Quark-Hadron Duality in L/T separated Structure functions

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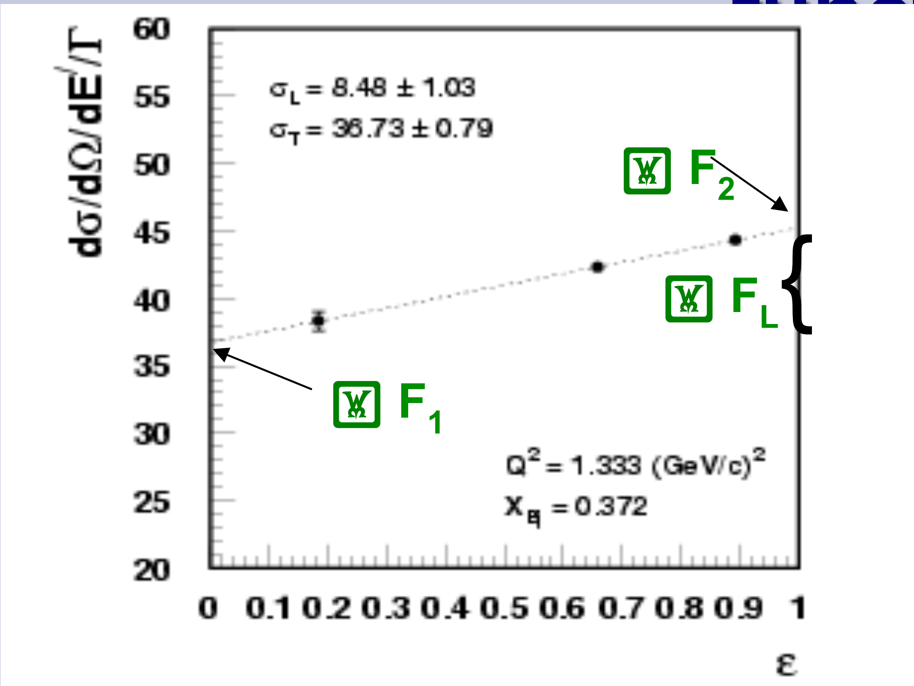
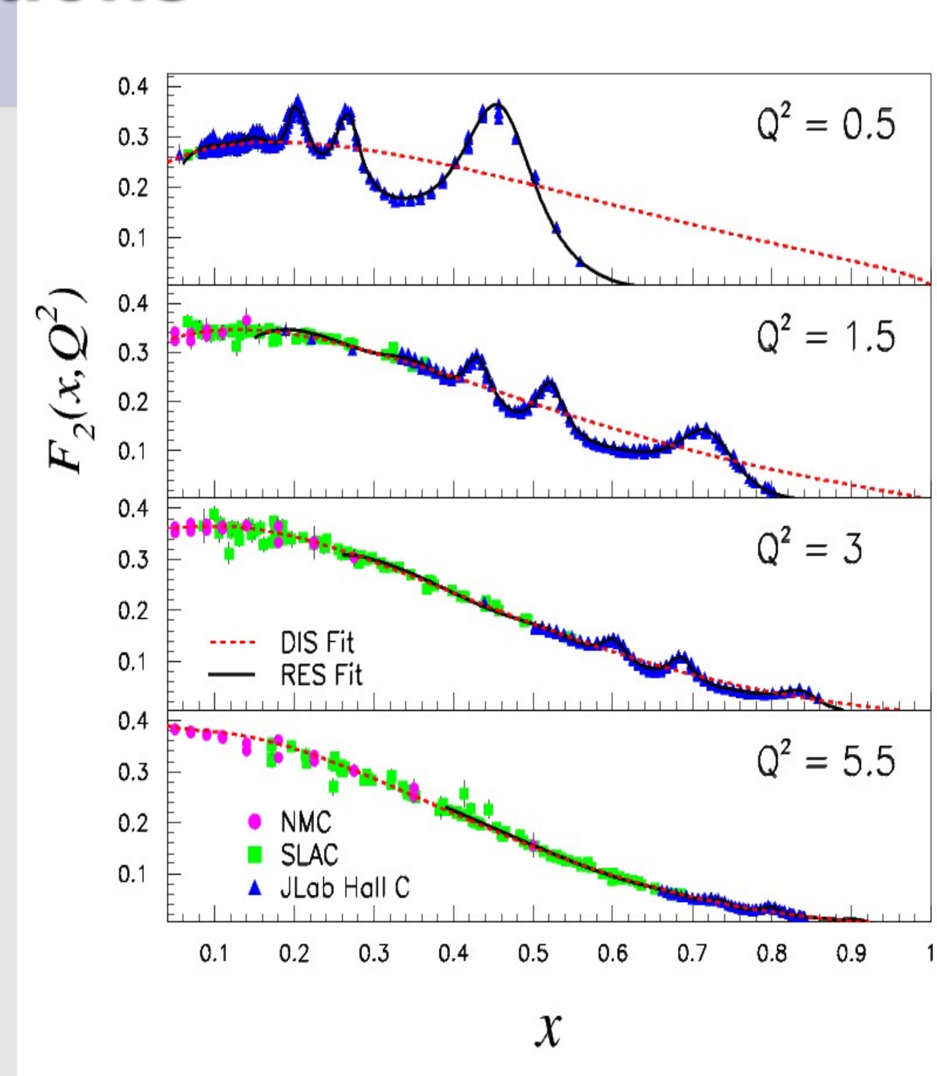
E94-110: proton separated structure functions





$$F_2 = (2xF_1+F_L)/(1+\nu^2/Q^2)$$

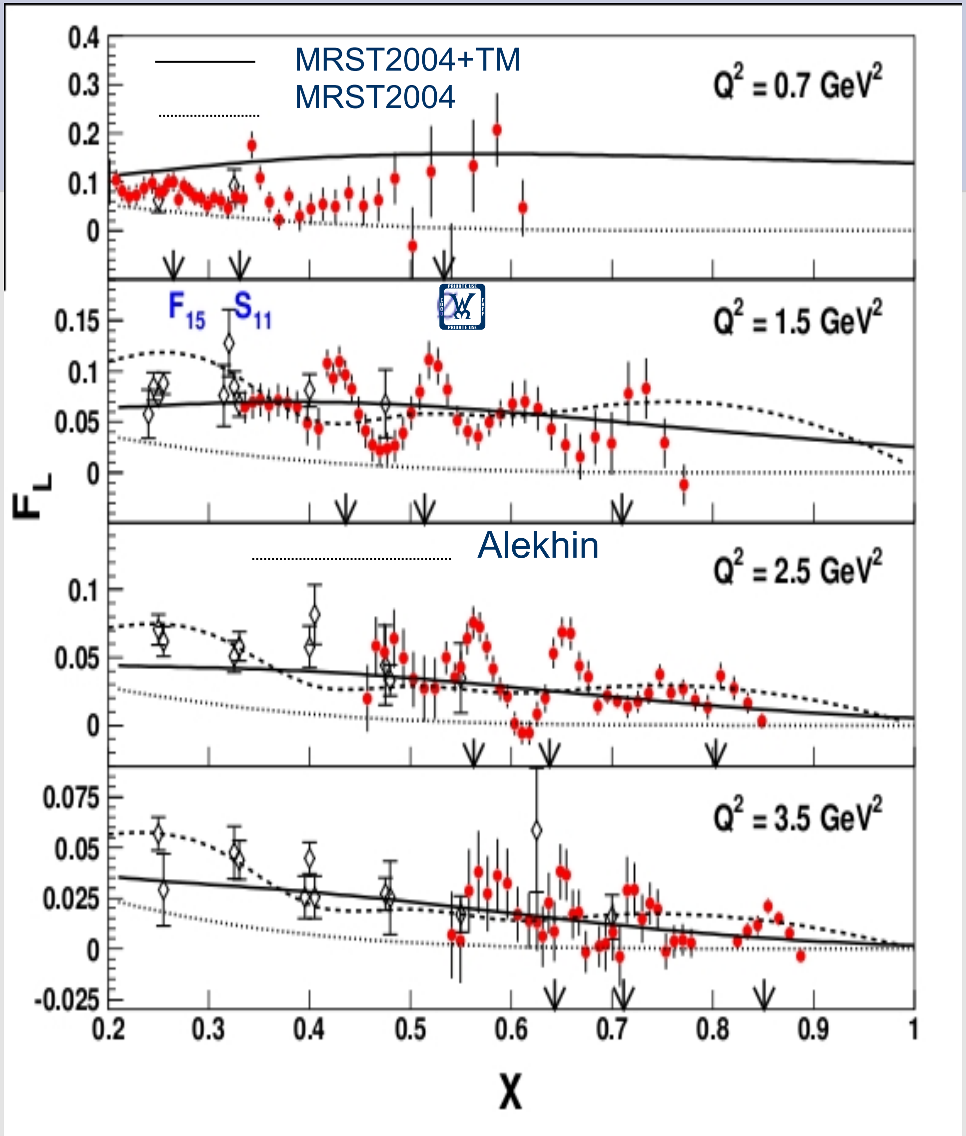
Just because duality holds well in sum (F_2) does not imply that It holds well *a priori* in individual F_1 or F_L

E94-110: proton F_L in resonance region

→ First observation of quark-hadron duality in F_L.

→ TM corrections are critical Component of scaling function.

