Scaling behavior and sea quark dependence of pion spectrum

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Outline

Introduction to staggered pion spectrum

Scaling behavior of pion spectrum

Sea quark dependence of pion spectrum

Cubic wall sources and Cubic U(1) sources

Summary
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Summary
Pion spectrum in staggered fermion formalism

\[
\text{SU}(4) \xrightarrow{\mathcal{O}(a^2)} \text{SO}(4) \xrightarrow{\mathcal{O}(a^2p^2)} \text{SW}_4
\]

- Pion tastes \((\gamma_5 \otimes \xi_T)\), \(\xi_T \in \{I, \xi_5, \xi_\mu, \xi_\mu 5, \xi_\mu \nu\}\).
- In continuum limit, they respect SU(4).
- S\chiPT: \(\mathcal{O}(a^2)\) terms break SU(4) down to SO(4).
- S\chiPT: \(\mathcal{O}(a^2p^2)\) terms break SO(4) down to SW_4.
Improved staggered fermions

- The taste-breaking comes from high momentum gluon exchange.
- Fat-links reduce the taste-breaking by suppressing these interactions.
- We have found that HYP staggered fermions reduce the taste symmetry breaking more efficiently than asqtad staggered fermions.
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Summary
Parameters of the MILC fine lattices and coarse lattices

1-loop tadpole-improved Symanzik gauge action, 2 + 1 flavors of Asqtad staggered sea quarks; Coulomb gauge fixing; HYP smeared staggered valence quarks

<table>
<thead>
<tr>
<th>parameters</th>
<th>MILC fine lattices</th>
<th>MILC coarse lattices</th>
</tr>
</thead>
<tbody>
<tr>
<td>sea quark masses</td>
<td>$am_l = 0.0062$, $am_s = 0.031$</td>
<td>$am_l = 0.01$, $am_s = 0.05$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>7.09</td>
<td>6.76</td>
</tr>
<tr>
<td>$a$</td>
<td>0.09fm</td>
<td>0.125fm</td>
</tr>
<tr>
<td>geometry</td>
<td>$28^3 \times 96$</td>
<td>$20^3 \times 64$</td>
</tr>
<tr>
<td># of confs</td>
<td>995</td>
<td>671</td>
</tr>
<tr>
<td>valence quark masses</td>
<td>0.003, 0.006, ..., 0.030</td>
<td>0.01, 0.02, ..., 0.05</td>
</tr>
</tbody>
</table>
Comparison between coarse lattices and fine lattices (I)

(a) Coarse lattices

Coarse: $\mathcal{O}(a^2) \approx \mathcal{O}(p^2)$. (S$\chi$PT)

(b) Fine lattices

Fine: $\mathcal{O}(a^2) \approx \mathcal{O}(p^4) \ll \mathcal{O}(p^2)$. (S$\chi$PT)
Comparison between coarse lattices and fine lattices (II)

- **Coarse**: \( \mathcal{O}(a^2) \approx \mathcal{O}(p^2) \). (S\(\chi\)PT)
- **Fine**: \( \mathcal{O}(a^2) \approx \mathcal{O}(p^4) \ll \mathcal{O}(p^2) \). (S\(\chi\)PT)
Scaling behavior of pion spectrum

Splittings of pion multiplet spectrum : $\Delta(\tau)$

$$m^2_\pi(\tau) = m^2_\pi(P) + \Delta(\tau)$$

<table>
<thead>
<tr>
<th>Taste$^1$</th>
<th>$\Delta(\tau)$ [($\text{GeV}^2$)]</th>
<th>$\Delta(\text{Fine}) / \Delta(\text{Coarse})$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a = 0.125\text{fm}$</td>
<td>$a = 0.09\text{fm}$</td>
</tr>
<tr>
<td>$A$</td>
<td>0.0278(6)</td>
<td>0.0087(3)</td>
</tr>
<tr>
<td>$T$</td>
<td>0.0540(13)</td>
<td>0.0168(4)</td>
</tr>
<tr>
<td>$V$</td>
<td>0.0783(17)</td>
<td>0.0250(6)</td>
</tr>
<tr>
<td>$S$</td>
<td>0.1005(84)</td>
<td>0.0300(31)</td>
</tr>
</tbody>
</table>

$^1$P : Pseudo-scalar($\xi_5$), A : Axial vector($\xi_{\mu 5}$), T : Tensor($\xi_{\mu \nu}$), V : Vector($\xi_\mu$), S : Scalar($I$)
Scaling behavior of pion spectrum

$\Delta(\tau)$ behave linearly as a function of $a^2\alpha^2_{\overline{MS}}$. 

$\Delta(S)$, $\Delta(V)$, $\Delta(T)$, $\Delta(A)$
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Cubic wall sources and Cubic U(1) sources

Summary
Lattice ensembles

- We used MILC coarse lattice ensembles ($a = 0.125\text{fm}$).

