Nucleon form factors with dynamical twisted mass fermions

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- definitions
- twisted mass fermions
- calculation of form factors
- results

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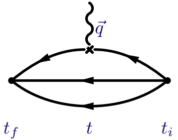
Matrix elements

We are interested in QCD matrix elements

$$\langle N(p_f, s_f)|j_\mu|N(p_i, s_i)\rangle$$



where

- $|N(p_f, s_f)\rangle$, $|N(p_i, s_i)\rangle$ are nucleon states with final (initial) momentum $p_f(p_i)$ and spin $s_f(s_i)$
- j_μ is a current, e.g.
  - Electromagnetic current $V_\mu^{EM}(x) = \frac{2}{3}\bar{u}(x)\gamma_\mu u(x) - \frac{1}{3}\bar{d}(x)\gamma_\mu d(x)$
  - Isovector current $V_\mu^a(x) = \bar{\psi}(x)\gamma_\mu \frac{\tau^a}{2}\psi(x)$
  - Axial current $A_\mu^a(x) = \bar{\psi}(x)\gamma_\mu\gamma_5 \frac{\tau^a}{2}\psi(x)$

Electromagnetic nucleon form factors

The electromagnetic matrix element of the nucleon can be expressed in terms of two form factors.

$$
\begin{aligned}
\langle N(p_f, s_f)|V_\mu(0)|N(p_i, s_i)\rangle &= \sqrt{\frac{m_N^2}{E_{N(\vec{p}_f)}E_{N(\vec{p}_i)}}}\,\bar{u}(p_f, s_f)\mathcal{O}_\mu u(p_i, s_i) \\
\mathcal{O}_\mu &= \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)
\end{aligned}
$$

$q = p_f - p_i$ is the momentum transfer
F_1, F_2 are the Dirac form factors.

$$
\begin{aligned}
G_E(q^2) &= F_1(q^2) - \frac{q^2}{(2m_N)^2}F_2(q^2) \\
G_M(q^2) &= F_1(q^2) + F_2(q^2)
\end{aligned}
$$

G_E, G_M are the electric and magnetic Sachs form factors.

Axial nucleon form factors

The axial current matrix element of the nucleon can be expressed in terms of the form factors G_A and G_p.

$$\langle N(p_f, s_f)|A^a_\mu(0)|N(p_i, s_i)\rangle = \sqrt{\frac{m_N^2}{E_{N(\vec{p}_f)}E_{N(\vec{p}_i)}}}\bar{u}(p_f, s_f)\mathcal{O}_\mu u(p_i, s_i)$$

$$\mathcal{O}_\mu = \left[-\gamma_\mu\gamma_5 G_A(q^2) + i\frac{q^\mu\gamma_5}{2m_N}G_p(q^2)\right]\frac{\tau^a}{2}$$

Wilson twisted mass QCD

two degenerate flavors $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$

change of variables: $\psi = \frac{1}{\sqrt{2}}[\mathbb{1} + i\tau^3\gamma_5]\chi \qquad \bar{\psi} = \bar{\chi}\frac{1}{\sqrt{2}}[\mathbb{1} + i\tau^3\gamma_5]$

mass term: $\bar{\psi}m\psi = \bar{\chi}i\gamma_5\tau^3 m\chi$

Other terms of the continuum action invariant $\Rightarrow$ QCD $=$ tmQCD

On the lattice with Wilson fermions the two formulations are not equivalent

$$S = S_g + a^4 \sum_x \bar{\chi}(x)\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - \frac{ar}{2}\nabla_\mu\nabla^*_\mu + m_{\rm crit} + i\gamma_5\tau^3\mu\right]\chi(x)$$

Wilson tmQCD at maximal twist

- Automatic $O(a)$ improvement for a class of observables
- $\det[D^2_{\rm tm}] = \det[D^2_W + \mu^2] \Rightarrow$ protection against small EVs
- Operator renormalization: mixing pattern is simplified
- Explicit flavor symmetry breaking

S_g: tree level Symanzik improved gauge action

Operators in twisted basis

Proton interpolating field

The standard nucleon interpolating field in the physical basis:

$$J(x) = \epsilon^{abc} \left[u^{a\top}(x) \mathcal{C}\gamma_5 d^b(x) \right] u^c(x)$$

