Stability of $P_{11}$ Resonances Extracted from $\pi N$ data

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**Introduction**

Extraction of $N^*$ from $\pi N$ data is important!

* Understanding spectrum and structure of $N^*$ within QCD

**Steps to extract $N^*$**

1. Construct a reaction model through analysis of data

2. From the constructed model, resonance properties (pole position, vertex form factor) are extracted with analytic continuation
Extraction of $N^*$ properties is inevitably model-dependent!

Several approaches: dynamical model (EBAC, Jülich ...)
K-matrix (GWU/VPI, Bonn-Gatchina ...)

Existence of some $N^*$ is controversial

Question!

How much extracted $N^*$ parameters depend on:

1. model
2. precision of data (amplitude)
We study stability of pole structure:

Roper resonance \([N(1440)P_{11}]\)

* Large variation of parameters of EBAC-DCC model
* Including bare nucleon state
* Variation of \(P_{11}\) amplitude for \(1.6 < W < 2\) GeV

Higher mass \(P_{11}\) resonances

* Fit to amplitude up to \(W \sim 2.5\) GeV
* Simultaneous fit to \(\pi N, \eta N, \pi \Delta\) channels
EBAC-DCC (Dynamical Coupled-Channel) model


Coupled-channel Lippmann-Schwinger equation

\[ T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb} \]

\[ \{a, b, c\} = \pi N, \eta N, \pi \pi N (\pi \Delta, \sigma N, \rho N) \]
EBAC-DCC (Dynamical Coupled-Channel) model


Coupled-channel Lippmann-Schwinger equation

\[ T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb} \]

\[ V_{ab} = \begin{array}{c}
\text{s} \\
\text{t} \\
\text{u} \\
\text{c}
\end{array} + \text{bare N*} \]
Bare Nucleon Model

Pearce and Afnan, PRC 34, 991 (1986)

Motivation: $P_{11}$ amplitude of EBAC-DCC model below $\pi N$ threshold, $E \sim m_N$

**EBAC-DCC**

$$T \sim \Gamma_{\pi NN}^* \left[ \frac{d_1}{E - m_N} + \frac{d_0}{(E - m_N)^2} + d_2 \right] \Gamma_{\pi NN}$$

**Jülich**

$$T \sim \Gamma_{\pi NN}^* \left[ \frac{d'_1}{E - m_N} \right] \Gamma_{\pi NN}$$

Question: Is Roper poles sensitive to the analytic structure at $E \sim m_N$?
Bare Nucleon Model

Pearce and Afnan, PRC 34, 991 (1986)

* Potentials

\[ V_{ab} = v'_{ab} + \frac{\Gamma_{N,a}^\dagger \Gamma_{N,b}}{E - m_N^0} + \sum_{N^*} \frac{\Gamma_{N^*,a}^\dagger \Gamma_{N^*,b}}{E - m_{N^*}^0} \]

\[ v' = v - \]

* Nucleon Pole Condition

(i) \[ T \left( E \sim m_N \right) \sim \frac{\bar{\Gamma} \bar{\Gamma}}{E - m_N} \]

(ii) \[ \bar{\Gamma} = F_{\pi NN}^{\text{phys}} \]
Extraction of $N^*$ information with analytic continuation

Suzuki et al., PRC 79, 025205 (2009)

$\pi N$ scattering amplitude near a pole ($E \sim M_R$)

$$F_{\pi N}(E) \sim \frac{\bar{\Gamma}(M_R) \bar{\Gamma}(M_R)}{E - M_R} + \text{(regular terms)}$$

Parameters characterizing Resonance

* Pole position of amplitude: $M_R$

* $N^* \rightarrow MB$ decay vertex: $\bar{\Gamma}(M_R)$
Results

Benchmarks

∗ SAID SES (single energy solution) (SP06)

⇐ Parameters of all models are fitted

∗ SAID EDS (energy-dependent solution) (SP06)

∗ JLMS (Julía-Díaz et al., PRC 76, 065201 (2007) )
* $P_{11}$ amplitude (benchmarks)

<table>
<thead>
<tr>
<th>Model</th>
<th>$u u u u p p$</th>
<th>$u p u p p p$</th>
<th>$w w w w u p$</th>
<th>$\chi^2/(# \text{ of data})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAID-EDS(SP06)</td>
<td>(1359, −81)</td>
<td>(1388, −83)</td>
<td></td>
<td>2.94</td>
</tr>
<tr>
<td>JLMS</td>
<td>(1357, −76)</td>
<td>(1364, −105)</td>
<td>(1820, −248)</td>
<td>3.55</td>
</tr>
</tbody>
</table>
* $P_{11}$ amplitude (benchmarks)

* Sheet assignment of the poles

$$\left(s_{\pi N}, s_{\eta N}, s_{\pi\pi N}, s_{\pi\Delta}, s_{\rho N}, s_{\sigma N}\right) = (u, p, u, u, p, p)$$
More Models 1

