Gapless Dirac spectrum at high temperature

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Background

- Just **above** $T_c$ $\langle \bar{\psi} \psi \rangle \neq 0$ if P-loop is complex
  (Chandrasekharan and Christ, hep-lat/9509095)
  $\Rightarrow$ Chiral symmetry is restored at $T_c$ only if P-loop real

- **Random matrix model** $\Rightarrow$ Chiral symmetry restoration occurs
  - at higher $T$ if P-loop complex for SU(3)
  - never if P-loop<0 for SU(2)  
  (Stephanov, PLB375 (1996) 249)

- **Lattice:**
  - SU(3): in all P-loop sectors spectral gap appears
    at the same $T = T_c$ (Gattringer et al. PRD66 (2002) 054502)
  - SU(2): $\rho(0) \neq 0$ up to $T = 2T_c$ (Bornyakov et al. arXiv:0807.1980)
Qualitative picture

- In quenched SU(N) YM Polyakov-loop Z(N) symmetry spontaneously broken above $T_c$ (deconfined phase).

- Chiral symmetry restoration above $T_c$ depends strongly on the Polyakov-loop sector.

- Banks-Casher:

  $\langle \bar{\psi} \psi \rangle = \pi \rho(0)$

  chiral symmetry breaking $\Leftrightarrow$ Dirac operator spectral density at 0

- Experience:
  
  - $(-1) \times P$ closer to 1 $\Rightarrow$ more low Dirac modes
  
  - $(-1) \times P$ effective boundary condition for quarks
SU(2) further questions

• Does $\rho(0) \neq 0$ persist at arbitrarily high $T$ in the P-loop<0 sector?

• Comparison of Dirac spectrum with random matrix theory (around and above $T_c$)

• Instantons $\Leftrightarrow \rho(0) \neq 0$ ?

• How do dynamical fermions select the correct P-loop sector?
SU(2) simulation parameters

• All runs at quenched $\beta = 2.6$ \ ($\beta_c$ for $N_T=10.4$)

• Vary $N_T$ to change temperature

• $T = 2.6T_c$ ($N_T = 4$), $T = 1.7T_c$ ($N_T = 6$)

• Spatial sizes: $N_S = 8, 10, 12, 16, 20$: $N_{Tc}/N_S = 0.52 - 1.30$

• Overlap Dirac operator

• Antiperiodic quark boundary condition in time
Density of low modes for different Polyakov loop sectors

\[ \rho_a^3 \]

- P-loop<0  T=2.6T_c
- P-loop>0  T=2.6T_c
- Lowest free mode

\[ \lambda_a \]
Density of low modes for different Polyakov loop sectors
Density of modes at zero

\[ \rho(0) a^3 \]

For different spatial box sizes and \( N_{Tc}/N_S \):

- Orange circles: \( T = 2.6 T_c \)
- Blue squares: \( T = 1.7 T_c \)
Cumulative distribution of scaled smallest eigenvalues for $Q=0$

$T = 2.6T_c$  $\Sigma = \langle \bar{\psi} \psi \rangle$: best one-parameter fit to random matrix prediction

![Graph showing cumulative distribution of scaled smallest eigenvalues for $Q=0$. The x-axis represents $\sum \lambda$, and the y-axis represents $P$. The graph includes a best-fit line and three curves for different values of $N_{Tc}/N_S$: 0.65, 1.04, and 1.30. The RMT prediction is also shown as a red line.](image-url)
Possible role of instantons?

• Common wisdom: instanton-antiinstanton 0-modes \( \Rightarrow \rho(0) \neq 0 \)

• As temperature goes up:
  
  – Topological susceptibility drops (instantons “squeezed out”)
  
  – \( \rho(0) \approx \langle \bar{\psi} \psi \rangle \) increases

• \( \Rightarrow \) At high \( T \) instantons cannot be responsible for \( \rho(0) \neq 0 \)
Why is $\langle \bar{\psi} \psi \rangle = 0$ above $T_c$ in the real world?

- Fermion determinant breaks P-loop $Z(N)$ symmetry
- Favors sector with the least number of low modes
- Effective boundary condition as far from periodic as possible
  - P-loop real for $SU(3)$
  - P-loop<0 for $SU(2)$
- Is it really only the low modes that matter?
Difference in fermion action between P-loop sectors
one quark flavor of mass $m$

![Graph showing difference in fermion action](image)
Conclusions

• In quenched SU(2) above $T_c$ chiral condensate has strong dependence on the P-loop average
  - If $\langle P \rangle > 0$ condensate vanishes at $T_c$
  - If $\langle P \rangle < 0$ condensate increases with $T$

• In the $\langle P \rangle < 0$ sector with chiral symmetry broken above $T_c$
  - Good agreement with random matrix theory
  - Topological charge fluctuations cannot account for low Dirac modes

• In the real world:
  - Fermion determinant suppresses “wrong” P-loop sector
  - Small fraction of lowest Dirac modes ($< 1\%$) responsible for that

• Picture should be qualitatively similar for other Dirac operators and $SU(3)$