And in the twisted basis at maximal twist:

$$J(x) = \frac{1}{\sqrt{2}} [\mathbb{1} + i\gamma_5] \epsilon^{abc} \left[{u^\chi}^{a\top}(x) \mathcal{C}\gamma_5 {d^\chi}^b(x) \right] {u^\chi}^c(x)$$

we use smeared fields for u^χ and d^χ

Operators

- $V_\mu^{0,3} = {V^\chi}_\mu^{0,3}$, use Noether currents $\rightarrow Z_V = 1$
  $\Rightarrow$ Electromagnetic form factors
- $A_\mu^3 = {A^\chi}_\mu^3$, use local current, need Z_A
  $\Rightarrow$ Axial form factors

Smearing

We use smeared quark fields for the construction of the interpolating fields
$\Rightarrow$ increase overlap with proton state / decrease overlap with excited states
The smearing is the same as for our calculation of baryon masses
[ETMC, C. Alexandrou et al. arXiv:0803.3190]

$$\mathbf{q}^a(t,\vec{x}) = \sum_{\vec{y}} F^{ab}(\vec{x},\vec{y};U(t))\, q^b(t,\vec{y})$$

$$F = (\mathbb{1} + \alpha H)^N \qquad H(\vec{x},\vec{y};U(t)) = \sum_{i=1}^{3} \Big(U_i(x)\delta_{x,y-\hat{\imath}} + U_i^\dagger(x-\hat{\imath})\delta_{x,y+\hat{\imath}} \Big)$$

[S. Gusken, Nucl.Phys.Proc.Suppl.17:361-364,1990]

[C. Alexandrou, S. Gusken, F. Jegerlehner, K. Schilling, R. Sommer, Nucl.Phys.B414:815-855,1994]

In addition: APE-smearing of the gauge fields U_μ entering H.

Correlation functions

Measure two-point and three-point functions

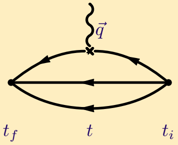
$$G(\vec{q},t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f\cdot\vec{q}}\,\Gamma^0_{\beta\alpha}\,\langle J_\alpha(t_f,\vec{x}_f)\bar{J}_\beta(0)\rangle$$

$$G^\mu(\Gamma^\nu,\vec{q},t) = \sum_{\vec{x},\vec{x}_f} e^{i\vec{x}\cdot\vec{q}}\,\Gamma^\nu_{\beta\alpha}\,\langle J_\alpha(t_f,\vec{x}_f)j^\mu(t,\vec{x})\bar{J}_\beta(0)\rangle$$

$\Gamma^0 = \frac{1}{4}(\mathbb{1}+\gamma_0)$ and $\Gamma^k = i\Gamma^0\gamma_5\gamma_k$

Kinematical setup

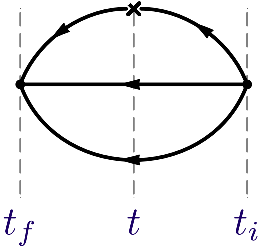
- Source at $t_i = 0$, $\vec{x} = 0$
- Sink at $t_f = 12(14)$, $\vec{p}_f = 0$
- Operator j^μ at t, $\vec{q} = -\vec{p}_i$



Sequential inversion through the sink

Connected contributions

- $\left[d^a_\alpha(x)\bar{d}^b_\beta(y)\right]_f \equiv \mathcal{D}^{ab}_{\alpha\beta}(x,y)$
- $\left[u^a_\alpha(x)\bar{u}^b_\beta(y)\right]_f \equiv \mathcal{U}^{ab}_{\alpha\beta}(x,y)$
- $\mathcal{D}^\dagger(y,x) = \gamma_5\mathcal{U}(x,y)\gamma_5$
- No new inversions for different operator $j^\mu(t,\vec{q})$
- But: new inversions necessary for different interpolating fields



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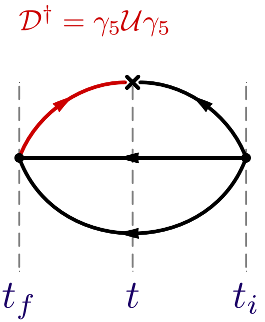

