Non-perturbative renormalization of $N_f = 2 + 1$ QCD with Schrödinger functional scheme

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Our ultimate purpose

- Determine the fundamental parameter of $N_f = 2+1$ QCD

$$\mathcal{L} = -\frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + \overline{\psi}_i \left(\gamma_\mu D_\mu + m_i\right) \psi_i$$

  - Strong coupling g: target of this talk
    - Low energy input r_0 is measured by PACS-CS (Namekawa)
  - Quark masses m_i
    - Bare quark masses are measured by PACS-CS (Kadoh, Kuramashi, Ukita)
    - NP renormalization factor will be needed.

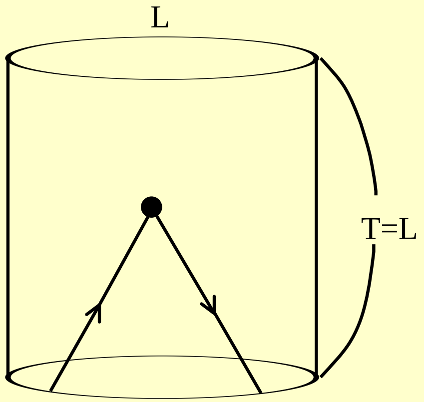
- We adopt input of low energy experimental values.
  - Comparison with high energy input (estimation of systematic error)
  - Need calculation from weak to strong coupling region.

Plan of this project

- Evaluate $\alpha_S(M_Z)$ by an input of low energy observable (r_0).
- NP renormalization factor of quark mass.
  - as a by product of $\alpha_S(M_Z)$ in this talk (inhomogeneous BC at $t = 0$, T)

Method

- Non-perturbative renormalization with Schrödinger functional



- Finite volume of L^4
- Appropriate boundary condition
- Renormalization scale $\sim 1/L$

  - Good compatibility with lattice.
  - Covers from low to high energy region.

Schrödinger functional scheme (Lüscher et al, Alpha)

- Dirichlet boundary condition at $t = 0$, T.

$$U_k(x)|_{x_0=0} = \exp\left(aC_k\right), \quad C_k = \frac{i}{L}\begin{pmatrix} \phi_1 & & \\ & \phi_2 & \\ & & \phi_3 \end{pmatrix}$$

  - Unique global minimum at background field B_μ.
  - Mass gap in fermionic mode (quark mass can be set to zero).

- Renormalized coupling

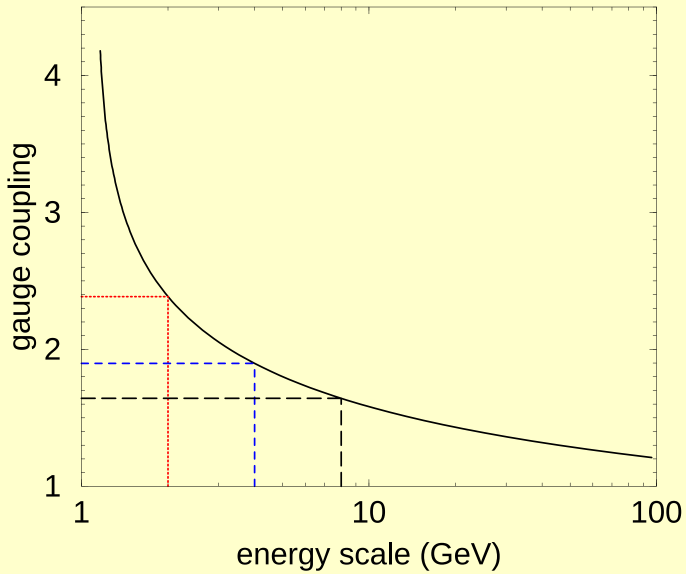
$$S_0 = \frac{1}{g_0^2}F_{\mu\nu}^2 \Rightarrow \Gamma_0[B_\mu] = \Gamma[B_\mu] = \frac{1}{g_R^2(L)}k[B_\mu]$$

