Rotor Spectra, Berry Phases, and Monopole Fields: From Antiferromagnets to QCD

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Outline

From Graphene to Ni_xCoO_2

Rotor Spectrum at Half-Filling

Rotor Spectrum in the Single-Hole Sector

Rotor Spectrum in the QCD Vacuum Sector

Rotor Spectrum in the Single-Nucleon Sector

Conclusions

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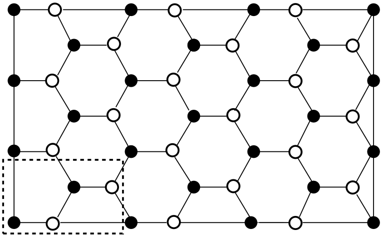
The Hubbard model on the honeycomb lattice

$$H = -t \sum_{\langle xy \rangle} (c_x^\dagger c_y + c_y^\dagger c_x) + U \sum_x (c_x^\dagger c_x - 1)^2, \quad c_x = (c_{x\uparrow}, c_{x\downarrow})$$



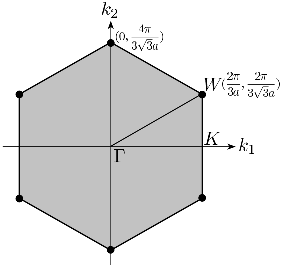
Local charge and spin operators

$$Q_x = c_x^\dagger c_x - 1, \quad \vec{S}_x = c_x^\dagger \frac{\vec{\sigma}}{2} c_x, \quad [S_x^a, S_y^b] = i\delta_{xy}\varepsilon_{abc} S_x^c$$

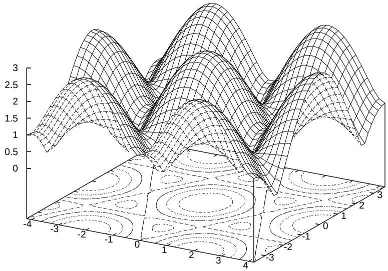
$U(1)_Q$ and $SU(2)_s$ symmetries

$$Q = \sum_x Q_x, \quad \vec{S} = \sum_x \vec{S}_x, \quad [H, Q] = [H, \vec{S}] = 0$$

Unbroken $SU(2)_s$ symmetric phase (graphene) at $U < U_c$



Brillouin zone



Dispersion relation

Effective Dirac Lagrangian for free graphene

$$\mathcal{L} = \sum_{\substack{f=\alpha,\beta \\ s=+,-}} \overline{\psi}_s^f \gamma_\mu \partial_\mu \psi_s^f$$

The t-J model for the antiferromagnetic $SU(2)_s$ broken symmetry phase (Ni_xCoO_2) at $U \gg U_c$

$$H = P\Big\{ - t \sum_{\langle xy \rangle} (c_x^\dagger c_y + c_y^\dagger c_x) + J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y \Big\} P.$$

reduces to the Heisenberg model at half-filling

$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

Effective Goldstone boson field in $SU(2)/U(1) = S^2$

$$\vec{e}(x) = (e_1(x), e_2(x), e_3(x)), \qquad \vec{e}(x)^2 = 1$$

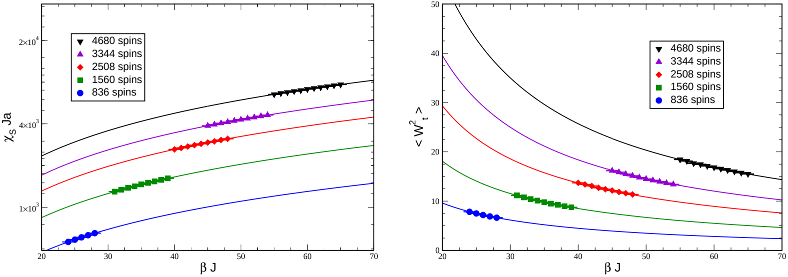
Low-energy effective action for magnons

$$S[\vec{e}] = \int d^2x \; dt \; \frac{\rho_s}{2} \left(\partial_i \vec{e} \cdot \partial_i \vec{e} + \frac{1}{c^2} \partial_t \vec{e} \cdot \partial_t \vec{e} \right)$$