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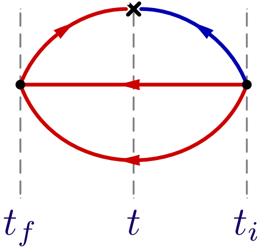

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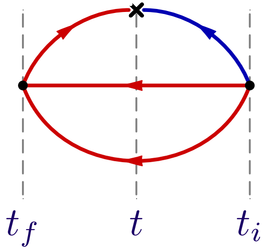

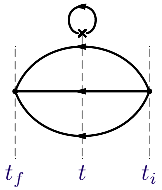

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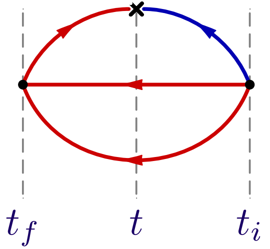

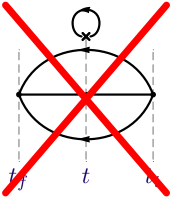

Ratios

Leading time dependence and overlap factors cancel in ratios:

$$R^\mu = \frac{G^\mu(\Gamma,\vec{q},t)}{G(\vec{0},t_f)}\sqrt{\frac{G(\vec{p}_i,t_f-t)G(\vec{0},t)G(\vec{0},t_f)}{G(\vec{0},t_f-t)G(\vec{p}_i,t)G(\vec{p}_i,t_f)}} \to \Pi^\mu(\Gamma,\vec{q})$$

If the separation between t_f and t_i sufficient: R^μ has a plateau

Optimal combinations

- Electric Sachs form factor from
  $\Pi^\mu(\Gamma^0,\vec{q}) = \frac{c}{2m}[(m+E)\delta_{0,\mu} + \sum_k iq_k\delta_{k,\mu}]\ G_E(q^2)$
- Magnetic Sachs form factor from
  $\Pi^i(\Gamma^1,\vec{q}) + \Pi^i(\Gamma^2,\vec{q}) + \Pi^i(\Gamma^3,\vec{q}) = \frac{c}{2m}\sum_{jkl}\epsilon_{jkl}q_j\delta_{l,i}\, G_M(q^2)$
- Axial form factors
  $\Pi^{5i}(\Gamma^1,\vec{q}) + \Pi^{5i}(\Gamma^2,\vec{q}) + \Pi^{5i}(\Gamma^3,\vec{q}) = \frac{ic}{4m}[(q_1+q_2+q_3)\frac{q_i}{2m}G_p - (E+m)G_A]$

$c = \sqrt{\frac{2m^2}{E(E+m)}}$

Ratios

Leading time dependence and overlap factors cancel in ratios:

alternatively: $$R^\mu = \frac{G^\mu(\Gamma,\vec{q},t)}{\sqrt{G(\vec{0},2(t_f-t))G(\vec{q},2(t-t_i))}} \to \Pi^\mu(\Gamma,\vec{q})$$

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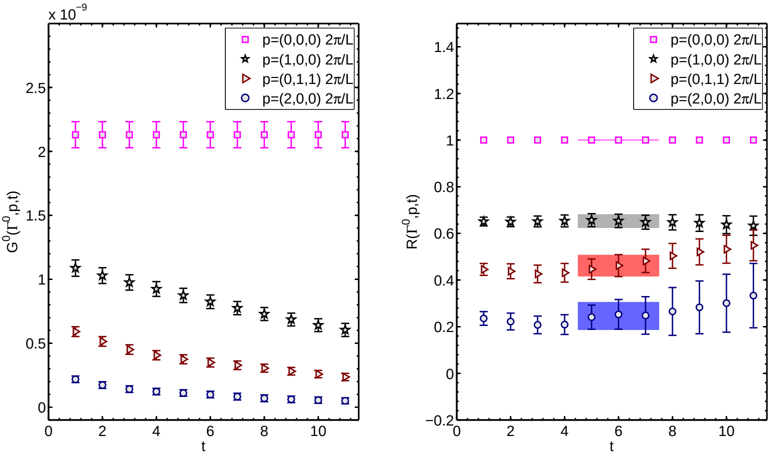
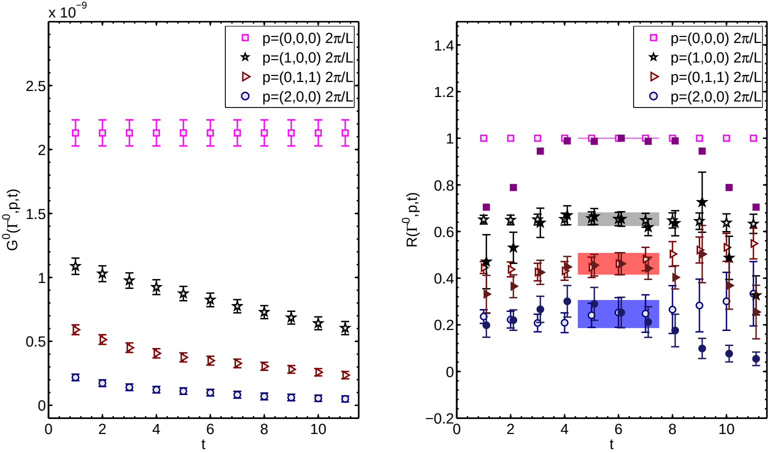
Plateaus