- Mass renormalization factor

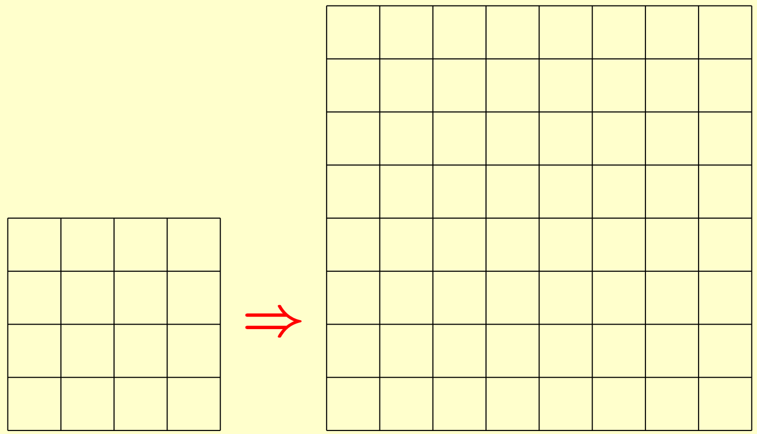
$$Z_m(L) = \frac{\langle P(t = L/2) \cdot \mathcal{O}_{\mathrm{boundary}}\rangle_{\mathrm{lattice}}}{\langle P(t = L/2) \cdot \mathcal{O}_{\mathrm{boundary}}\rangle_{\mathrm{tree}}}$$

Step Scaling Function

- Renormalization group flow $g(L) \longrightarrow g(2L)$
  when one changes the renormalization scale $L \longrightarrow 2L$



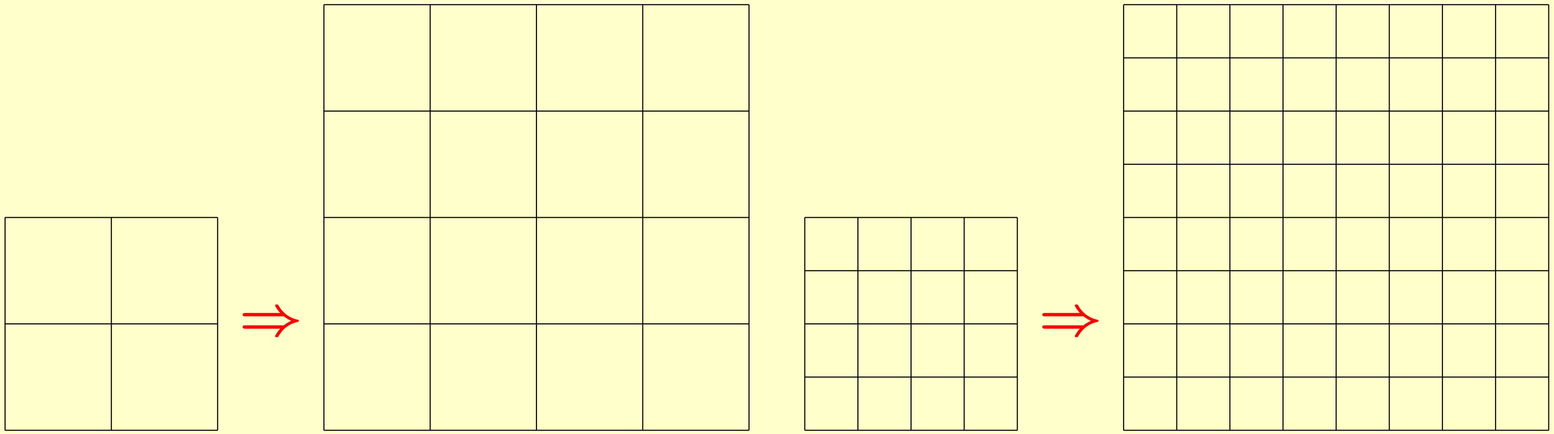
implemented easily on lattice



- Follow the renormalization group flow in discretized way.

Step scaling function

- The point is:
  - To take continuum limit for every step of RG flow.
    - $\boldsymbol{L \to 2L}$, $\quad \boldsymbol{g(L,a) \to g(2L,a)}$



- Obtain the RG flow $\boldsymbol{g(L) \to g(2L)}$ in the continuum.

