Electromagnetic Structure Functions and Neutrino Nucleon Scattering

Hallsie Reno
University of Iowa
mary-hall-ren@uiowa.edu

May 5, 2006
Introduction

- It has been known for a long time that in the few GeV energy region, the quasi-elastic, few pion and inclusive contributions to the cross section are nearly equal. Lipari, Lusignoli and Sartogo, 1995 made the standard plot.

- All components important to understand neutrino oscillation experiments, the balance of which depends on e.g., the minimum invariant mass of the final hadronic state, $W_{\text{min}}^2$. Recent work by Kuzmin, Lyubushkin, Naumov, hep-ph/0511308 attempts to find the $W_{\text{min}}$ so that the components best represent current neutrino measurements.

- The inelastic component is not currently well calculated in this energy regime because of the necessity of low-$Q^2$ structure functions.
This talk is about extrapolations to low-$Q^2$ of structure functions for $W^2 > W_{\text{min}}^2$.

I’ll assume local quark-hadron duality.

Plan

- Brief review neutrino scattering in NLO QCD with target mass corrections (TMC) and the importance of the low-$Q^2$ contribution to the cross section.

- Comparison of NLO+TMC with a parameterization of $F_{2}^{ep}$. (NLO+TMC overestimates $F_{2}$ at low $Q^2$.)

- The Capella, Kaidalov, Merino and Thanh Van (CKMT) parameterization of $F_{2}^{ep}$ and the Bodek-Yang-Park parameterization.

- The translation to $\nu N$ scattering.
• Reevaluated cross sections with these two extrapolations at low-$Q^2$.

• Summary.
Mass Corrections

Differential cross section (CC) \( m = \) muon mass, \( M = \) nucleon mass:

\[
\frac{d\sigma}{dx\ dy} = \frac{G_F^2 ME}{\pi(1 + Q^2/M_W^2)^2} \left[ \left( xy^2 + \frac{m^2 y}{2ME} \right) F_1^{TMC} 
+ \left( 1 - \frac{m^2}{4E^2} - y - \frac{M xy}{2E} \right) F_2^{TMC} + \left( xy - \frac{xy^2}{2} - \frac{m^2 y}{4ME} \right) F_3^{TMC} 
+ \left( \frac{m^2 y}{2ME} + \frac{m^4}{4M^2 E^2 x} \right) F_4^{TMC} - \frac{m^2}{ME} F_5^{TMC} \right]
\]
TMC corrections come from:

- $x \rightarrow \xi$ with
  \[
  \frac{1}{\xi} = \frac{1}{2x} + \sqrt{\frac{1}{4x^2} + \frac{M^2}{Q^2}} \quad \iff \quad \xi = \frac{2x}{1 + \sqrt{1 + \frac{4M^2x^2}{Q^2}}}
  \]

- A “mismatch” between quark momentum $p$ and nucleon momentum $P$: proton momentum $P^2 = M^2$ and incident parton momentum $p^2 = 0$, then $p^+ = \xi P^+$, but $p^- \neq \xi P^-$. 

- Including non-collinear partons in the nucleon, with $k_T < M$. R.K. Ellis et al.
With the identifications:

\[ \rho^2 = 1 + \frac{4M^2x^2}{Q^2} \]

\[ F_2 = q(\xi, Q^2) + \bar{q}(\xi, Q^2) \]

Georgi and Politzer [PRD 14 (1976)], Barbieri et al., and Georgi, Politzer and deRujula [Ann. Phys. 103 (1977), where $2xF_1 = F_2$ is not assumed]. The results, for example, for $F_2$:

\[ F_{2}^{TM C}(x, Q^2) = 2 \frac{x^2 F_2(\xi, Q^2)}{\rho^3} \xi + 12 \frac{M^2 Q^2}{x^3} \frac{x^2}{\rho^4} \int_\xi^1 d\xi' \frac{F_2(\xi', Q^2)}{\xi'} \]

\[ + 24 \frac{M^4 Q^4}{x^4} \frac{x^2}{\rho^5} \int_\xi^1 d\xi' \int_{\xi'}^1 \frac{F_2(\xi'', Q^2)}{\xi''} \]

\[ + 24 \frac{M^4 Q^4}{x^4} \frac{x^2}{\rho^5} \int_\xi^1 d\xi' \int_{\xi'}^1 \frac{F_2(\xi'', Q^2)}{\xi''} \]

\[ + \cdots \]
DIS CC cross sections

- Neutrino-nucleon CC cross section for $Q^2 > Q_{\text{min}}^2$ normalized to the $\nu N$ cross section.
- Calculated using NLO+TMC.
- Half the cross section comes from $Q^2 < 1 \text{ GeV}^2$. 