- Duality is considerably broken for $Q^2 < 4$ without this contribution

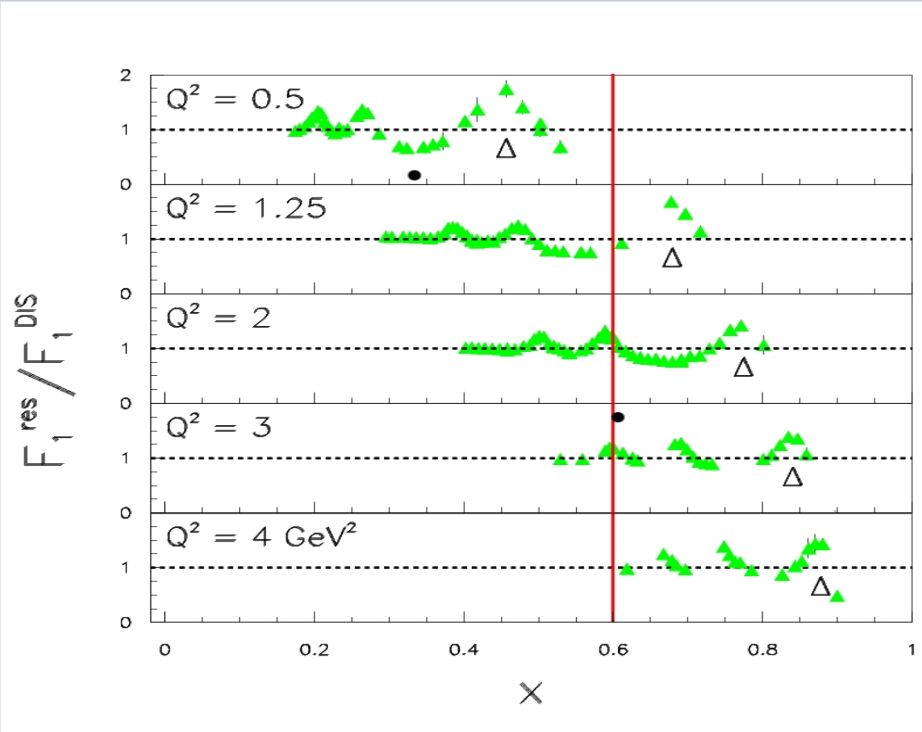


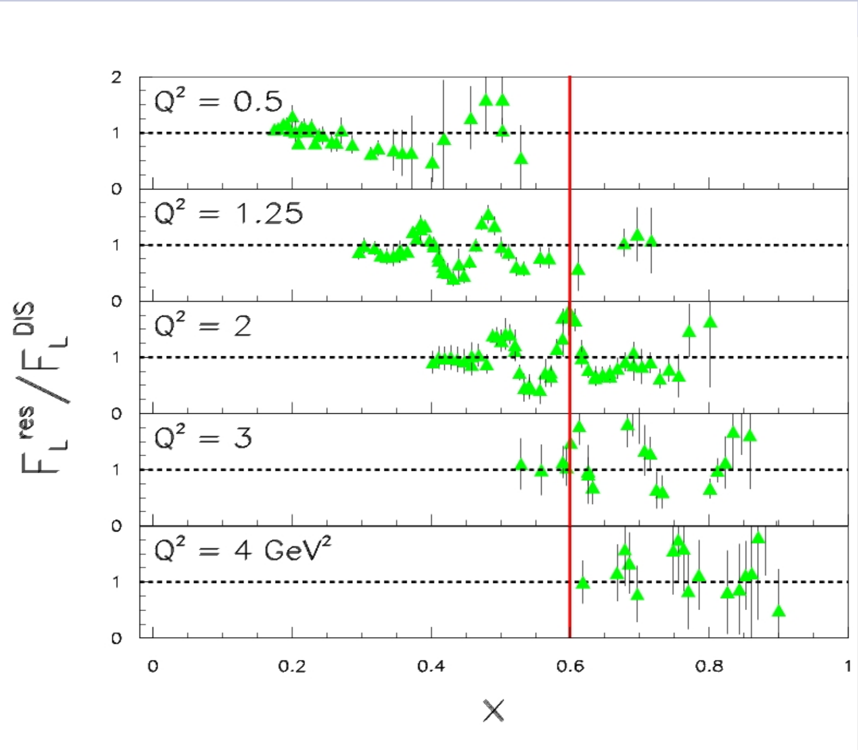
Comparison L/T separated data to empirical DIS fits

DIS fit: F_2ALLM fit to F_2 + $R = \sigma_L / \sigma_T$

H.Abramowicz and A.Levy, Hep-ph/9712415

K. Abe et.al Phys.Lett.B452:194-200,1999





→ **Duality well obeyed when comparing to empirical scaling curve.**

→ **In principle these fits contain non-perturbative contributions such as target mass (TM) corrections and higher-twist (HT)**

Several methods have been utilized for quantification, including:

(i) Compare Q^2 dependence of integral over local W^2 ranges to DIS scaling curve (or pQCD curve) – see talks by Simona M. and Ioana N.

(ii) Compare structure function moments at different orders (n=2,4,6,...) to the Q^2 dependence expected from pQCD+TM (scaling predictions).

(iii) Compare *truncated* moments defined over local W^2 ranges to pQCD evolution.

To compared Data to QCD Moments using PDFs, must correct for known TM effects

In massless limit only operators with spin = n contributes to n^{th} Cornwall-Norton (CN) moments,

Massless limit SF

This is not true for finite M^2/Q^2. However,

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3}\left[\frac{3+3(n+1)r+n(n+2)r^2}{(n+2)(n+3)}\right] F_2^{TMC}(x,Q^2)$$

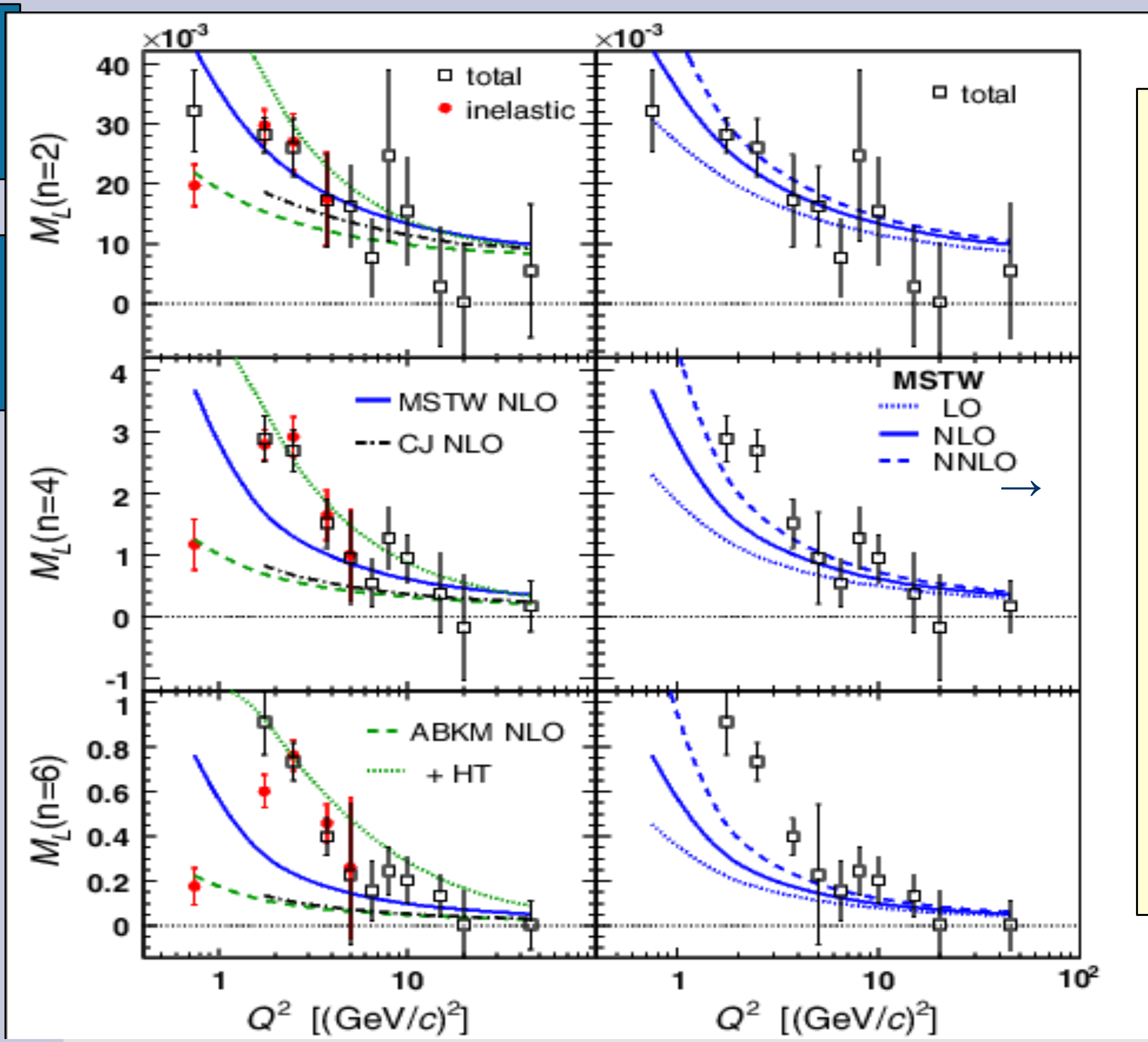
projects out pure spin n contribution – *Nachtmann (1973)*

Here F_2^{TMC} is the *experimental* structure functions.

For consistency, it should be true that

Nachtmann Moments of proton F_L

P. Monaghan, A. Accardi, MEC, C.E. Keppel, W. Melnitchouk, L. Zhu, PRL 110, 15202 (2013).



→ TM contributions have been removed from data via Nachtmann moments.

