S-parameter & pseudo-NG boson mass from lattice QCD

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Lattice Gauge Theory in the LHC era

- Lattice Gauge Theory (LGT) has been successfully applied to a wide range of physics.
- What can we do using LGT in the LHC era?
Technicolor (TC) [Weinberg(1979), Susskind(1979)]

- Strongly interacting gauge theory

- “χ-symmetry” of TC is dynamically broken at $\Lambda_{TC}$ (as in QCD).
  ➡ Triggers EW symmetry breaking
  ➡ Weak bosons acquire their masses.

- Typically, $m_{W^\pm} = g_2 F_{TC}/2 \iff F_{TC} \sim 250$ GeV
  ($F_{TC}$: technipion decay constant)
  ➡ $\Lambda_{TC} \sim (F_{TC} / f_\pi) \times \Lambda_{QCD} \sim 2600 \times \Lambda_{QCD}$

- Elementary scalar is not necessary.
  ➡ No “hierarchy problem”

- Attractive candidate for the Higgs sector in the SM
Two key observables in TC

- **S-parameter** [Peskin, Takeuchi (1990, 1992)]
  - tends to be sizably affected in TC.

- **Light pseudo-NG bosons**
  - often appear with a mass detectable in LHC (sometimes appear in the excluded region).
**S-parameter** [Peskin, Takeuchi (1990, 1992)]

- Parameterizes "potential new physics contributions" to the EW gauge bosons' self-energy. "Oblique correction"
- Useful for New Physics search using the EW precision data

\[
S = 16\pi \left[ \frac{\partial}{\partial q^2} \left( q^2 (\Pi^{(1)}_{VV} - \Pi^{(1)}_{AA}) \right) \right]_{q^2=0}
\]

\[
i \int d^4 x \, e^{i q \cdot x} \langle 0 | T [J_{A,\mu}(x) J_{B,\nu}(0)] | 0 \rangle = \left( g_{\mu\nu} q^2 - q_{\mu} q_{\nu} \right) \Pi^{(1)}_{AB}(q^2) - q_{\mu} q_{\nu} \Pi^{(0)}_{AB}(q^2)
\]

\[
\langle VV-AA \rangle \Rightarrow S\text{-parameter}
\]
$S$-parameter and $L_{10}$

Interesting scale $\sim \Lambda_{TC} \Leftrightarrow$ Low energy TC $\Rightarrow$ ChPT in TC

- In ordinary QCD ChPT [Gasser and Leutwyler (1984, 1985)]

\[
\Pi^{(1)}_{V-A}(q^2) = \Pi^{(1)}_{VV}(q^2) - \Pi^{(1)}_{AA}(q^2)
\]

\[
= -\frac{f^2}{q^2} - 8 L_{10}^r(\mu) - \frac{\ln \left( \frac{m^2_\pi}{\mu^2} \right) + \frac{1}{3} - H(4m^2_\pi/q^2)}{24\pi^2}
\]

\[
H(x) = (1 + x) \left[ \sqrt{1 + x} \ln \left( \frac{\sqrt{1 + x} - 1}{\sqrt{1 + x} + 1} \right) + 2 \right]
\]

$L_{10}$: one of LEC's in ChPT

- Reinterpret QCD $\rightarrow$ TC, and substitute the result

\[
S = -16\pi \left[ L_{10}^r(\mu) - \frac{1}{192\pi^2} \left\{ \ln \left( \frac{\mu^2}{m^2_H} \right) - \frac{1}{6} \right\} \right]
\]

Therefore determining $L_{10}$ is equivalent to determining $S$-parameter.
Pseudo NG Boson Mass \cite{Peskin1980, Preskill1981}

- TC models ⇒ too many NG bosons.
- One standard way out: introduce extra gauge symmetry which explicitly breaks $\chi$-symmetry.
- Then NG bosons acquire the mass, and become pseudo-NG.

$$m_{PNG}^2 = G \int_0^\infty dq^2 q^2 \left[ \Pi_T^{(1)}(q^2) - \Pi_X^{(1)}(q^2) \right]$$

$G$: model dependent coefficient
$\Pi_T$: VP of currents corresponding to unbroken generators
$\Pi_X$: VP of currents corresponding to broken generators

Once the underlying TC theory is specified, the NP part is independent of further details.
Pseudo NG Boson Mass \cite{Peskin1980, Preskill1981}

- A well known example is the charged pion in QED+QCD theory.
- QED interaction explicitly breaks chiral symmetry of QCD.

