B and *D* Meson Decay Constants

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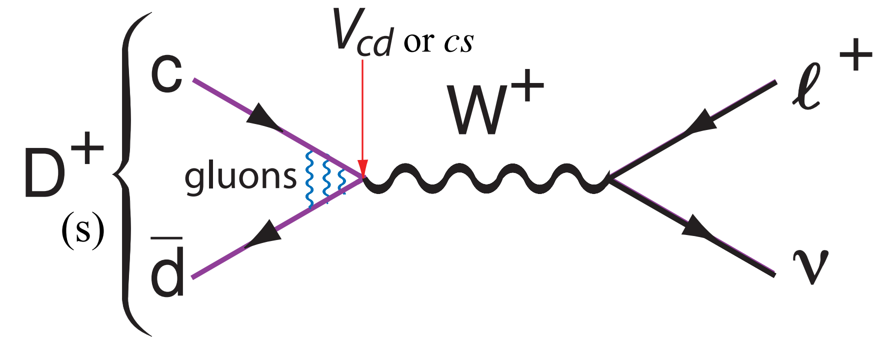
For the Fermilab Lattice
and MILC Collaborations

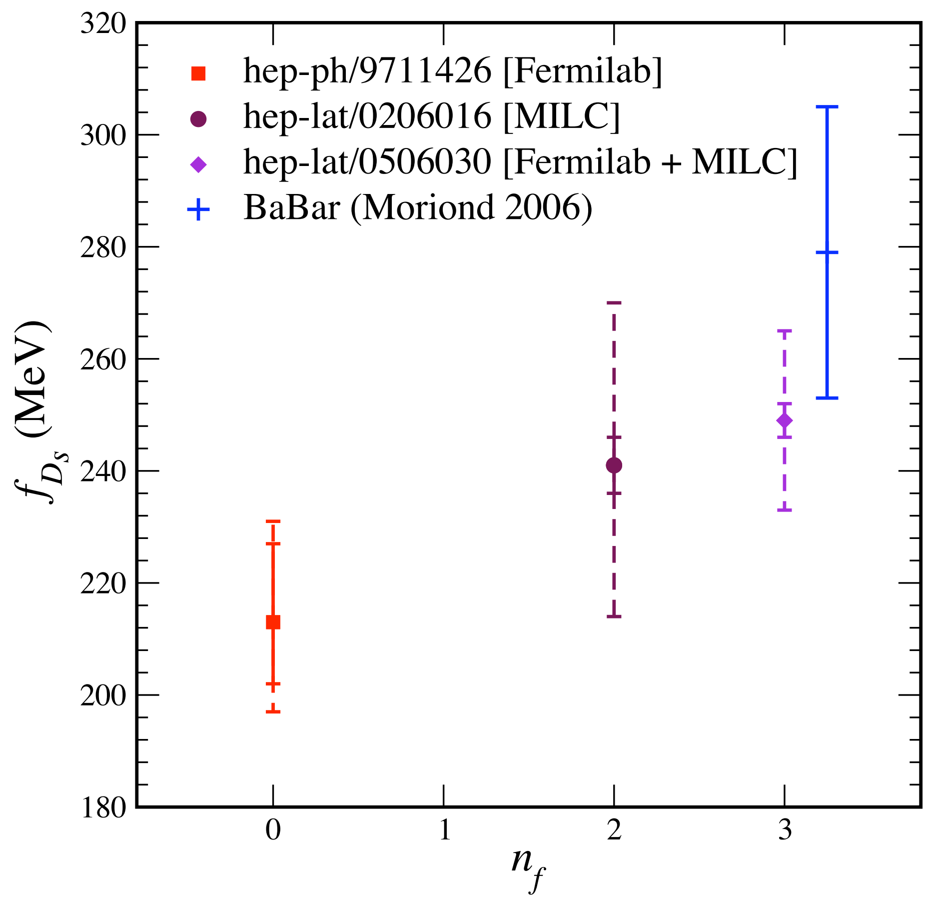
Lattice 2008
Williamsburg
July 14-19, 2008

2005:



- The D and D_s decay constants were predicted by Fermilab/MILC to 10% before the experiments were done to that accuracy.

f_{D_s}



Fermilab/MILC, Phys. Rev. Lett. **95**: 122002, 2005.

$$f_{D^+} = \begin{cases} 201 \pm 03 \pm 17 \ \text{MeV [lattice]} \\ 223 \pm 17 \pm 03 \ \text{MeV [CLEO]} \end{cases}$$

$$f_{D_s} = \begin{cases} 249 \pm 03 \pm 16 \ \text{MeV [lattice]} \\ 279 \pm 17 \pm 20 \ \text{MeV [BaBar]} \end{cases}$$

$$\frac{\sqrt{m_{D^+}} f_{D^+}}{\sqrt{m_{D_s}} f_{D_s}} = \begin{cases} 0.786 \pm 0.042 \ \text{MeV [lattice]} \\ 0.779 \pm 0.093 \ \text{MeV [expt]} \end{cases}$$

Caveat: We claimed a success, but as calculations become increasingly accurate, at some point we do not expect perfect agreement between the Standard Model and experiment.