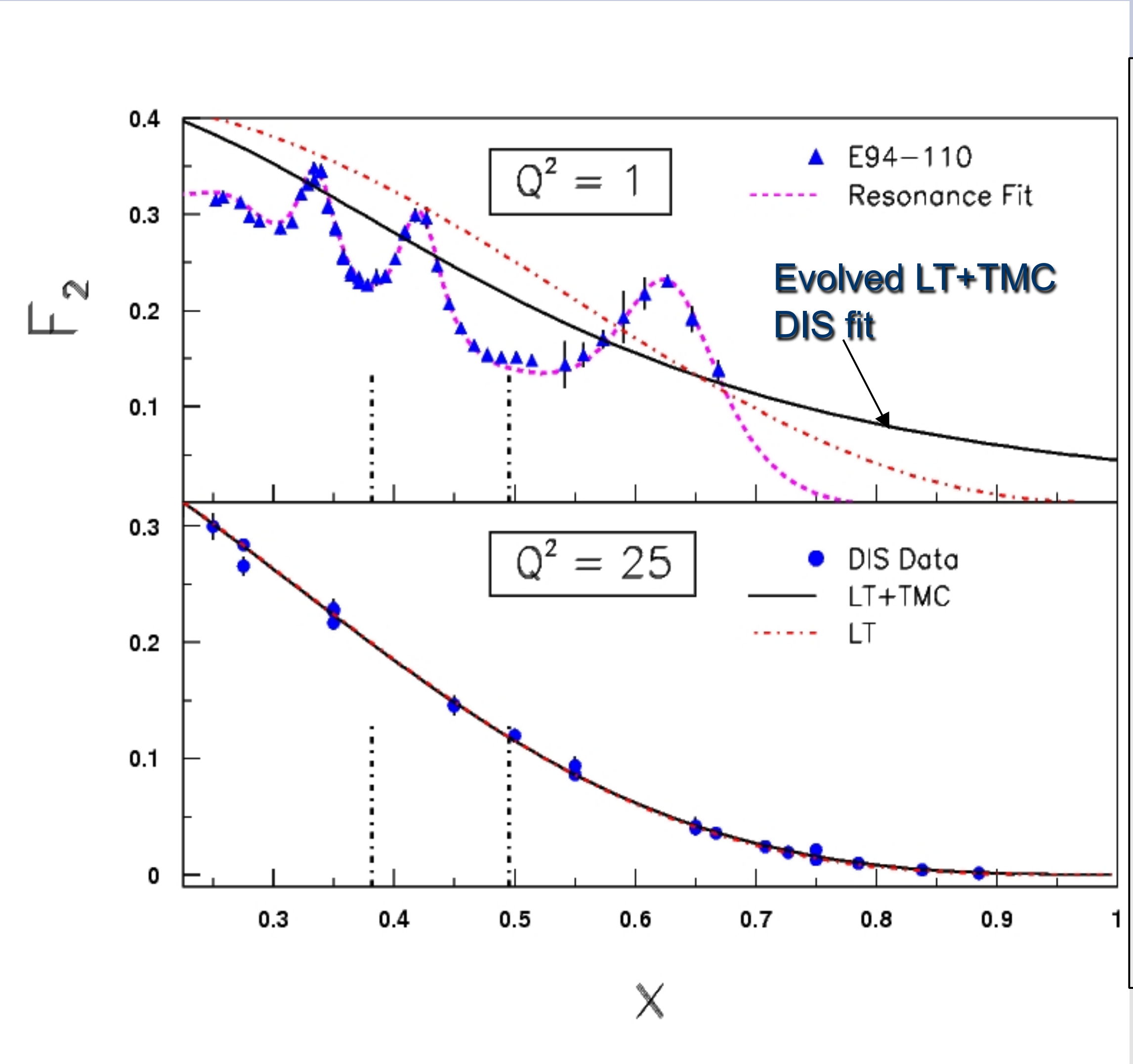
→ Different PDF NLO results are similar at Q^2 > 20, but are significantly different at Low Q^2.

→ Note that only ABKM includes H-T terms in fit. Contribution partially absorbed in MRST gluon?

→ Differences in higher moments likely due to underestimated Gluon strength At high *x* and/or H-T contributions.

Truncated Moment Analysis: basic idea

Allows study of ***regions in W*** within pQCD framework

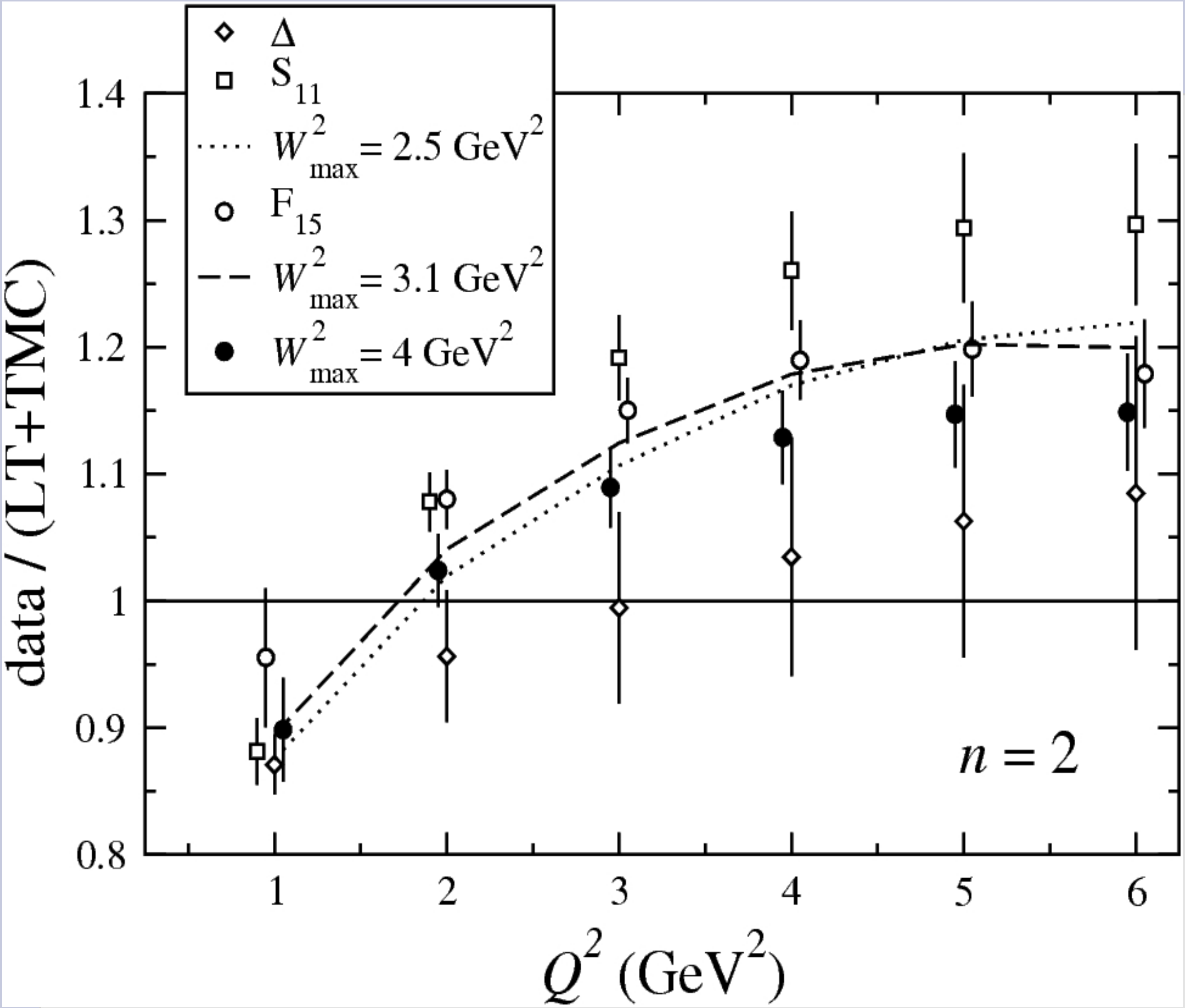


→ Compare integral over select resonance regions to evolved scaling curve + TM

→ Scaling curve is empirical Fit to data at Q^2 = 25, where TM contribution has been separated from leading-twist via an unfolding Procedure.

→ LT Scaling curve is then evolved to lower Q^2 before recalculating the TM contributions at the lower Q^2.

Q^2 Dependence of Truncated Moments, *x* Regions Defined by Resonances



- Consider now individual and total resonance region
- Large Q^2 dependence below ~3 GeV^2 - decreases at higher Q^2
- Below Q^2 = 0.75 GeV^2 the applicability of pQCD analysis doubtful
- Facilitates careful Higher Twist analysis….

Truncated moment analysis can also be applied to F_1 and F_L

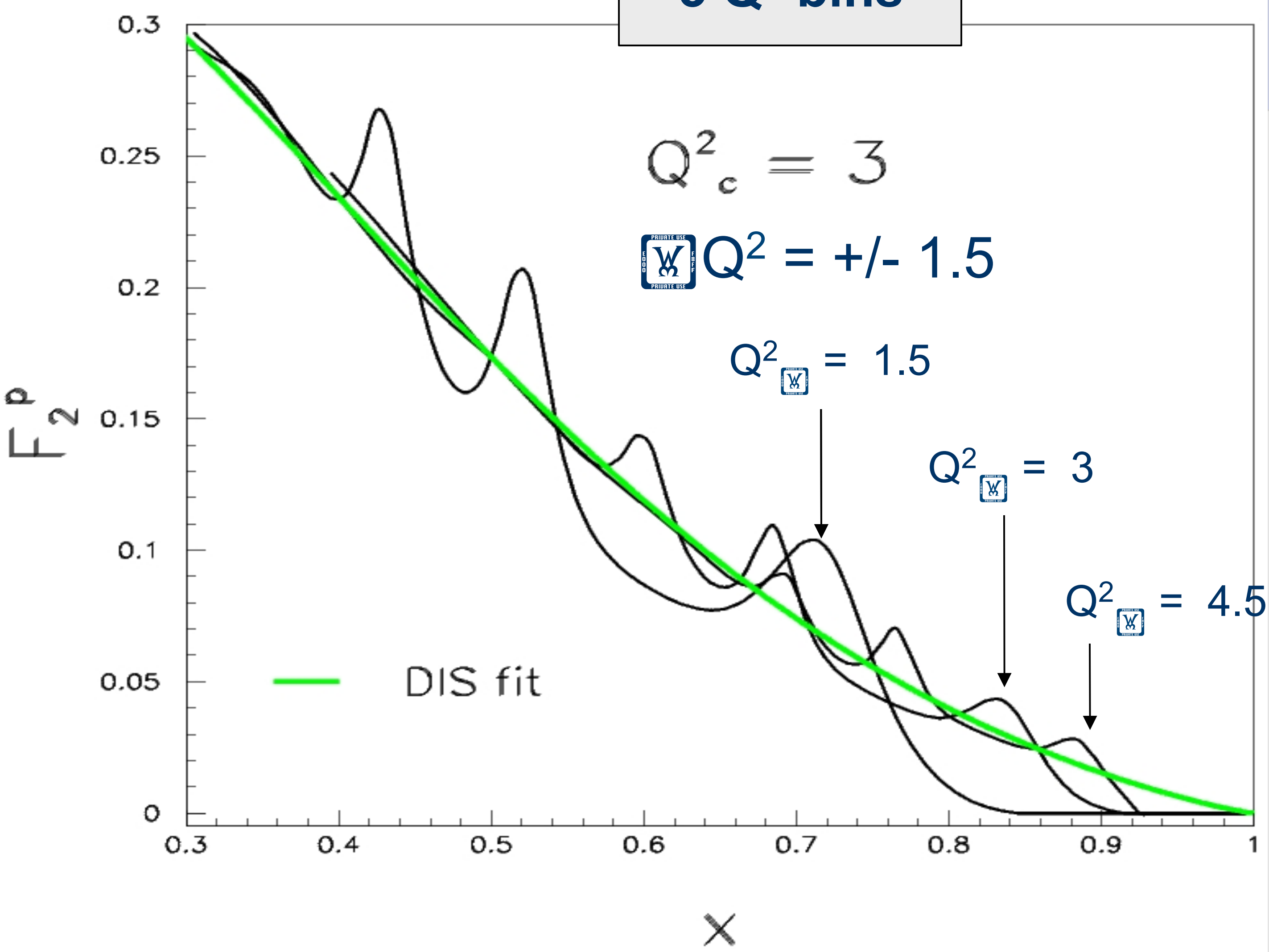
Can we use duality to help constrain large-*x PDFs*?