Numerical setup

- Iwasaki gauge action $\beta = 2.1 \sim 6.2$
  - tree level boundary improvement
  - inhomogeneous DBC, $\theta = \pi/5$
- Wilson fermion with clover term.
  - non-perturbative c_{SW}
  - one loop boundary improvement
- RHMC/HMC algorithm for 3rd/two flavour(s)
- CPS++ code
- Machines
  - PC cluster kaede at Tsukuba: (∼180 PU)
  - SR11000 at Tokyo: (∼64 PU)
  - PACS-CS: (256 PU)× 1 month
  - RSCC at Riken (128 PU)
  - T2K at Tsukuba: (2560 cores)× 10 days
  - T2K at Tokyo: (128 cores)

Current status

- Take the continuum limit by three box sizes.

L/a	4	6	8
$2L/a$	8	12	16

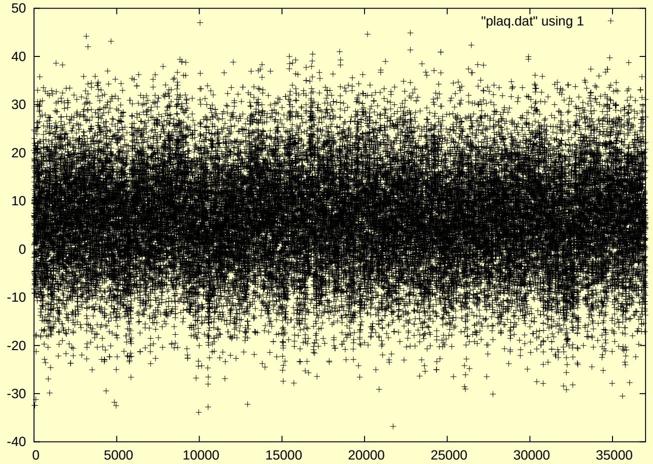
- Tuning of β and κ is finished for fixed physical box size.

$\bar{g}^2$	1.001	1.249	1.524	1.840	2.129	2.632	3.418
4^4	$110K$	$170K$	$230K$	$170K$	$210K$	$320K$	$100K$
8^4	$40K$	$40K$	$86K$	$134K$	$50K$	$74K$	$308K$
6^4	$153K$	$150K$	$50K$	$170K$	$110K$	$144K$	$120K$
12^4	$42K$	$51K$	$42K$	$35K$	$28K$	$21K$	$38K$
8^4	$98K$	$86K$	$122K$	$98K$	$74K$	$122K$	$122K$
16^4	$16.2K$	$18K$	$116K$	$3.2K$	$4K$	$5.1K$	$3.2K$

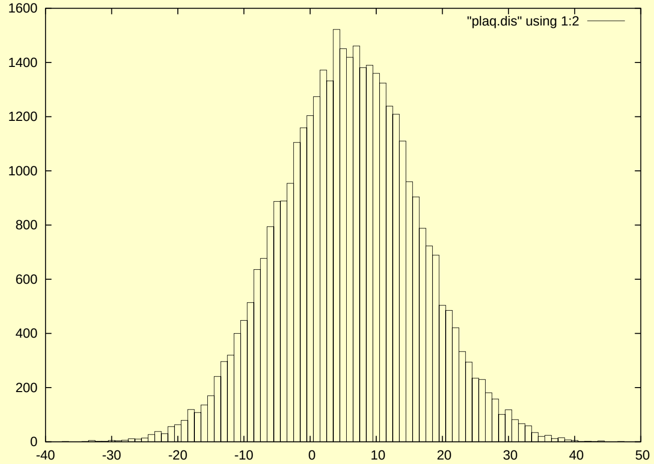
- Now performing simulation for larger box of $2L$
  - $L/a = 8 \longrightarrow L/a = 16$: Now going on

Distribution of data

- Distribution of $\partial S/\partial \eta \propto 1/\bar{g}^2$ ($\mathbf{12^4}$ at strong coupling)



