The hadronic light-by-light contribution to the anomalous magnetic moment of the muon: a lattice approach

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Introduction

- Muon anomalous magnetic moment, $g-2$
- Vacuum polarisation contribution to $g-2$
- Light-by-Light: Introduction
- Light-by-Light: method
- Light-by-Light results (intermediate).
- Conclusions

Muon Magnetic Moment

Consider a classical charged body. If it rotates it has angular momentum

$$\mathbf{J} = m\langle r_m^2 \rangle \omega$$

and a magnetic moment

$$\mathbf{M} = \frac{1}{2} q \langle r_q^2 \rangle \omega$$

If the mass-radius r_m and charge-radius r_q are the same,

$$\mathbf{M} = \frac{q}{2m} \mathbf{J}$$

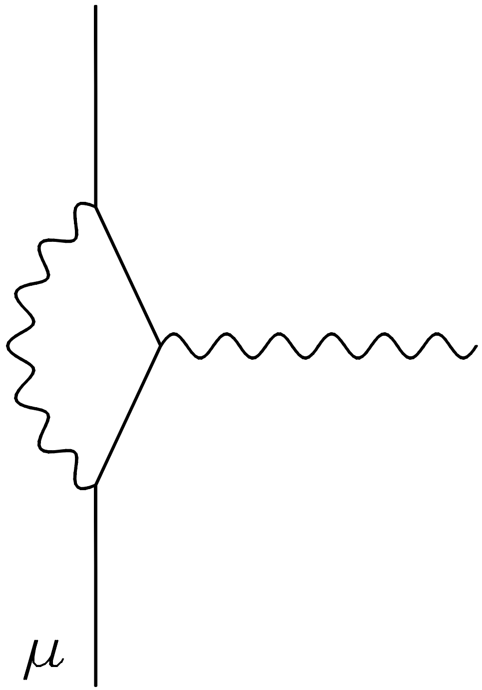
Muon Magnetic Moment

If you solve the Dirac equation in a magnetic field, you find that a fermion has a magnetic moment twice as big as this classical model:

$$\mathbf{M} = g \, \frac{q}{2m} \mathbf{J}$$

with $g = 2$ from the (tree-level) Dirac equation.

Muon Anomalous Magnetic Moment



Quantum corrections change the result:

$$a_\mu \equiv (g-2)/2 = \frac{\alpha_e}{2\pi} + \cdots$$

a_μ is called the anomalous magnetic moment.

Muon Anomalous Magnetic Moment

Can be measured to great accuracy

$$a_\mu = 116\,592\,082\,(55)\ \times\ 10^{-11}$$

Can also be calculated:
5 loop QED
2 loop electro-weak
hadronic effects

Possible mismatch, $2.7\ \sigma$.
Main theory uncertainty, hadronic effects.

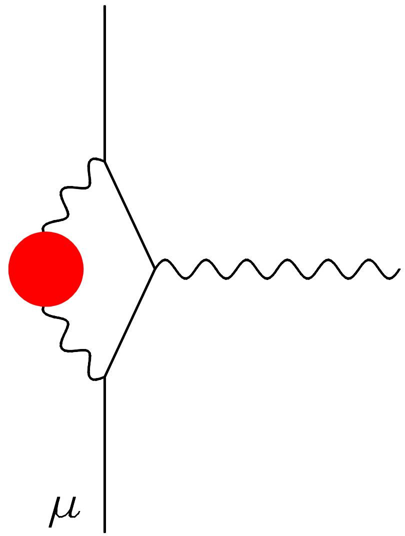
Muon Anomalous Magnetic Moment

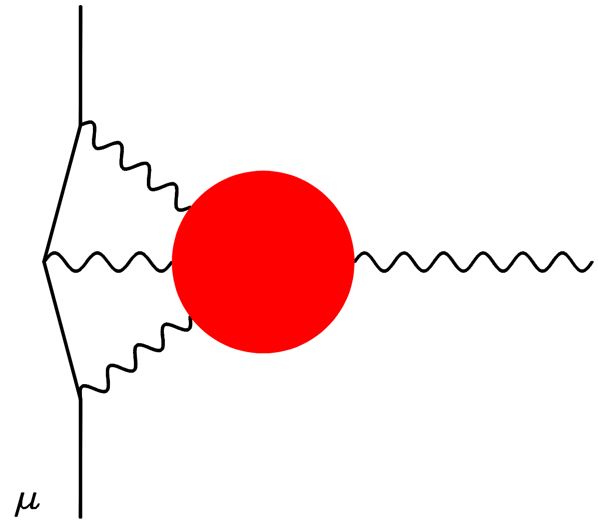
Is this mismatch of $2.7\ \sigma$ a sign of new physics? Expect

$$\sim \frac{m_\mu^2}{M_{new}^2}$$

Or is it a problem on the theory side?

