QCD Equation of State at Non-zero Chemical Potential


[MILC Collaboration]

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Outline

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Motivation

Experiments at RHIC start with a baryon rich environment; hence they naturally have a non-zero chemical potential.

Finite temperature field theory formalism easily admits a chemical potential, but we are left with a complex action and can no longer use importance sampling.

If the chemical potential is small, we can employ the Taylor expansion method:

Methodology

Physical quantities of interest are Taylor expanded in the chemical potentials for light and strange quarks. For example:

\[
\frac{p}{T^4} = \frac{\ln Z}{VT^3} = \sum_{n,m=0}^{\infty} c_{nm}(T) \left( \frac{\bar{\mu}_l}{T} \right)^n \left( \frac{\bar{\mu}_h}{T} \right)^m.
\] (1)

Only terms with \( n + m \) even appear due to \( CP \) symmetry.

\[
c_{nm}(T) = \frac{1}{n!} \frac{1}{m!} \frac{N_T^3}{N_\sigma^3} \frac{\partial^{n+m} \ln Z}{\partial (\mu_l N_T)^n \partial (\mu_h N_T)^m} \bigg|_{\mu_l, h=0}.
\] (2)

For the interaction measure,

\[
\frac{I}{T^4} = -\frac{N_t^3}{N_s^3} \frac{d \ln Z}{d \ln a} = \sum_{n,m} b_{nm}(T) \left( \frac{\bar{\mu}_l}{T} \right)^n \left( \frac{\bar{\mu}_h}{T} \right)^m.
\] (3)
Temperature dependent coefficients $c_{nm}(T)$ and $b_{nm}(T)$ are combinations of observables that can be calculated on non-zero $T$ ensembles, but with zero chemical potential.

We Taylor expand up to 6th order.

40 fermionic observables have to be determined using stochastic estimators, as well as several gluonic observables.

Details can be found in C. Bernard et al., Phys. Rev. D 77,014503 (2008); arXiv:0710.1330 [hep-lat].

Ensembles are generated on a line of constant physics with $m_{l} = 0.1m_{s}$ and $m_{s}$ approximately the physical strange quark mass.
Our previous work used lattices with $N_t = 4$. We now use $N_t = 6$ and compare with coarser lattices.

It is interesting to compare the free theory for different $N_t$ to see how the continuum limit is approached.

Next let’s compare $N_t = 4$ (black) and 6 (red) for the interacting theory.
Unmixed coefficients for pressure

\[ T \text{ [MeV]} \]

- \( c_{20} \)
- \( c_{40} \)
- \( c_{60} \)
- \( c_{02} \)
- \( c_{04} \)
- \( c_{06} \)
Comments on coefficients

- There is considerable structure at low $T$ and then an approach to SB limit above the cross-over temperature.
- Higher coefficients are small.
- Errors grow rapidly for higher order terms.
- Errors are better controlled for $N_t = 6$. 
Mixed coefficients for pressure

\[ c_{11}, c_{12}, c_{13}, c_{22}, c_{23}, c_{33}, c_{31}, c_{32}, c_{34}, c_{42}, c_{51}, c_{52} \]

Temperature \( T \) in MeV ranges from 100 to 700.

Unmixed coefficients for I
Mixed coefficients for I
Results

With the coefficients in hand, we can calculate interesting quantities, such as
- pressure
- interaction measure
- energy density
- quark number density
- quark number susceptibility

Due to non-zero $C_{n1}(T)$ terms a non-zero strange quark density is induced even with $\mu_h = 0$. To study the $n_S = 0$ plasma, we must tune $\mu_h$ as a function of $\mu_l$ and $T$.

We show change in pressure, interaction measure and energy density for various $\mu_l$ as a function of $T$. 
Pressure with $\mu = 0$

![Graph showing pressure $p/T^4$ vs. temperature $T$ (in MeV)](image)

- $N_t = 6$, $m_{ud} = 0.1m_s$
- $N_t = 6$, $m_{ud} = 0.2m_s$
- $N_t = 4$, $m_{ud} = 0.1m_s$
Interaction measure with $\mu = 0$
Δ Interaction Measure

![Graph showing the interaction measure as a function of temperature (T) for different chemical potentials (μ_i/T) and lattice sizes (N_t = 4, N_t = 6). The graph plots ΔI/T⁴ vs. T [MeV] and includes error bars for each data point. Different lines represent different values of μ_i/T: μ_i/T = 0.6, μ_i/T = 0.4, μ_i/T = 0.2, and μ_i/T = 0.1.]
Energy density with $\mu = 0$

![Graph showing the relationship between energy density ($\varepsilon$) and temperature ($T$) with different values of $N_t$ and $m_{ud}$.