DGMLY sum rule in the chiral limit \cite{Das1967}

\[
m_{\pi^\pm}^2 = -\frac{3\alpha}{4\pi} \int_0^\infty dq^2 q^2 \frac{\Pi_{V-A}^{(1)}(q^2)}{f^2} \bigg|_{m_q=0}
\]

\(\langle VV-AA \rangle\) comes in again.

With different method,
Duncan, Eichten , Thacker\cite{1998}, Blum, Doi, Hayakawa, Izubuchi, Yamada\cite{2007},
Namekawa, Kikukawa\cite{2006}
In this work

- Consider two-flavor QCD as TC, and calculate $\Pi_{V-A}(q^2)$ on the lattice.
- Evaluate
  - $L_{10}$ (or S-parameter) through
    \[
    \Pi_{V-A}^{(1)}(q^2) = -\frac{f_\pi^2}{q^2} - 8 L_{10}(\mu) - \frac{\ln \left( \frac{m_\pi^2}{\mu^2} \right) + \frac{1}{3} - H(4m_\pi^2/q^2)}{24\pi^2}
    \]
  - $m_{\pi^\pm}^2$ (or pseudo-NG boson mass) from
    \[
    m_{\pi^\pm}^2 = -\frac{3\alpha}{4\pi} \int_0^\infty dq^2 \frac{q^2 \Pi_{V-A}^{(1)}(q^2)|_{m_q=0}}{f^2}
    \]
- Compare with their experimental values.

Demonstrate the feasibility of the lattice technique for these quantities.
In continuum, WT Identity guarantees that $\langle VV- AA \rangle$ vanishes if there is no spontaneous nor explicit $\chi$-sym breaking.

If the lattice formulation explicitly breaks $\chi$-sym, it is difficult to disentangle the effect of the $S\chi$SB from the explicit breaking due to the lattice artifact.

Exact $\chi$-sym is required in this calculation to extract the physic from $\langle VV- AA \rangle$. [Sharpe(2007)]

Overlap fermion formalism
## Simulation Parameters

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\( r_0 = 0.49 \text{ fm} \Rightarrow a = 0.1184(12)(11) \text{ fm} \quad (1/a = 1.67(2)(2) \text{ GeV}) \)

\( (L/a)^3 \times (T/a) = 16^3 \times 32 \Rightarrow V \approx (1.9 \text{ fm})^3 \)

- lightest pion \( \Rightarrow m_\pi \approx 290 \text{ MeV}, \ m_\pi L \approx 2.8. \)
- Calculation is done in a fixed topological sector \( Q_{\text{top}} = 0. \)
Current correlator in continuum

\[ i \int d^4 x \ e^{iq \cdot x} \langle 0 \mid T \left[ J_\mu(x) J_\nu^+(0) \right] \mid 0 \rangle = \left( q^2 g_{\mu\nu} - q_\mu q_\nu \right) \Pi^{(1)}_J(q^2) - q_\mu q_\nu \Pi^{(0)}_J(q^2), \]

\[ J_\mu(x) = \begin{cases} 
V_\mu(x) = \bar{q}_1(x) \gamma_\mu q_2(x), \\
A_\mu(x) = \bar{q}_1(x) \gamma_\mu \gamma_5 q_2(x), 
\end{cases} \]
Current correlator on the lattice