x 10^-9
G^0(Γ^0,p,t)
p=(0,0,0) 2π/L
p=(1,0,0) 2π/L
p=(0,1,1) 2π/L
p=(2,0,0) 2π/L
t
R(Γ^0,p,t)
p=(0,0,0) 2π/L
p=(1,0,0) 2π/L
p=(0,1,1) 2π/L
p=(2,0,0) 2π/L
t

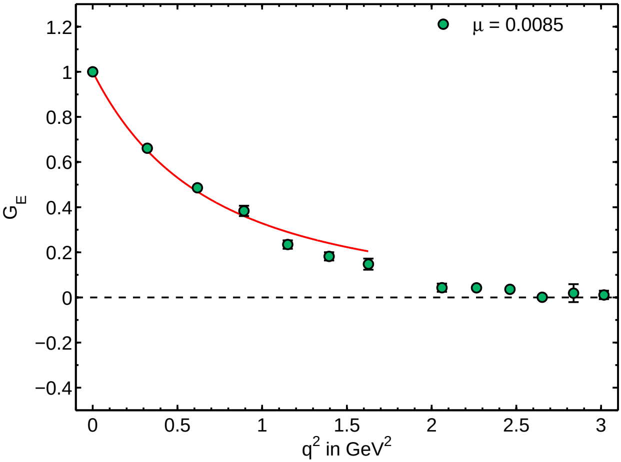
Plateaus



⇒ compatible with simpler ratio

Electric Sachs form factor

- connected parts of V_μ^{EM}
- $a = 0.089(1)\ fm$ from m_N
- $L = 2.13\ fm$
- $m_\pi = 447\ MeV$
- $m_\pi = 313\ MeV$



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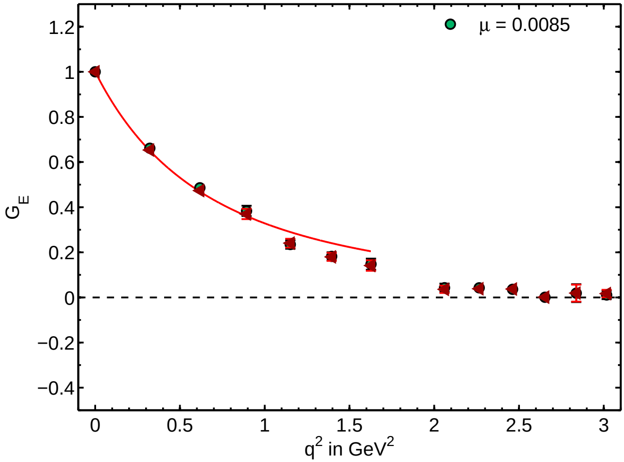

local vector current ⇒ $Z_V = 0.608(1)$
compatible with determination from WI and pion f.f.