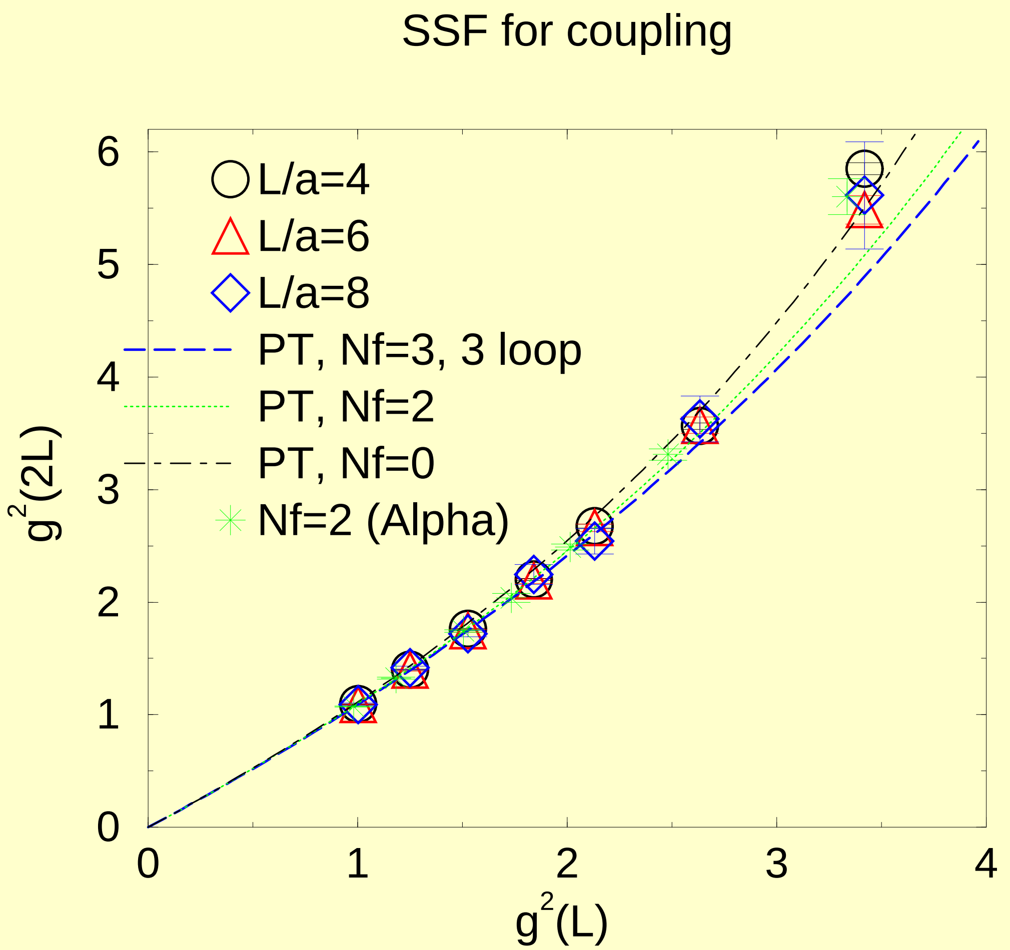
$\partial S/\partial \eta$ vs τ



Its distribution

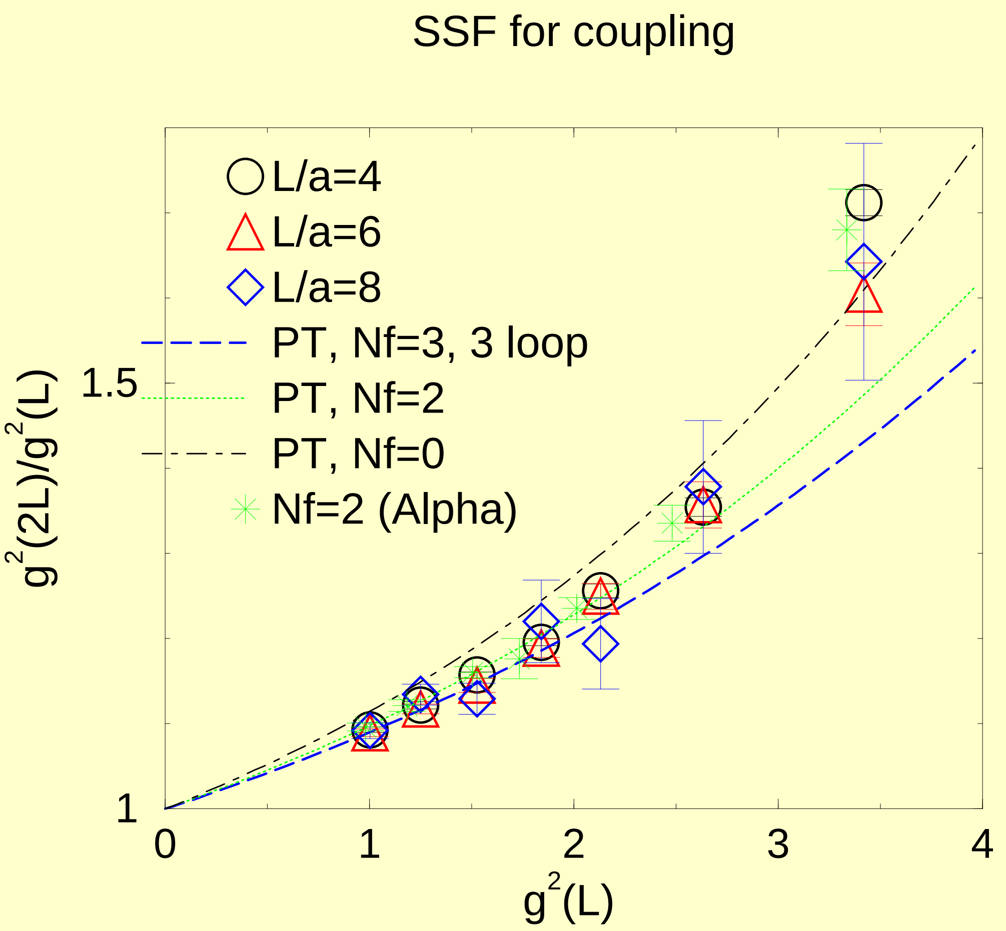
SSF (preliminary)

- $\sigma(u) = \bar{g}^2(2L)|_{u=\bar{g}^2(L)}$



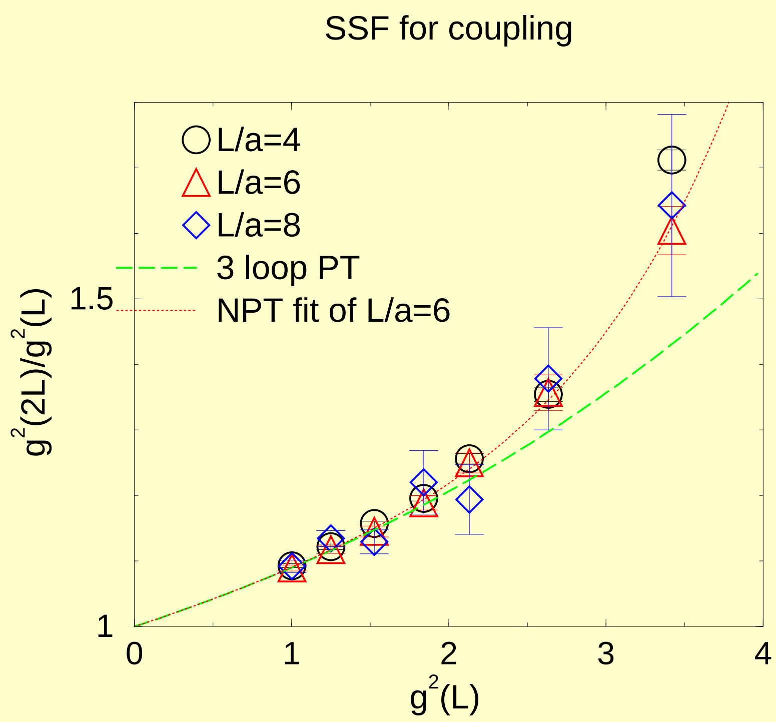
SSF (preliminary)

- $\sigma(u)/u$



SSF (preliminary)

- $\sigma(u)/u$



- Polynomial fit

$$\sigma(u) = u + s_0 u^2 + s_1 u^3 + s_2 u^4 + f_3 u^5 + f_4 u^6$$
$$\chi^2/\mathrm{dof} \sim 0.8$$

Introduction of scale

- r_0 at vanishing PCAC mass ($m_u = m_d = m_s \to 0$)