Hadronic Effects





Vacuum Polarisation

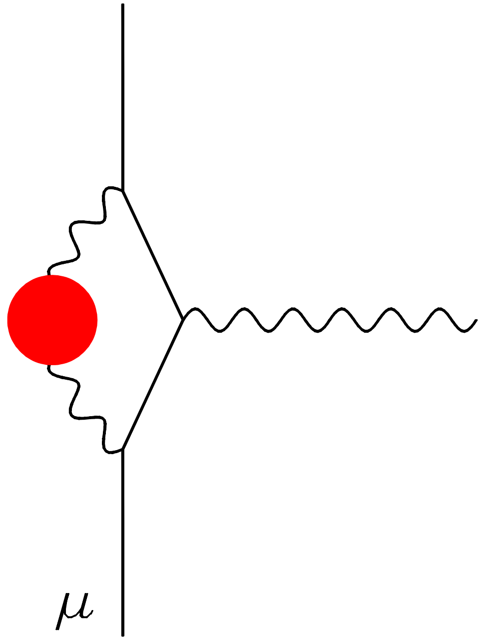
Light-by-Light

α_e^2 α_e^3 α_e^3

$6934(63) \times 10^{-11}$ $-98(1) \times 10^{-11}$ $136(25) \times 10^{-11}$

(Melnikov + Vainshtein)

Vacuum Polarisation

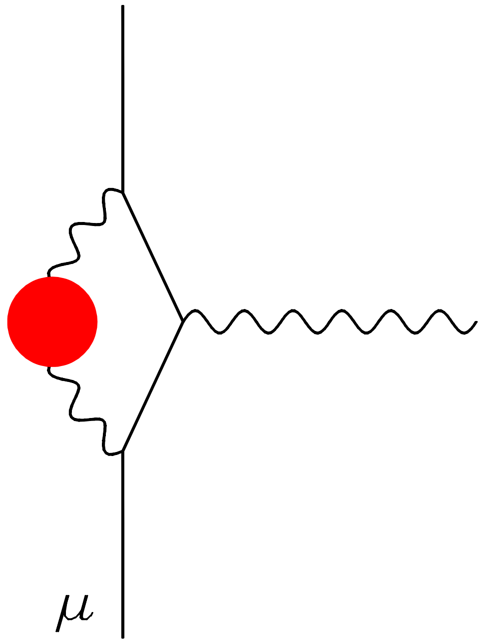


Related to cross-section

$$e^+ \, e^- \rightarrow hadrons$$

via dispersion relation and electromagnetic kernel.

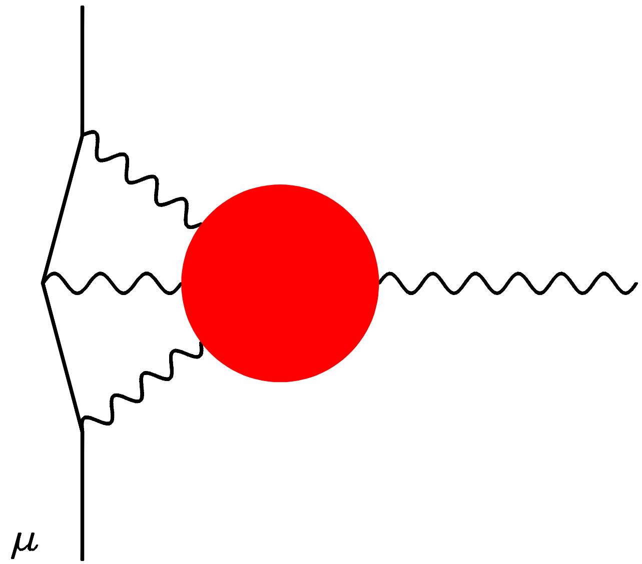
Vacuum Polarisation



Calculating the vacuum polarisation in lattice gauge theory.