Fit to predictions in the ε-regime of magnon chiral perturbation theory with $\beta c \approx L$, $l = (\beta c/L)^{1/3}$

$$\chi_s = \frac{\mathcal{M}_s^2 L^2 \beta}{3} \left\{ 1 + 2\frac{c}{\rho_s L l}\beta_1(l) + \left(\frac{c}{\rho_s L l}\right)^2 \left[\beta_1(l)^2 + 3\beta_2(l)\right] \right\}$$

$$\chi_u = \frac{2\rho_s}{3c^2} \left\{ 1 + \frac{1}{3}\frac{c}{\rho_s L l}\widetilde{\beta}_1(l) + \frac{1}{3}\left(\frac{c}{\rho_s L l}\right)^2 \left[\widetilde{\beta}_2(l) - \frac{1}{3}\widetilde{\beta}_1(l)^2 - 6\psi(l)\right] \right\}$$



$$\widetilde{\mathcal{M}}_s = 0.2689(4), \quad \rho_s = 0.102(2)J, \quad c = 1.297(16)Ja$$

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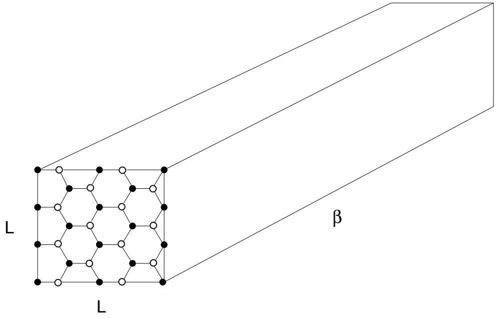
Effective Lagrange function in the δ-regime of magnon chiral perturbation theory with $\beta c \gg L$

$$\mathcal{L} = \int d^2x \, \frac{\rho_s}{2} \left(\partial_i \vec{e} \cdot \partial_i \vec{e} + \frac{1}{c^2} \partial_t \vec{e} \cdot \partial_t \vec{e} \right) = \frac{\Theta}{2} \partial_t \vec{e} \cdot \partial_t \vec{e}$$

Moment of inertia

$$\Theta = \frac{\rho_s L^2}{c^2} \left[1 + \frac{3.900265}{4\pi} \frac{c}{\rho_s L} + \mathcal{O}\left(\frac{1}{L^2} \right) \right]$$

P. Hasenfratz and F. Niedermayer, Z. Phys. B92 (1993) 91



Spherical coordinates for the staggered magnetization

$$\vec{e} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$$

Effective Lagrange function

$$\mathcal{L} = \frac{\Theta}{2}\partial_t\vec{e}\cdot\partial_t\vec{e} = \frac{\Theta}{2}\left[(\partial_t\theta)^2 + \sin^2\theta(\partial_t\varphi)^2\right]$$

Canonically conjugate momenta

$$p_\theta = \frac{\delta\mathcal{L}}{\delta\partial_t\theta} = \Theta\ \partial_t\theta,\ \ p_\varphi = \frac{\delta\mathcal{L}}{\delta\partial_t\varphi} = \Theta\ \sin^2\theta\ \partial_t\varphi$$

Quantum mechanical rotor Hamiltonian

$$H = -\frac{1}{2\Theta}\left(\frac{1}{\sin\theta}\partial_\theta[\sin\theta\partial_\theta] + \frac{1}{\sin^2\theta}\partial_\varphi^2\right) = \frac{\vec{S}^2}{2\Theta}$$

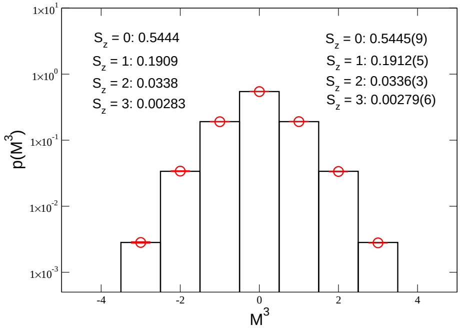
Rotor spectrum

$$E_S = \frac{S(S+1)}{2\Theta}$$

Probability distribution of magnetization $M^3 = S^3$

$$p(M^3) = \frac{1}{Z} \sum_{S \geq |M^3|} \exp(-\beta E_S), \quad Z = \sum_{S=0}^{\infty} (2S+1) \exp(-\beta E_S)$$

