The electric dipole moment of the nucleon from lattice QCD with imaginary vacuum angle theta

Yoshifumi Nakamura(NIC/DESY)

for the theta collaboration

S. Aoki(RBRC/Tsukuba), R. Horsley(Edinburgh), YN, D. Pleiter(NIC/DESY), P.E.L. Rakow(Liverpool), G. Schierholz(NIC/DESY), T. Izubuchi(Kanazawa/RBRC) and J. Zanotti(Edinburgh)

Electric Dipole Moment (EDM)

- Permanent EDM is a signature of T (Time reversal symmetry, = CP) violation
- Various candidates of CP violations
  - Electro Weak: CKM phase in quark mass matrix
    very small
  - New Physics: SUSY, left-right, multi Higgs
  - Vacuum angle $\theta$
    In QCD gauge invariant CP odd terms are allowed

\[
S_\theta = i\theta \frac{1}{32\pi^2} \int d^4 x F_{\mu\nu}(x) \tilde{F}_{\mu\nu}(x) = i\theta Q
\]
Neutron EDM

Since 50 years ago

<table>
<thead>
<tr>
<th>$d_n$</th>
<th>$&lt; 2.9 \times 10^{-13} e \times fm$</th>
</tr>
</thead>
</table>

Baker et al. (2007)

Harris (2007)
## NEDM on lattice

<table>
<thead>
<tr>
<th>Year</th>
<th>Collaboration</th>
<th>$N_f$</th>
<th>Lattice</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>89</td>
<td>Aoki-Gocksch</td>
<td>0</td>
<td>Wilson</td>
<td>Electric Field</td>
</tr>
<tr>
<td>04</td>
<td>Berruto et al.</td>
<td>0</td>
<td>DWF</td>
<td>$F_3$</td>
</tr>
<tr>
<td>05</td>
<td>Berruto et al.</td>
<td>2</td>
<td>DWF</td>
<td>$F_3$</td>
</tr>
<tr>
<td>06</td>
<td>CP-PACS</td>
<td>0</td>
<td>DWF</td>
<td>$F_3$</td>
</tr>
<tr>
<td>06</td>
<td>CP-PACS</td>
<td>0</td>
<td>overlap</td>
<td>$F_3$</td>
</tr>
<tr>
<td>06</td>
<td>QCDSF</td>
<td>0</td>
<td>overlap</td>
<td>$F_3$</td>
</tr>
<tr>
<td>06</td>
<td>Blum, Izubuchi, Doi</td>
<td>2</td>
<td>DWF</td>
<td>Electric Field</td>
</tr>
<tr>
<td>08</td>
<td>CP-PACS</td>
<td>2</td>
<td>Clover</td>
<td>Electric Field</td>
</tr>
</tbody>
</table>

- **2 methods (so far)**
  - External Electric Field
    - Measure a spin splitting of energy
  - Electric form factor $F_3$ \(\leftarrow\) in this talk
    - Measure 3pt function with momenta $p$, and $p \to 0$

- **Dynamical simulations** are important
  - NEDM is very sensitive to sea quark mass, $d_n = 0$ in the chiral limit

- **Reweighting method** have been used \(\leftarrow\) noisy (needs large statistics)
  - In usual QCD simulations, $\theta = 0$
  - In the real world $\theta$ is real
  - But one can not do Full QCD HMC simulations with real $\theta
LQCD with imaginary $\theta$
Motivation

- To calculate $d_N/\theta$

In lattice QCD, $\theta$ is one of the input parameters

\[ d_N/\theta \text{ from lattice} \]
\[ |d_n| \text{ from ex.} \]
\[ \rightarrow |\theta| < ? \]

- To check feasibility of lattice QCD simulations at imaginary $\theta$
Simulations with imaginary $\theta$

There are 2 choices

- **gluonic:** $-\theta F \tilde{F}$
  
  needed smearing/cooling

- **fermionic:** $m\theta \bar{\psi} \gamma_5 \psi$
  
  by using anomalous chiral WT relation

rotation by $\theta$

$$m \rightarrow me^{i \frac{\theta}{N_f} \gamma_5}$$
**Action with \( \theta \)**

We choose fermionic way

\[
S_F + S_\theta = \bar{\psi} \left\{ D + [\cos(\theta/N_f) + i \sin(\theta/N_f) \gamma_5] m \right\} \psi
\]

\[
\bar{m} = \cos(\theta/N_f) m \\
\bar{\theta} = \tan(\theta/N_f) N_f
\]

\[
S_F = \bar{\psi} \left\{ D + \bar{m} + i (\bar{\theta}/N_f) \gamma_5 \bar{m} \right\} \psi
\]
Lattice action

\[ S_F = \bar{\psi} \left\{ D + \bar{m} + i \left( \frac{\theta_R}{N_f} \right) Z_m Z_P \gamma_5 \bar{m} \right\} \psi \]