Where will that point be?

2008:

- Uncertainties in f_{Ds} from experiment and in the Fermilab/MILC calculation have been reduced, theory has stayed low and experiment has stayed high.

- New calculation from HPQCD:

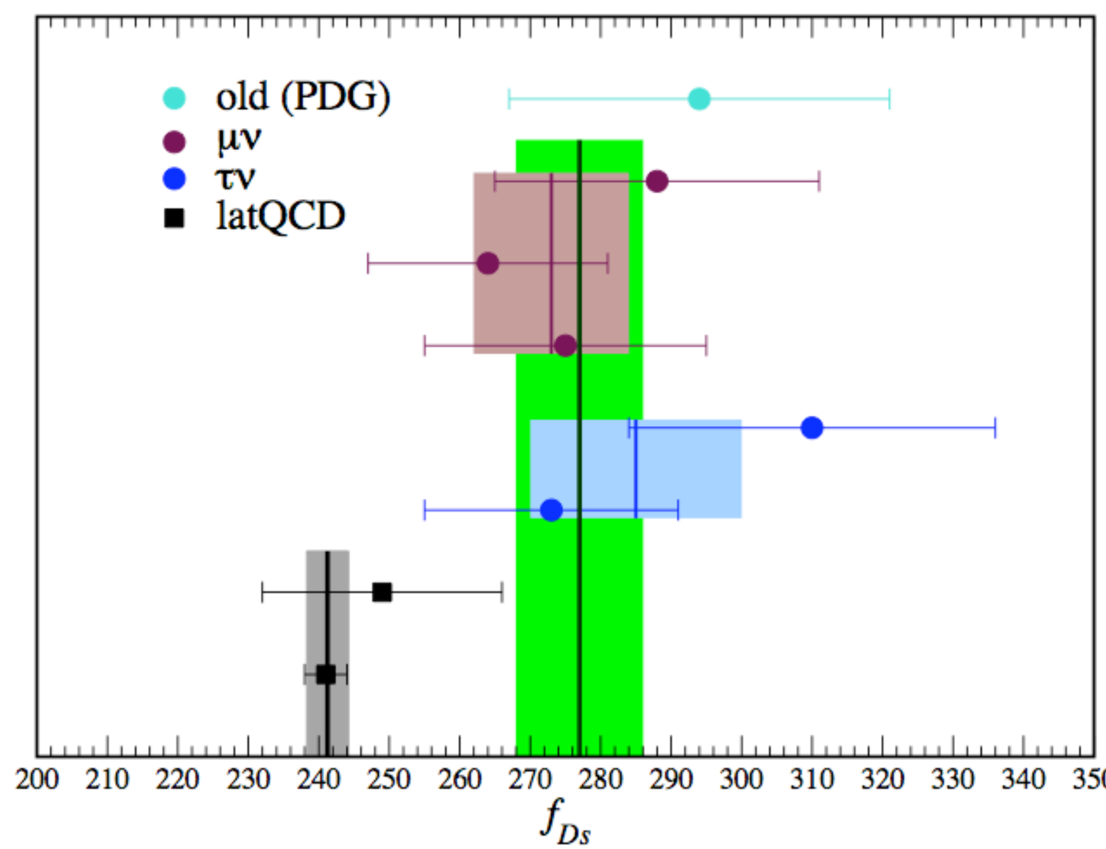
$$f_K = 157(2)\ \mathrm{MeV} \quad f_D = 207(4)\ \mathrm{MeV}$$

$$f_K/f_\pi = 1.189(7) \quad f_{Ds} = 241(3)\ \mathrm{MeV}$$

(f_π, f_K, and f_D correct to 2%.)

- 3.5 σ in f_{Ds}.

HPQCD, 2008



BaBar
CLEO
Belle

CLEO
CLEO

Fermilab/MILC '05

HPQCD '08

Fermilab/MILC D, D_s, B, and B_s decay constants

- Improved staggered (asqtad) light quarks,
- Clover/Fermilab $O(a)$ improved heavy quarks.
- MILC 2+1 flavor Symanzik improved gauge configurations (Phys. Rev. D70:114501, 2004).

