Axial properties of N and N* resonances

Ki-Seok Choi, W. Plessas, and R.F. Wagenbrunn

Karl-Franzens-Universität Graz, Austria
Weak Transitions

Decay parameter of n

\[ \lambda \equiv \frac{g_A}{g_V} = -1.2694 \pm 0.0028 \]

Particle Data Group (PDG) 2010

How about resonant states N*?

Recently, attracted much interest in studies of the strong interactions (QCD).
Chiral Restoration Scenario of QCD

Motivation

Formalism

Results

Summary
Chiral Restoration Scenario of QCD

Chiral doublets
Chiral Restoration Scenario of QCD

Goldberger-Treiman Relation

\[ g_{\pi NN} = \frac{g_A M_N}{f_\pi} \]

- **Axial charge**
- **Nucleon mass**
- **Pion decay constant**
- **Pion Nucleon coupling**
Chiral Restoration Scenario of QCD

Motivation

Formalism

Results

Summary

Chiral doublets

Restoration of the chiral symmetry:

\[ g_{\pi NN} \simeq 0 \]

Goldberger-Treiman Relation

\[ g_{\pi NN} = \frac{g_A M_N}{f_\pi} \]

Axial charge

Nucleon mass

Pion Nucleon coupling

Pion decay constant

Chiral Restoration Scenario of QCD

Motivation

Chiral doublets

Restoration of the chiral symmetry:

\[ g_{\pi NN} \approx 0 \]

Requirement for restoration:

\[ g_{A} \approx 0 \]

Goldberger-Treiman Relation

\[ g_{\pi NN} = \frac{g_{A} M_{N}}{f_{\pi}} \]

Analytic and Lattice Results

Non-relativistic SU(6) wave function

\[ \frac{1^+}{2}, \, N(939) : \quad G^A = \frac{5}{3} \quad \sim 1.66 \]

\[ \frac{1^+}{2}, \, N(1440) : \quad G^A = \frac{5}{3} \quad \sim 1.66 \]

\[ \frac{1^+}{2}, \, N(1710) : \quad G^A = \frac{1}{3} \quad \sim 0.33 \]

\[ \frac{1^-}{2}, \, N(1535) : \quad G^A = -\frac{1}{9} \quad \sim -0.11 \quad \text{Small} \]

\[ \frac{1^-}{2}, \, N(1650) : \quad G^A = \frac{5}{9} \quad \sim 0.55 \]

Lattice results

- for N(939): \(\sim 1.26\)
- for N(1650): \(\sim 0.55\)
- for N(1535): \(\sim 0.0\)


Relativistic Constituent Quark Model

- **Framework: Relativistic Quantum Mechanics (RQM)**
  
i.e. quantum theory respecting Poincaré invariance
  (theory on $\mathcal{H}$ corresponding to a finite number of particles, not a field theory)

- **Invariant mass operator**

  \[
  \hat{M} = \hat{M}_{\text{free}} + \hat{M}_{\text{int}} = \sqrt{\hat{H}_0^2 - \hat{P}_{\text{free}}^2} + \sum_{i<j} \left[ \hat{V}_{ij}^{\text{conf}} + \hat{V}_{ij}^{\text{hf}} \right]
  \]

  Eigenvalue equation: \( \hat{M} | P, J, \Sigma \rangle = m | P, J, \Sigma \rangle \)

  Relativistic kinetic energy: \( H_0 = \sum_{i=1}^{3} \sqrt{p_i^2 + m_i^2} \)

  Confinement: \( \hat{V}_{ij}^{\text{conf}} = V_0 + Cr_{ij} \)

  Hyperfine interaction \( \hat{V}_{ij}^{\text{hf}} : \) OGE, psGBE, EGBE
Axial Form Factors

Matrix elements of the axial current operator

\[ \langle P, \frac{1}{2}, \Sigma' | \hat{A}_a^\mu | P, \frac{1}{2}, \Sigma \rangle = \bar{U}(P, \Sigma') g_A \gamma^\mu \gamma_5 \frac{\tau_a}{2} U(P, \Sigma) \]

\[ \mathcal{J}_A : \text{Axial charge of } N \text{ and } N^* \]