$$\langle J_\mu(0) J_\nu(x) \rangle$$

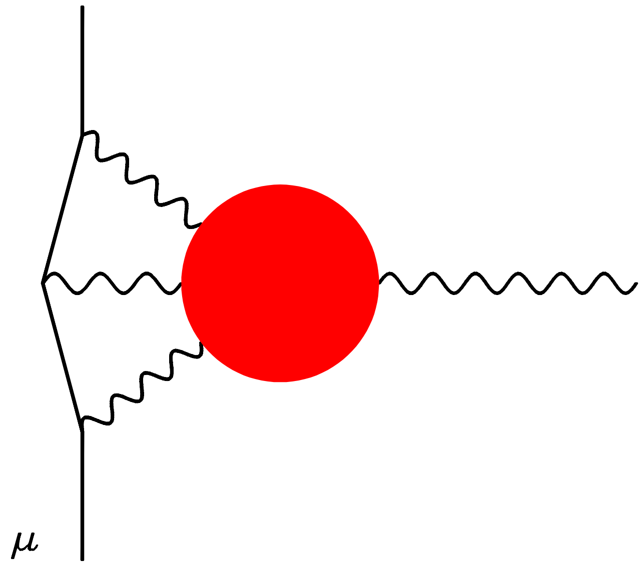
Light-by-Light



Called light-by-light because ‘blob’ represents photon-photon scattering.

Light-by-light contribution: Lattice could be useful. (Current uncertainty from phenomenology is 20% or more.)
Lattice proposal: Hayakawa, Blum, Izubuchi, Yamada; Lattice 2005, Dublin

Light-by-Light



For light-by-light 'blob', we need QCD contribution to

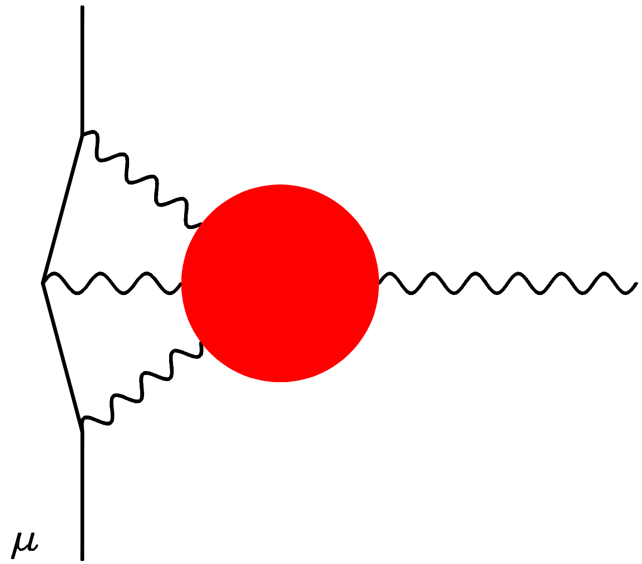
$$\langle J_{\nu_1}(x_1) J_{\nu_2}(x_2) J_{\nu_3}(x_3) J_{\mu}(x_q) \rangle_{con}$$

or its Fourier transform.

Rest of calculation by QED perturbation theory.

More complicated than $\langle JJ \rangle$, which just depends on a single momentum.

Light-by-Light



For a given q the light-by-light amplitude depends on two internal photon momenta (eg k_1, k_2). Final momentum fixed by momentum conservation.

Our plan is to measure $\langle JJJJ \rangle$ for enough values of the photon momentum to be able to calculate the light-by-light contribution to $g-2$. ("Direct Method")

Light-by-Light: Results

Calculations in Jülich.

Clover fermions, two-flavour simulation.

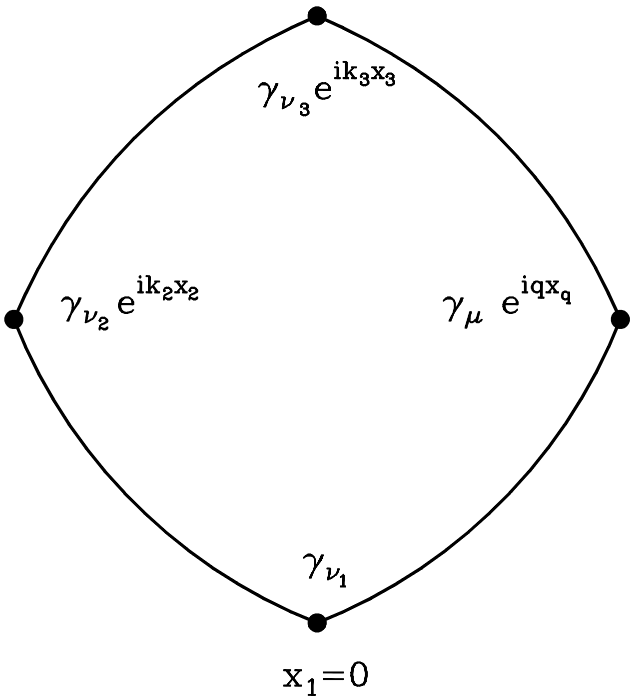
$24^3 \times 48$ lattice. Currently 24 configurations measured.