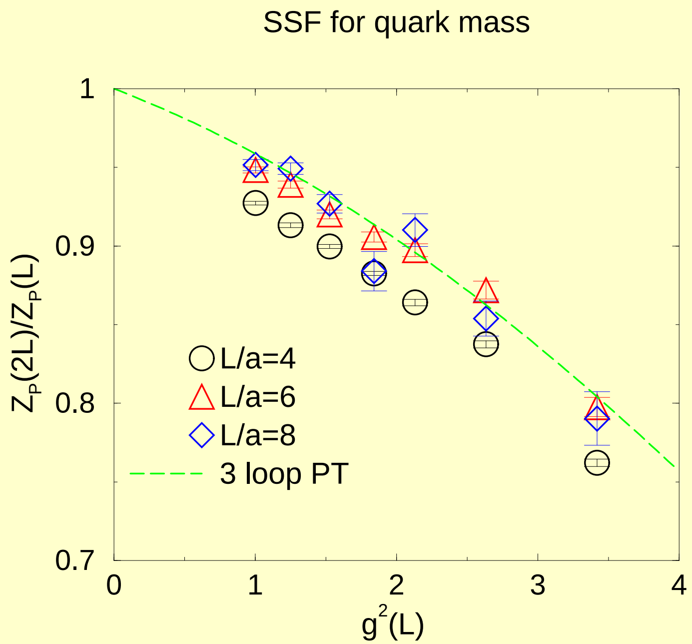
β	$(r_0/a)_{m=0}$
1.83	4.342(26)
1.90	6.45(27)(34)
2.05	7.787(66)

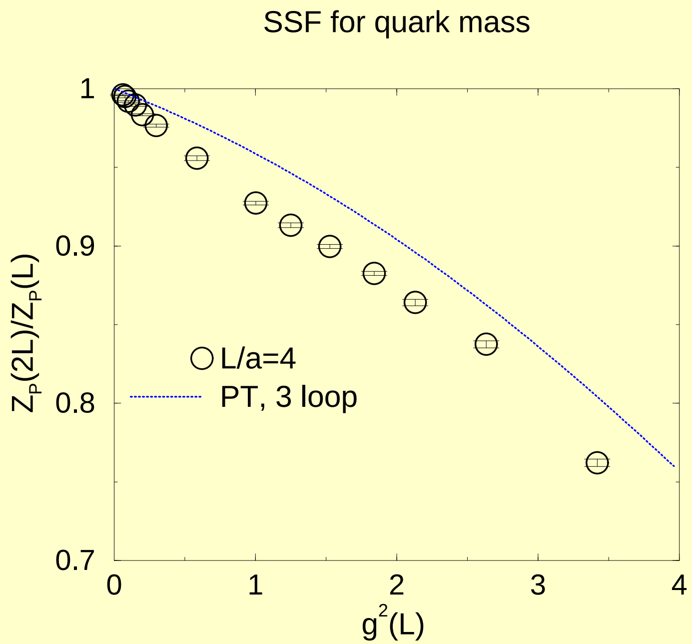
L^4	β	$\bar{g}^2$	m_{AWT}
4^4	1.90	4.695(23)	−0.00039(28)
4^4	2.05	3.808(17)	0.00010(31)
6^4	2.05	4.763(70)	−0.00810(20)

$$\Lambda = \mu \left(b_0 \bar{g}\right)^{-\frac{b_1}{2b_0^2}} \exp\left(-\frac{1}{2b_0\bar{g}}\right) \exp\left(-\int_0^{\bar{g}} dg \left(\frac{1}{\beta} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right)\right)$$

$$\Lambda_{\overline{\mathrm{MS}}} = 2.612 \Lambda_{\mathrm{SF}}$$

SSF of Z_m (preliminary)





- Scaling behaviour is not good for $L/a = 4$.
- May be able to take the continuum limit by two data points.
- PT improvement may not work for 4^4 but may be for 6^4.