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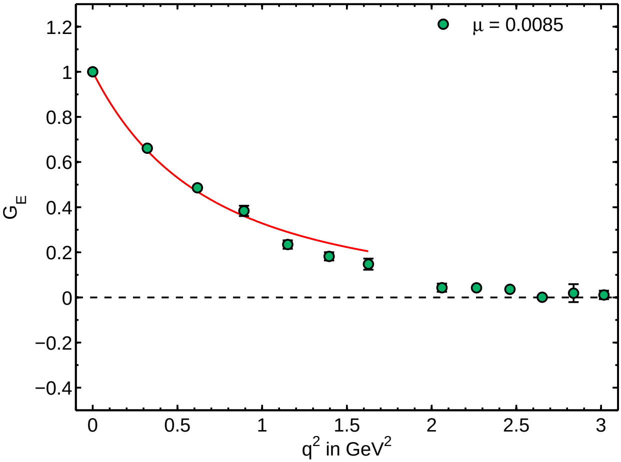

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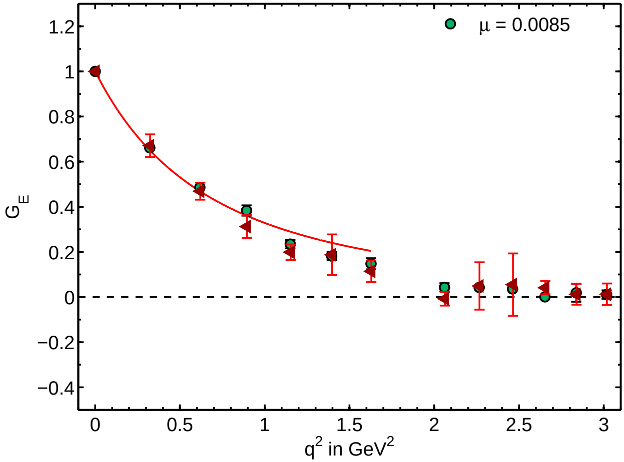

$t_{\text{sink}} = 14$

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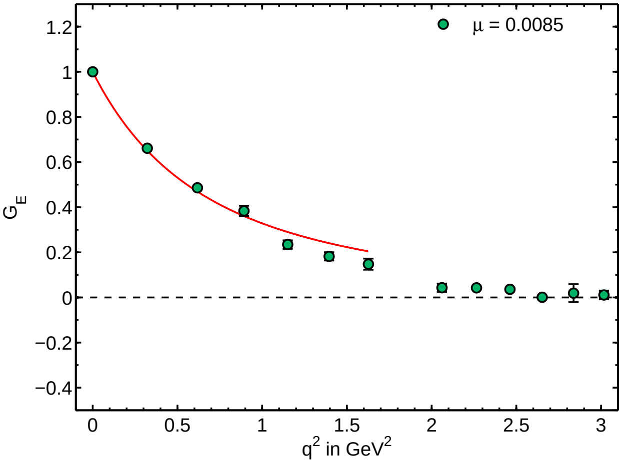

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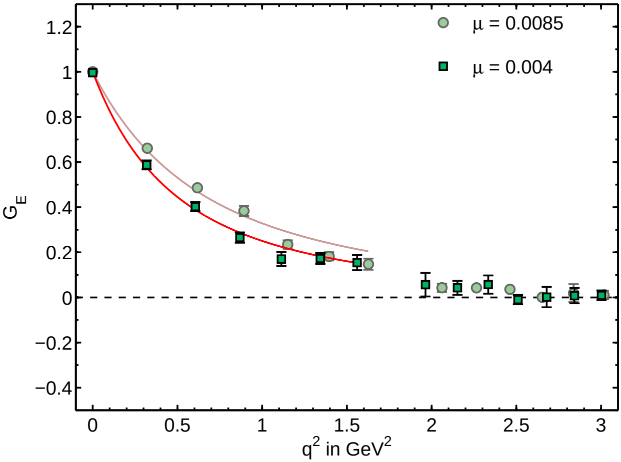

Magnetic Sachs form factor

- $m_\pi = 447\ MeV$
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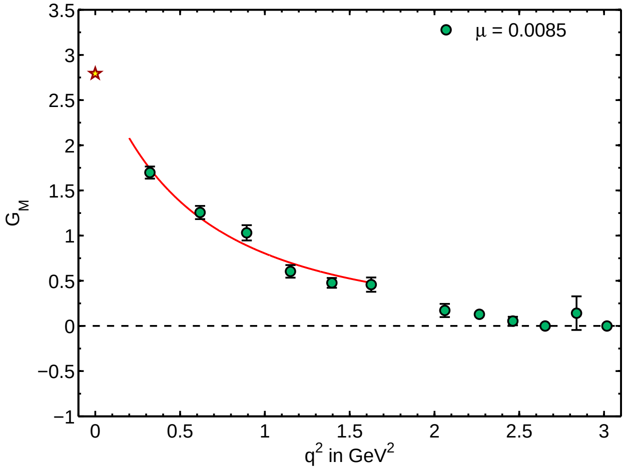

