Self-consistency of the Dressed Electromagnetic Nucleon Current

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Language

Whenever I say “photoprocess,” the photon may be real or virtual. The formalism to be presented here applies to either case.

How to Read the Diagrams

Time runs from right to left in all diagrams, i.e., the same direction as in matrix elements:

$$\langle \text{final} | (\text{some operator}) | \text{initial} \rangle$$

$$\Leftarrow \quad \text{time} \quad \Leftarrow$$
Introduction

\[ \gamma N \rightarrow \pi N \]

\[ \gamma N \rightarrow \pi \pi N \]

\[ NN \rightarrow NN\gamma \]

\[ \gamma N \rightarrow \gamma N \]
What is the common feature of these photo reactions?
Introduction

\[ \gamma N \rightarrow \pi N \]

\[ \gamma N \rightarrow \pi\pi N \]

\[ \gamma N \rightarrow \gamma N \]

\[ N N \rightarrow N N \gamma \]

e.m. nucleon current
Electromagnetic Current $J^\mu$ of the Nucleon

How does one describe the current in a Lorentz-covariant Bethe-Salpeter-type approach?
Electromagnetic Current $J^\mu$ of the Nucleon

How does one describe the current in a Lorentz-covariant Bethe-Salpeter-type approach?

- The most general Lorentz-covariant structure of $J^\mu$ requires **12 form factors**.
- Applying gauge invariance, this reduces to **8 form factors**.
- Applying time-reversal invariance, this reduces further to **6 form factors**.
Electromagnetic Current $J^\mu$ of the Nucleon

How does one describe the current in a Lorentz-covariant Bethe-Salpeter-type approach?

- The most general Lorentz-covariant structure of $J^\mu$ requires 12 form factors.
- Applying gauge invariance, this reduces to 8 form factors.
- Applying time-reversal invariance, this reduces further to 6 form factors: $F_1, F_2, f_1, f_2, g_1, g_2$

$$J^\mu(p', p) = e \left[ \delta_N \gamma^\mu + \delta_N \gamma_T^\mu (F_1 - 1) + \frac{i \sigma^{\mu\nu} k_\nu}{2m} \kappa_N F_2 ight.$$  
$$\left. + \frac{S^{-1}(p')}{2m} \left( \gamma_T^\mu f_1 + \frac{i \sigma^{\mu\nu} k_\nu}{2m} \kappa_N f_2 \right) + \left( \gamma_T^\mu f_1 + \frac{i \sigma^{\mu\nu} k_\nu}{2m} \kappa_N f_2 \right) S^{-1}(p) \frac{S^{-1}(p)}{2m} \right]$$

Constraints:
- No kinematic singularity: $f_1(k_2) \rightarrow 0$ and $g_1(k_2) \rightarrow 0$
- Chiral-symmetry limit: $f_1 \rightarrow g_A - G_A(k_2)$ and $f_2 \rightarrow \frac{1}{2}$

(Approximation)
Electromagnetic Current $J^\mu$ of the Nucleon

How does one describe the current in a Lorentz-covariant Bethe-Salpeter-type approach?

- The most general Lorentz-covariant structure of $J^\mu$ requires **12 form factors**.
- Applying gauge invariance, this reduces to **8 form factors**.
- Applying time-reversal invariance, this reduces further to **6 form factors**:

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$$+ \frac{S^{-1}(p')}{2m} \left( \gamma_T^\mu f_1 + \frac{i \sigma^{\mu\nu} k_\nu}{2m} \kappa_N f_2 \right) + \left( \gamma_T^\mu f_1 + \frac{i \sigma^{\mu\nu} k_\nu}{2m} \kappa_N f_2 \right) \frac{S^{-1}(p)}{2m} \right.$$  
$$+ \frac{S^{-1}(p')}{2m} \left( \gamma_T^\mu g_1 + \frac{i \sigma^{\mu\nu} k_\nu}{2m} \kappa_N g_2 \right) \frac{S^{-1}(p)}{2m} \right]$$

(Approximation)

**Constraints:**

- no kinematic singularity: $f_1(k^2) \xrightarrow{k^2=0} 0$ and $g_1(k^2) \xrightarrow{k^2=0} 0$
- chiral-symmetry limit: $f_1 \rightarrow \frac{g_A - G_A(k^2)}{g_A}$ and $f_2 \rightarrow 1$
Implications of off-shell structure: Pion photoproduction