Ensembles:

a [fm]	am_h	am_l	β	r_1/a	configs	# m_q
0.09	0.031	0.0031	7.08	3.69	435	11
		0.0062	7.09	3.70	557	10
		0.0124	7.11	3.72	518	8
0.12	0.05	0.005	6.76	2.64	529	12
		0.007	6.76	2.63	833	12
		0.01	6.76	2.62	592	12
		0.02	6.79	2.65	460	12
		0.03	6.81	2.66	549	12
0.15	0.0484	0.0097	6.572	2.13	631	9
		0.0194	6.586	2.13	631	9
		0.029	6.600	2.13	440	9

We are finishing a reanalysis of our existing data and preparing for new runs this year with four times the statistics.

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Partially quenched staggered chiral perturbation theory used to extrapolate to the chiral and continuum limits.

The decay constants are defined by

$$\left\langle 0 \mid A_\mu \mid H_q(p) \right\rangle = i f_{H_q} p_\mu$$

The combination

$$\phi_{H_q} = f_{H_q} \sqrt{m_{H_q}}$$

is obtained from a combined fit to

$$C_O(t) = \left\langle O^\dagger_{H_q}(t) \;\; O_{H_q}(0) \right\rangle$$
$$C_{A_4}(t) = \left\langle \, A_4(t) \;\;\; O_{H_q}(0) \right\rangle \, ,$$

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Nonperturbative.

Very nearly unity.

Staggered chiral perturbation theory.

Aubin and Bernard.

$$M^2_{ab,\xi} = (m_a + m_b)\mu + a^2\Delta_\xi$$

$$\phi_{Hq} = \Phi_H\left[1 + \Delta f_H(m_q, m_l, m_h) + p_H(m_q, m_l, m_h)\right]$$

$$\Delta f_H = -\frac{1+3g^2_{H^*H\pi}}{2(4\pi f_\pi)^2}\left[\bar{h}_q + h^I_q + a^2\left(\delta'_A h^A_q + \delta'_V h^V_q\right)\right]$$

$$\begin{aligned} p &= \frac{1}{2(4\pi f_\pi)^2}\left[p_1(m_l, m_h) + p_2(m_q)\right] \\ p_1 &= f_1(\Lambda_\chi)\left[\frac{11}{9}\mu(2m_l + m_h) + a^2\left(\frac{3}{2}\bar{\Delta} + \frac{1}{3}\Delta_I\right)\right] \\ p_2 &= f_2(\Lambda_\chi)\left[\frac{5}{3}\mu m_q + a^2\left(\frac{3}{2}\bar{\Delta} - \frac{2}{3}\Delta_I\right)\right], \end{aligned}$$

Global fit to all partially quenched data at all lattice spacings used to obtain results.

$$\bar{\Delta} = \frac{1}{16}\sum_\xi n_\xi \Delta_\xi$$

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Taste breaking effects in meson masses.

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Analytic and nonanalytic taste-breaking effects in decay constants.

$$\Delta f_H = -\frac{1 + 3g^2_{H^*H\pi}}{2(4\pi f_\pi)^2}\left[\bar{h}_q + h^I_q + a^2\left(\delta'_A h^A_q + \delta'_V h^V_q\right)\right]$$