* $2N^*-3p \quad v \neq v_{JLMS}$ smooth fit

* $2N^*-4p \quad v \neq v_{JLMS}$ fitted to oscillated behavior
<table>
<thead>
<tr>
<th>Model</th>
<th>$^{uuu}u$ppp</th>
<th>$^{uuu}u$ppp</th>
<th>$u$uuu$u$pp</th>
<th>$u$uuuu$u$pp</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAID-EDS(SP06)</td>
<td>(1359, −81)</td>
<td>(1388, −83)</td>
<td>—</td>
<td>—</td>
<td>2.94</td>
</tr>
<tr>
<td>JLMS</td>
<td>(1357, −76)</td>
<td>(1364, −105)</td>
<td>—</td>
<td>(1820, −248)</td>
<td>3.55</td>
</tr>
<tr>
<td>$2N^*-3p$</td>
<td>(1368, −82)</td>
<td>(1375, −110)</td>
<td>—</td>
<td>(1810, −82)</td>
<td>3.28</td>
</tr>
<tr>
<td>$2N^*-4p$</td>
<td>(1370, −81)</td>
<td>(1384, −115)</td>
<td>(1635, −68)</td>
<td>(1960, −214)</td>
<td>3.36</td>
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</tbody>
</table>

* Roper two poles are stable!

* Additional pole in $2N^* - 4p$!

* Different high $W$ amplitude ⇒ Different higher mass pole position
More Models 2 (Bare Nucleon Model)

<table>
<thead>
<tr>
<th>Model</th>
<th>( u_{p_{1}u_{1}p_{2}p_{2}p} )</th>
<th>( u_{p_{1}u_{1}p_{2}p_{3}p_{3}} )</th>
<th>( u_{u_{1}u_{1}u_{2}u_{2}p} )</th>
<th>( u_{u_{1}u_{1}u_{2}u_{2}u} )</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAID-EDS(SP06)</td>
<td>(1359, −81)</td>
<td>(1388, −83)</td>
<td>—</td>
<td>—</td>
<td>2.94</td>
</tr>
<tr>
<td>JLMS</td>
<td>(1357, −76)</td>
<td>(1364, −105)</td>
<td>—</td>
<td>(1820, −248)</td>
<td>3.55</td>
</tr>
<tr>
<td>( 1N_01N^{\ast}-3p )</td>
<td>(1364, −81)</td>
<td>(1377, −129)</td>
<td>—</td>
<td>(1769, −132)</td>
<td>2.51</td>
</tr>
</tbody>
</table>

\* Roper two poles are stable, not sensitive to the analytic structure at \( E \sim m_N \)
Fit to Different data

* **SAID-SES** for $W \leq 1.55$ GeV

* **CMB** for $W \geq 1.55$ GeV
Roper two poles are stable, not sensitive to the amplitude in $W > 1.5$ GeV!
Fit up to high $W$ & multi-channel $\pi N \rightarrow MB$

**Question**

* Help stabilize higher mass pole positions?

* Simultaneous fit to multi-channel amplitudes
  
  $(\pi N \rightarrow \pi N, \pi N \rightarrow \eta N, \pi N \rightarrow \pi \Delta \ldots)$ helps?

  cf Ceci et al, PRL 97, 062002 (2006)
Fit to $\pi N \rightarrow \pi N$ up to $W \sim 2.5$ GeV

* 4 models, fitted to $\pi N$ amplitude up to 2.5 GeV
Simultaneous fit to multi-channel $\pi N \rightarrow MB$ amplitudes
<table>
<thead>
<tr>
<th>Model</th>
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<th>врпккк</th>
</tr>
</thead>
<tbody>
<tr>
<td>black</td>
<td>(1360,−87)</td>
<td>(1368,−116)</td>
<td>—</td>
<td>(1702, −93)</td>
<td>(1967, −110)</td>
</tr>
<tr>
<td>blue</td>
<td>(1367,−80)</td>
<td>(1378,−101)</td>
<td>(1554,−68)</td>
<td>(1810,−179)</td>
<td>—</td>
</tr>
<tr>
<td>purple</td>
<td>(1367,−76)</td>
<td>(1381,−103)</td>
<td>(1542,−44)</td>
<td>(1798,−109)</td>
<td>—</td>
</tr>
<tr>
<td>red</td>
<td>(1364,−86)</td>
<td>(1375,−106)</td>
<td>—</td>
<td>(1778, −142)</td>
<td>(1999, −138)</td>
</tr>
</tbody>
</table>

* Roper two poles are stable

* Higher mass poles are still not stabilized

* Pole $W \sim 1550 - 50i$ has very small coupling to $\pi N$
Summary

Stability of $P_{11}$ poles

* Roper two poles are stable against

- Large variation of parameters within EBAC-DCC ($v \neq v_{JLMS}$)
- Inclusion of bare nucleon state (different analytic structure at $E \sim m_N$)
- Fitting to different amplitude

provided amplitudes are precisely fitted to SAID SES for $W < 1.5$ GeV

$$E_{\text{pole}} = 1363^{+6}_{-6} - i 79^{+3}_{-3} \ (s_{\pi\Delta} = u), \ 1373^{+10}_{-10} - i 114^{+15}_{-9} \ (s_{\pi\Delta} = p)$$
Summary

Fit to higher $W$ & simultaneous fit to $\pi N \rightarrow \pi N$, $\pi N \rightarrow \eta N$, $\pi N \rightarrow \pi \Delta$

* Roper two poles are stable

* Higher mass poles are still unstable

$\Rightarrow$ amplitudes of all channels need to be fitted?