Revisiting strong coupling QCD at finite baryon density and temperature

M. Fromm, Ph. de Forcrand
ETH, Zürich

Lattice’08, July 16th, 2008
Motivation

Strong coupling QCD (SCQCD) under investigation for > 25 years...

**Analytical (Mean Field 1/d)**
- Mass spectrum: Kawamoto and Smit '81, Kluberg-Stern, Morel, Petersson '82
- Phase diagram, Damgaard, Kawamoto, Shigemoto ’84
- Phase diagram with 1/g^2 corrections: Faldt and Petersson ’86, Bilić et al.’92
- Latest: Nishida ’04, Kawamoto et al. ’05, Miura and Ohnishi ’08 (next talk)

**Numerical**
- Karsch and Mütter ’89, MDP-approach (T ≈ 0, µ ≈ µ_c)
- Boyd et al. ’92, MDP at T ≈ T_c, µ = 0
- Azcoiti et al. ’99 (MDP under scrutiny)
- de Forcrand and Kim ’06, HMC, mass spectrum
Some Definitions:

\[
Z = Z(m, \mu, \beta) = \int \mathcal{D}U \mathcal{D}\bar{\chi} \mathcal{D}\chi \ e^{-S_F - \beta S_G},
\]

\(\mu\) chemical potential, \(m\) staggered quark mass, \(\beta = \frac{6}{g_0^2}\) inverse gauge coupling

\[
S_G = \sum_P \left( 1 - \frac{1}{3} \text{Re} \text{tr}[U_P] \right)
\]

\[
S_F = \sum_{x, \nu} \bar{\chi}_x \left[ \eta_{x\nu} U_{x\nu} \chi_{x+\nu} - \eta_{x\nu}^{-1} U^\dagger_{x-\nu\nu} \chi_{x-\nu} \right] + 2m \sum_x \bar{\chi}_x \chi_x
\]

\[
\eta_{x\nu} = e^\mu \ (\nu = 0) \text{ and } (-1)^{\sum_{\rho < \nu} x_\rho} \text{ otherwise.}
\]
In Strong (infinite) coupling limit, $\beta = 0$ - can do integral in links $U_{xy}$ first [Rossi & Wolff]:

$$Z(m, \mu) = \int \mathcal{D}\bar{\chi}\mathcal{D}\chi e^{2m \sum_x \bar{\chi}_x \chi_x} \prod_{\langle xy \rangle} F_{xy}$$

where $F_{xy} = \sum_{k=0}^{3} (-1)^k \alpha_k (M_x M_y)^k + \kappa/6 \left[ (\bar{B}_x B_y)^3 + (\bar{B}_y B_x)^3 \right]$ and

$$\kappa = \begin{cases} 
0, & \text{for } U(3) \\
1, & \text{for } SU(3) 
\end{cases}$$

New degrees of freedom are color singlet

- **Monomers** $M_x = \sum_{a,x} \bar{\chi}_{ax} \chi_{ax}, (\bullet)$, monomers per site $n_x = 0, \ldots, 3$
- **Dimers** $D_{k,xy} = \frac{1}{k!} (M_x M_y)^k \left( -, =, \equiv \right)$, bond number $n_b = 0, \ldots, 3$
- **(Anti-)Baryons** $B_x = \chi_{1x} \chi_{2x} \chi_{3x}$, $\bar{B}_x = \bar{\chi}_{3x} \bar{\chi}_{2x} \bar{\chi}_{1x}$, $- - -$
Self-avoiding loops $C$ of $\bar{B}B_x$ pairs are formed, with signed weights $\rho(C)$,

$$Z(m, \mu) = \sum_{\{n_x, n_b, \Box\}} \prod_b \frac{(3 - n_b)!}{3! n_b!} \prod_x \frac{3!}{n_x!} (2m)^{n_x} \prod_{\text{loops } C} \rho(C),$$

constraint $n_x + \sum_{b_x} n_{b_x} = 3$.

Example

- MDP description [Karsch & Mütter, 1989]: Signed baryonic loops are associated with polymer loops. Mapping the weight

$$\rho_B(C) \rightarrow w_{\text{polymer}}(C)$$

$$\pm \cosh \frac{3\mu}{T} \rightarrow 1 \pm \cosh \frac{3\mu}{T}$$

softens the sign problem.
Self-avoiding loops $C$ of $\bar{B}B_x$ pairs are formed, with \textit{signed} weights $\rho(C)$,

$$Z(m, \mu) = \sum_{\{n_x, n_b, \square\}} \prod_b \frac{(3 - n_b)!}{3! n_b!} \prod_x \frac{3!}{n_x!} (2m)^{n_x} \prod_{\text{loops } C} \rho(C),$$

constraint $n_x + \sum_{b_x} n_{b_x} = 3$.