ie. Are 'duality averaged' data in the resonance region consistent with Q^2,*x* dependence of structure functions at Larger W^2?

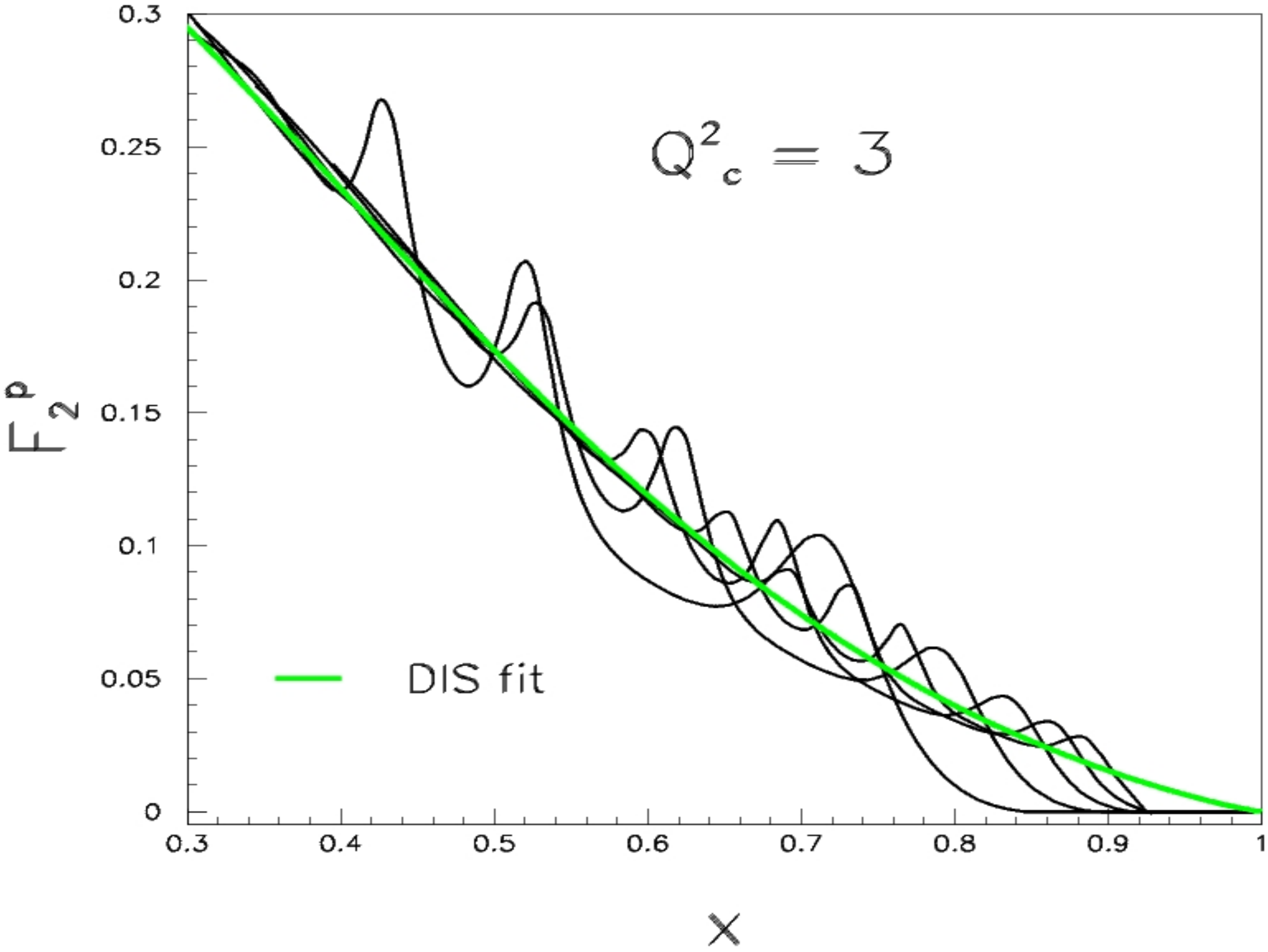
If so then how do we *average*?