$$\beta = 5.40, \quad \kappa = 0.13625$$

$$a = 0.07 \text{ fm}, \quad a^{-1} = 2.75 \text{ GeV}$$

Local current $J_\mu(x) = \overline{\psi}(x)\gamma_\mu\psi(x)$
Disconnected contributions neglected.

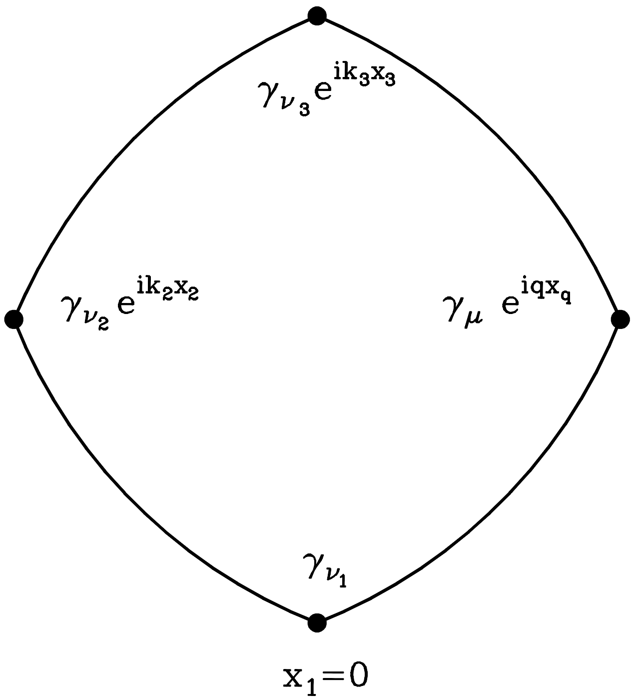
Light-by-Light: Results



$$\langle J_{\nu_1}(k_1) J_{\nu_2}(k_2) J_{\nu_3}(k_3) J_\mu(q) \rangle$$

Now have measurements of the 4-current tensor for all polarisations and a respectable range of momenta.

Light-by-Light: Results

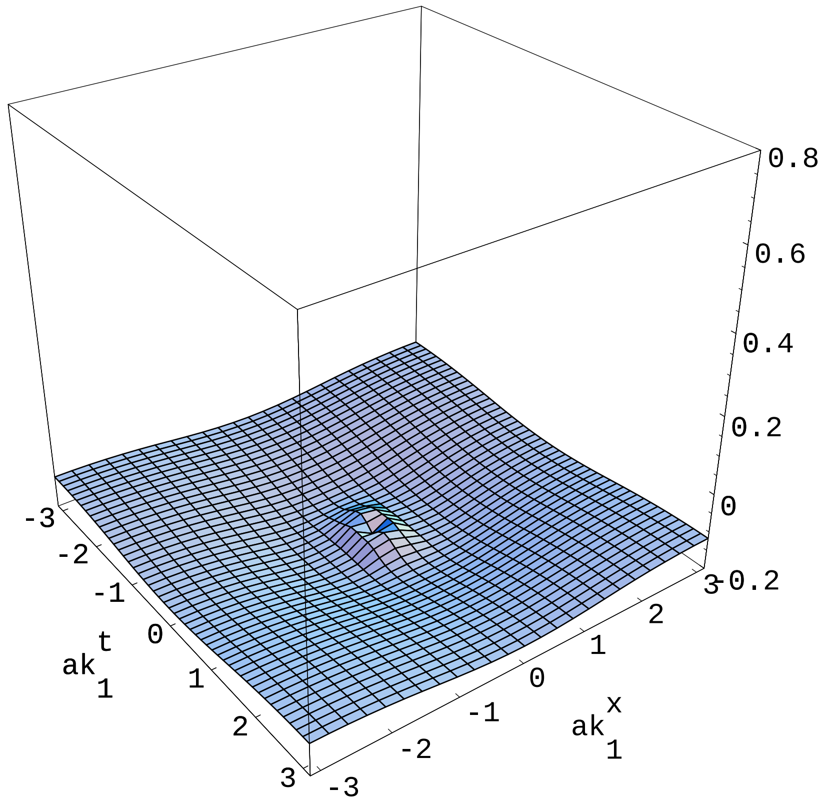


$$\langle J_{\nu_1}(k_1) J_{\nu_2}(k_2) J_{\nu_3}(k_3) J_{\mu}(q) \rangle$$

Picture: Fix polarisations; q and k_2; plot as a function of k_1. (Have all k_1, but just plot as function of x and t components).