**Example**

MDP description [Karsch & Mütter, 1989]: Signed baryonic loops are associated with polymer loops. Mapping the weight

$$\rho_B(C) \rightarrow w_{\text{polymer}}(C) \quad \pm \cosh 3\mu/T \rightarrow 1 \pm \cosh 3\mu/T$$

softens the sign problem.
The worm algorithm in strong coupling QCD

Starting over from initial formulation of \( Z = \int \ldots \) introduce two additional meson fields \( M_{bx_1}, M_{cx_2} \), i.e.

\[
\langle M_{bx_1} M_{cx_2} \rangle = \int D\bar{\chi} D\chi \, M_{bx_1} M_{cx_2} e^{2m \sum_x \bar{\chi}_x \chi_x} \prod_{\langle xy \rangle} F_{xy}.
\]

Example

- Corresponds to introduction of additional monomers ("worm" head and tail)
The worm algorithm in strong coupling QCD

Starting over from initial formulation of $Z = \int \ldots$ introduce two additional meson fields $M_{bx_1}, M_{cx_2}$, i.e.

$$\langle M_{bx_1} M_{cx_2} \rangle = \int D\bar{\chi} D\chi \ M_{bx_1} M_{cx_2} e^{2m \sum_x \bar{\chi}_x \chi_x} \prod_{\langle xy \rangle} F_{xy}.$$ 

Example

- Corresponds to introduction of additional monomers ("worm" head and tail)
- MC-update via heatbath shift of head, sample configs contributing to $\langle M_{bx_1} M_{cx_2} \rangle$
Starting over from initial formulation of $Z = \int \ldots$ introduce two additional meson fields $M_{bx_1}, M_{cx_2}$, i.e.

$$\langle M_{bx_1} M_{cx_2} \rangle = \int D\bar{\chi} D\chi \ M_{bx_1} M_{cx_2} e^{2m \sum_x \bar{\chi}_x \chi_x} \prod_{\langle xy \rangle} F_{xy}.$$ 

**Example**

- Corresponds to introduction of additional monomers ("worm" head and tail)
- MC-update via heatbath shift of head, sample configs contributing to $\langle M_{bx_1} M_{cx_2} \rangle$
- No restrictions in mass range (e.g. can take $m \to 0$) in contrast to local monomer changing Metropolis step $\leftrightarrow \bullet \bullet$
The worm algorithm in strong coupling QCD

Starting over from initial formulation of $Z = \int \ldots$ introduce two additional meson fields $M_{bx_1}, M_{cx_2}$, i.e.

$$\langle M_{bx_1} M_{cx_2} \rangle = \int D\bar{\chi} D\chi M_{bx_1} M_{cx_2} e^{2m \sum_x \bar{\chi}_x \chi_x} \prod_{\langle xy \rangle} F_{xy}.$$ 

Example

- Corresponds to introduction of additional monomers ("worm" head and tail)
- MC-update via heatbath shift of head, sample configs contributing to $\langle M_{bx_1} M_{cx_2} \rangle$
- No restrictions in mass range (e.g. can take $m \to 0$) in contrast to local monomer changing Metropolis step $- \leftrightarrow \bullet\bullet$
- See $U(N)$ [Adams & Chandrasekharan '03]
The worm algorithm in strong coupling QCD

Starting over from initial formulation of $Z = \int \ldots$ introduce two additional meson fields $M_{bx_1}, M_{cx_2}$, i.e.

$$\langle M_{bx_1} M_{cx_2} \rangle = \int D\bar{\chi} D\chi M_{bx_1} M_{cx_2} e^{2m \sum_x \bar{\chi}_x \chi_x} \prod_{\langle xy \rangle} F_{xy}.$$ 

Example

- Corresponds to introduction of additional monomers ("worm" head and tail)
- MC-update via heatbath shift of head, sample configs contributing to $\langle M_{bx_1} M_{cx_2} \rangle$
- No restrictions in mass range (e.g. can take $m \to 0$) in contrast to local monomer changing Metropolis step $\leftrightarrow$ ⊗
- See $U(N)$ [Adams & Chandrasekharan ’03]

M. Fromm

Revisiting SCQCD at finite $\mu$ and $T$
Simple efficiency test

Local Metropolis, $4^3\times 2$ at $\mu_c, m_q = 0.025$

Worm, same parameter set

M. Fromm
Revisiting SCQCD at finite $\mu$ and $T$
Consistency check with HMC

Worm–MDP vs. HMC (Forcrand and Kim ’06) $\beta = 0$, same volume ($\mu = T = 0$)

$\pi$, HMC
$\sigma / N_c$
$\sigma / N_c$, Mean–Field
$m_\pi$
$\sigma / N_c$, Worm
$m_\pi$
Consistency check with HMC