Conclusion

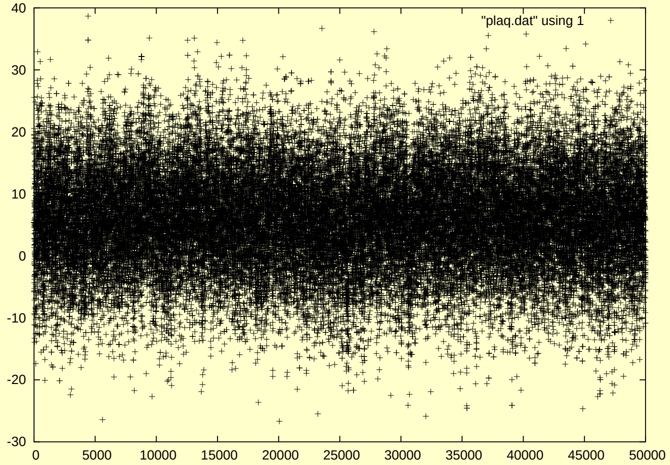
- Calculation for the running coupling is going on
- Scaling behaviour seems to be good.
- Scaling behaviour of quark mass SSF is not so good.
  - We may need perturbative improvement.

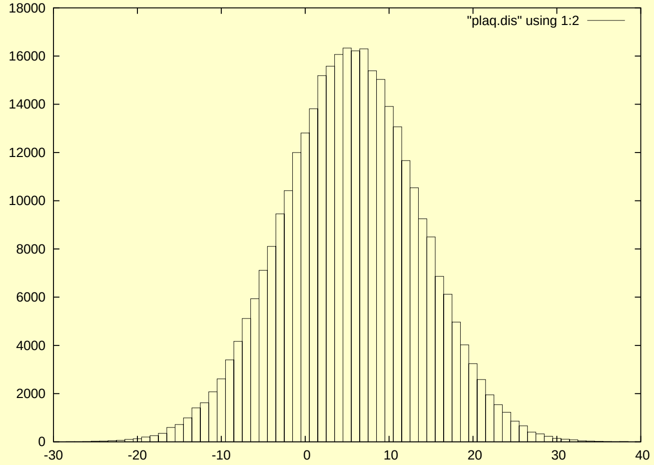
Future work

- Take the continuum limit
- Take data for $L/a = 16$
  - Much expectation on T2K machine!
- Adopt appropriate setup for Z_m and repeat the calculation
  - Homogeneous BC, $\theta = 0.5$

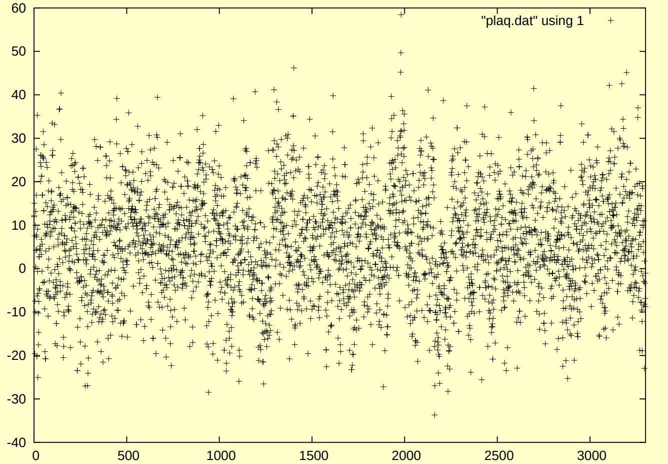
Distribution of data

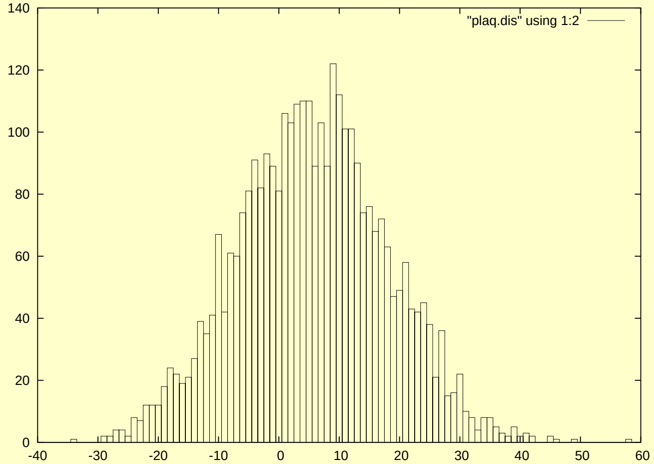
- (8^4 at strong coupling)





- (16^4 at strong coupling)





Scaling of SSF

- For one loop improved gauge boundary term (condition B)