Incoming / Outgoing baryon states: \( |V, M, J, \Sigma\rangle = |P, J, \Sigma\rangle \)

\[ \langle V, M, J, \Sigma | \hat{A}_a^\mu | V, M, J, \Sigma \rangle = \frac{2}{MM'} \sum_{\sigma_i} \sum_{\sigma'_i} \sum_{\mu_i, \mu'_i} \int d^3 \vec{k}_2 d^3 \vec{k}_3 d^3 \vec{k}'_2 d^3 \vec{k}'_3 \]

\[ \times \frac{\left( \sum_i \omega_i' \right)^3}{\prod_i 2 \omega_i} \prod_{\sigma'_i} D_{\sigma_i \mu_i}^* \left\{ R_W[k'_i; B(V')] \right\} \Psi_{M', J', \Sigma'}^* \left( \vec{k}'_1, \vec{k}'_2, \vec{k}'_3; \mu'_1, \mu'_2, \mu'_3 \right) \langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{A}_{a, rd}^\mu | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle \]

where \( p_i = B_c(V) k_i, \ p'_i = B_c(V') k'_i \)

and \( \omega_i = \sqrt{k_i^2 + m_i^2} \)
Point Form Spectator Model

- **Point form spectator model**

\[
\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{O}_{rd} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle = \bar{u}(p'_1, \sigma'_1) \left[ g_A^q \gamma^\mu + \frac{2f_\pi}{Q^2 + m_\pi^2} g_{qq \pi} \tilde{q}^\mu \right] \gamma_5 \frac{\tau_a}{2} u(p_1, \sigma_1)
\]

- **Axial current**

\[
\langle p'_1, \sigma'_1 | \hat{A}_\mu^a, \text{spec} | p_1, \sigma_1 \rangle = \bar{u}(p'_1, \sigma'_1) \left[ g_A^q \gamma^\mu + \frac{2f_\pi}{Q^2 + m_\pi^2} g_{qq \pi} \tilde{q}^\mu \right] \gamma_5 \frac{\tau_a}{2} u(p_1, \sigma_1)
\]

\[g_A^q = 1 \quad \text{(pointlike constituent quark)}\]
Results

- Axial form factors of N
- Axial charges of N and some N* relative to other results
- Axial charges of all N* resonances below 2 GeV
- Axial form factors of N*
Axial and Pseudo-scalar Form Factors of nucleon

\[ g_A^{EXP} = 1.2695 \pm 0.0029 \]
\[ g_A^{psGBE} = 1.15 \]

## Comparison

<table>
<thead>
<tr>
<th>State</th>
<th>$J^P$</th>
<th>EGBE</th>
<th>Lattice QCD</th>
<th>GN</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(939)$</td>
<td>$\frac{1}{2}^+$</td>
<td>1.15</td>
<td>1.10$\sim$1.40</td>
<td>1.66</td>
<td>1.65</td>
</tr>
<tr>
<td>$N^*(1440)$</td>
<td>$\frac{1}{2}^+$</td>
<td>1.16</td>
<td>–</td>
<td>1.66</td>
<td>1.61</td>
</tr>
<tr>
<td>$N^*(1535)$</td>
<td>$\frac{1}{2}^-$</td>
<td>0.02</td>
<td>$\sim$0.00</td>
<td>-0.11</td>
<td>-0.20</td>
</tr>
<tr>
<td>$N^*(1710)$</td>
<td>$\frac{1}{2}^+$</td>
<td>0.35</td>
<td>–</td>
<td>0.33</td>
<td>0.42</td>
</tr>
<tr>
<td>$N^*(1650)$</td>
<td>$\frac{1}{2}^-$</td>
<td>0.51</td>
<td>$\sim$0.55</td>
<td>0.55</td>
<td>0.64</td>
</tr>
</tbody>
</table>