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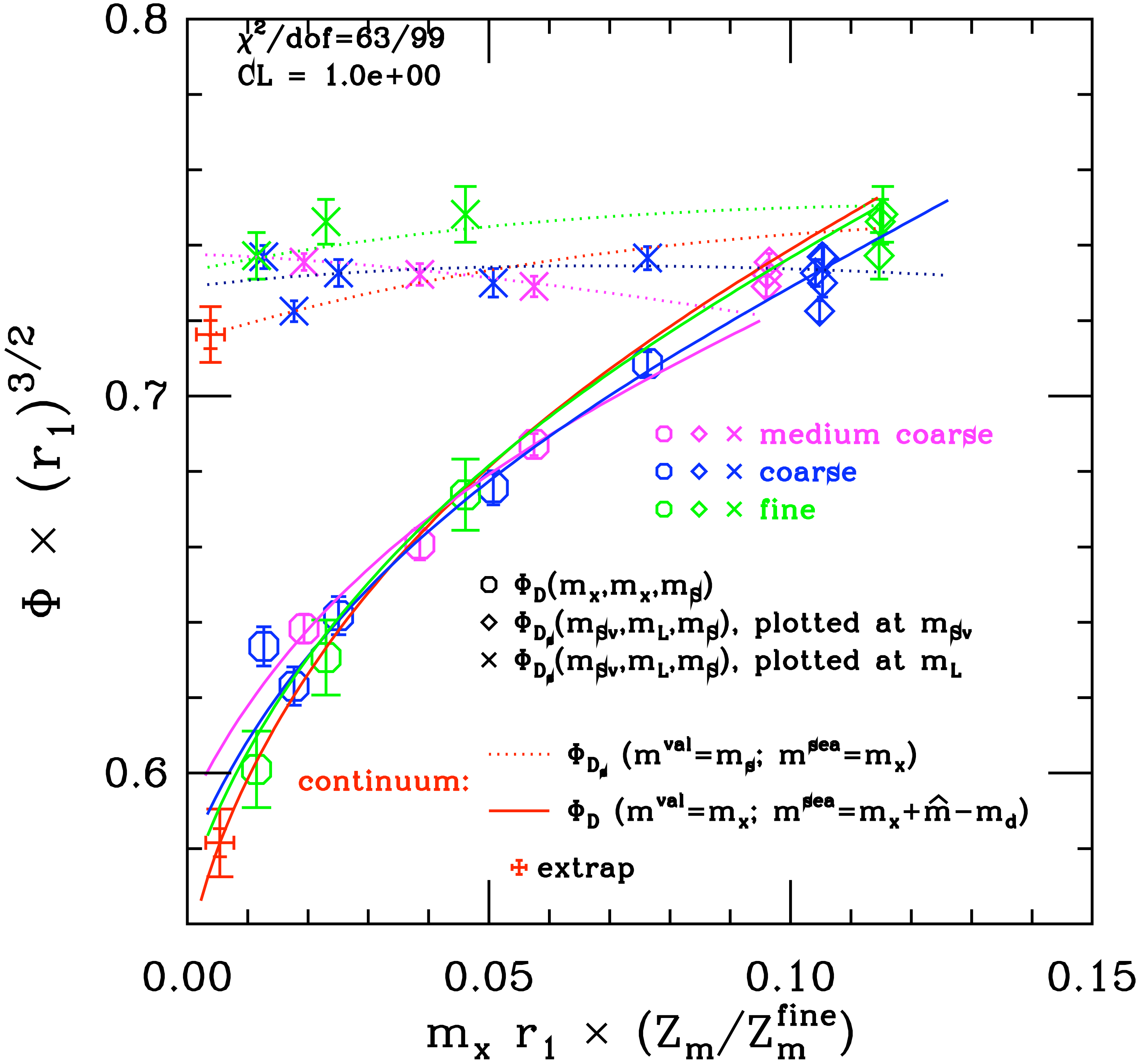
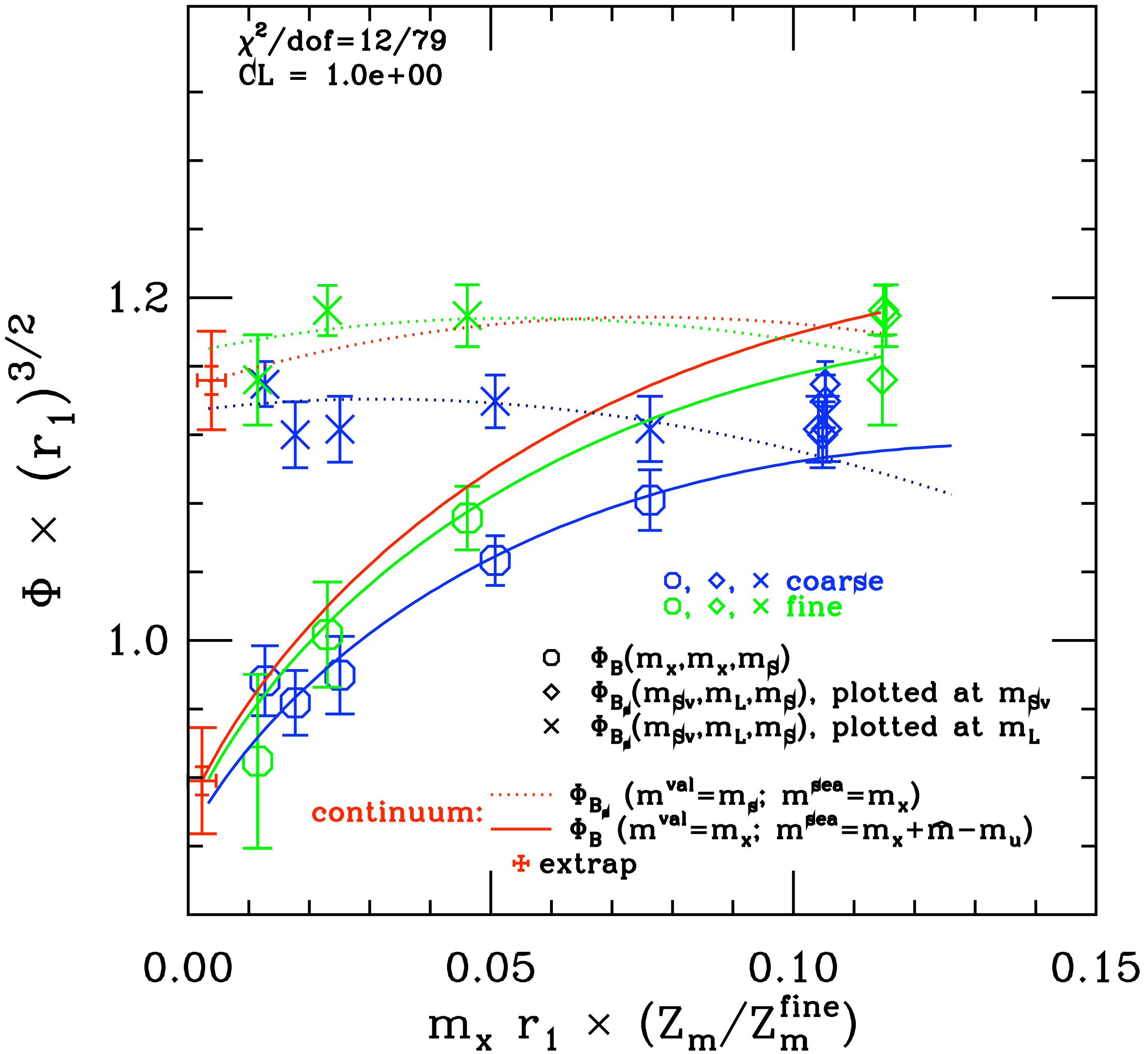
Φ_D and Φ_{Ds} chiral extrapolation