Honeycomb Lattice, 836 Spins, βJ = 60



Perfect agreement without additional adjustable parameters

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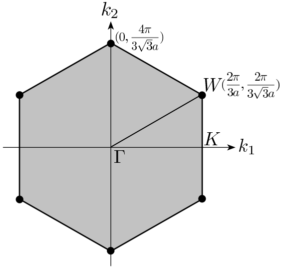
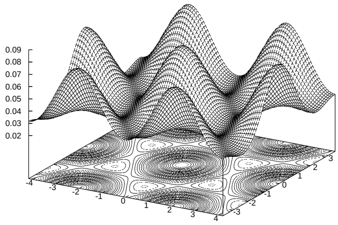
Hole dispersion in the t-J model





Effective Lagrangian for Holes

$$\mathcal{L} = \sum_{\substack{f=\alpha,\beta \\ s=+,-}} \left[M\psi_s^{f\dagger}\psi_s^f + \psi_s^{f\dagger} D_t \psi_s^f + \frac{1}{2M'} D_i\psi_s^{f\dagger} D_i \psi_s^f \right]$$

Covariant derivative coupling to composite magnon gauge field

$$D_\mu \psi_\pm^f(x) = \left[\partial_\mu \pm i v_\mu^3(x)\right] \psi_\pm^f(x)$$

Effective Lagrange function for quantum mechanical rotor

$$\mathcal{L} = \frac{\Theta}{2}\partial_t\vec{e}\cdot\partial_t\vec{e} + \sum_{f=\alpha,\beta}\Psi^{f\dagger}\left[E(\vec{p}) - i\partial_t + v_t^3\sigma_3\right]\Psi^f, \Psi(t) = \left(\begin{array}{c}\psi_+^f(t)\\ \psi_-^f(t)\end{array}\right)$$

Spherical coordinates for the staggered magnetization

$$\vec{e} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta) \;\Rightarrow\; v_t^3 = \sin^2\frac{\theta}{2}\partial_t\varphi$$

Canonically conjugate momenta

$$\Theta\;\partial_t\theta = p_\theta, \quad \Theta\;\partial_t\varphi = \frac{1}{\sin^2\theta}(p_\varphi + iA_\varphi)$$

Abelian monopole Berry gauge field

$$A_\theta = 0, \quad A_\varphi = i\sin^2\frac{\theta}{2}\sigma_3, \quad F_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta = \frac{i}{2}\sin\theta\;\sigma_3$$

Rotor Hamiltonian in the single-hole sector

$$\begin{aligned} H &= -\frac{1}{2\Theta}\left\{\frac{1}{\sin\theta}\partial_\theta[\,\sin\theta\partial_\theta] + \frac{1}{\sin^2\theta}(\partial_\varphi - A_\varphi)^2\right\} + E(\vec{p}) \\ &= \frac{1}{2\Theta}\left(\vec{J}^2 - \frac{1}{4}\right) + E(\vec{p}) \end{aligned}$$

Angular momentum operators

$$J_\pm = \exp(\pm i\varphi)\left(\pm\,\partial_\theta + i\cot\theta\;\partial_\varphi - \frac{1}{2}\tan\frac{\theta}{2}\sigma_3\right), \quad J_3 = -i\partial_\varphi - \frac{\sigma_3}{2}$$

Energy spectrum

$$E_j = \frac{1}{2\Theta}\left[j(j+1) - \frac{1}{4}\right] + E(\vec{p}), \quad j \in \{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots\}$$

Wave functions are monopole harmonics

$$Y^{\pm}_{\frac{1}{2},\pm\frac{1}{2}}(\theta,\varphi) = \frac{1}{\sqrt{2\pi}}\sin\frac{\theta}{2}\exp(\pm i\varphi), \quad Y^{\pm}_{\frac{1}{2},\mp\frac{1}{2}}(\theta,\varphi) = \frac{1}{\sqrt{2\pi}}\cos\frac{\theta}{2}$$

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Rotor Lagrange function in massless $N_f = 2$ QCD

$$\mathcal{L} = \int d^3x \, \frac{F_\pi^2}{4} \mathrm{Tr}\left[\partial_\mu U^\dagger \partial_\mu U\right] = \frac{\Theta}{4} \mathrm{Tr}\left[\partial_t U^\dagger \partial_t U\right], \quad \Theta = F_\pi^2 L^3$$