$\sigma_{\text{CC}}(Q_{\text{min}}^2)/\sigma_{\text{CC}}$ vs. $Q_{\text{min}}^2/\text{GeV}^2$

- $W_{\text{min}} = M + m_\pi$
- $E_\nu = 5 \text{ GeV}$
- $W_{\text{min}} = 1.4 \text{ GeV}$
What values of $x$?

- $W^2 = Q^2(\frac{1}{x} - 1) + M^2$
- $x > \frac{Q^2}{(2ME_\nu)}$
\[ F_2^{ep}, \ Q^2 = 4 \ GeV^2 \]

Use the Abramowicz, Levin, Levy and Maor (ALLM) parameterization (solid) of \( F_2 \) represent \( ep \) data. ALLM, Phys. Lett. 1991, AL hep-ph/9712415. This has 23 parameters.

Also shown, NLO+TMC and NNLO+TMC and SLAC data for \( Q^2 = 3.7 - 4.3 \ GeV^2 \). L. Whitlow et al., Phys. Lett. B (1990).
\[ F_{2e}^{ep}, \quad Q^2 = 0.5 \text{ GeV}^2 \]

ALLM (solid), data from E665M. Adams et al., Phys. Rev. D 54 (1996) with \( Q^2 = 0.43, \ 0.59 \text{ GeV}^2 \) NLO+TMC, NNLO+TMC.
Capella, Kaidalov, Merino and Thanh Van


\[ F_2(x, Q^2) = F_2^{\text{sea}}(x, Q^2) + F_2^{\text{val}}(x, Q^2) \]

\[ = Ax^{-\Delta(Q^2)}(1 - x)^n(Q^2) + 4 \left( \frac{Q^2}{Q^2 + a} \right)^{1+\Delta(Q^2)} \]

\[ + Bx^{1-\alpha_R}(1 - x)^n(Q^2) \left( \frac{Q^2}{Q^2 + b} \right)^{\alpha_R} \]

\[ \times \left( 1 + f(1 - x) \right) \]
CKMT Valence in $ep$ scattering

CKMT fit $\alpha_R = 0.4250$ and $b = 0.6452$ GeV$^2$.

$$F_2^{val}(x, Q^2) = B x^{1-\alpha_R} (1 - x)^{n(Q^2)} \left( \frac{Q^2}{Q^2 + b} \right)^{\alpha_R} \left( 1 + f(1 - x) \right)$$

$B = B_u$ is calculated to be 1.2064, $f = B_d/B_u = 0.15$ is also calculated. They are calculated invoking valence counting rules at $Q^2 = 2$ GeV$^2$. Also fit is $c = 3.5489$ GeV$^2$ in

$$n(Q^2) = \frac{3}{2} \left( 1 + \frac{Q^2}{Q^2 + c} \right)$$
CKMT “Sea” in $e^p$ scattering

CKMT fit $A = 0.1502$ and $a = 0.2631$ GeV$^2$.

$$F_{2}^{sea}(x, Q^2) = Ax^{-\Delta(Q^2)}(1 - x)^{n(Q^2)+4}\left(\frac{Q^2}{Q^2 + a}\right)^{1+\Delta(Q^2)}$$

Also fit is $\Delta_0 = 0.07684$ and $d = 1.1170$ GeV$^2$ in

$$\Delta(Q^2) = \Delta_0 \left(1 + \frac{2Q^2}{Q^2 + d}\right)$$

$\Delta_0$ is similar to power law in generalized vector meson dominance at low $Q^2$, where it is pomeron dominated.
Comparison: ALLM and CKMT in $ep$ scattering

ALLM (solid), and CKMT (dashed).
CKMT in $\nu N$ scattering

See CKMT Moriond Proceedings.

- $F^\text{sea}_2$ changes only overall normalization: $A \rightarrow A_\nu = 0.60$, which I fixed at $Q^2 = 10$ GeV$^2$ to match reasonably well with the NLO+TMC evaluation.