- \( Z_m \): the renormalization constant of the quark mass
- \( Z_P \): the renormalization constant the pseudoscalar density
- \( \theta_R \): renormalized vacuum angle

\[ \theta_R = (Z_m Z_P)^{-1} \bar{\theta} \]

**chiral fermion:** nicer, definitely, extremely expensive
- \( Z_m Z_P = 1 \) and \( \theta_R = \bar{\theta} \)

**clover fermion:** relatively cheap
- \( Z_m Z_P = 1 \) and \( \theta_R = \bar{\theta} \) in the continuum limit
We employ \( N_f = 2 \) flavors of dynamical clover fermions (chiral symmetry is violated)

\[
a(D + \bar{m}) \rightarrow D^{lat} = D^{Wilson} + T^{clover}
\]

\[
a \bar{m} \rightarrow \frac{1}{2\kappa} - \frac{1}{2\kappa_c} \quad \text{VWI quark}
\]

\[
D^{Wilson}_{x,y} = \delta_{x,y} - \kappa \sum_{\mu} \left\{ (1 - \gamma_{\mu})U_{\mu}(x)\delta_{x+\mu,y} + (1 + \gamma_{\mu})U^{\dagger}_{\mu}(x - \mu)\delta_{x-\mu,y} \right\}
\]

\[
T^{clover}_{x,y} = \left\{ \frac{i}{2} c_{SW} \kappa \sigma_{\mu\nu} F_{\mu\nu}(x) + i(1 - \frac{\kappa}{\kappa_c}) \frac{\theta_R}{2} Z_m Z_P \gamma_5 \right\} \delta_{x,y}
\]

In simulations with dynamical quarks, \( \gamma_5 D \gamma_5 = D^\dagger \) is required

The vacuum angle \( \bar{\theta} \) is taken to be purely imaginary

\[
\bar{\theta} = -i |\bar{\theta}| \equiv -i \bar{\theta}^I
\]

\[\downarrow\]

The Boltzmann weight positive definite
Simulation

Lattice Size: $16^3 \times 32$

\[ \beta: 2.1 \text{(Iwasaki)} \]
\[ \kappa: 0.1357 \]
\[ \kappa_c: 0.138984 \]

Lattice Spacing: $a=0.1076(13) \text{ fm}$
\[ am_\pi: 0.6309(8) \]

same action as CP-PACS

<table>
<thead>
<tr>
<th>$\theta^I$</th>
<th># of traj.</th>
<th>$\langle P \rangle$</th>
<th>$\tau_{int}^P$</th>
<th>$\langle Q \rangle$</th>
<th>$\langle Q^2 \rangle - \langle Q \rangle^2$</th>
<th>$e^{-\Delta H}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>9000</td>
<td>0.598059(13)</td>
<td>3.24(68)</td>
<td>-0.06(31)</td>
<td>24.9(14)</td>
<td>1.0008(25)</td>
</tr>
<tr>
<td>0.2</td>
<td>9000</td>
<td>0.598045(11)</td>
<td>3.30(73)</td>
<td>-3.52(46)</td>
<td>24.1(15)</td>
<td>1.0029(35)</td>
</tr>
<tr>
<td>0.4</td>
<td>7000</td>
<td>0.598045(17)</td>
<td>3.02(41)</td>
<td>-7.35(36)</td>
<td>22.7(17)</td>
<td>1.0001(37)</td>
</tr>
<tr>
<td>1.0</td>
<td>6000</td>
<td>0.598078(16)</td>
<td>4.1(19)</td>
<td>-18.38(30)</td>
<td>21.7(15)</td>
<td>1.0001(14)</td>
</tr>
<tr>
<td>1.5</td>
<td>6000</td>
<td>0.598081(16)</td>
<td>2.56(61)</td>
<td>-27.84(37)</td>
<td>18.1(13)</td>
<td>0.9950(28)</td>
</tr>
</tbody>
</table>
\[ \theta_v = \theta_s \]

\[ \theta_v = 0 \]
Topological charge distribution

\[ P(Q) \text{ is changed by } \bar{\theta}^I \]
**Effective value of \( \theta \)**

\[
\theta^{\text{input}} = (1 - \frac{\kappa}{\kappa^c}) \bar{\theta}^I R Z^\theta m Z^\theta P
\]

ex.