'DIS-like' duality averaging procedure

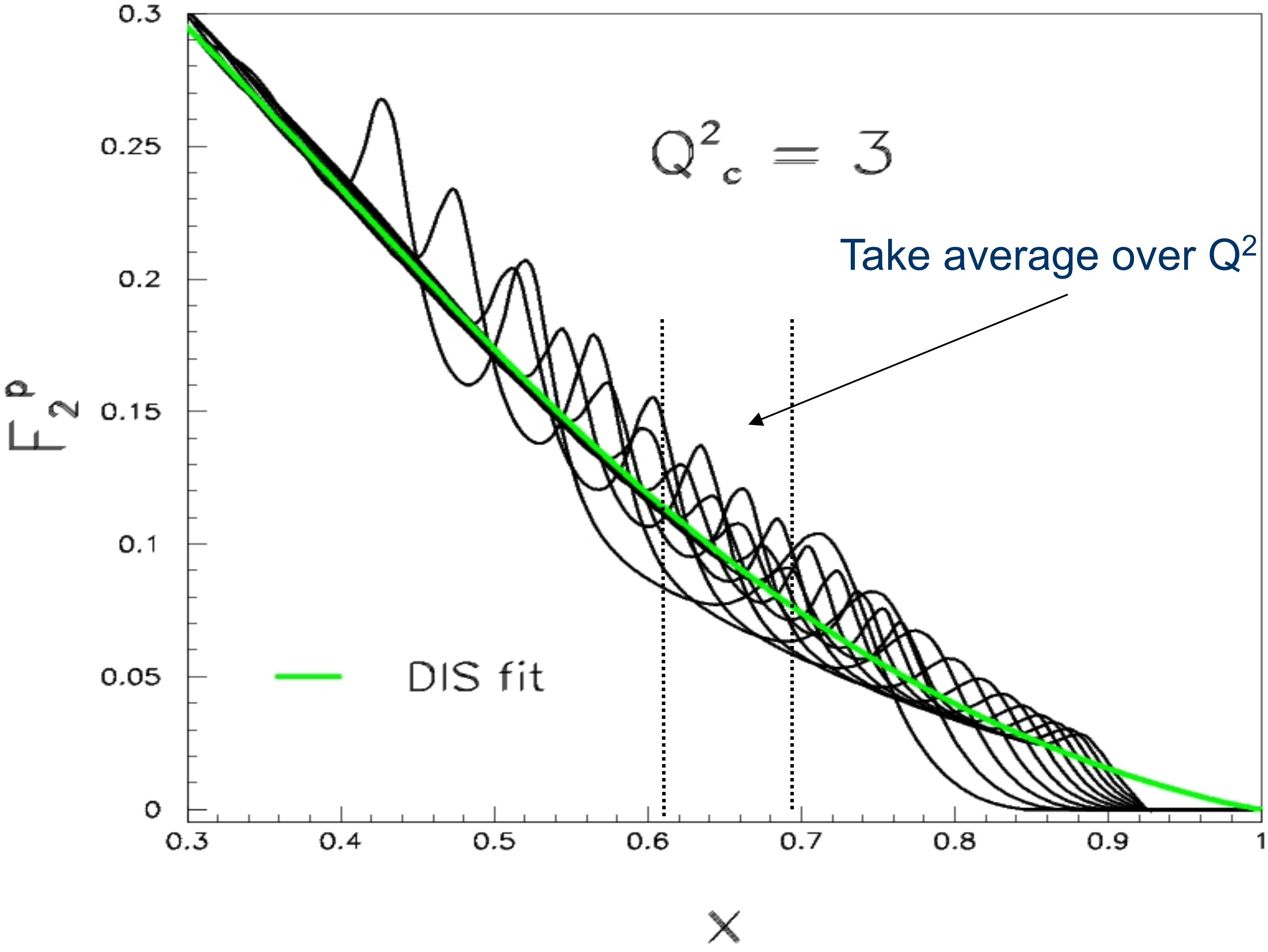
3 Q² bins



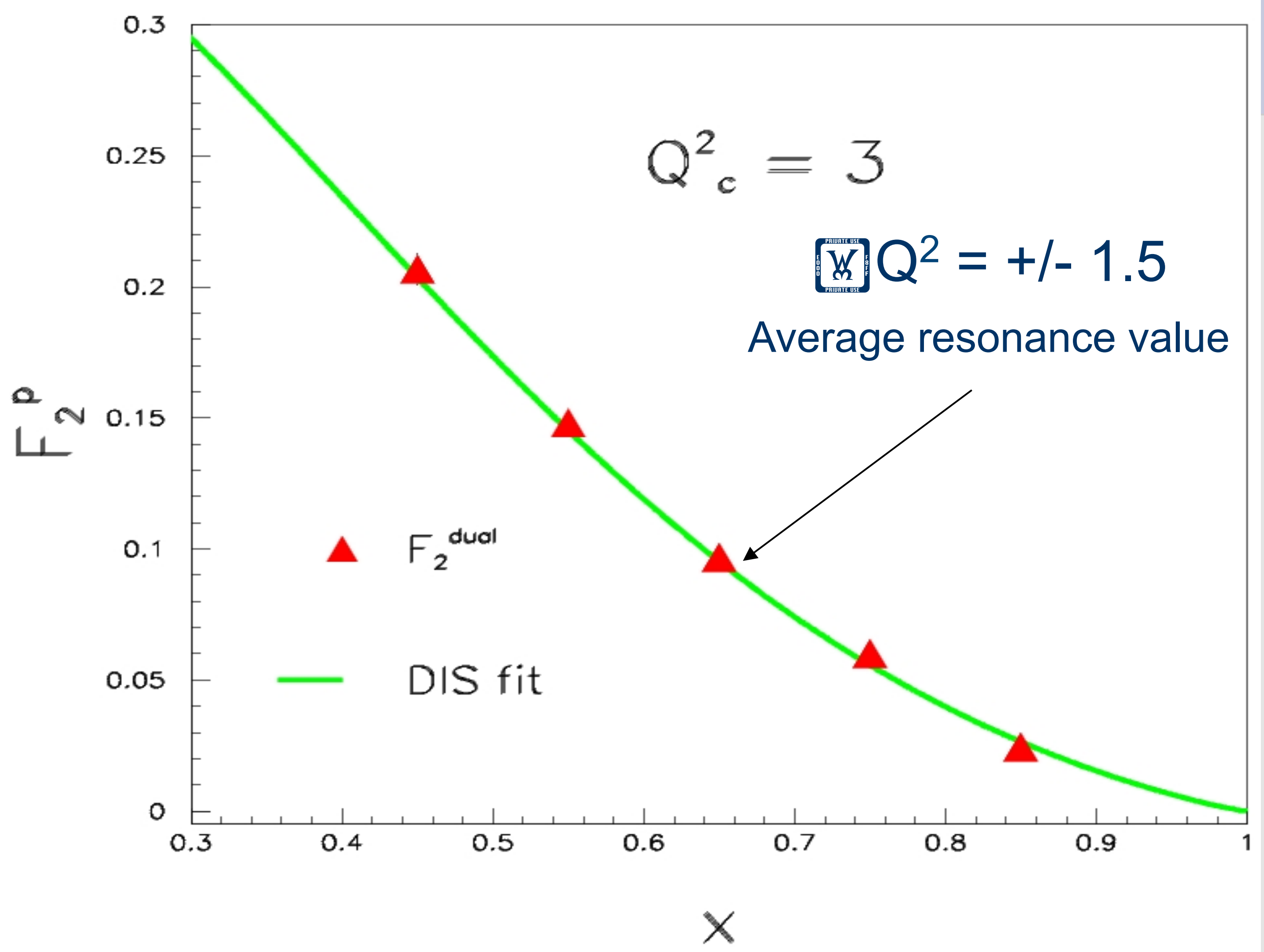
5 Q^2 bins



9 Q^2 bins



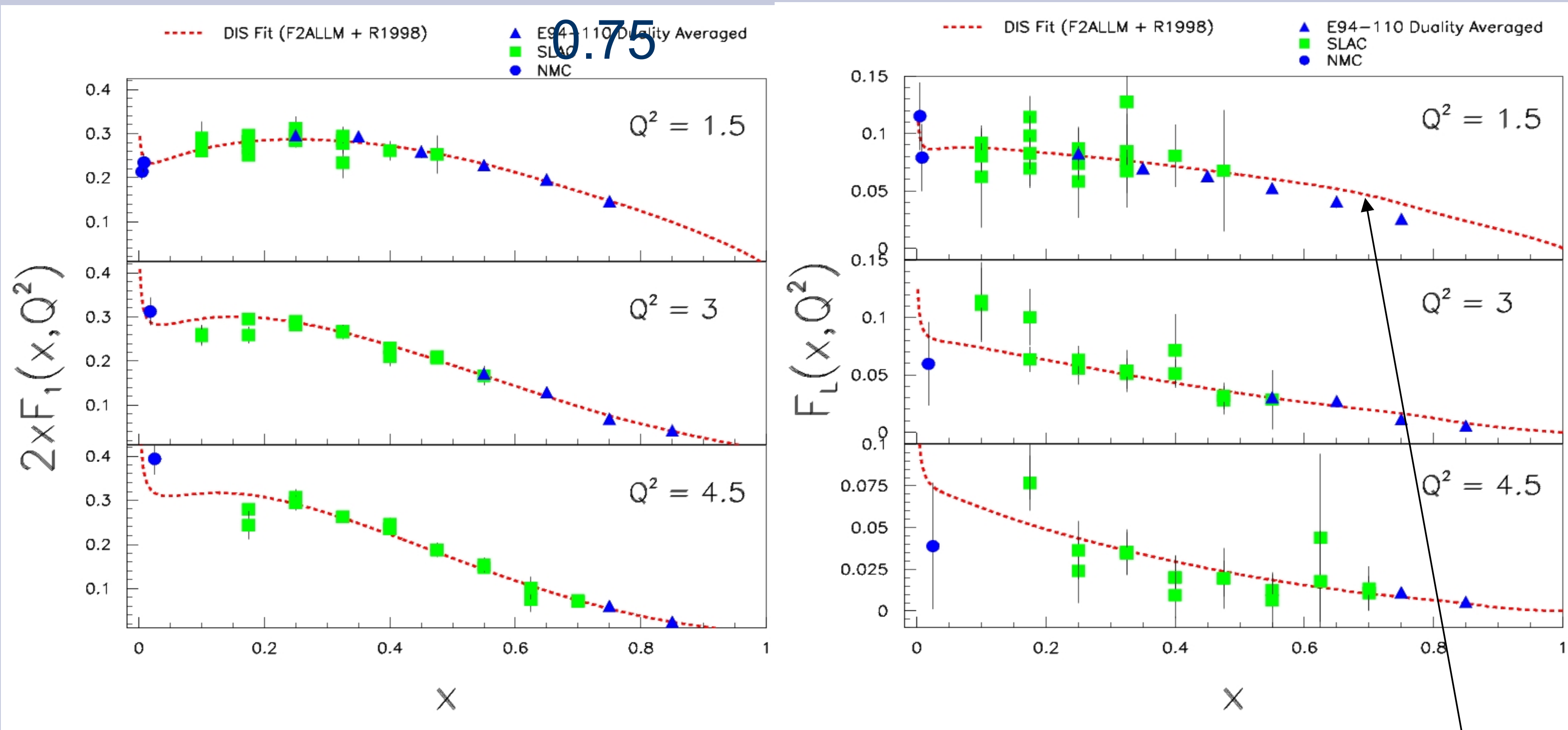
0.3
0.25
0.2
0.15
0.1
0.05
0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
F_2^p
x
$Q^2_c = 3$
Take average over Q^2
DIS fit



$Q^2_c = 3$
ΔQ^2 = +/- 1.5
Average resonance value
F_2^{dual}
DIS fit
F_2^p
x
0.3
0.25
0.2
0.15
0.1
0.05
0
0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

Duality averaging results for F_1 and F_L

$Q^2 = +/-$ 0.75



- **Good consistency with DIS and relatively smooth *x* dependence.**
- **Note different Q^2 dependence in averaged F_L from fit at lowest Q^2.**

Resonances have same Q^2 dependence as scaling curve.

But what scaling curve?

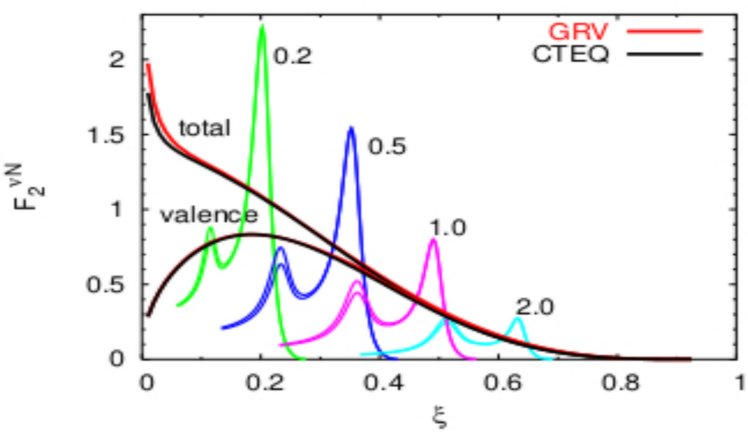
pure pQCD curve?

or defined by data (LT+TM+HT)?