Slope is larger in the continuum limit. Taste breaking effects suppress the logs at finite *a*.

Φ_B and Φ_{Bs} chiral extrapolation



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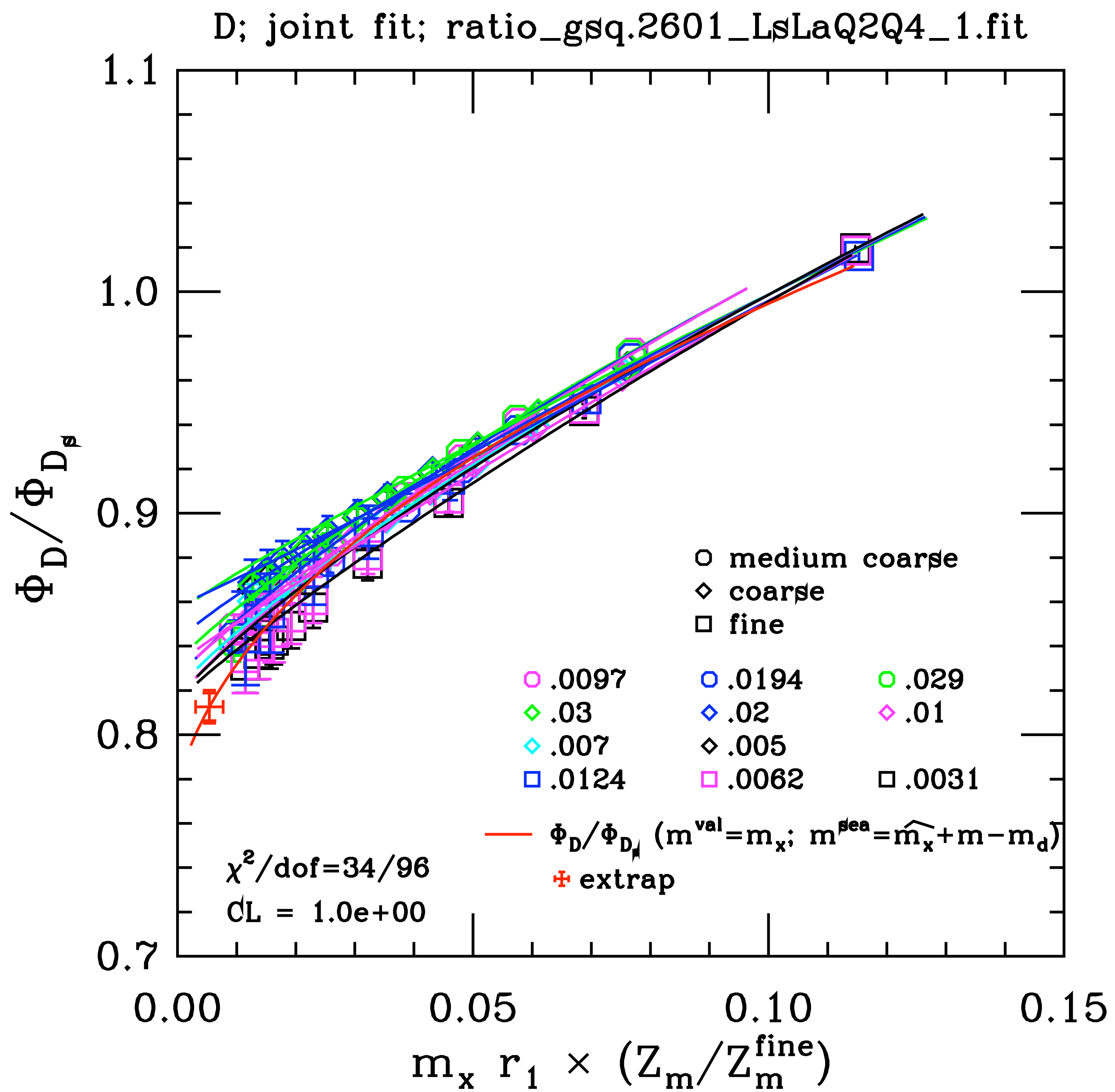

Error budgets

Improved this year.