Input parameter \( \theta^{\text{input}} = 0.00472572 \)

If \( \kappa^c = 0.138984 \) and \( Z^\theta m Z^\theta P = 1 \) \( \rightarrow \) \( \bar{\theta}^I R = 0.4 \)

- We could define renormalized \( \theta \) by \( Q(\theta) \)

\[
\theta^I R \sim \theta^I \times 0.75
\]

- One could also check the effect of \( \theta \) in other hadronic observable

\[
m_{\pi}(\theta) = m_{\pi}(0) \cos(i\theta/N_f)?
\]

Brower et al. (2003), Aoki et al. (2007)
Nucleon form factors

The electromagnetic current between nucleon states

\[ \langle p', s' | J_\mu | p, s \rangle = \bar{u}_\theta(p', s') \mathcal{J}_\mu u_\theta(p, s) \]

\[ \mathcal{J}_\mu = \gamma_\mu F_1^\theta(q^2) + \sigma_{\mu\nu}q_\nu \frac{F_2^\theta(q^2)}{2m_N^\theta} \]

\[ + i \theta \left[ (\gamma q_\mu - \gamma_\mu q^2) \gamma_5 F_A^\theta(q^2) + \sigma_{\mu\nu}q_\nu \gamma_5 \frac{F_3^\theta(q^2)}{2m_N^\theta} \right] \]

\[ q = p' - p, \quad q^2 = -(E' - E)^2 + (\vec{p}' - \vec{p})^2, \quad \gamma p = iE\gamma_4 + \vec{\gamma}\vec{p} \]

Electric dipole moment

\[ d_N^\theta = \lim_{q^2 \to 0} \frac{e F_3^\theta(q^2)}{2m_N^\theta} \]

The Dirac spinors are modified by a phase in the \( \theta \) vacuum

\[ u_\theta(p, s) = e^{i\alpha(\theta)\gamma_5} u(p, s) \]

\[ \bar{u}_\theta(p, s) = \bar{u}(p, s) e^{i\alpha(\theta)\gamma_5} \]
Spinor relation is modified to

\[ \sum_{s',s} u_{\theta}(\vec{p}, s') \bar{u}_{\theta}(\vec{p}, s) = e^{i\alpha(\theta)\gamma_5} \left( \frac{-i\gamma p + m_{\theta}^N}{2E_N^\theta} \right) e^{i\alpha(\theta)\gamma_5} \]

The lowest order in $\theta$

\[ \sum_{s',s} u_{\theta}(\vec{p}, s') \bar{u}_{\theta}(\vec{p}, s) = \frac{-i\gamma p + m_{N}(1 - 2\alpha'\bar{\theta}^I\gamma_5)}{2E_N}. \]

We are primarily interested in the electric dipole moment in the limit $\theta \to 0$, it is sufficient to consider the lowest order expansion only

\[ d_n \text{ for } \bar{\theta}^I \leq 0.4 \text{ in this work} \]
\[ \text{Tr} \left[ G^\theta_{NN}(t; 0) \Gamma_4 \right] \simeq \frac{1}{2} |Z_N|^2 e^{-m_N t}, \]
\[ \text{Tr} \left[ G^\theta_{NN}(t; 0) \Gamma_4 \gamma_5 \right] \simeq -\alpha' \bar{\theta}^I \frac{1}{2} |Z_N|^2 e^{-m_N t}, \]
\[ \Gamma_4 = \frac{1 + \gamma_4}{2} \]

taking ratio \( R(t) = \frac{\text{Tr} \left[ G^\theta_{NN}(t, 0) \Gamma_4 \gamma_5 \right]}{\text{Tr} \left[ G^\theta_{NN}(t, 0) \Gamma_4 \right]} \simeq -\alpha' \bar{\theta}^I \]

\[ \bar{\theta}^I = 0.4 \]
Nucleon form factors

\[
R_{\mu}(t', t; \vec{p}', \vec{p}) = \frac{G_{NJ\mu N}^{\theta}(t', t; \vec{p}', \vec{p})}{\text{Tr}[G_{NN}^{\theta}(t'; \vec{p}')] \Gamma_4} \times \left\{ \frac{\text{Tr}[G_{NN}^{\theta}(t; \vec{p}) \Gamma_4] \text{Tr}[G_{NN}^{\theta}(t'; \vec{p}' \Gamma_4] \text{Tr}[G_{NN}^{\theta}(t' - t; \vec{p}) \Gamma_4] \text{Tr}[G_{NN}^{\theta}(t' - t; \vec{p}') \Gamma_4]}{\text{Tr}[G_{NN}^{\theta}(t; \vec{p}) \Gamma_4] \text{Tr}[G_{NN}^{\theta}(t'; \vec{p}) \Gamma_4] \text{Tr}[G_{NN}^{\theta}(t' - t; \vec{p}) \Gamma_4]} \right\}^{1/2} = \sqrt{\frac{E^{'\theta} E^{\theta}}{(E^{\theta} + m^{\theta}_N)(E^{'\theta} + m^{\theta}_N)}} F(\Gamma, \mathcal{J}_\mu),
\]

