Why (light) meson decays are interesting

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Outline

1. Motivation
2. Theoretical tools
3. Pseudo-scalars
4. Vectors
5. Axial-vectors
6. Others
7. Conclusions
The big picture

- This talk: introduction and pep talk
- Three main motivations:
  - Understanding QCD (almost 40 years old now)
    \[
    \mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_q \bar{q} \gamma^\mu \left( \partial_\mu - \frac{i}{2} g_S \lambda^a G^a_\mu \right) q
    \]
    \[
    G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - i g_S f^{abc} G^b_\nu G^c_\mu
    \]
    - Determining standard model parameters precisely
    - Testing/finding effects beyond the standard model
- Some examples of all will be mentioned
- Understanding QCD needed for the other two
- I will not talk about $D$, $B$, $J/\psi$, $\Upsilon$, …
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Mathematical equations:

\[ \mathcal{L}_{QCD} = -\frac{1}{4} G_{\mu \nu}^a G^{a \mu \nu} + \sum_q \bar{q} \gamma^\mu \left( \partial_\mu - \frac{i}{2} g_S \lambda^a G^{a}_\mu \right) q \]

\[ G_{\mu \nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - ig_S f^{abc} G^b_\nu G^c_\mu \]
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Mesons are \textit{simple}

- Made of a quark and anti-quark
- So these we should really understand in detail
- In order of difficulty:
  - Static properties: mass, . . .
  - Dynamic properties: formfactors
  - Dynamic properties: decays and scattering
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- quark-antiquark
- add gluons
- Formfactor
- Two body decay
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Which mesons?

- Pseudo-scalar octet (nonet)
  - $\pi^\pm$, $\pi^0$
  - $K^\pm$, $K^0$, $\bar{K}^0$
  - $\eta(, \eta')$

- Vector nonet
  - $\rho^0$, $\rho^\pm$, $\omega$
  - $K^*$, $\phi$

- Scalars
  - $f_0(500)$ or $\sigma$, $f_0(980)$, $a_0(980)$, $K_0(800)$ or $\kappa$

- Axial-vectors
  - $a_1(1260)$, $f_1(1285)$, $b_1(1235)$, $K_1(1270)$, $K_1(1400)$, …

- Others
Theoretical tools: some general comments

Theory
- Start from first principles
- In principle improvable to any precision
- But always beware of assumptions, approximations, . . .
- An uncontrolled approximation turns a theory into a model
- Sometimes the theory has many free parameters (or functions)
- Example: Chiral Perturbation Theory or perturbative QCD

Models
- Needed whenever cannot be done from the full theory
- Can be useful to summarize/understand results
- Can be a first step towards finding a good theory
- Example: Nambu-Jona-Lasinio model

In between
- Typically a theory with uncontrolled approximations
- Requires a certain finesse to get reliable results
- Example: Schwinger-Dyson equations
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Theoretical tools: QCD only

Perturbative QCD: not directly useful but input as constraints on other methods

- Structure functions
- Light-cone wave functions
- Formfactors at large $Q^2$
- ...
Theoretical tools: QCD only

Lattice gauge theory:
- Discretize space time and integrate out the quarks and gluons numerically
- From first principles (in principle)
- Extrapolations needed (see talks next week)
- Static properties: good
- Formfactors of stable particle: starting
- Decays and scattering of stable particles: difficult
- Resonances: very difficult
- Main reason for all the difficulties: everything is in Euclidean space (imaginary time)
- Why: Minkowski space: integrands are oscillatory: $\int dt \, e^{i\omega t}$ rather than $\int dt \, e^{-\omega t}$
Theoretical tools: QCD only

Light cone quantization  
[hep-ph/9705477]

- Only physical degrees of freedom (ie no ghosts . . . )
- Wave functions are expanded in Fock states: partons directly visible
- The perturbative vacuum is the physical vacuum
- In principle allows for a competing numerical nonperturbative method
- Was a very active field 1990s
- Main (unsolved) difficulty: dealing with the zero mode
  This is where all the trouble of spontaneous symmetry breaking and confinement hides in this approach

Note: this not quark models on the light cone
Why (light) meson decays are interesting

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Motivation

Theoretical tools

Pseudo-scalars

Vectors

Axial-vectors

Others

Conclusions

Theoretical tools: QCD only

QCD, Finite Energy Sum Rules, ...