Light-by-Light: Results

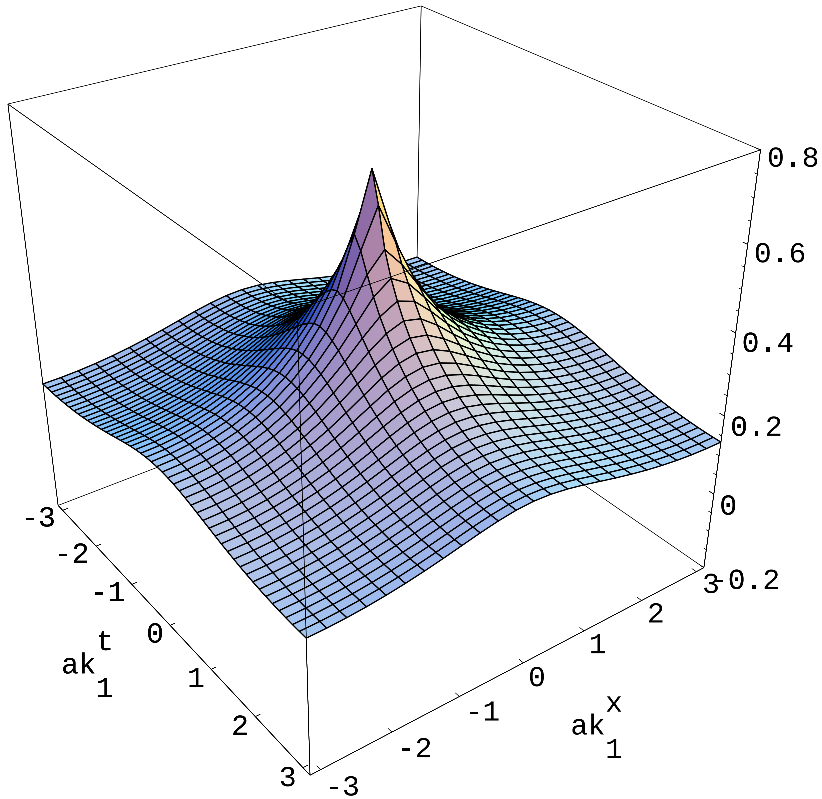
Parallel polarisations: | | | |



Statistical errors, a few %.