Rotor spectrum

$$E_I = \frac{j_L(j_L+1) + j_R(j_R+1)}{\Theta} = \frac{I(I+2)}{2\Theta}$$

Rotor quantum numbers

$$j_L = j_R, \quad I = j_L + j_R \in \{0, 1, 2, ...\}$$

Degeneracy

$$g = (2j_L+1)(2j_R+1) = (I+1)^2$$

H. Leutwyler, Phys. Lett. B189 (1987) 197

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Effective Lagrange function for quantum mechanical rotor

$$\mathcal{L} = \frac{\Theta}{4}\mathrm{Tr}\left[\partial_t U^\dagger \partial_t U\right] + \Psi^\dagger\left[E(\vec{p}) - i\partial_t - iv_t - i\frac{g_A}{M}(\vec{\sigma}\cdot\vec{p})a_t\right]\Psi$$

Gauge and vector fields composed of pion fields $U = u^2$

$$v_t = \frac{1}{2}\left(u\partial_t u^\dagger + u^\dagger\partial_t u\right), \quad a_t = \frac{1}{2i}\left(u\partial_t u^\dagger - u^\dagger\partial_t u\right)$$

Spherical coordinates for the pion field

$$U = \cos\alpha + i\sin\alpha\vec{e}_\alpha\cdot\vec{\tau}, \quad \vec{e}_\alpha = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta),$$
$$\vec{e}_\theta = (\cos\theta\cos\varphi, \cos\theta\sin\varphi, -\sin\theta), \quad \vec{e}_\varphi = (-\sin\varphi, \cos\varphi, 0)$$

Concrete form of gauge and vector fields

$$v_t = i\sin^2\frac{\alpha}{2}\left(\partial_t\theta\ \vec{e}_\varphi - \sin\theta\ \partial_t\varphi\ \vec{e}_\theta\right)\cdot\vec{\tau},$$
$$a_t = \left(\frac{\partial_t\alpha}{2}\vec{e}_\alpha + \sin\alpha\frac{\partial_t\theta}{2}\vec{e}_\theta + \sin\alpha\sin\theta\frac{\partial_t\varphi}{2}\vec{e}_\varphi\right)\cdot\vec{\tau}$$

Rotor Hamiltonian in the single-nucleon sector

$$H = E(\vec{p}) - \frac{1}{2\Theta}\left\{\frac{1}{\sin^2\alpha}(\partial_\alpha - A_\alpha)[\,\sin^2\alpha(\partial_\alpha - A_\alpha)]\right.$$
$$\left.+\frac{1}{\sin^2\alpha\sin\theta}(\partial_\theta - A_\theta)[\,\sin\theta(\partial_\theta - A_\theta)] + \frac{1}{\sin^2\alpha\sin^2\theta}(\partial_\varphi - A_\varphi)^2\right\}$$

Non-Abelian monopole Berry gauge field ($\Lambda = g_A|\vec{p}|/M$)

$$\begin{aligned}
A_\alpha &= i\frac{\Lambda}{2}(\vec{\sigma}\cdot\vec{e}_p)\vec{e}_\alpha\cdot\vec{\tau},\ A_\theta = i\left(\sin^2\frac{\alpha}{2}\ \vec{e}_\varphi + \frac{\Lambda}{2}(\vec{\sigma}\cdot\vec{e}_p)\sin\alpha\ \vec{e}_\theta\right)\cdot\vec{\tau}, \\
A_\varphi &= i\left(-\sin^2\frac{\alpha}{2}\sin\theta\ \vec{e}_\theta + \frac{\Lambda}{2}(\vec{\sigma}\cdot\vec{e}_p)\sin\alpha\sin\theta\ \vec{e}_\varphi\right)\cdot\vec{\tau} \\
F_{\alpha\theta} &= \partial_\alpha A_\theta - \partial_\theta A_\alpha + [A_\alpha, A_\theta] = i\frac{1-\Lambda^2}{2}\sin\alpha\ \vec{e}_\varphi\cdot\vec{\tau}, \\
F_{\theta\varphi} &= \partial_\theta A_\varphi - \partial_\varphi A_\theta + [A_\theta, A_\varphi] = i\frac{1-\Lambda^2}{2}\sin^2\alpha\ \sin\theta\ \vec{e}_\alpha\cdot\vec{\tau}, \\
F_{\varphi\alpha} &= \partial_\varphi A_\alpha - \partial_\alpha A_\varphi + [A_\varphi, A_\alpha] = i\frac{1-\Lambda^2}{2}\sin\alpha\ \sin\theta\ \vec{e}_\theta\cdot\vec{\tau}
\end{aligned}$$