<table>
<thead>
<tr>
<th>Geometry</th>
<th>$m_l a$ (light quark mass)</th>
<th>$m_s a$ (strange quark mass)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$24^3 \times 64$</td>
<td>0.005</td>
<td>0.05</td>
</tr>
<tr>
<td>$20^3 \times 64$</td>
<td>0.007</td>
<td>0.05</td>
</tr>
<tr>
<td>$20^3 \times 64$</td>
<td>0.010</td>
<td>0.05</td>
</tr>
<tr>
<td>$20^3 \times 64$</td>
<td>0.020</td>
<td>0.05</td>
</tr>
<tr>
<td>$20^3 \times 64$</td>
<td>0.030</td>
<td>0.05</td>
</tr>
</tbody>
</table>

- We study the dependence of pion spectrum on light sea quark masses.
Sea quark dependence of pion spectrum

- Various light quark mass data points are on top of each other.
- Slopes are parallel to each other.
- So there is no dependence of pion spectrum on light sea quark mass.
Comparing splittings in the chiral limit

- Except for $m_\ell a = 0.005$, $\Delta(\tau)$ does not depend on the sea quark mass within statistical uncertainty.
- Note that the ensemble for $m_\ell a = 0.005$ has a larger volume of $24^3$ compared to other ensembles.
- This could come from a finite volume effect but is also consistent with others within two $\sigma$. 
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Summary
Sources

- In order to select a specific pion taste we must choose sources and sinks that belong to a specific irrep of the time slice group.
- Propagators are obtained by solving the Dirac equation with source $h$

$$\begin{align*}
(D + m)\chi &= h \\
\chi(x, a; t; \vec{A}) &= \sum_{y, b} G(x, a; y, b)h(y, b; \vec{A})
\end{align*}$$

- $G$ is the point-to-point quark propagator, $x$, $y$ label lattice sites, and $a$, $b$ are color indices.
Cubic wall sources and Cubic U(1) sources

**Cubic wall sources**

\[ h(y, b; t; \vec{A}) = \delta_{y, t} \sum_{\vec{n}} \delta_{\vec{y}, 2\vec{n} + \vec{A}} \eta(b) \]

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{\eta} \eta(c) \eta^*(c') = \delta_{c, c'}
\]

- \( \vec{n} \) is a vector labeling \( 2^3 \) cubes in the time slice.
- \( \vec{A} \) labels points within the cubes.
- \( \eta \)'s are U(1) noise vectors normalized as in the above formulae.

**Cubic U(1) sources**

\[ h(y, b; t; \vec{A}) = \delta_{y, t} \sum_{\vec{n}} \delta_{\vec{y}, 2\vec{n} + \vec{A}} \eta(\vec{n}, b) \]

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{\eta} \eta(\vec{n}, c) \eta^*(\vec{n}', c') = \delta_{\vec{n}, \vec{n}'} \delta_{c, c'}
\]
Comparison between CW and CU1 sources (I)

$20^3 \times 64, m_la = 0.01, m_sa = 0.05$

(e) Cubic wall [CW]

(f) Cubic U(1) [CU(1)]

There is no difference between CW and CU(1) for LT tastes.
In the case of NLT tastes, statistical uncertainties for CW are smaller than those for CU(1).

We prefer CW to CU(1) for our future numerical study.
Comparison between CW and CU1 sources (III)

<table>
<thead>
<tr>
<th>Taste</th>
<th>$\Delta(\tau)$ [(GeV)$^2$]</th>
<th>Cubic wall [CW]</th>
<th>Cubic U(1) [CU(1)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_i\xi_5$</td>
<td>0.0278(6)</td>
<td>0.0274(5)</td>
<td></td>
</tr>
<tr>
<td>$\xi_4\xi_i$</td>
<td>0.0540(13)</td>
<td>0.0535(11)</td>
<td></td>
</tr>
<tr>
<td>$\xi_4$</td>
<td>0.0783(17)</td>
<td>0.0779(24)</td>
<td></td>
</tr>
<tr>
<td>$\xi_4\xi_5$</td>
<td>0.0253(33)</td>
<td>0.0240(49)</td>
<td></td>
</tr>
<tr>
<td>$\xi_i\xi_j$</td>
<td>0.0500(43)</td>
<td>0.0486(61)</td>
<td></td>
</tr>
<tr>
<td>$\xi_i$</td>
<td>0.0740(57)</td>
<td>0.0773(101)</td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>0.1005(84)</td>
<td>0.1014(134)</td>
<td></td>
</tr>
</tbody>
</table>

- As predicted by $S\chi$PT, the data respect SO(4) symmetry.
- Statistical gains for CW are about twice compared to CU(1) for NLT tastes.
- Therefore we prefer CW to CU(1).
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- With HYP staggered valence quarks, the taste breaking is reduced by factor of 0.3 on fine lattices ($a = 0.09\text{fm}$) than coarse lattices ($a = 0.125\text{fm}$).
- There is no dependence of pion spectrum on sea quark masses.
- We prefer using cubic wall sources since statistical errors are smaller for cubic wall sources than for cubic U(1) sources.