}(t - \tau, \vec{p})G_{\mu\mu}(\tau, \vec{p}')G_{\mu\mu}(t, \vec{p}')}{G_{\nu\nu}(\tau, \vec{p})G_{\mu\mu}(t - \tau, \vec{p}')G_{\nu\nu}(t, \vec{p})}}
\]

is independent of \(\tau\)

\((\mu, \nu = 1 \ldots 3)\)

t fixed; potential problems with \(\sqrt{\text{, argument can be negative}}\)
Details of the lattice calculation

- QCDSF-UKQCD configurations
- 2 dynamical flavours of Wilson fermions
- non-perturbatively improved Dirac operator
  \[ \frac{i}{4} c_{SW} a g^2 \bar{\psi}(x) \sigma_{\mu\nu} F_{\mu\nu}(x) \psi(x), \quad c_{SW}(g) \text{ (ALPHA coll.)} \]
- (Jacobi) smeared sources and sinks
- local vector current \( \rightsquigarrow \) renormalisation \( Z_V = 1 / G_1^{\text{unren}}(0) \)
- compute for 3 values of \( \vec{p}' \) and 17 values of \( \vec{p} \) and all polarisation combinations
- no disconnected contribution \( (G^{\text{disc}}(U) = - G^{\text{disc}}(U^*)) \), both have equal weight
## Lattices

<table>
<thead>
<tr>
<th>Volume</th>
<th>$\beta$</th>
<th>$\kappa$</th>
<th>$m_\pi$ [MeV]</th>
<th>$a$ [fm]</th>
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<tbody>
<tr>
<td>$16^3$ 32</td>
<td>5.29</td>
<td>0.13500</td>
<td>929(2)</td>
<td>0.089</td>
</tr>
<tr>
<td>$16^3$ 32</td>
<td>5.29</td>
<td>0.13550</td>
<td>784(3)</td>
<td>0.084</td>
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<tr>
<td>$24^3$ 48</td>
<td>5.29</td>
<td>0.13590</td>
<td>591(2)</td>
<td>0.080</td>
</tr>
<tr>
<td>$24^3$ 48</td>
<td>5.29</td>
<td>0.13620</td>
<td>406(6)</td>
<td>0.077</td>
</tr>
</tbody>
</table>

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Vector meson electromagnetic form factors
Introduction

Results

Conclusion/Outlook

two-point function example

\[ \beta = 5.29 \quad \kappa = 0.13620 \quad \text{Vol}24^3 48 \quad m_\pi = 406(5)\text{MeV} \]

\[ m_\rho = 900(1)\text{MeV} \]

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Vector meson electromagnetic form factors
\[ Q^2 = 0, \quad \beta = 5.29, \quad \kappa = 0.13590, \quad \text{Vol}=24^{348}, \quad \text{iop}=4 \]
Example for form factor fits

\[ \beta = 5.29 \quad \kappa = 0.13620 \quad \text{Vol}24^348 \quad m_\pi = 406\text{MeV} \]

\[ Q^2[\text{GeV}^2] \]

- \( G_C \)
- \( G_M \)
- \( G_Q \)
Example for form factor fits

\[ \beta = 5.29 \quad \kappa = 0.13620 \quad \text{Vol}24^348 \quad m_\pi = 406\text{MeV} \]

\[ \langle r^2 \rangle = 0.49 \pm 0.04 \text{ fm}^2 \]

\[ G_C(t), \text{ monopole fit} \]
Example for form factor fits

\[ \beta = 5.29 \quad \kappa = .13620 \quad \text{Vol}24^348 \quad m_\pi = 406\text{MeV} \]

\[ \mu_M = 1.69 \pm 0.25 \]

\[ \langle r^2 \rangle = 0.49 \pm 0.04 \text{ fm}^2 \]

\[ Q^2[\text{GeV}^2] \]
Example for form factor fits

\[ \beta = 5.29 \quad \kappa = 0.13620 \quad \text{Vol}\,24^3\,48 \quad m_\pi = 406\text{MeV} \]

\[ \langle r^2 \rangle = 0.49 \pm 0.04 \text{ fm}^2 \]

\[ \mu_M = 1.69 \pm 0.25 \]

\[ \mu_Q = -0.015 \pm 0.004 \text{ fm}^2 \]
Charge radii

\[ \langle r^2 \rangle \text{[fm}^2 \text{]} \]

\[ m_{\pi}^2 \text{[GeV}^2 \text{]} \]

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Charge radii

\[ \langle r^2 \rangle \text{[fm}^2\text{]} \]

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Vector meson electromagnetic form factors
Magnetic moment

$g_\rho$

$m_\pi^2$ [GeV$^2$]

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Vector meson electromagnetic form factors
Magnetic moment

$\mu_M$, linear fit

Vector meson electromagnetic form factors
Quadrupole moment

\[ \mu_Q [\text{fm}^2] \]

\[ m_{\pi}^2 [\text{GeV}^2] \]

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Vector meson electromagnetic form factors
Quadrupole moment

\[ \mu_Q, \text{linear fit} \]

\[ \mu_Q \text{[fm}^2\text{]} \]

\[ m^2_{\pi}[\text{GeV}^2] \]

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Vector meson electromagnetic form factors
first unquenched direct computation of the vector meson e.m. form factors

still preliminary

Charge radii

- slightly larger than found by Hedditch et al (larger $Q^2$ range)
- growing towards smaller $m_q$

$g$-factor

- $\sim 2$; close to quark model expectation
- chiral curvature?

quadrupole moment

- $\sim 0$ at large pion masses
- decreasing quark mass: increasingly negative $\Rightarrow$ oblate shape

next: axial/tensor form factors; GFF
When does the $\rho$ decay?

![Graph showing the relationship between $E$ (in MeV) and $m_\pi^2 [GeV]^2$. The graph includes data points and a line of best fit.](image)
Another ratio example

\[ Q^2 =, \beta = 5.29, \kappa = 0.13590, \text{Vol}=24^348, \text{iop}=1 \]