**s-channel:**

\[ F_s S(p + k) J_i^\mu (p + k, p) = F_s S(p + k) \left( e\delta_i^\gamma \gamma^\mu + \frac{i\sigma^{\mu\nu} k_\nu e\kappa_i}{2m} \right) + F_s \frac{i\sigma^{\mu\nu} k_\nu e\kappa_i}{2m} \frac{f_{2i}}{2m} \]

**u-channel:**

\[ J_f^\mu (p', p' - k) S(p' - k) F_u = \left( e\delta_f^\gamma \gamma^\mu + \frac{i\sigma^{\mu\nu} k_\nu e\kappa_f}{2m} \right) S(p' - k) F_u + \frac{i\sigma^{\mu\nu} k_\nu e\kappa_f}{2m} \frac{f_{2f}}{2m} F_u \]

**contact terms**
Electromagnetic Current $J^\mu$ of the Nucleon

- Photoprocesses, in general, require a more detailed description of $J^\mu$.

- The dynamical structures of the current $J^\mu$ can be determined by requiring self-consistency.
Introduction

\[ \gamma N \rightarrow \pi N \]

\[ \gamma N \rightarrow \pi\pi N \]

\[ \gamma N \rightarrow \gamma N \]

e.m. nucleon current

\[ NN \rightarrow NN \gamma \]
Dynamical Links between Photoprocesses

\[ \gamma N \rightarrow \pi N \]

\[ \gamma N \rightarrow \pi \pi N \]

e.m. nucleon current

\[ NN \rightarrow NN\gamma \]
Pions, Nucleons, and Photons

\[ T(a) = \pi \rho + X \]

\[ T(b) = V + V T \]

\[ T(c) = U + U \]

\[ T(d) = U + U X \]

\[ T(e) = \cdots \]

- **\( \pi N T \) matrix**
  - \( T(a) = \pi \rho + X \)
  - \( T(b) = V + V T \)
  - \( T(c) = U + U \)
  - \( T(d) = U + U X \)
  - \( T(e) = \cdots \)

- **dressed nucleon propagator**
  - \( = \)
  - \( = \)

- **dressed \( \pi NN \) vertex**
  - \( = \)
  - \( = \)

- **Tower of nonlinear Dyson-Schwinger-type equations**
**Nucleon Current** $J^\mu$

\[
\begin{align*}
\text{(a)} & \quad \quad = \quad + \\
\text{(b)} & \quad \quad = \quad + 
\end{align*}
\]

- **Couple photon to dressed propagator:**

\[
\begin{align*}
\text{(a)} & \quad \quad = \quad + \\
\text{(b)} & \quad \quad = \quad + 
\end{align*}
\]

- **Tower of nonlinear Dyson-Schwinger-type equations**
Pion Photoproduction

Pion-production current $M^\mu$:

\[ M = b + X b \] (a)

\[ b = U \] (b)

Nucleon current $J^\mu$:

\[ \Rightarrow \text{The internal structures of the dressed nucleon current can be understood by the dynamics of the pion production current.} \]

Tower of *nonlinear* Dyson-Schwinger-type equations
Rewriting the Production Current

**Pion-production current** $M^\mu$:

(a) $M = \epsilon + B + X B_T$

(b) $M = \epsilon + B + T B_T$

(c) $B = \epsilon + U + U_L + U_R$

**Contact-type current** $M^\mu_c$:

$M^\mu_c = \epsilon + U + U_L + U_R$

**Tower of nonlinear Dyson-Schwinger-type equations**
Rewriting the Production Current

- **Pion-production current** $M^\mu$: 
  \[
  \begin{align*}
  M^\mu &= B + B + X B_T & (a) \\
  M^\mu &= B + B + T B_T & (b) \\
  B &= \text{(irrelevant for gauge invariance)} 
  \end{align*}
\]

- **Contact-type current** $M_c^\mu$: 
  \[
  \begin{align*}
  \text{partial integral equation} \\
  \end{align*}
\]

- **Tower of nonlinear** Dyson-Schwinger-type equations
Pion-production current $M^\mu$:

\begin{align*}
M^\mu &= \begin{array}{c}
\text{Diagram (a)}
\end{array} + \begin{array}{c}
\text{Diagram (b)}
\end{array} + \begin{array}{c}
\text{Diagram (c)}
\end{array} \\
M^\mu &= \begin{array}{c}
\text{Diagram (a)}
\end{array} + \begin{array}{c}
\text{Diagram (b)}
\end{array} + \begin{array}{c}
\text{Diagram (c)}
\end{array} \\
B^\mu &= \begin{array}{c}
\text{Diagram (a)}
\end{array} + \begin{array}{c}
\text{Diagram (b)}
\end{array} + \begin{array}{c}
\text{Diagram (c)}
\end{array}
\end{align*}

