Quark Propagators at the confinement and deconfinement phases

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Main Results

* Quark propagators have negative norm contributions at confinement phase.
* This feature remains at deconfinement phase.
Introduction

Quark propagator

Quark confinement

Chiral symmetry breaking

Quark propagators relate to two phase transitions in QCD.

Quark confinement

Pole mass, Asymptotic state

Chiral symmetry breaking

Order parameter: $\langle \psi(0) \bar{\psi}(x) \rangle_{x \to 0}$
At finite temperature

**low**
- Quark: confined
- Chiral symmetry: breaking

**high**
- Quark: deconfined
- Chiral symmetry: restored

**How do behaviors of quark propagators change?**

Do quark propagators have no pole at confinement phase and one or some pole(s) at deconfinement?
Formulation

**Quark propagator**

\[ G(x, y) = \langle \psi(x) \bar{\psi}(y) \rangle = \langle W^{-1}(x, y; U) \rangle \]

\[ S_f = \sum_{x, y} \bar{\psi}(x) W(x, y; U) \psi(y) \]

in our calculations, **Clover fermion**

**Time-time correlation function**

\[ G(x_4 - y_4) = \langle \psi(x_4) \bar{\psi}(y_4) \rangle = \sum_{\vec{x}, \vec{y}} \langle W^{-1}(x, y; U) \rangle \]

when \( \vec{p} = 0 \)

\[ G(t) = G_4(t) \gamma_4 + G_s(t) \]
Time-time correlation function (One pole case)

\[ G(t) = \frac{Z_1}{2V \cosh(m\beta/2)} \left[ \cosh(m(t - \beta/2))\gamma_4 - \sinh(m(t - \beta/2)) \right] \]

Effective mass

\[ \frac{G_4(t)}{G_4(t + 1)} = \frac{\cosh(m(t - \beta/2))}{\cosh(m(t + 1 - \beta/2))} \]

\[ \frac{G_s(t)}{G_s(t + 1)} = \frac{\sinh(m(t - \beta/2))}{\sinh(m(t + 1 - \beta/2))} \]
Numerical Conditions

- Quenched approximation
- Plaq. gauge action + Wilson fermions with Clover
- Gauge Fixing: Landau Gauge
- Thermalization: 1000
- Sweeps between measurements: 1000
- # of Configuration: 20 - 50
- $\beta = 6.10$, $\kappa = 0.1345559, 0.1353591$
- $\beta = 6.25$, $\kappa = 0.1346226, 0.1352633$
- Confinement phase: $N_t = 16, N_s = 24, 32$
- Deconfinement phase: $N_t = 8, N_s = 24, 32$
Numerical Result 1

Time-time correlation function at confinement phase

\begin{align*}
G_4(n_t) & \quad \text{for } T/T_c = 0.54499 \\
G_6(n_t) & \quad \text{for } T/T_c = 0.54499
\end{align*}

* beta = 6.10, kappa = 0.1345559
* Nt = 16, Ns = 24
Numerical Result 2

Time-time correlation function at deconfinement phase

\[ G_{4}(n) \]

\[ G_{5}(n) \]

\[ T/T_c = 1.08998 \]

\[ \beta = 6.10, \kappa = 0.1345559 \]

\[ N_t = 8, N_s = 24 \]
Numerical Result 3

Effective mass at confinement phase

\( \kappa = 0.1345559 \)

\( \kappa = 0.1353591 \)

\( \frac{T}{T_c} = 0.54499 \)

\( \frac{T}{T_c} = 0.68325 \)

\( \beta = 6.10 \)

\( N_t = 16, N_s = 24 \)

\( \beta = 6.25 \)

\( N_t = 16, N_s = 24 \)
For details

\[ G(t) = \rho_1 e^{-m_1 t} + \rho_2 e^{-m_2 t} \quad (m_1 < m_2) \]

\[ m_{\text{eff}} = \ln \frac{G(t)}{G(t+1)} = \ln \frac{\rho_1 e^{-m_1 t}}{\rho_1 e^{-m_1 (t+1)}} = m_1 \quad (t \to \infty) \]

\[ m'_{\text{eff}} = \ln \frac{G(t)}{G(t+1)} = \ln \frac{\rho_1 e^{-m_1 t} + \rho_2 e^{-m_2 t}}{\rho_1 e^{-m_1 (t+1)} + \rho_2 e^{-m_2 (t+1)}} \]

\[ = m_1 + \ln \frac{1 + \frac{\rho_2}{\rho_1} e^{-(m_2-m_1)t}}{1 + \frac{\rho_2}{\rho_1} e^{-(m_2-m_1)(t+1)}} \quad (t \to 0) \]

\[
\frac{\rho_2}{\rho_1} > 0 \quad \Rightarrow \quad \frac{1 + \frac{\rho_2}{\rho_1} e^{-(m_2-m_1)t}}{1 + \frac{\rho_2}{\rho_1} e^{-(m_2-m_1)(t+1)}} > 1 \quad \Rightarrow \quad m_{\text{eff}} < m'_{\text{eff}}
\]

\[
\frac{\rho_2}{\rho_1} < 0 \quad \Rightarrow \quad \frac{1 + \frac{\rho_2}{\rho_1} e^{-(m_2-m_1)t}}{1 + \frac{\rho_2}{\rho_1} e^{-(m_2-m_1)(t+1)}} < 1 \quad \Rightarrow \quad m_{\text{eff}} > m'_{\text{eff}}
\]
namely violation of positivity. If a certain degree of freedom has negative norm contributions in its propagator, it cannot describe a physical asymptotic state, i.e., there is no Källén–Lehmann spectral representation for its propagator.
**Numerical Result 4**

**Effective mass at denfinement phase**

![Graphs showing effective mass at different values of beta and T/Tc]

- **Beta = 6.10**
- **Nt = 8, Ns = 24**
- **Beta = 6.25**
- **Nt = 8, Ns = 24**

- \(\kappa = 0.1345559\)
- \(\kappa = 0.1353591\)

- \(\frac{T}{T_c} = 1.08998\)
- \(\frac{T}{T_c} = 1.36650\)
Numerical Result 5

Volume dependence of effective mass

\[ T/T_c = 0.54499 \]

\[ T/T_c = 1.08998 \]

\* beta = 6.10, kappa = 0.1345559
Numerical Result 6

Temperature dependence of quark mass

\[ \beta = 6.10, \kappa = 0.1345559 \]

\[ N_t = 16, 14, 12, 10, 8, 6, N_s = 32 \]
Summary

* Effective mass shows that quark propagators include negative norm state.
* This feature remains at deconfinement phase.
* We can not fit the scalar part of time-time correlation functions.
* These behavior do not depend quark mass or spatial volume.
* Effective mass at confinement phase twice larger than it at deconfinement phase.