Worm–MDP vs. HMC (Forcrand and Kim ’06) \( \beta = 0 \), same volume (\( \mu = T = 0 \))

\[ m_\pi, \text{HMC} \] vs. \[ m_\pi, \text{Mean–Field} \] vs. \[ m_\pi, \text{Worm} \]

Revisiting SCQCD at finite \( \mu \) and \( T \)
Phase diagram in the chiral limit as obtained by *mean-field* calculations [Damgaard et al. '85, Nishida '04, Kawamoto et al. '05]

\[ T_c = \frac{5}{3}, \mu_c(T=0) = 0.57 - 0.66. (\sigma \propto \langle \bar{\chi}_a \chi_a \rangle) \]
SCQCD chiral restoration transition

Puzzle

- Strong coupling MC-simulations [Karsch & Mütter 1989] at finite quark mass and $T = 1/4$ confirm 1st order finite $\mu$ transition and extrapolate to $\mu_c(T \approx 0, m = 0) = 0.63$ (in agreement with mean-field).
- However: Expect $(T = 0)$-phase transition when

$$3\mu \geq F_B \approx M_{\text{Nucleon}} \approx 3,$$

i.e.

$$\mu_c \approx 1$$

- Nuclear attraction strong, $O(300\text{ MeV})$?
- Or: Finite $T$ effects (MC), extrapolation in $m$ (MC) or mean field approach inaccurate?
  → Check with worm-MC in the chiral limit, $T \approx 0$.
- Note: Mean field calculations with $1/g^2$ corrections [Bilić et al. 1992] show that $\mu_c \rightarrow M_N/3$. 
Preliminary Results, varying mass

Baryon number density $n_B$, $L^3 \times 2 \times (4)$, $m_q = 0.1$, Worm vs. Metropolis

- $N_t = 4, L = 8$
- $N_t = 2, L = 10$
- $N_t = 4, L = 8$, Karsch '89

M. Fromm  Revisiting SCQCD at finite $\mu$ and $T$
Preliminary Results, varying mass

Baryon number density $n_B$, $L^3 \times 2$ (4), $m_q = 0.025$

Phase coexistence $N_t = 4$
Preliminary Results, varying mass

Chiral condensate $\sigma$, $L^3 \times 2 (4)$, $m_q = 0.025$

- $N_t = 2, L = 4$
- $N_t = 4, L = 4$
- $N_t = 4, L = 10$

Phase coexistence $N_t = 4$
Preliminary Results, varying mass

Baryon number density \( n_B \), \( L^3 \times 2 \) (4), \( m_q = 0 \)

\begin{align*}
\text{Phase coexistence} & \quad N_t = 4 \\
\text{Nt = 2, L = 4} & \\
\text{Nt = 4, L = 4} & \\
\text{L = 10} & \\
\text{L = 10} &
\end{align*}
Preliminary Results, varying mass

Baryon number density $n_B$, $10^3 \times 2$, varying $m_q$

$\mu$ $n_B$, $10^3 \times 2$, varying $m_q$

$10^3 \times 2$, $m_q = 0.1$
$m_q = 0.025$
$m_q = 0$

M. Fromm
Revisiting SCQCD at finite $\mu$ and $T$
M. Fromm
Revisiting SCQCD at finite $\mu$ and $T$
c.f. Mean-Field phase diagram, Nishida ’04: qualitative agreement
Preliminary Results, $m = 0$

Baryon number density $n_B$, $L^3 \times 2 (4)$, $m_q = 0$

- $N_t = 2, L = 4$
- $N_t = 4, L = 4$
- $L = 10$

Phase coexistence $N_t = 4$
Preliminary Results, $m = 0$

Take $T_c = 5/3$, c.f. MC $T_c \approx 1.4$ [Boyd et al.'92]
Preliminary Results, \( m = 0 \)

Take \( T_c = 5/3 \), c.f. MC \( T_c \approx 1.4 \) [Boyd et al.’92]
Summary & Outlook

- Can locate $T = 1/4$ transition with $\mu_c \approx 0.62$ in the chiral limit ($< m_B/3$), $T = 1/2, \mu_c \approx 0.54$
- Observe smoothening of finite $T$ transition with increasing mass - in accord with mean-field

"Assignment"

- Extrapolation $T \to 0$ remains open,
  - Study 1st order PT with multicanonical algorithm
  - Include asymmetry $\gamma$ to vary $T$ continuously
  - Check mean-field predicted relation $T = \gamma^2/N_t$
- SCQCD Phase diagram
  - In the chiral limit: Locate TCP
  - CEP for varying quark mass
  - Flavor dependence of phase diagram
- Consistency check $\mu \to i\mu$
Revisiting SCQCD at finite $\mu$ and $T$