	Φ_{Ds}	Φ_{Dd}	R^D	Φ_{Bs}	Φ_{Bd}	R^B	
Statistics	3.1 1.0	3.8 1.5	1.0 1.0	2.1 2.5	3.1 3.4	1.8 2.2	2007 2008
Inputs r_1, m_s, m_l	1.4	2.1	0.6	1.8	2.5	0.6	
Input m_c or m_b	2.7	2.7	0.1	1.1	1.1	0.1	
Z	1.4	1.4	<0.1	1.4	1.4	<0.1	
Higher order ρ_{A4}	0.1	0.1	<0.1	0.4	0.4	<0.1	
Heavy q discretization	2.7	2.7	0.3	1.9	1.9	0.2	
Light q disc. & χ extr.	1.2	2.6	1.6	2.0	2.4	2.4	
V	0.2	0.6	0.6	0.2	0.6	0.6	
Total systematic	4.5	5.3	1.8	3.8	4.4	2.6	

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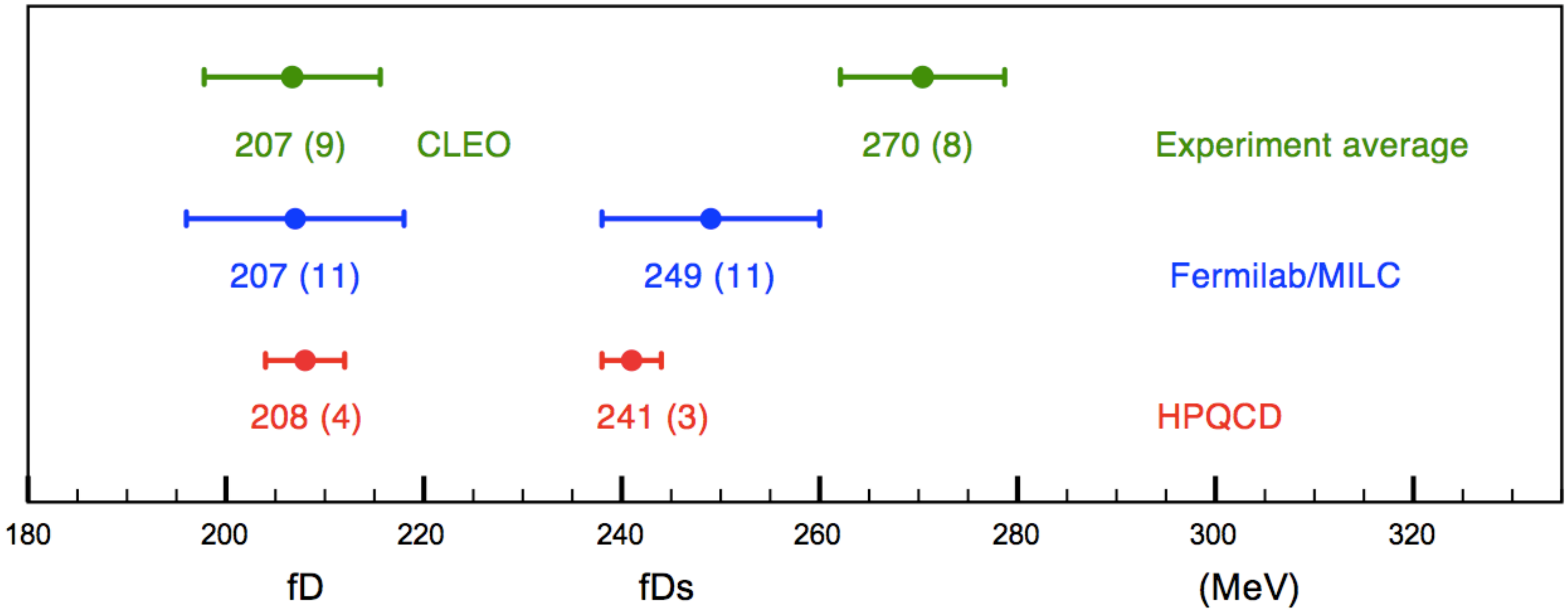
Results

f_D = 207 (11) MeV
f_{Ds} = 249 (11) MeV
f_B = 195 (11) MeV
f_{Bs} = 243 (11) MeV