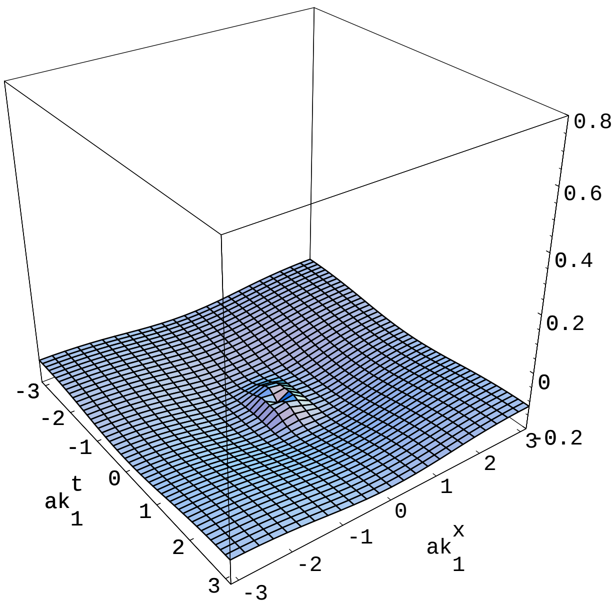
Light-by-Light: Results

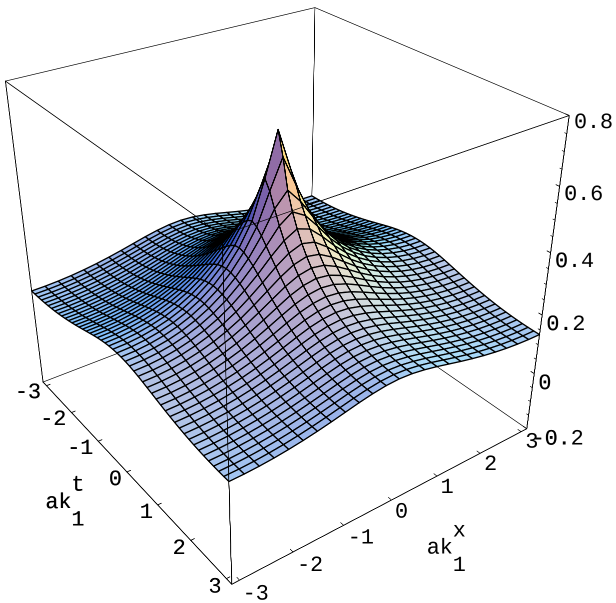
Crossed polarisations: | — | —



Light-by-Light: Results

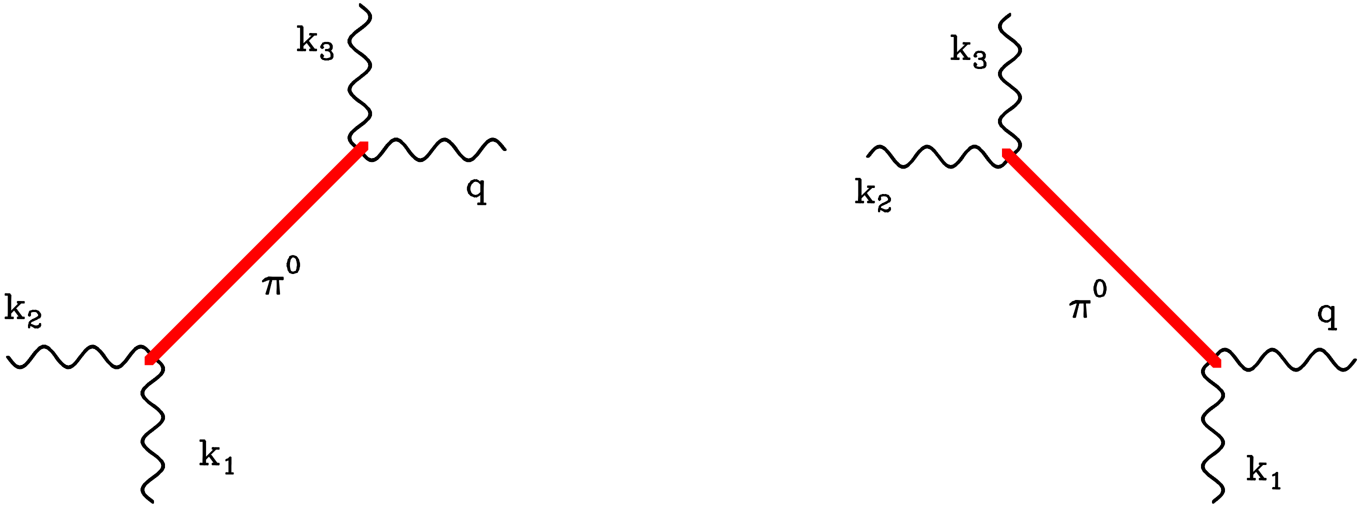
Why the dramatic difference between crossed polarisation and parallel?