Magnetic Sachs form factor

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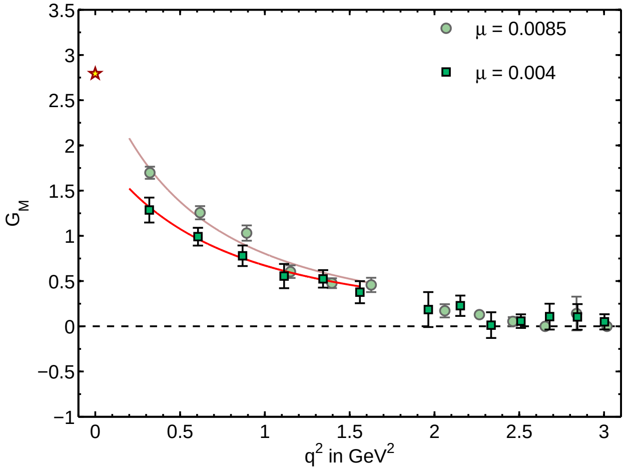
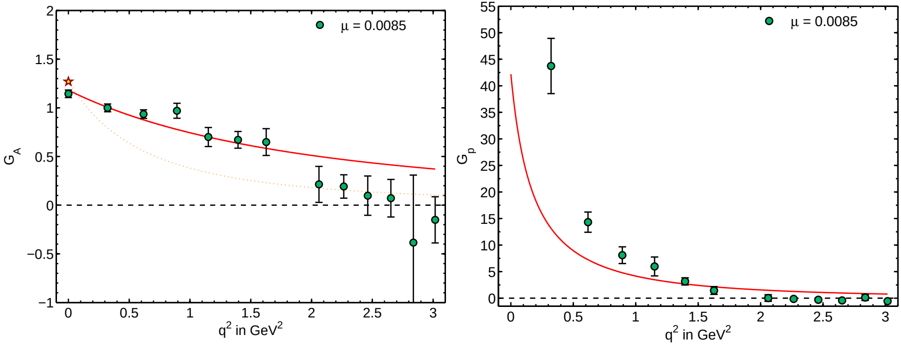


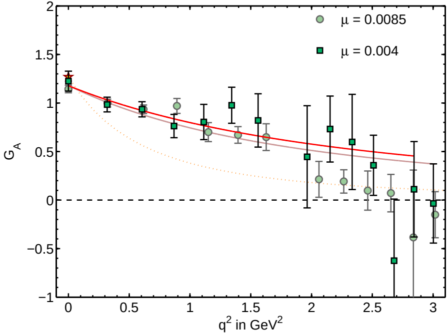
Axial form factors

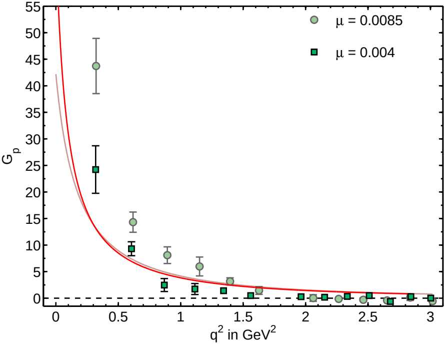


- We use $Z_A = 0.76(1)$

  [ETMC, P. Dimopoulos et al. PoS LATTICE2007:241,2007]

Axial form factors





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Final remarks

- G_E, G_M, G_A, G_p calculable with twisted mass fermions
- 2 sequential sources + 1 point source
  ($(4+2) \times 12$ inversions / conf)
- reasonable results
  - r.m.s. radius $\sqrt{\langle r^2 \rangle} \sim 0.6 - 0.7\ fm$
  - magnetic moment $\mu \sim 2.0 - 2.9\ \mu_N$
  - axial coupling $g_A \sim 1.1 - 1.2$
- need to:
  - increase statistics
  - estimate finite volume effects
  - continuum extrapolation
  - chiral extrapolation
  - calculate disconnected pieces