\[
F(\Gamma, \mathcal{J}_\mu) = \frac{1}{4} \text{Tr} \Gamma \left[ e^{i\alpha(\theta)\gamma_5} \frac{E^{\theta} \gamma_4 - i \vec{\gamma} \vec{p}'}{E^{\theta}'} + m^{\theta}_N e^{i\alpha(\theta)\gamma_5} \right] \times \mathcal{J}_\mu \left[ e^{i\alpha(\theta)\gamma_5} \frac{E^\theta \gamma_4 - i \vec{\gamma} \vec{p} + m^{\theta}_N e^{i\alpha(\theta)\gamma_5}}{E^\theta} \right]
\]

\[
\mathcal{J}_\mu = \gamma_\mu F_1^\theta(q^2) + \sigma_{\mu\nu} \frac{F_2^\theta(q^2)}{2m^{\theta}_N} + i \theta \left[ (\gamma q q_\mu - \gamma_\mu q^2) \gamma_5 F_A^\theta(q^2) + \sigma_{\mu\nu} q_\nu \gamma_5 \frac{F_3^\theta(q^2)}{2m^{\theta}_N} \right]
\]
momenta: quantized in units of $2\pi/L$

- conventional (periodic) boundary condition (BC)
  
  $p = \frac{2\pi}{16} \sim 0.4$ is not small $\to$ noisier & large extrap.


\[
\psi(x_k + L) = e^{i\alpha_k} \psi(x_k), \quad k = 1, 2, 3.
\]

the dispersion relation for the nucleon

\[
E = \sqrt{m_N^2 + (\vec{p} + \vec{\alpha})^2}
\]

choice of twist angles

\[
\vec{\alpha} = \frac{2\pi}{L} (0, 0, 0)
\]
\[
\vec{\alpha} = \frac{2\pi}{L} (0.36, 0, 0)
\]
\[
\vec{\alpha} = \frac{2\pi}{L} (0.36, 0.36, 0)
\]
\[
\vec{\alpha} = \frac{2\pi}{L} (0.36, 0.36, 0.36)
\]
Preliminary results

\[ \bar{\theta}^I = 0.2 \]

\[ \bar{\theta}^I = 0.4 \]

Fit using a dipole ansatz

\[ F_3^\theta(q^2) = F_3^\theta(0) \frac{1}{(1 + q^2/M^2)^2} \]

The renormalization constant \( Z_V \) is needed
\[ F_3^\theta(0) = \lim_{q^2 \to 0} \frac{F_3^\theta(q^2)}{F_1^p(q^2)} \]

\( Z_V \) cancels
Anapole form factor

$\bar{\theta}^I = 0.2$

$\bar{\theta}^I = 0.4$
fit

\[ d^\theta_N = \frac{\partial d^\theta_N}{\partial \bar{\theta}^I} \bar{\theta}^I + c \bar{\theta}^I^3 \]
gives at $\bar{\theta}^I = 0$

$$\frac{\partial d_N^\theta}{\partial \theta^I} = 0.080(10)_{\text{stat+fit}}^{\text{(?)}_{\text{sys}}} \, [e \times \text{fm}] \quad \text{Proton}$$

$$\frac{\partial d_N^\theta}{\partial \theta^I} = -0.049(5)_{\text{stat+fit}}^{\text{(?)}_{\text{sys}}} \, [e \times \text{fm}] \quad \text{Neutron}$$

\[ \Downarrow \]

$$|\theta| < 6 \times 10^{-12}$$

preliminary
$\bar{\theta}_s^I = 0.4$, $\bar{\theta}_v^I = 0.0$
Charge distribution and $\theta$ vacuum

$$\langle Q^n \rangle_c \equiv i^n \frac{\partial^n}{\partial \theta^n} \ln Z(\theta)$$
\[ S = \frac{\langle Q^3 \rangle_c}{\langle Q^2 \rangle_c} \]

\[ K = \frac{\langle Q^4 \rangle_c}{\langle Q^2 \rangle_c} \]
Conclusions and future plans

• Have performed simulations of QCD with $N_f = 2$ flavors of dynamical quarks at imaginary vacuum angle $\theta$

• The use of partially twisted boundary conditions has allowed us to compute the relevant neutron form factor $F_3(q^2)$ with high precision over the entire range of momenta down to $(aq)^2 \approx 0.02$
  → greatly facilitated the extrapolation to $q^2 = 0$

• Successfully obtain signal disentangled from statistical noise

• Plan
  – Sea quark mass dependence ($\chi$PT)
    EDM is zero in the chiral limit
    In this work $m_\pi$ is very heavy $\sim 1$ GeV
  – Volume dependence
    In this work $V \sim (1.7 \text{ fm})^3$ → small for baryon
  – Dependence of the boundary conditions
    We used periodic BC for sea quark TBC for valence quark
  – Gluonic operator instead of the pseudoscalar operator