Light-by-Light: Results

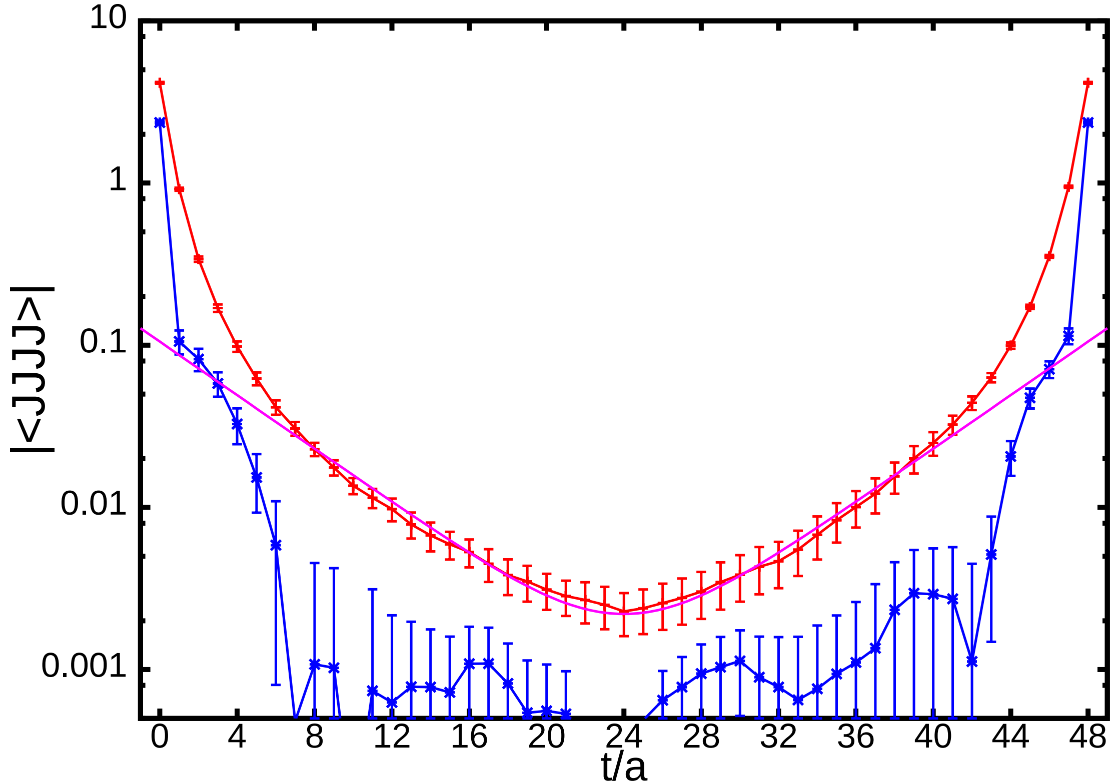
π^0 decays to two cross-polarised photons (coupling to $F^{\mu\nu}F^{\star}_{\mu\nu}$).



Possible in crossed case, not in parallel case.

Light-by-Light: Results

Look at same data in position space:

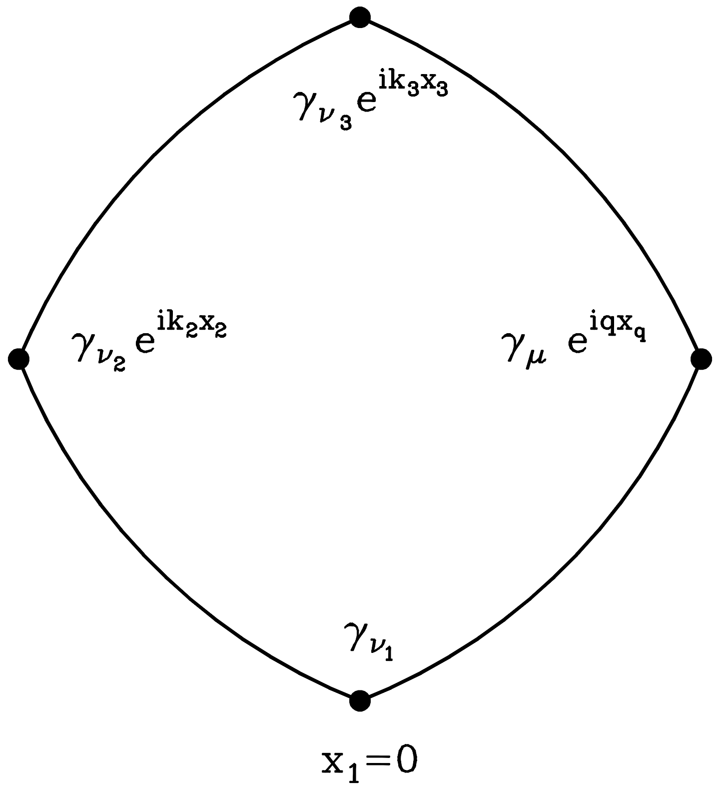


Parallel polarisation: Almost a δ function.

Crossed polarisation: Long tail, matches known pion mass.