and further lattice QCD calculation
# Axial Charges of Chiral Partners

<table>
<thead>
<tr>
<th>State</th>
<th>$J^P$</th>
<th>Mass</th>
<th>$g_A$</th>
<th>Mass</th>
<th>$g_A$</th>
<th>Mass</th>
<th>$g_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(939)$</td>
<td>$\frac{1}{2}^+$</td>
<td>939</td>
<td>1.15</td>
<td>939</td>
<td>1.15</td>
<td>939</td>
<td>1.11</td>
</tr>
<tr>
<td>$N^*(1520)$</td>
<td>$\frac{3}{2}^-$</td>
<td>1524</td>
<td>-0.64</td>
<td>1519</td>
<td>-0.21</td>
<td>1520</td>
<td>-0.15</td>
</tr>
<tr>
<td>$N^*(1440)$</td>
<td>$\frac{1}{2}^+$</td>
<td>1464</td>
<td>1.16</td>
<td>1459</td>
<td>1.13</td>
<td>1578</td>
<td>1.10</td>
</tr>
<tr>
<td>$N^*(1535)$</td>
<td>$\frac{1}{2}^-$</td>
<td>1498</td>
<td>0.02</td>
<td>1519</td>
<td>0.09</td>
<td>1520</td>
<td>0.13</td>
</tr>
<tr>
<td>$N^*(1680)$</td>
<td>$\frac{5}{2}^+$</td>
<td>1689</td>
<td>0.89</td>
<td>1728</td>
<td>0.83</td>
<td>1858</td>
<td>0.70</td>
</tr>
<tr>
<td>$N^*(1675)$</td>
<td>$\frac{5}{2}^-$</td>
<td>1676</td>
<td>0.84</td>
<td>1647</td>
<td>0.83</td>
<td>1690</td>
<td>0.80</td>
</tr>
<tr>
<td>$N^*(1710)$</td>
<td>$\frac{1}{2}^+$</td>
<td>1757</td>
<td>0.35</td>
<td>1776</td>
<td>0.37</td>
<td>1860</td>
<td>0.32</td>
</tr>
<tr>
<td>$N^*(1650)$</td>
<td>$\frac{1}{2}^-$</td>
<td>1581</td>
<td>0.51</td>
<td>1647</td>
<td>0.46</td>
<td>1690</td>
<td>0.44</td>
</tr>
<tr>
<td>$N^*(1720)$</td>
<td>$\frac{3}{2}^+$</td>
<td>1746</td>
<td>0.35</td>
<td>1728</td>
<td>0.34</td>
<td>1858</td>
<td>0.25</td>
</tr>
<tr>
<td>$N^*(1700)$</td>
<td>$\frac{3}{2}^-$</td>
<td>1608</td>
<td>-0.10</td>
<td>1647</td>
<td>-0.50</td>
<td>1690</td>
<td>-0.47</td>
</tr>
</tbody>
</table>
## Classification

<table>
<thead>
<tr>
<th>$(LS)J^P$</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0\frac{1}{2})\frac{1}{2}^+$</td>
<td>$N(939)^{100}$</td>
<td>$\Lambda(1116)^{100}$</td>
<td>$\Sigma(1193)^{100}$</td>
<td>$\Xi(1318)^{100}$</td>
</tr>
<tr>
<td>$(0\frac{1}{2})\frac{1}{2}^+$</td>
<td>$N(1440)^{100}$</td>
<td>$\Lambda(1600)^{96}$</td>
<td>$\Sigma(1660)^{100}$</td>
<td>$\Xi(1690)^{100}$</td>
</tr>
<tr>
<td>$(0\frac{1}{2})\frac{1}{2}^+$</td>
<td>$N(1710)^{100}$</td>
<td>$\Sigma(1880)^{99}$</td>
<td>$\Sigma(1560)^{94}$</td>
<td>$\Xi(1820)^{97}$</td>
</tr>
<tr>
<td>$(1\frac{1}{2})\frac{1}{2}^-$</td>
<td>$N(1535)^{100}$</td>
<td>$\Lambda(1670)^{72}$</td>
<td>$\Sigma(1670)^{94}$</td>
<td>$\Xi(1820)^{97}$</td>
</tr>
<tr>
<td>$(1\frac{3}{2})\frac{1}{2}^-$</td>
<td>$N(1650)^{100}$</td>
<td>$\Lambda(1800)^{100}$</td>
<td>$\Sigma(1620)^{100}$</td>
<td>$\Xi(1820)^{97}$</td>
</tr>
<tr>
<td>$(1\frac{1}{2})\frac{3}{2}^-$</td>
<td>$N(1520)^{100}$</td>
<td>$\Lambda(1690)^{72}$</td>
<td>$\Sigma(1775)^{100}$</td>
<td>$\Sigma(1950)^{100}$</td>
</tr>
<tr>
<td>$(1\frac{3}{2})\frac{3}{2}^-$</td>
<td>$N(1700)^{100}$</td>
<td>$\Sigma(1940)^{100}$</td>
<td>$\Sigma(1950)^{100}$</td>
<td>$\Sigma(1950)^{100}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$(LS)J^P$</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0\frac{3}{2})\frac{3}{2}^+$</td>
<td>$\Delta(1232)^{100}$</td>
<td>$\Sigma(1385)^{100}$</td>
<td>$\Xi(1530)^{100}$</td>
<td>$\Omega(1672)^{100}$</td>
</tr>
<tr>
<td>$(0\frac{3}{2})\frac{3}{2}^+$</td>
<td>$\Delta(1600)^{100}$</td>
<td>$\Sigma(1690)^{99}$</td>
<td>$\Sigma(1750)^{94}$</td>
<td>$\Omega(1672)^{100}$</td>
</tr>
<tr>
<td>$(1\frac{1}{2})\frac{1}{2}^-$</td>
<td>$\Delta(1620)^{100}$</td>
<td>$\Sigma(1750)^{94}$</td>
<td>$\Omega(1672)^{100}$</td>
<td>$\Omega(1672)^{100}$</td>
</tr>
<tr>
<td>$(1\frac{3}{2})\frac{3}{2}^-$</td>
<td>$\Delta(1700)^{100}$</td>
<td>$\Sigma(1750)^{94}$</td>
<td>$\Omega(1672)^{100}$</td>
<td>$\Omega(1672)^{100}$</td>
</tr>
</tbody>
</table>