Predictions for neutrino scattering from a number of groups (see talk by O. Lalakulich NuInt 2009)

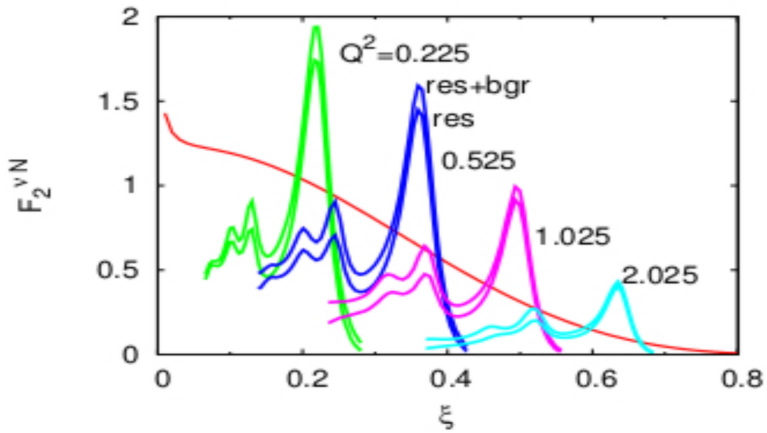
$F_2^{\nu p, \nu n}$: Duality HOLDS for the averaged structure functions

Duality: on average the resonances appear to oscillate around and slide down the leading twist function



OL, Melnitchouk, Paschos, PRC 75

included: 4 resonances
F_2 calculated analytically
investigation of F_3 and $2xF_1$ is also done



Giessen BUU

included: 12 resonances + phenomenological 1-pion background
F_2 is restored from xsec

Predictions for neutrino scattering from a number of groups (see talk by O. Lalakulich NuInt 2009)

$F_2^{\nu p, \nu n}$: In neutrino–nucleon scattering duality does NOT hold for proton and neutron targets separately

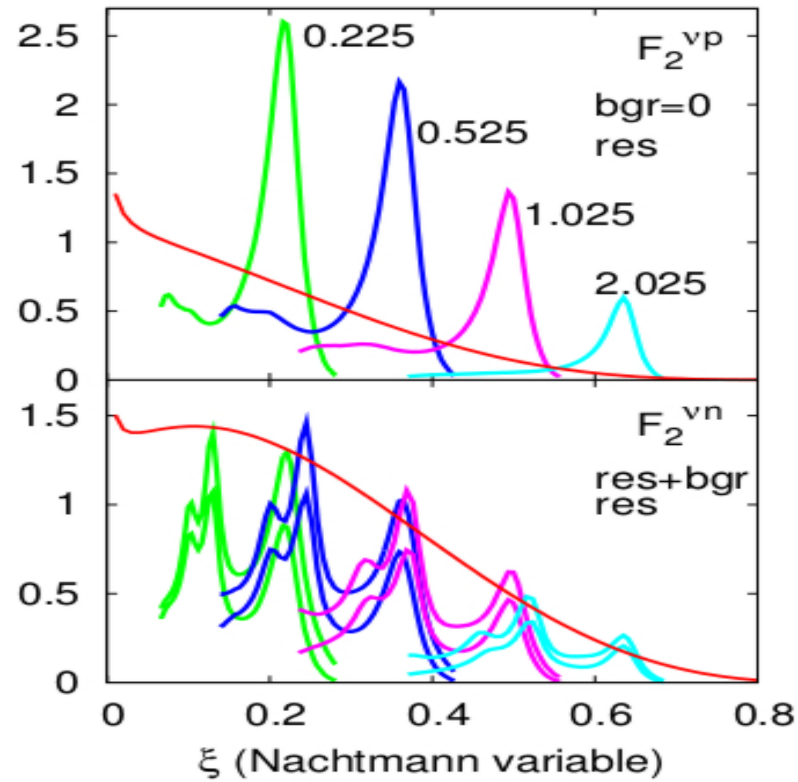
Low-lying resonances: $F_2^{\nu n(res)} < F_2^{\nu p(res)}$
neutron<proton

DIS: $F_2^{\nu n(DIS)} > F_2^{\nu p(DIS)}$
neutron>proton

$F_2^{\nu p(res-3/2)} = 3 F_2^{\nu n(res-3/2)}$
$F_2^{\nu p(res-1/2)} \equiv 0$

$F_2^{\nu n(res)}$: finite contributions from isospin-3/2 and -1/2 resonances

Interplay between the resonances with different isospins: isospin-3/2 resonances give strength to the proton structure functions, while isospin-1/2 resonances contribute to the neutron structure function only



Olga Lalakulich (Justus–Liebig University, Giessen) Duality and neutrinos NuInt 09, Sitges, 21 May 2009 6 / 22

→

Important consequences for non-isoscalar targets such as 56Fe.

ν scattering could provide important new information on duality, but is currently unmeasured.

In fact, very little experimental in resonance production currently exists.

No 'free' nucleon target data for decades with none in sight.

Many experimental challenges:

- → **Unknown beam energy event by event.**
  Must construct inclusive kinematics using measurement of hadronic energy

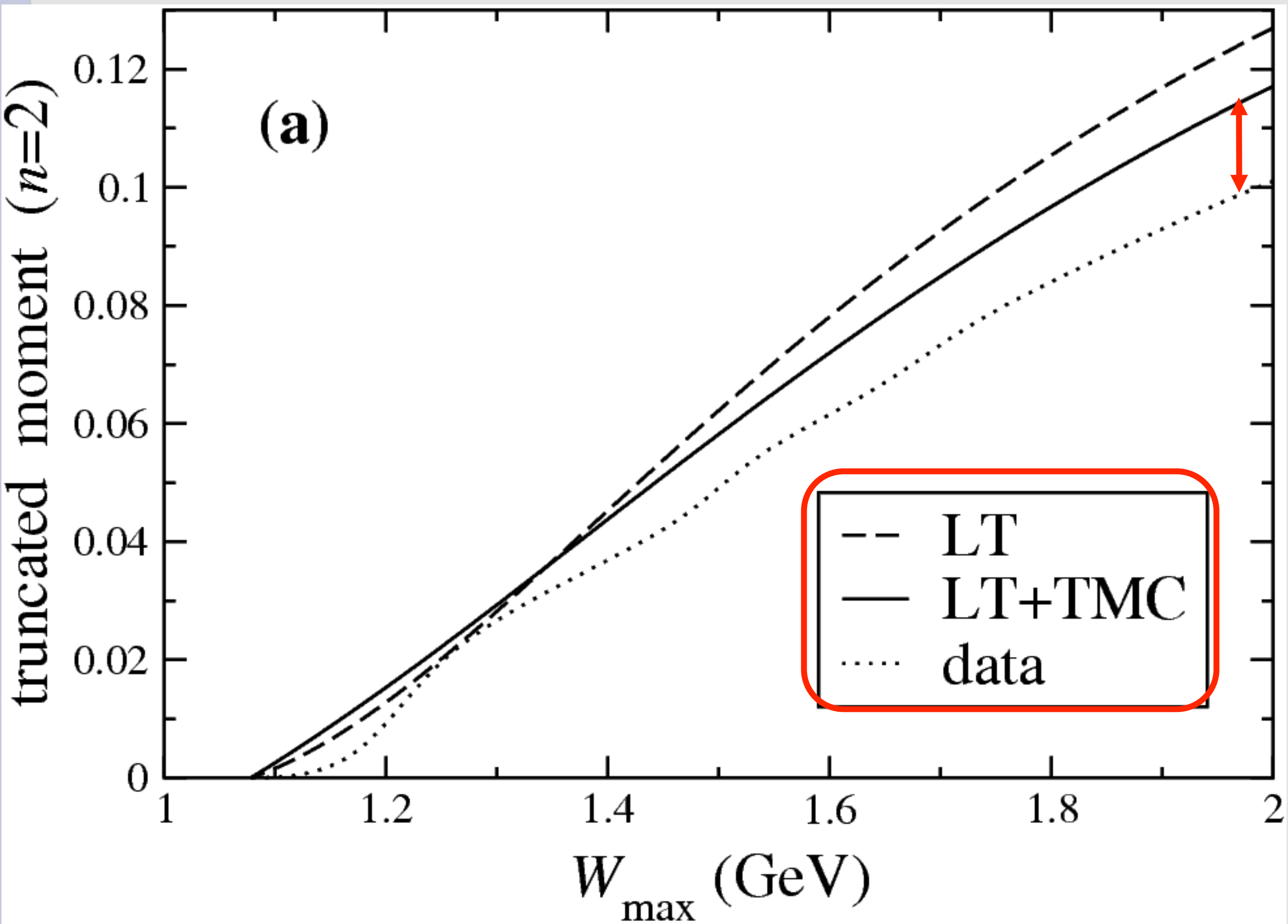
- → **Experiments such as MINERvA use nuclear targets, where separation of isoscalar effects from medium modifications are difficult to separate.**

  to extract isoscalar effects one must assume **the medium modifications**

Backup

Truncated Moment Analysis (NLO) of Hall C F_2 Data

- *Assume* data at highest Q^2 (25 GeV^2) is entirely leading twist
- Evolve (target mass corrected fit) as **NS**, with uncertainty evaluated, from Q^2 = 25 GeV^2 down to lower Q^2



This difference quantifies the higher twist.

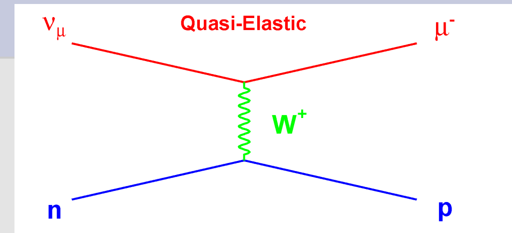
smallest x (low x = high W), largest integration range

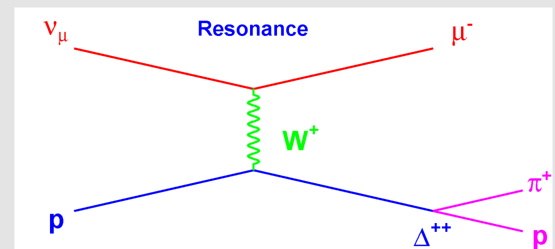
highest x, smallest integration range

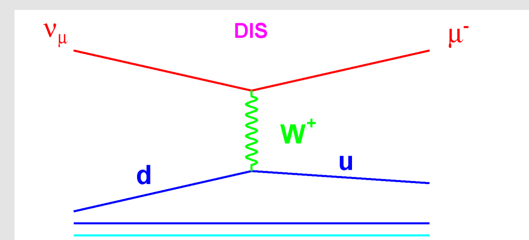
In principle, inclusive ν scattering can provide unique information on duality.