Contact-type current $M_c^\mu$:

\begin{align*}
M_c^\mu &= \begin{array}{c}
\text{Diagram (a)}
\end{array} + \begin{array}{c}
\text{Diagram (b)}
\end{array} \\
M_c^\mu &= \begin{array}{c}
\text{Diagram (a)}
\end{array} + \begin{array}{c}
\text{Diagram (b)}
\end{array} \\
B^\mu &= \begin{array}{c}
\text{Diagram (a)}
\end{array} + \begin{array}{c}
\text{Diagram (b)}
\end{array}
\end{align*}

Tower of *nonlinear* Dyson-Schwinger-type equations
Nucleon Current $J^\mu$

$\begin{array}{c}
\begin{align*}
&= \quad \quad + \\
&= \quad \quad + \\
&= \quad \quad +
\end{align*}
\end{array}$

(a)

(b)

Tower of *nonlinear* Dyson-Schwinger-type equations
Nucleon Current $J^\mu$

\[ J^\mu = J^\mu_T + J^\mu_L \] (a) transverse

\[ J^\mu_s = J^\mu_{sT} + J^\mu_{sL} \] (b) longitudinal

- Tower of \textit{nonlinear} Dyson-Schwinger-type equations
Nucleon Current $J^\mu$

\[ J^\mu = \text{Diagram (a)} \]

\[ J^\mu_s = \text{Diagram (b)} \]

Gauge Invariance: Ward-Takahashi Identity (WTI)

\[ k_\mu J^\mu(p', p) = k_\mu J^\mu_s(p', p) = S^{-1}(p')Q_N - Q_NS^{-1}(p) \]

$S$: dressed nucleon propagator
Problems?

- Everything is exact!

- Everything is nonlinear!

- Everything is hideously complicated!
Everything is exact!

Everything is nonlinear!

Everything is hideously complicated!

But...
Let's cut the Gordian knot!

\[ M = B + X_B \] (a)

Do not use $X$. Work with full $T$.

\[ M = B + T_B \] (b)

\[ B = U + L \] (c)

\[ = \] (a)

\[ = + + \] (b)
Cutting the Gordian Knot


\[ \mathcal{M} = \mathcal{D} + \mathcal{X} B_T \] 
\[ \mathcal{M} = \mathcal{B} + \mathcal{T} B_T \] 
\[ \mathcal{B} = \mathcal{U} + \mathcal{L} \]

\[ J_\mu^S \]

not the full nucleon current

determine approximation by WTI for the nucleon current \( J_\mu \)
Cutting the Gordian Knot


(a)

(b)

(c)

determine approximation of $M_\mu^c$ by generalized WTI for the photoproduction current $M^\mu$

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Approximating $M_\mu$ for $M_\mu^\mu$

\begin{align*}
\text{(a)}
\begin{array}{c}
\text{M} \\
\end{array}
&= \begin{array}{c}
\text{B} \\
\end{array} + \begin{array}{c}
\text{X} \\
\end{array} + \begin{array}{c}
\text{B_T} \\
\end{array} \\
\end{align*}

\begin{align*}
\text{(b)}
\begin{array}{c}
\text{M} \\
\end{array}
&= \begin{array}{c}
\text{B} \\
\end{array} + \begin{array}{c}
\text{T} \\
\end{array} + \begin{array}{c}
\text{B_T} \\
\end{array} \\
\end{align*}

\begin{align*}
\text{(c)}
\begin{array}{c}
\text{B} \\
\end{array}
&= \begin{array}{c}
\text{B} \\
\end{array} + \begin{array}{c}
\text{B} \\
\end{array} + \begin{array}{c}
\text{B_T} \\
\end{array} \\
\end{align*}

\text{Lowest-order approximation in terms of phenomenological form factors:}

\begin{equation}
M_\mu^\mu = g e \gamma_5 \frac{i \sigma^{\mu\nu} k_\nu \kappa_N}{4m^2} - (1 - \lambda) g \frac{\gamma_5 \gamma^\mu}{2m} \tilde{F}_t e_\pi - G \chi \left[ e_i \frac{(2p + k)^\mu}{s - p^2} \left( \tilde{F}_s - \tilde{F} \right) + e_f \frac{(2p' - k)^\mu}{u - p'^2} \left( \tilde{F}_u - \tilde{F} \right) + e_\pi \frac{(2q - k)^\mu}{t - q^2} \left( \tilde{F}_t - \tilde{F} \right) \right]
\end{equation}