- All rely on analyticity and Cauchy’s theorem

\[ \frac{1}{2\pi i} \oint_C dz f(z) = \sum_{\text{poles}} \text{residues} \]

- a typical curve

- Circle and residue points: perturbative QCD

- Axis: data and/or resonance saturation
Theoretical tools: other theory

Dispersion relations and unitarity

- Again Cauchy’s theorem
- But now choose $f(z)$ e.g. a decay or scattering amplitude
- $s, t, u$: moire parameters
- Unitarity $1 = S^\dagger S = 1 + T^\dagger T + i(T - T^\dagger)$
- Due to the cuts: phases provide constraints
- Integral equations for the amplitudes
- Questions: subtraction constants, experimental input for phases, asymptotic behaviour
Theoretical tools: other theory

Chiral Perturbation Theory
- see my talk tomorrow
- Well defined effective field theory
- Might or might not converge
- Often many parameters (Low-Energy-Constants)
- All (in principle) measurable
- In addition models can/must be used to estimate parameters
- State of the art: 2-loop (most needed processes done)
- For other than pseudo-scalars: quite limited
Theoretical tools: other theory

Schwinger-Dyson equations

- Idea ($\phi^3$): \[ \frac{\partial}{\partial \phi} \langle \phi \rangle = \frac{\partial}{\partial \phi} \langle \phi \rangle + \text{interaction terms} \]

- Full three-point function involves full four-point function
- Four involves five, \ldots
- An infinite set of consistency equations
- Need to truncate
- Need for a starting ansatz to make life bearable (usually a full gluon propagator)
- Usually kept at the “quenched” approximation
Models with “Quarks”

- Nonrelativistic constituent quark models: understanding the spectrum (fill up octets and nonets)
- Chiral quark model: quarks plus pseudo-scalars, no confinement
- Nambu-Jona-Lasinio models: Quarks with a four quark interaction
  - Has spontaneous chiral symmetry breaking
  - Produces a constituent quark mass from a gap equation:

\[
\begin{align*}
\text{Nambu-Jona-Lasinio model} & = \text{quarks} + \text{four quark interaction} \\
& = \text{mesons from a bubble sum} \\
& = \text{Mesons but no confinement}
\end{align*}
\]
Models with “Quarks”

- Add vector-like four quark interactions \( \rightarrow \) vectors and axial-vertex
- Add a ‘t Hooft vertex to get \( \eta' \) better (or a variation on that vertex)
- make the vertex non-local
- Add Polyakov loop
- Many more variations possible
- Usually large \( N_c \) or tree level at the “meson” level
- Some attempts to go beyond that: many difficulties and not clear if it ever yielded something useful
Effective Lagrangians

- Basic degrees of freedom: hadron fields
- Beware of product (mis)labelling
  - Chiral Perturbation Theory
  - Chiral Effective Theory
  - Are very popular names and “de vlag dekt niet altijd de lading” since they are not protected names (free flag doesn’t make free bottom)
- Note field redefinitions: same Lagrangian can look very different
- Hope: find a simple Lagrangian and then refine it
- A full classification attempt: Resonance chiral theory ($R\chi T$), also attempts to go to one-loop.
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\[ \pi^- \text{-decays} \]

- **Lightest hadrons**
  - \( \pi^0 \): main decay electromagnetic
    - \( \rightarrow \gamma\gamma \): test of the anomaly
    - \( \rightarrow e^+e^- \): existing discrepancy (KTeV) with standard model? extra contributions?
    - \( \rightarrow \gamma e^+e^- \): \( F_V \)
    - \( \rightarrow \bar{\nu}\nu \): looking for new stuff
  - \( \pi^+ \) (or \( \pi^- \) to the charge conjugate state)
    - main decay: weak leptonic
      - \( \rightarrow \mu^+\nu(\gamma) \): \( F_\pi \)
      - \( \rightarrow e^+\nu\gamma \): \( F_V \) and \( F_A \): CVC
      - \( \rightarrow e^+\nu \): lepton universality
      - \( \rightarrow \pi^0 e^+\nu \): \( V_{ud} \)
\[ \pi^0 \rightarrow e^+ e^- \]

- KTeV: \( BR = (7.48 \pm 0.29 \pm 0.25) \cdot 10^{-8} \)

- \( F_{\pi \gamma \gamma}(q_1^2, q_2^2) \) is an object we also like to know for \( g - 2 \) of muon.