## Classification

<table>
<thead>
<tr>
<th>((LS)J^P)</th>
<th>(N(939)^{100})</th>
<th>(\Lambda(1116)^{100})</th>
<th>(\Sigma(1193)^{100})</th>
<th>(\Xi(1318)^{100})</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0\frac{1}{2})_2^{1+})</td>
<td>((0\frac{1}{2})_2^{1+})</td>
<td>((0\frac{1}{2})_2^{1+})</td>
<td>((0\frac{1}{2})_2^{1+})</td>
<td>((0\frac{1}{2})_2^{1+})</td>
</tr>
<tr>
<td>((0\frac{1}{2})_2^{1+})</td>
<td>((0\frac{1}{2})_2^{1+})</td>
<td>((0\frac{1}{2})_2^{1+})</td>
<td>((0\frac{1}{2})_2^{1+})</td>
<td>((0\frac{1}{2})_2^{1+})</td>
</tr>
<tr>
<td>((1\frac{1}{2})_2^{1-})</td>
<td>(N(1535)^{100})</td>
<td>(\Lambda(1670)^{72})</td>
<td>(\Sigma(1560)^{94})</td>
<td>(-0.15)</td>
</tr>
<tr>
<td>((1\frac{3}{2})_2^{1-})</td>
<td>(N(1650)^{100})</td>
<td>(\Lambda(1800)^{100})</td>
<td>(\Sigma(1620)^{100})</td>
<td>(-0.15)</td>
</tr>
<tr>
<td>((1\frac{1}{2})_2^{3-})</td>
<td>(N(1520)^{100})</td>
<td>(\Lambda(1690)^{72})</td>
<td>(\Sigma(1670)^{94})</td>
<td>(\Xi(1820)^{97})</td>
</tr>
<tr>
<td>((1\frac{3}{2})_2^{3-})</td>
<td>(N(1700)^{100})</td>
<td>(\Sigma(1940)^{100})</td>
<td>(\Sigma(1950)^{100})</td>
<td>(-0.15)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>((LS)J^P)</th>
<th>(\Delta(1232)^{100})</th>
<th>(\Sigma(1385)^{100})</th>
<th>(\Xi(1530)^{100})</th>
<th>(\Omega(1672)^{100})</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0\frac{3}{2})_2^{3+})</td>
<td>((0\frac{3}{2})_2^{3+})</td>
<td>((0\frac{3}{2})_2^{3+})</td>
<td>((0\frac{3}{2})_2^{3+})</td>
<td>((0\frac{3}{2})_2^{3+})</td>
</tr>
<tr>
<td>((1\frac{1}{2})_2^{1-})</td>
<td>(\Delta(1620)^{100})</td>
<td>(\Sigma(1750)^{94})</td>
<td>(-0.08)</td>
<td></td>
</tr>
<tr>
<td>((1\frac{3}{2})_2^{1-})</td>
<td>(\Delta(1700)^{100})</td>
<td>(-0.76)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Axial Form Factors of N(1440) and N(1710)
Summary

- Performed a consistent study of $g_A$ for $N$ and $N^*$ within the RCQMs.

- Covariant predictions of RCQMs show generally considerable relativistic effects and describe experimental data well.

- In cases, where no experiments exist, there is reasonable agreement with lattice QCD results.

- The issue regarding chiral symmetry restoration remains open; the sizes of the $g_A$ for baryon resonances are consistent with a recent classification into flavor multiplets.

- It will be interesting to extend these relativistic studies to electromagnetic and weak transition form factors.
Nucleon Spectra

**Motivation**

**Formalism**

**Results**

**Summary**


Green zones - Experimental data in PDG (2010)