\[ \Pi_{J_{\mu\nu}}(\hat{q}) = \sum_{x} e^{i\hat{q} \cdot x} \langle 0 | T \left[ J_{\mu}^{(21)}(x) J_{\nu}^{(12)}(0) \right] | 0 \rangle \]

\[ = \sum_{n=0}^{\infty} B_{J}^{(n)}(\hat{q}_{\mu})^{2n} \delta_{\mu\nu} + \sum_{n,m=1}^{\infty} C_{J}^{(n,m)}(\hat{q}_{\mu})^{2n-1}(\hat{q}_{\nu})^{2m-1} \]

\[ V_{\mu}^{(12)} = Z \bar{q} \gamma_{\mu} (1 - a D/2m_{0}) q_{2} \text{ and similarly defined } A_{\mu}^{(12)} \]

\[ Z = 1.3842(3) \text{ is common, and determined nonperturbatively.} \]

- The currents are not conserved ones. c.f. [Kikukawa, A. Yamada (1999)]
- Many terms representing lattice artifacts show up.
  (only \( B_{J}^{(0)} \) & \( C_{J}^{(1,1)} \) are physically relevant.)
- But the exact symmetry between \( V_{\mu} \) and \( A_{\mu} \) simplifies the analysis!
Cancellation of the artifacts in $\Pi_{V-A}$

With our $V_\mu$ and $A_\mu$, $\langle VV- AA \rangle$ exactly vanishes in the absence of both explicit and spontaneous breakings as in continuum.

The artifacts arising in short distance vanishes in $\langle VV- AA \rangle$.

The artifacts coupling to long distance physics are numerically investigated, and found to be negligibly small in $\langle VV- AA \rangle$.

Therefore, we write $\langle VV- AA \rangle$ as

$$\Pi_{V\mu\nu} - \Pi_{A\mu\nu} = (\hat{q}^2 \delta_{\mu\nu} - \hat{q}_\mu \hat{q}_\nu) \Pi_{V-A}^{(1)} - \hat{q}_\mu \hat{q}_\nu \Pi_{V-A}^{(0)}$$

By considering $\mu=\nu$ and $\mu\neq\nu$, we extract $\Pi_{V-A}^{(0)}(q^2)$, $\Pi_{V-A}^{(1)}(q^2)$.
$L_{10}$ from $\Pi^{(1)}_{V-A}(q^2)$

- ChPT predicts [Gasser & Leutwyler (1984)]

$$\Pi^{(1)}_{V-A}(q^2) = -\frac{f_\pi^2}{q^2} - 8 L_{10}(\mu_\chi) - \frac{\ln \left( \frac{m_\pi^2}{\mu_\chi^2} \right) + \frac{1}{3} - H(x)}{24\pi^2}$$

($x=4m_\pi^2/q^2$, $H(x)$ is known function.)

Fit the data to the ChPT prediction using the measured $f_\pi$ and $m_\pi$.

$$L_{10}(m_\rho) = -5.2(2)^{(+0)_{-3}}(+5)_{-0} \times 10^{-3}$$

($\chi^2/dof=0.5, 2.3$)

$$L_{10}(\text{Exp}) = -5.09(47) \times 10^{-3}$$
Pseudo-NG boson mass

\[ m_{\pi^\pm}^2 = -\frac{3\alpha}{4\pi} \int_0^\infty dq^2 \frac{q^2 \Pi^{(1)}_{V-A}(q^2)|_{m_q=0}}{f^2} \]

Small \( q^2 \) region: Integrate fit func.