- Note: $A_\nu/A \simeq 4$ in sea part. For electromagnetic case

\[
\frac{1}{9}x(d + \bar{d} + s + \bar{s}) + \frac{4}{9}x(u + \bar{u}) \simeq \left(\frac{3}{9} + \frac{8}{9}\right)xq_{\text{sea}} = \frac{11}{9}xq_{\text{sea}}
\]

For CC case, with $\bar{u} = \bar{d} \simeq 2s = q_{\text{sea}}$,

\[
2x(d + s + \bar{u}) \simeq 5xq_{\text{sea}}
\]
CKMT in $\nu N$ scattering

- Expect that the underlying non-perturbative process is governed by the same $\Delta(Q^2)$ and form factor $\left(\frac{Q^2}{Q^2 + a}\right)^{1+\Delta}$.

- For the valence part, recalculate $B$ and $f$ at $Q^2 = 2$ GeV$^2$. I get

$$B_\nu = 2.695 \quad f_\nu = 0.595$$

- Valence $x$ and $Q^2$ dependence shouldn’t change between electromagnetic and charged current scattering.
CKMT for $F_1$

For $F_1$, use

$$R = \frac{F_2}{2xF_1} \left( 1 + \frac{4M^2x^2}{Q^2} \right) - 1$$

with a parameterization of $R$ from Whitlow et al., Phys. Lett. 1990. Below $Q^2 = 0.3$ GeV$^2$, rescale the value at 0.3 GeV$^2$ by $Q^2/(0.3$ GeV$^2)$.
CKMT for $F_3$

For $F_3$, use

$$F_3(x, Q^2) = \left[ \frac{A_\nu}{15} x^{-\Delta(Q^2)} (1 - x)^n(Q^2) + 4 \left( \frac{Q^2}{Q^2 + a} \right)^{1+\Delta(Q^2)} \right]$$

$$+ \quad B_\nu x^{1-\alpha_R}(1 - x)^n(Q^2) \left( \frac{Q^2}{Q^2 + b} \right)^{\alpha_R}$$

$$\times \quad \left( 1 + f_\nu(1 - x) \right) \right] \times (1.1x) \right].$$

- The denominator of 1.1 adjusts the integral of the valence part to give a Gross-Llewellyn-Smith sum rule result of $3 \times 0.9$ (QCD corrected).
• The normalization of the sea term is a little ad-hoc. It should be the $s$ quark contribution. The choice above is not bad in comparison to NLO+TMC at $Q^2 = 4 \text{ GeV}^2$. (I did not try to fine tune this parameter.)
Comparison: BYP and CKMT in $\nu N$ scattering

Bodek-Yang-Park (BYP) (solid), and CKMT (dashed).
Strategy for cross sections

- Use NLO+TMC in for $Q^2 > Q_0^2$. Attach a parameterization for $Q^2 < Q_0^2$. Should be insensitive to $Q_0^2$.

- Results shown for $Q_0^2 = 4$ GeV$^2$. 
\( \nu N \) CC cross section

- Solid lines, \( W_{\text{min}}^2 = 4 \text{ GeV}^2 \), dashed lines for \( W_{\text{min}}^2 = 2 \text{ GeV}^2 \).

- Upper solid and dashed are NLO+TMC, lower two are CKMT and BYP extrapolations below \( Q_0^2 \).

- Dotted lines show LO+TMC.
$\bar{\nu}N$ CC cross section

- Solid lines, $W_{\text{min}}^2 = 4$ GeV$^2$, dashed lines for $W_{\text{min}}^2 = 2$ GeV$^2$.
- Upper solid and dashed are NLO+TMC, lower two are CKMT and BYP extrapolations below $Q_0^2$.
- Dotted lines show LO+TMC.
Summary

- The CKMT and BYP extrapolations yield similar results on the cross sections. CKMT is slightly larger.

- The neutrino cross section is reduced by 7-8% for $W_{\text{min}}^2 = 2$ GeV$^2$ at 10 GeV, 11-13% at 5 GeV, relative to the NLO+TMC result.

- Antineutrino scattering is impacted more, with changes of order 20% at 10 GeV.

- CKMT parameterization has a simple interpretation. One can rescale the standard sea and valence PDFs by the same $Q^2$ dependent factors in the CKMT parameterization and get essentially the same results.
- Calculate $F_i(x, Q^2)$ using NLO+TMC above $Q_0^2$.
- For $Q^2 < Q_0^2$, rescale the separate sea and valence components of $F_i(x, Q_0^2)$ by e.g. $F_2^{sea}(x, Q^2)/F_2^{sea}(x, Q_0^2)$.

• I look forward to more measurements of neutrino structure functions and cross sections!