Don’t try to read the details. What is important is that this is a simple expression, easy to evaluate, and that it helps preserve gauge invariance of the entire production current.
Approximating $J_s^\mu$

\[ J_s^\mu(p', p) = (p' + p)^\mu \frac{S^{-1}(p')Q_N - Q_NS^{-1}(p)}{p'^2 - p^2} + \left[ \gamma^\mu - \frac{(p' + p)^\mu}{p'^2 - p^2} k \right] Q_N A(p'^2) + A(p^2) \]

- **Approximate $J_s^\mu$ by the minimal current that reproduces the WTI:**

  \[ S(p) = \frac{1}{\hat{p} + \frac{k}{2} - m^2} j_1^\mu + \frac{2m}{s^2} j_2^\mu \]

- **Half on-shell:**

  \[ SJ_s^\mu u = \left( \frac{1}{\hat{p} + \frac{k}{2} - m^2} j_1^\mu + \frac{2m}{s^2} j_2^\mu \right) Q_N u(p) \quad \text{with} \quad s = (p + k)^2 \]

- **Auxiliary currents:**

  \[ j_1^\mu = \gamma^\mu (1 - \kappa_1) + \frac{i\sigma^{\mu\nu}k_\nu}{2m} \kappa_1 \quad j_2^\mu = \frac{(2p + k)^\mu}{2m} \kappa_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m} \kappa_2 \]

Two parameters!
Does it work? — Yes!

- Preliminary results for $\gamma N \rightarrow \pi N$

$\gamma p \rightarrow \pi^+ n$

$\gamma p \rightarrow \pi^0 p$

$\gamma n \rightarrow \pi^- p$

On the importance of maintaining gauge invariance

Preliminary results for $\gamma N \rightarrow \pi N$:

Dashed green curves: w/o $M_c^\mu$

Dynamical Links between Photoprocesses — Bremsstrahlung

\[ \gamma N \rightarrow \pi N \]

\[ \pi N \rightarrow \pi \pi N \]

Bremsstrahlung

\[ N N \rightarrow N N \gamma \]
Bremsstrahlung \( NN \rightarrow NN\gamma \)

**Bremsstrahlung Current:**

\[
J_B^\mu = (TG_0 + 1) J_T^\mu (1 + G_0 T)
\]

\( T: \) NN \( T \)-matrix

\[J_T = \begin{align*}
\text{transverse} + d\mu G_0 V + VG_0 d\mu &+ V^\mu + J_T^\mu \end{align*}\]

**Compare the photon processes along the top nucleon line above to the meson production diagrams below.**

\[\text{Essential parts of the process can be described as a meson capture process — i.e., as an inverse photoproduction process — in the presence of a spectator nucleon.}\]
Application to KVI data. — Or: Resolving a longstanding problem:

Inclusion of the four-point interaction current from meson photoproduction brings about a dramatic improvement.
Dynamical Links between Photoprocesses — Two-Pion Production

\[ \gamma N \rightarrow \pi N \]

\[ \gamma N \rightarrow \pi\pi N \]

\[ \gamma N \rightarrow \gamma N \]

\[ NN \rightarrow NN\gamma \]

Two-Pion Production

\[ e.m. \text{ nucleon current} \]
Basic Two-pion Production Mechanisms

(a)  

(b)  

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Dynamical Links between Photoprocesses — Compton Scattering

\[ \gamma N \rightarrow \pi N \]

\[ \gamma N \rightarrow \pi\pi N \]

\[ NN \rightarrow NN\gamma \]

Compton Scattering

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37
Compton Scattering $\gamma N \rightarrow \gamma N$

\[
\begin{align*}
  k' \mu & \quad k' \nu \\
  p' & \quad p
\end{align*}
\]

\[
= \quad \text{\textbullet} \quad \text{\textbullet} + \quad \text{\textbullet} \quad \text{\textbullet} + \quad \text{\textbullet} \quad \text{\textbullet}
\]

- $s$- and $u$-channel terms employ dressed current just described.
- Contact term constrained by gauge invariance.
Conclusions

- There exists a very close relationship between the dressed nucleon current and the pion photoproduction current.
- Exploiting this relationship suggests physically meaningful approximations that work, despite the enormous complexity of the exact formalism.
- Maintaining full gauge invariance (as opposed to mere current conservation) is not a luxury but a necessity for the correct microscopic description of the reaction dynamics.
- Requiring gauge invariance in the form of off-shell (generalized) Ward-Takahashi identities for each subprocess provides a powerful tool for constraining the contributing mechanisms and ensuring overall gauge invariance as a matter of course.

Case in point:
Conclusions

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Case in point:

Thank you!