Rotor Hamiltonian with $\Lambda = g_A|\vec{p}|/M$

$$H = \frac{1}{2\Theta}\left(\vec{J}^{\,2} + \vec{K}^{\,2} - \frac{3}{4}\right) + \frac{1}{2\Theta}\left(\Lambda C + \frac{3}{4}\Lambda^2\right),$$

$$C = i(\vec{\sigma}\cdot\vec{e}_p)\left(\vec{e}_\alpha\partial_\alpha + \frac{1}{\sin\theta}\vec{e}_\theta\partial_\theta + \frac{1}{\sin\alpha\sin\theta}\vec{e}_\varphi\partial_\varphi - \tan\frac{\alpha}{2}\vec{e}_\alpha\right)\cdot\vec{\tau}$$

commutes with chiral rotations

$$\vec{J}_L = \frac{1}{2}\left(\vec{J} - \vec{K}\right), \quad \vec{J}_R = \frac{1}{2}\left(\vec{J} + \vec{K}\right), \quad C^2 = \vec{J}^{\,2} + \vec{K}^{\,2} + \frac{3}{4}$$

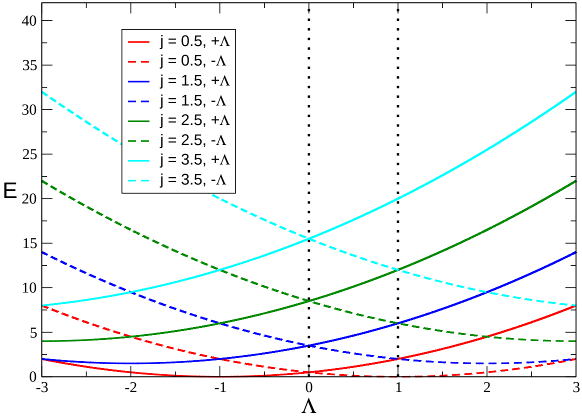
Energy spectrum

$$E_j = \frac{1}{2\Theta}\left[j'(j'+2) + \frac{\Lambda^2 - 1}{2}\right] + E(\vec{p}), \quad j' = j \pm \frac{\Lambda}{2}$$

Rotor quantum numbers and degeneracies

$$j_L = j_R \pm \frac{1}{2}, \quad j = j_L + j_R \in \left\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots\right\}, \quad g = 2\left(j + \frac{1}{2}\right)\left(j + \frac{3}{2}\right)$$

Rotor Spectrum as a function of $\Lambda = g_A|\vec{p}|/M$



Remarkably, for $\Lambda = \pm 1$ the non-Abelian field strength vanishes and $E_j(\pm 1) = \frac{1}{2\Theta} j'(j'+2)$ with $j' = j \pm \frac{1}{2}$. The QCD rotor spectrum then looks like the one of in the vacuum sector, although the system now has baryon number one.

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- There are intriguing analogies between antiferromagnets and QCD.
- Fermions have characteristic effects on the rotor spectrum.
- The rotor problem tests the effective theory nonperturbatively.
- Perturbative matching of Λ to the infinite volume effective theory is necessary before g_A could be extracted from the rotor level splitting.

Interesting related work

A. Ali-Khan et al., Nucl. Phys. B689 (2004) 175

W. Detmold and M. Savage, Phys. Lett. B599 (2004) 32

P. F. Bedaque, H. W. Griesshammer, and G. Rupak, Phys. Rev. D71 (2005) 054015

G. Colangelo, A. Fuhrer, and C. Haefeli, Nucl. Phys. Proc. Suppl. 153 (2006) 41

BGR collaboration, P. Hasenfratz et al., PoS (LATTICE 2007) 077

JLQCD collaboration, H. Fukaya et al.,
PoS (LATTICE 2007) 077, Phys. Rev. D76 (2007) 054503, Phys. Rev. Lett. 98 (2007) 172001