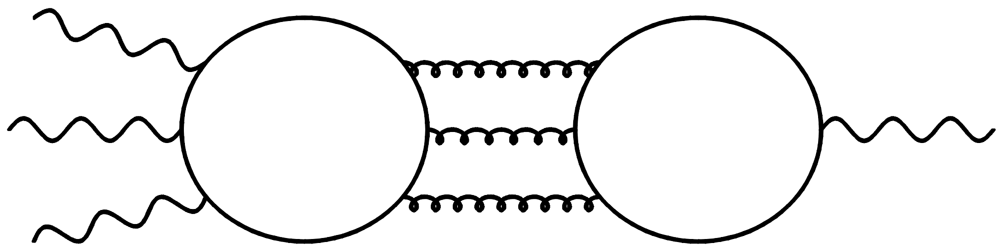
Magenta curve: π mass (measured directly).

Light-by-Light: Fermion Lines



Not quite full: Current calculation only considers the diagrams with all 4 photons attached to the same quark line, $\sum_f e_f^4$

Light-by-Light: Fermion Lines



Flavour $SU(3)$ suppresses most other diagrams, because there are quark loops with a single photon attached.

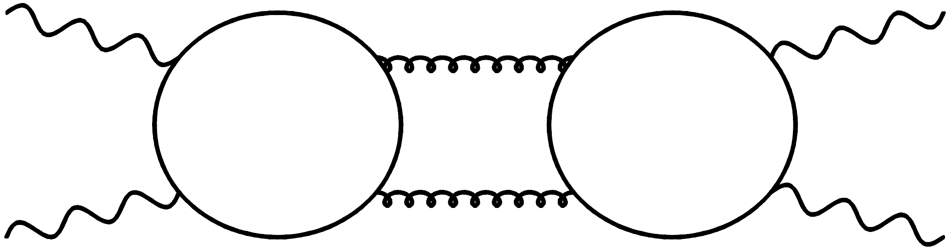
$$\left(\sum_{f'} e_{f'}^3\right) \times \left(\sum_f e_f\right)$$

$$\mathbf{e_u + e_d + e_s = \frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0}$$

$\sum_f e_f$ argument, or V-spin.

Argument suppresses all 3-line and 4-line diagrams, some 2-line diagrams.

Light-by-Light: Fermion Lines

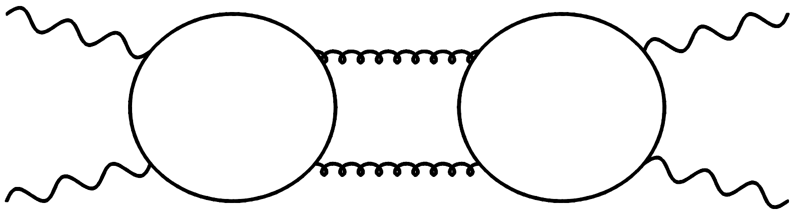


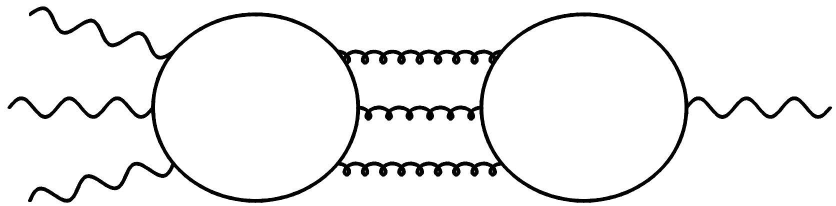
However there are diagrams with 2 photons on one quark line, and the other two on another quark line, which we are missing, and which aren't suppressed by interference between flavours.

$$\left(\sum_f e_f\right)^2$$

Some hope from large N_c. Should think about ways to include neglected diagrams.

Light-by-Light: Fermion Lines





Can write down a ‘direct’ method that produces all two-fermion-line diagrams at a cost-per-configuration about the same as the one-fermion-line contribution.

Extrapolations

Even after measuring $\langle JJJJ \rangle$ will be faced with extrapolations before we have a physical number.

- Lattice spacing to zero.
- Sea quark mass to m_u, m_d, m_s.
- Momentum extrapolations to $k \sim m_\mu$.

Conclusions

- Have interesting results for the hadronic tensor $\langle JJJJ \rangle$.
- Statistical Errors seem small (a few percent).
- π^0 contribution dramatic.
- Working on convolution of $\langle JJJJ \rangle$ tensor with QED part to give $g-2$.
- Currently just looking at the single quark-line connected contribution. Is this sufficient to give a useful result?
- Should try to find ways to include the other diagrams – two-quark-line diagrams feasible, all others suppressed by flavour SU(3).
- Limitations in computer time still mean that lattice results need to be extrapolated to physical points.
- Making good progress.