→ ν scattering is flavor sensitive

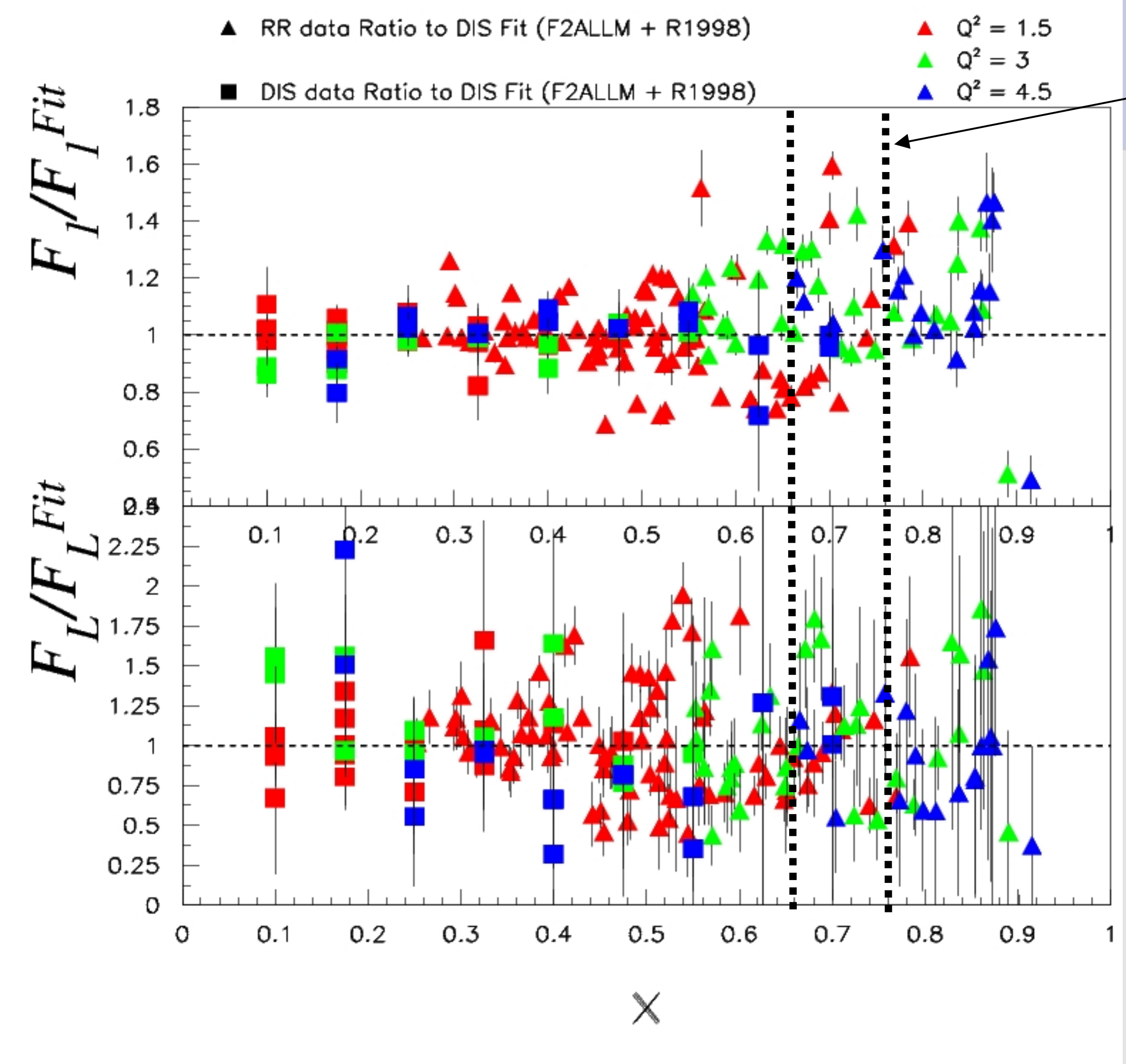
→







A closer look at Res/DIS ratios ...

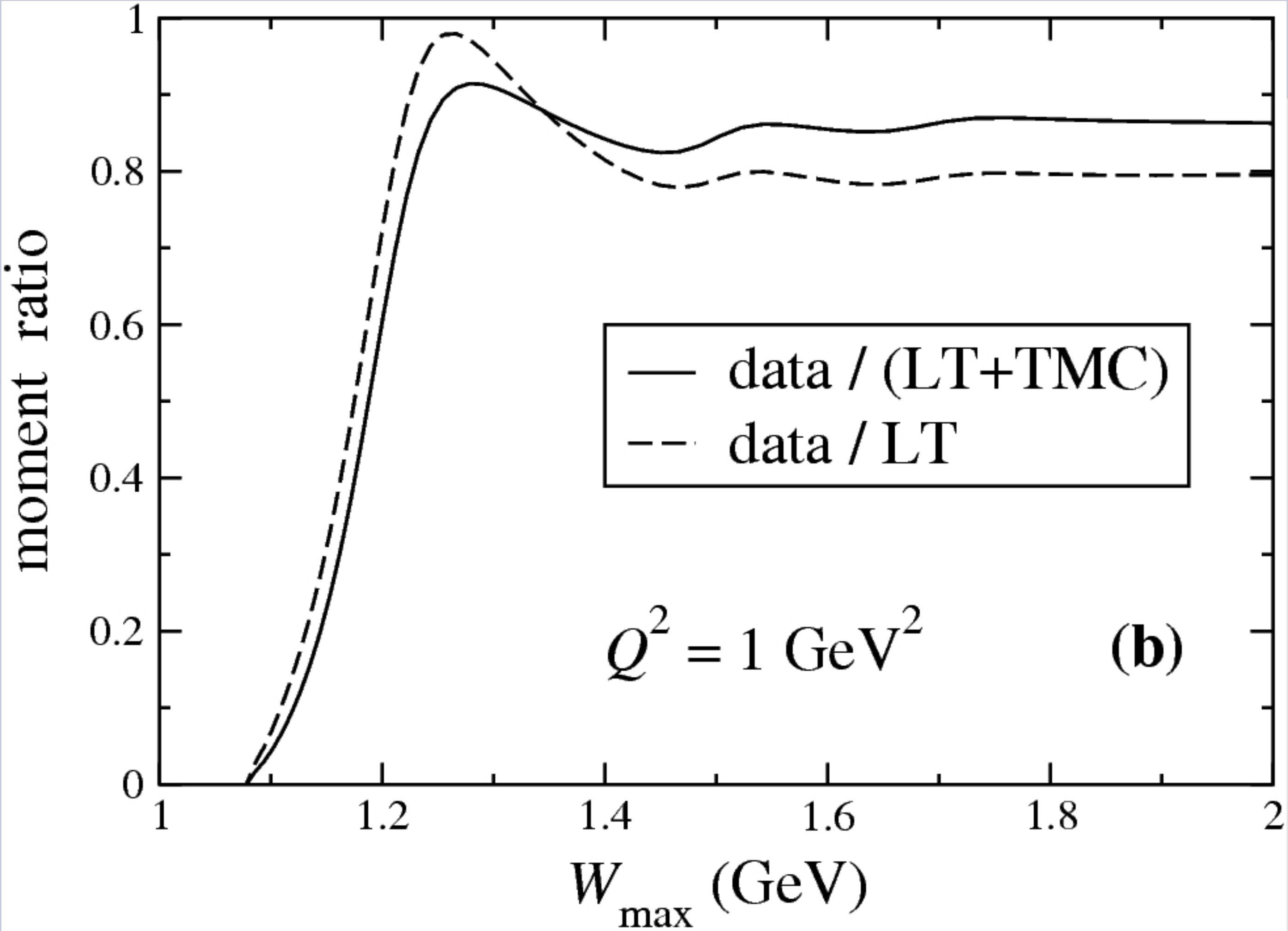


Averaging RR measurements for $0.65 < x < 0.75$ gives nearly same F_1 and F_L as DIS!

...and Q^2 dependence at fixed x is the same.

How can we use this observation to determine **duality averaged data**?

Quantified Higher Twist - ratio of curves on last plot



about 12% at $Q^2 = 1$ GeV^2

target mass corrections crucial

What about the Q^2 dependence?....

Unfolding TM Contributions from data

In the Operator Product Expansion

$$F_2^{TM}(x,Q^2) = \frac{x^2}{r^3}\frac{F_2^{(0)}(\xi,Q^2)}{\xi^2} + 6\frac{M^2}{Q^2}\frac{x^3}{r^4}\int_\xi^1 dx' \frac{F_2^{(0)}(x',Q^2)}{x'^2} + 12\frac{M^4}{Q^4}\frac{x^4}{r^5}\int_\xi^1 dx' \int_{x'}^1 dx'' \frac{F_2^{(0)}(x'',Q^2)}{x''^2}$$

$$F_1^{TM}(x,Q^2) = \frac{x}{r}\frac{F_1^{(0)}(\xi,Q^2)}{\xi} + \frac{M^2}{Q^2}\frac{x^2}{r^2}\int_\xi^1 dx' \frac{F_2^{(0)}(x',Q^2)}{x'^2} + \frac{2M^4}{Q^4}\frac{x^3}{r^3}\int_\xi^1 dx' \int_{x'}^1 dx'' \frac{F_2^{(0)}(x'',Q^2)}{x''^2}$$

$$2xF_1^{TM} = \frac{F_2^{TM} - F_L^{TM}}{r^2}$$

$$2xF_1^{(0)} = F_2^{(0)} - F_L^{(0)}$$

$$r = 1 + \nu^2/Q^2 = \sqrt{1 + \frac{4M^2x^2}{Q^2}}$$

$$\xi = 2x/(1+r)$$

Parameterize $F_{2,L}^{M=0}$ (x,Q^2) and fit $F^{TM}_{2,L}$(x,Q^2) to world data set => determine TMCs directly from data.