- Unitarity: on-shell \( \gamma \gamma \) \( BR \geq (4.75 \pm 0.02) \cdot 10^{-8} \)

- Typical VMD \( 6.4 \cdot 10^{-8} \)

- ChPT: fix coefficient from \( \eta \rightarrow \mu^+ \mu^- \), compatible (sign ambiguity)

- Dorokhov-Ivanov \( BR = (6.2 \pm 0.1) \cdot 10^{-8} \) (3.3 \( \sigma \))

- (Reasonable?) assumption \( F_{\pi \gamma \gamma}(t, t) = 1/(1 + t/s_1) \)
Main decay: weak (nonleptonic, semileptonic, leptonic)
Interplay of weak and strong interaction
\( K \to \pi\pi: \Delta l = 1/2 \) enhancement
\( K \to 3\pi: \)
  - Dynamics: Dalitzplot distributions
  - \( CP \)-violation
  - \( \pi\pi \)-scattering lengths
\( K \to l\nu: \) Lepton universality, \( F_K \)
\( K \to \pi l\nu: \) \( V_{us} \), formfactors
\( K \to \pi\pi l\nu: \) formfactors (\( L - i^r \)), \( \pi\pi \)-scattering lengths
\( K \to \pi l^+l^-, K \to \pi\nu\nu: \) get \( V_{CKM} \) fully from \( K \) system, strong constraints on new physics models
$\eta$-decays

- $\eta \to 3\pi$: $m_u - m_d$ or $Q$: Lanz on Tuesday
- $\eta \to \gamma^*\gamma^*$: anomaly, formfactors
- $\eta \to \pi^0\gamma\gamma$ dominated by high orders in ChPT
- $\eta \to \pi^+\pi^-\gamma^*$: anomaly, formfactors
Vectors: Theoretical tools/questions

- Effective Lagrangians: does there exist a consistent power-counting?
- Is there a preferred way to describe vectors? Massive Yang-Mills, Hidden Local Symmetry, Antisymmetric tensor, . . . ?
- Does there exist something like Vector Meson Dominance beyond the pion electromagnetic formfactor?
- Large $N_c$ says that a tree level Lagrangian should exist, but not that it must be a simple one: how well does this really work?
- Lattice has difficulties with resonances, can be done but not very accurate at present (if quark masses such that resonance is above threshold)
- For static properties and decay constants ChPT can be done
- KSRF relation between $m_V$, $g_V$ and $f_\pi$
Examples of decays

- \( \omega \rightarrow 3\pi \)
  - Can this be modelled via \( \omega \rightarrow \rho \pi \rightarrow 3\pi \)?
- \( \rho-\omega-\phi \) mixing
- \( \rho^0-\rho^+ \) mass difference: \( e^+e^- \) and \( \tau \), also in other production modes?
- Radiative decays
  - \( \rho, \omega \rightarrow \pi \gamma, \phi \rightarrow \eta \gamma, \eta' \rightarrow \rho \gamma \eta' \rightarrow \omega \gamma \)
  - Same but with the photon off-shell, i.e. to \( e^+e^-, \mu^+\mu^- \)
  - Are these related via (naive) VMD (no from NA60)
\[ \omega \rightarrow \pi^0 \mu^+ \mu^- \]

\[ F_\omega = \frac{1}{1 - m_{\mu\mu}^2/\Lambda^2_\omega} \]

\[ F_{\omega}^{\text{exp}} \neq \frac{1}{1 - m_{\mu\mu}^2/m_V^2} \]

Terschlüsen-Leupold

\[ F_{\omega}^{\text{exp}} = \frac{1 + m_{\mu\mu}^2/m_V^2}{1 - m_{\mu\mu}^2/m_V^2} \]
\[ \omega \rightarrow \pi^0 \mu^+ \mu^- \]

corresponding differential decay rate:

Terschlüsen-Leupold achieve this with lowest order terms in the antisymmetric tensor representation, other representations need higher derivative interactions.
Axial vectors

- Are they the chiral partners of the $\rho$?
- How do the axial nonets mix in the strange and singlet sectors?
  Relevant question for muon $g - 2$
- $a_1 \rightarrow 3\pi$ via $a_1 \rightarrow \rho\pi$ or additional contributions
- $a_1 \rightarrow \pi\gamma$: this vanishes in almost simple theoretical models
- Mass: Weinberg sum rules predict $m_{a_1} \approx \sqrt{2}m_{\rho}$
- Width (PDG) 250 to 600 MeV our estimate
- is there a problem with continuum/resonance separation due to the large width?
Many more resonances

Questions similar:
- Does it really exist
- mass
- width
- Main decay modes
- Mixing with other particles

On the theory side
- nature of the particle
- What is the best way to describe it
- If used in resonance saturation models we need its couplings
An overview of theoretical tools
A few examples of questions and decays