Functional forms adopted:

\[ \hat{q}^2 \Pi^{(1),\text{fit}}_{V-A}(\hat{q}^2) = -\hat{f}_\pi^2 + \frac{\hat{q}^2 \hat{f}_V^2}{\hat{q}^2 + \hat{m}_V^2} - \frac{\hat{q}^2 \hat{f}_A^2}{\hat{q}^2 + \hat{m}_A^2} - \frac{\hat{q}^2}{24\pi^2} \frac{X(\hat{q}^2)}{1 + x_5 (Q_\rho^2)^4}, \]

where \( Q_\rho^2 = \hat{q}^2/(a^2m_\rho^2) \)

- 1st and 2nd Weinberg sum rules are imposed.

\[ \hat{f}_\pi^2 = \hat{f}_V^2 - \hat{f}_A^2, \quad \hat{f}_A \hat{m}_A = \hat{f}_V \hat{m}_V, \]
\[ \hat{f}_V = x_1 + x_3 \hat{m}_\pi^2, \quad \hat{m}_V = x_2 + x_4 \hat{m}_\pi^2 \]

- \( X(q^2) \) are chosen to be consistent with the ChPT (OPE) prediction in small (large) \( q^2 \).

\[
X(q^2) = \begin{cases} 
\ln \left( \frac{\hat{m}_\pi^2}{\hat{m}_\rho^2} \right) + \frac{1}{3} - H(4\hat{m}_\pi^2/\hat{q}^2) + x_6 Q_\rho^2 \\
x_6 Q_\rho^2 \ln(Q_\rho^2).
\end{cases}
\]

Integral region is separated at \( q^2=2 \) to avoid discretization effects.
Pseudo-NG boson mass

- **Small** $q^2$ region: Integrate fit func.
  \[ \Delta m^2_{\pi} |_{q^2 \leq 2.0} = 676(50) \text{ and } 811(12) \text{ MeV}^2 \]

- **Large** $q^2$ region: OPE predicts
  \[ \Pi^{(1)}_{V-A}(q^2) \sim a_6/(q^2)^3 \]
  \[ a_6 = [-0.001, -0.01] \text{ GeV}^6 \]
  \[ \Delta m^2_{\pi} |_{q^2 \geq 2.0} = 182(149) \text{ MeV}^2 \]
  \[ \Delta m^2_{\pi} = 993(12)(^{+0}_{-135})(149) \text{ MeV}^2 \]

Errors: (statistical)(chiral extrapolation)(large $q^2$)

Exp: $\Delta m^2_{\pi} = 1261.2 \text{ MeV}^2$
Summary

- We used overlap fermion to calculate the S-parameter and pNG boson mass in 2-flavor QCD. Chiral symmetry on the lattice is essential in this calculation.

- Both the calculations reasonably reproduced the experimental values. Thus the feasibility of the LGT to calculate these quantities is demonstrated.

- The study of more realistic TC models is an interesting extension.

- LGT may be able to directly investigate physical quantities relevant for the LHC phenomenology.
Cancellation of the artifacts in $VV$-$AA$

- Define a measure of artifacts

$$\Delta_J = \sum_{\mu, \nu} \hat{q}_\mu \hat{q}_\nu \left( \frac{1}{\hat{q}^2} - \frac{\hat{q}_\nu}{\sum_\lambda (\hat{q}_\lambda)^3} \right) \Pi_{J \mu \nu}$$

$$\Delta_J = \sum_{n=1}^{\infty} B_J^{(n)} \left( \frac{Q^{(2n+2)}}{q^2} - \frac{Q^{(2n+3)}}{Q^{(3)}} \right)$$

$$+ \sum_{n,m=1}^{\infty} C_J^{(n,m)} Q^{(2n)} \left( \frac{Q^{(2m)}}{q^2} - \frac{Q^{(2m+1)}}{Q^{(3)}} \right)$$

$(n = m = 1$ is not included$)$

$\Delta_J$ entirely consists of lattice artifacts!

In the difference $\langle VV-AA \rangle$, irrelevant terms cancel!
\[ \Pi_{V-A}^{(0)}(q^2) \]

In the spectral representation,

\[ q^2 \Pi_{V-A}^{(0)}(q^2) = \frac{f_\pi^2 m_\pi^2}{q^2 + m_\pi^2} + (\text{excited states } \sim O(m_q^2)) \]

- The obtained \( \Pi_{V-A}^{(0)}(q^2) \) is compared to the spectral rep.
- For \( f_\pi \) and \( m_\pi \), the measured values are used.