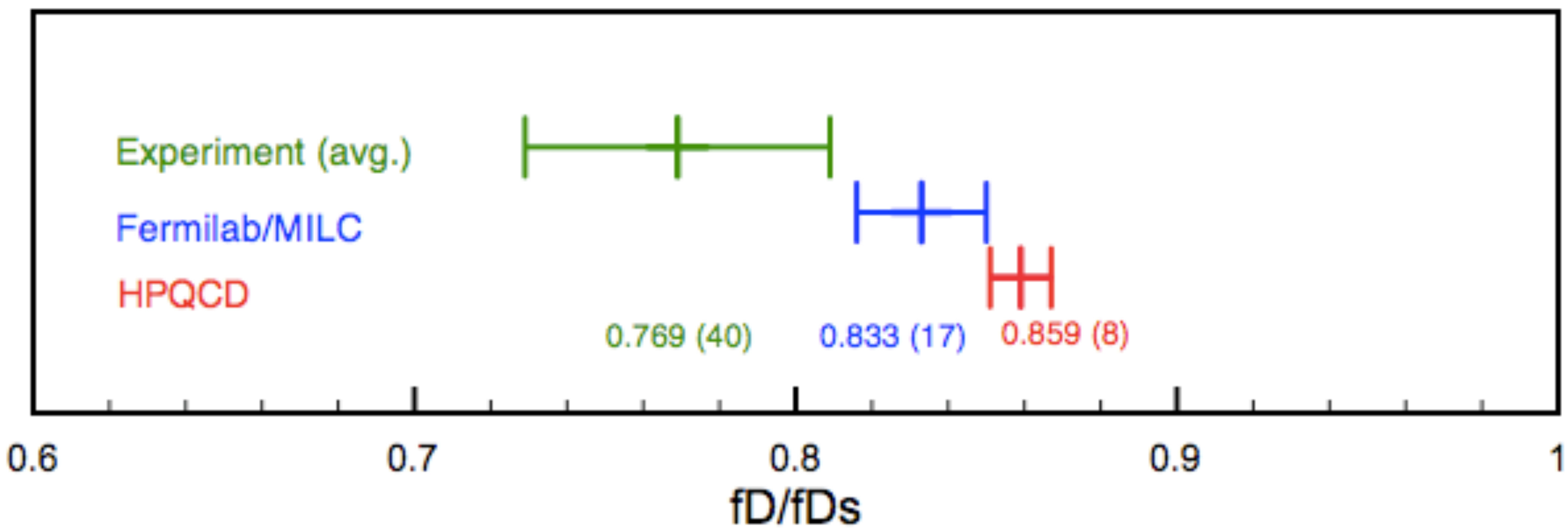
f_D/f_{Ds} = .833 (8)(17),
f_B/f_{Bs} = .803 (18)(21).

Comparison of f_{Ds} with experiment



- For f_D, good agreement between experiment, HPQCD and Fermilab/ MILC.
- For f_{Ds},
  - Agreement between HPQCD and Fermilab/MILC,
  - 1.6 σ disagreement between Fermilab/MILC and experiment,
  - 3.5 σ disagreement between HPQCD and experiment.

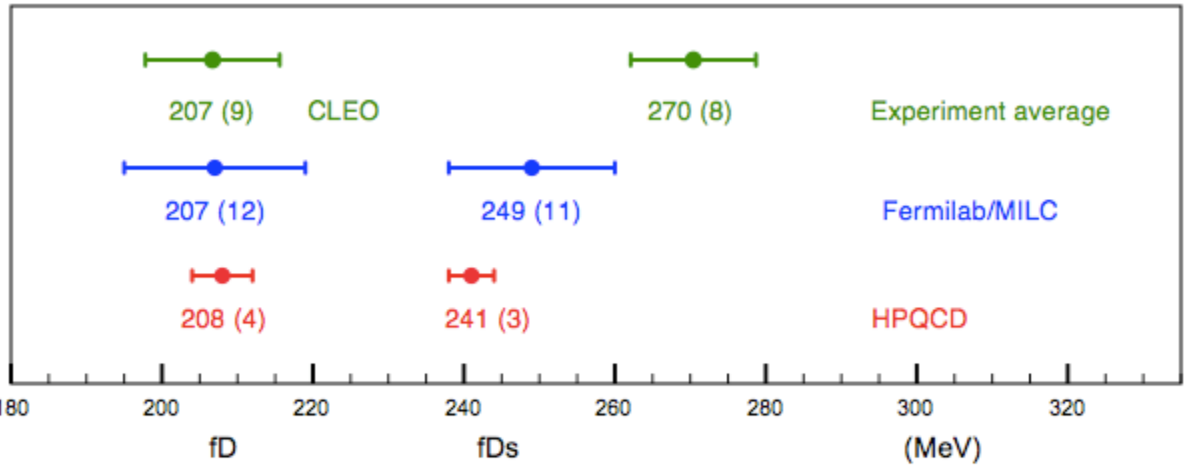
Comparison of f_D/f_{Ds} with experiment



- For now, looking at f_D/f_{Ds} doesn't clean up the picture.
  - A slight disagreement between HPQCD and FNAL/MILC develops.
  - Experimental uncertainties are independent, and add in quadrature.