- $N_t = 6, m_{ud} = 0.1m_s$
- $N_t = 6, m_{ud} = 0.2m_s$
- $N_t = 4, m_{ud} = 0.1m_s$]
△ Energy Density

![Graph showing the energy density for different temperatures and chemical potentials.](image)
Light quark density

\[ \frac{n_{ud}}{T^3} \text{ vs. } T [\text{MeV}] \]

- \( \mu / T = 0.6 \)
- \( \mu / T = 0.4 \)
- \( \mu / T = 0.2 \)
- \( \mu / T = 0.1 \)

- \( N_t = 4 \)
- \( N_t = 6 \)
Light quark susceptibility

\[ \chi_{uu}/T^2 \]

\[ T \text{ [MeV]} \]

\[ \chi_{uu} \text{ vs } T \]

\[ N_t = 4 \]
\[ N_t = 6 \]

\[ \mu_t/T = 0.6 \]
\[ \mu_t/T = 0.4 \]
\[ \mu_t/T = 0.2 \]
\[ \mu_t/T = 0.1 \]
Strange quark susceptibility

\[ \chi_{ss}/T^2 \]

\[ T \text{ [MeV]} \]

- \( N_t = 4 \)
- \( N_t = 6 \)

- \( \mu_f/T = 0.6 \)
- \( \mu_f/T = 0.4 \)
- \( \mu_f/T = 0.2 \)
- \( \mu_f/T = 0.1 \)
**Isentropic EOS**

- In a heavy-ion collision, after thermalization the system expands and cools with constant entropy.
- We would like to find the EOS with fixed ratio of entropy to baryon number.
- We calculate EOS with appropriate ratios for AGS, SPS, RHIC:

<table>
<thead>
<tr>
<th>Expt</th>
<th>$s/n_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGS</td>
<td>30</td>
</tr>
<tr>
<td>SPS</td>
<td>45</td>
</tr>
<tr>
<td>RHIC</td>
<td>300</td>
</tr>
</tbody>
</table>

- This requires finding trajectories in the $(\mu_l, \mu_s, T)$ space with $n_s = 0$ and $s/n_B$ as in the table.
Isentropic Pressure

\( m_{ud} = 0.1 m_s \)

- \( S/N_B = 30 \)
- \( S/N_B = 45 \)
- \( S/N_B = 300 \)
- \( S/N_B = \infty \)

Filled: \( N_t = 6 \)
Empty: \( N_t = 4 \)
Isentropic interaction measure
Isentropic energy density

Filled: \( N = 6 \)
Empty: \( N_t = 4 \)

Spectroscopy

\( \varepsilon / T^4 \)

Temperature \( T \) [MeV]

- \( S/N_B = 30 \)
- \( S/N_B = 45 \)
- \( S/N_B = 300 \)
- \( S/N_B = \infty \)
Isentropic light quark density

\[ \frac{n_{ud}}{T^3} \]

- \( m_{ud} = 0.1 m_s \)
- Filled: \( N_t = 6 \)
- Empty: \( N_t = 4 \)
- \( S/N_B = 30 \)
- \( S/N_B = 45 \)
- \( S/N_B = 300 \)
Isentropic light quark susceptibility

Absence of a peak indicates that we are far from a critical endpoint.
Conclusions

- We have extended our sixth order Taylor expansion study of thermodynamics with chemical potential toward the continuum limit by going from $N_t = 4$ to 6.

- We compute the coefficients relevant for both pressure $p$ and interaction measure $I$.

- We observe modest lattice spacing effects, with the effect of chemical potential smaller on the pressure and quark densities and susceptibilities at the smaller lattice spacing.

- We have calculated the isentropic equation of state, which is interesting for phenomenology.

- It would be interesting to extend this work to yet smaller lattice spacing and to go to lighter quark mass.

- Happy birthday, Carleton!