Theory vs. experiment for f_{D_s}

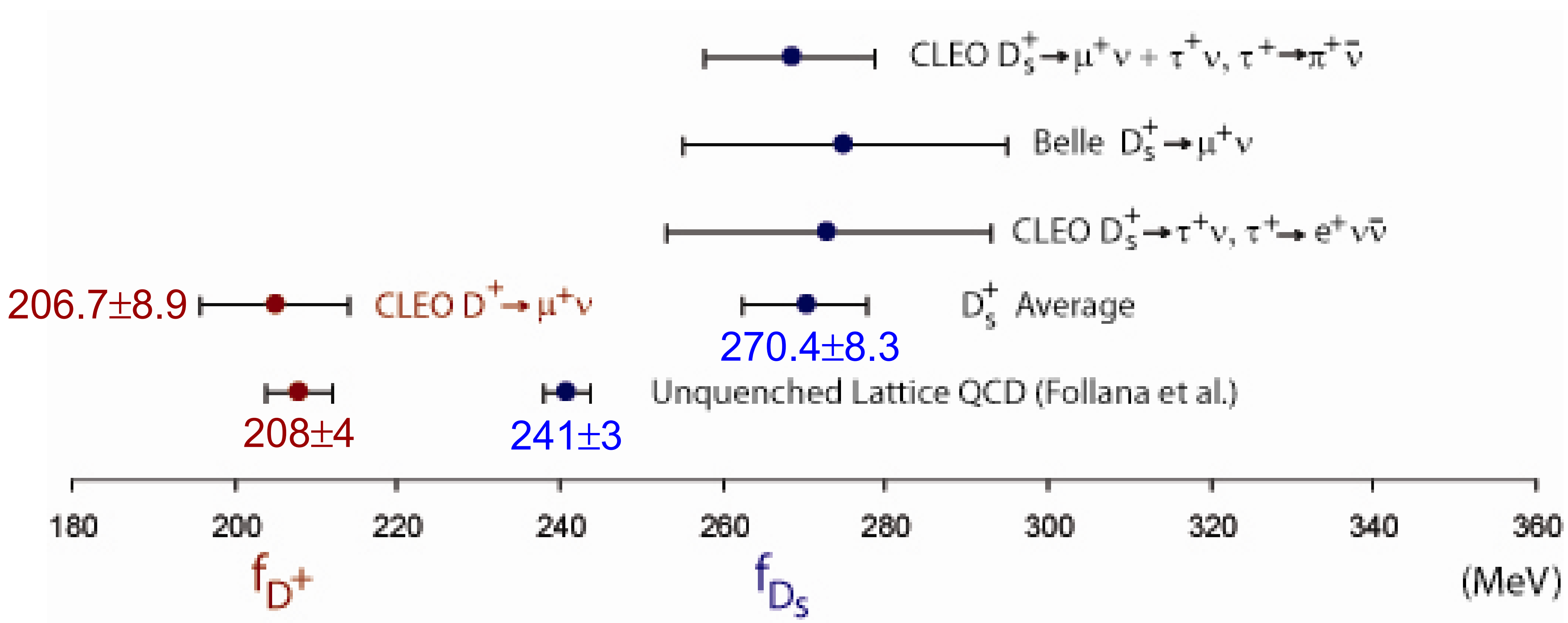
- 3.5 σ discrepancy is dominated by experimental *statistical* error(!).
  - Double HPQCD theory error bar, discrepancy → 3.3σ.
  - Triple HPQCD error (and include Fermilab/MILC 2005 value) → 3.1σ.
  - f_{D_s} should be easier than f_D, but f_D, f_K, and f_π come out fine to 2%.
- What if the discrepancy is real (Kronfeld talk Friday)?
  - Kronfeld and Dobrescu, effect could be caused by:
    - Charged Higgs (in a new 2HDM)
    - Leptoquarks (of two ilks)



The view from CLEO (Sheldon Stone):

Conclusions

- We are in close agreement with the Follana et al calculation for f_{D^+}. This gives credence to their methods
- The disagreement with f_{D_s} is enhanced



Outlook

- Reanalysis of our existing data is being completed.
  - Bringing down several of biggest uncertainties.
- New runs are starting to quadruple the current statistics.
  - Should help with most of the uncertainties.
- f_{Ds} theory vs. experiment remains a puzzle.
  - A good target for other fermion methods.