- ***Not* a perturbative expansion**
- ***Assume that higher twist operators obey same formalism.***

*Proton charged lepton data on F_2 and F_L fit for **$0.3 < Q^2 < 250$** and **$x > 1x10^{-4}$***

Truncated Moments

Originally developed to address lack of low *x* data

Forte and Magnea, PLB 448, 295 (1999); Forte, Magnea, Piccione, and Ridolfi, NPB 594, 46 (2001); Piccione PLB 518, 207 (2001); Kotlorz and Kotlorz, PLB 644, 284 (2007).

Idea: construct doubly truncated moments from

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx\ x^{n-2}\ F_2(x, Q^2)$$

Truncated moments follow ***DGLAP-like evolution*** equations.

$$\frac{d\overline{M}_n(\Delta x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\left(P'_{(n)} \otimes \overline{M}_n\right)(\Delta x, Q^2)$$

With modified splitting functions given by

$$P'_n(z, \alpha_S(Q^2)) = z^n P(z, \alpha_S(Q^2))$$

Allows study of ***regions in W*** within pQCD in well-defined, systematic way.

F_L^p Data Sets

Data Set	Q^2_{Min} (GeV^2)	x_{min}	Q^2_{Max} (GeV^2)	x_{max}	# Data Points
BCDMS [1]	15	0.07	50	0.65	10
EMC [2]	15	0.041	90	0.369	28
NMC [3]	1.31	0.0045	20.6	0.11	10
SLAC (Whitlow [18])	0.63	0.1	20	0.86	90
SLAC (E140x [19])	0.5	0.1	3.6	0.50	4
H1 [?]	25	0.00062	90	0.0036	5
E99-118 [20]	0.273	0.077	1.67	0.320	7

Fit Form

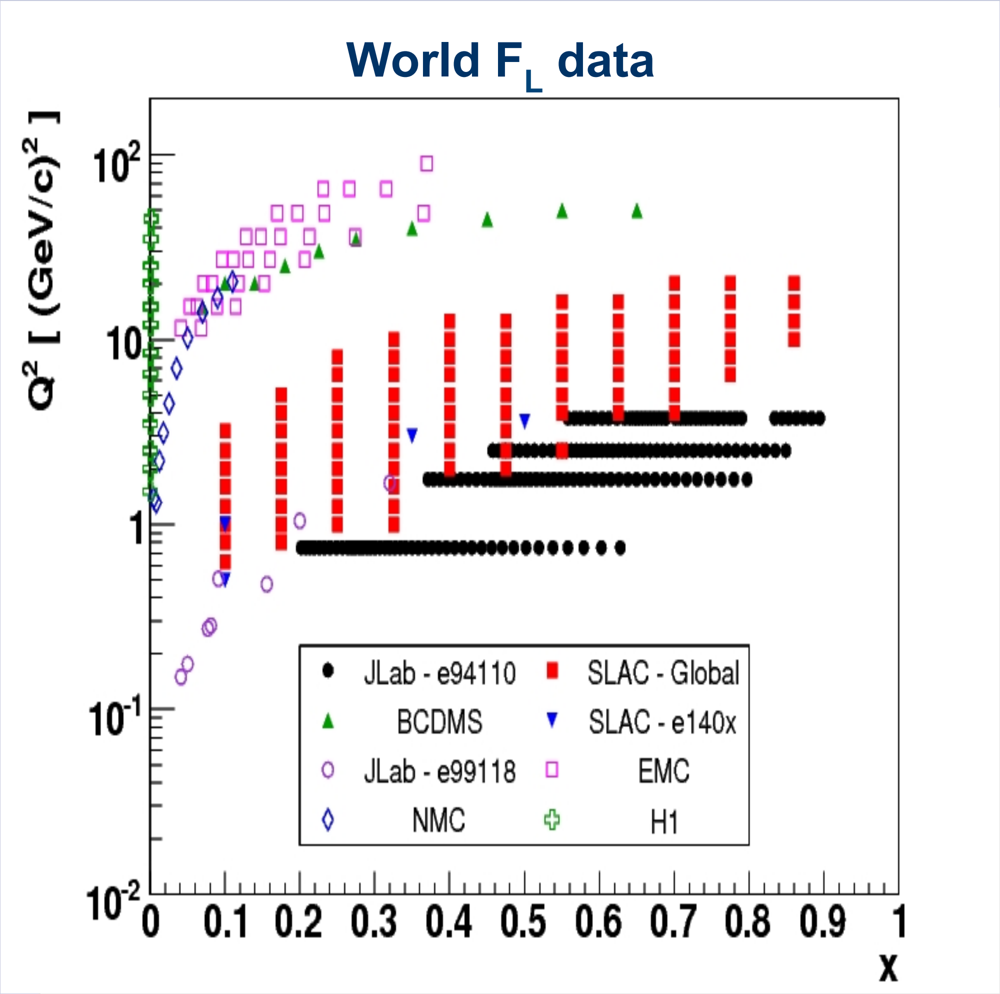
$$F^{(0)}_{2,L}(x) = Ax^B(1-x)^C(1+D\sqrt{x}+Ex),$$

F_2 parameter Q^2 dependence

$$A(Q^2) = A_1 + A_2 e^{-Q^2/A_3} + A_4 \log(0.3^2 + Q^2)$$

*Same form for **A**, **B**, **C**, **D**, and **E***

Hall C E94-110: proton L/T separated structure

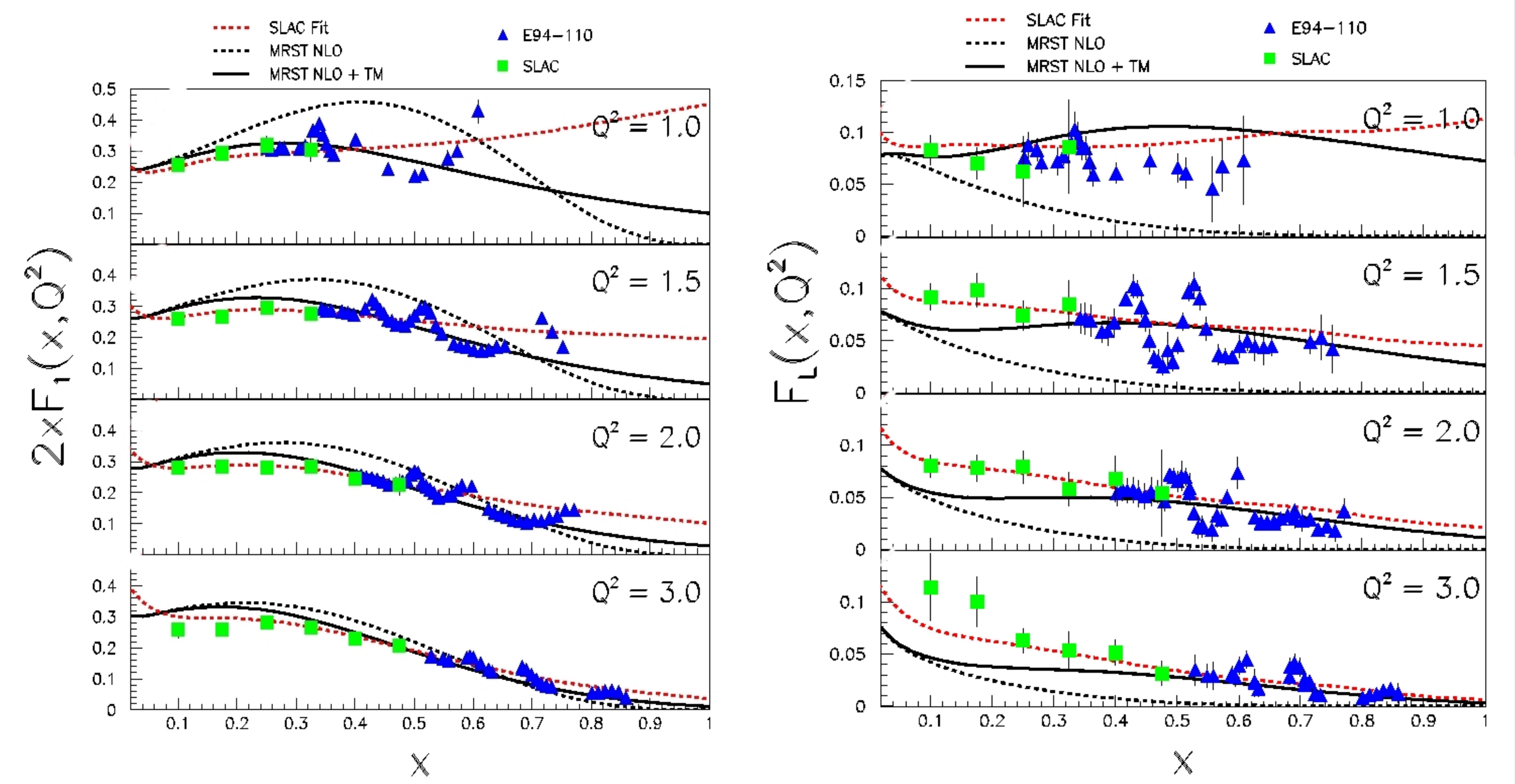


Extracted resonance region
F_1, F_L, F_2 for $0.3 < Q^2 < 5$
via Rosenbluth separations.

→ ~200 individual L/T separations

→ Allow for study of Q-H duality in *separated* structure functions.

Hall C E94-110: proton L/T separated structure



→ Duality observed to hold at 10-20% level depending on the scaling curve chosen

→ Target Mass (TM) contributions can be significant at low Q^2, especially in F_L

=> These are *necessary* for duality to hold at a reasonable level

Moments in pQCD

- Moments of Structure Functions

$$M_n^{2,L}(Q^2) \equiv \int_0^1 dx \;\; x^{n-2} \; F_{2,L}(x, Q^2)$$

$$M_n^1(Q^2) \equiv \int_0^1 dx \;\; x^{n-1} \; F_1(x, Q^2).$$

Mellin Transforms

If n = 2 → Bloom-Gilman duality integral!

- Operator Product Expansion (OPE)

$$M_n(Q^2) = \Sigma \; (nM_0^2/ Q^2)^{k-1} B_{nk}(Q^2)$$

higher twist pQCD

N = 2, 4, 6, ...

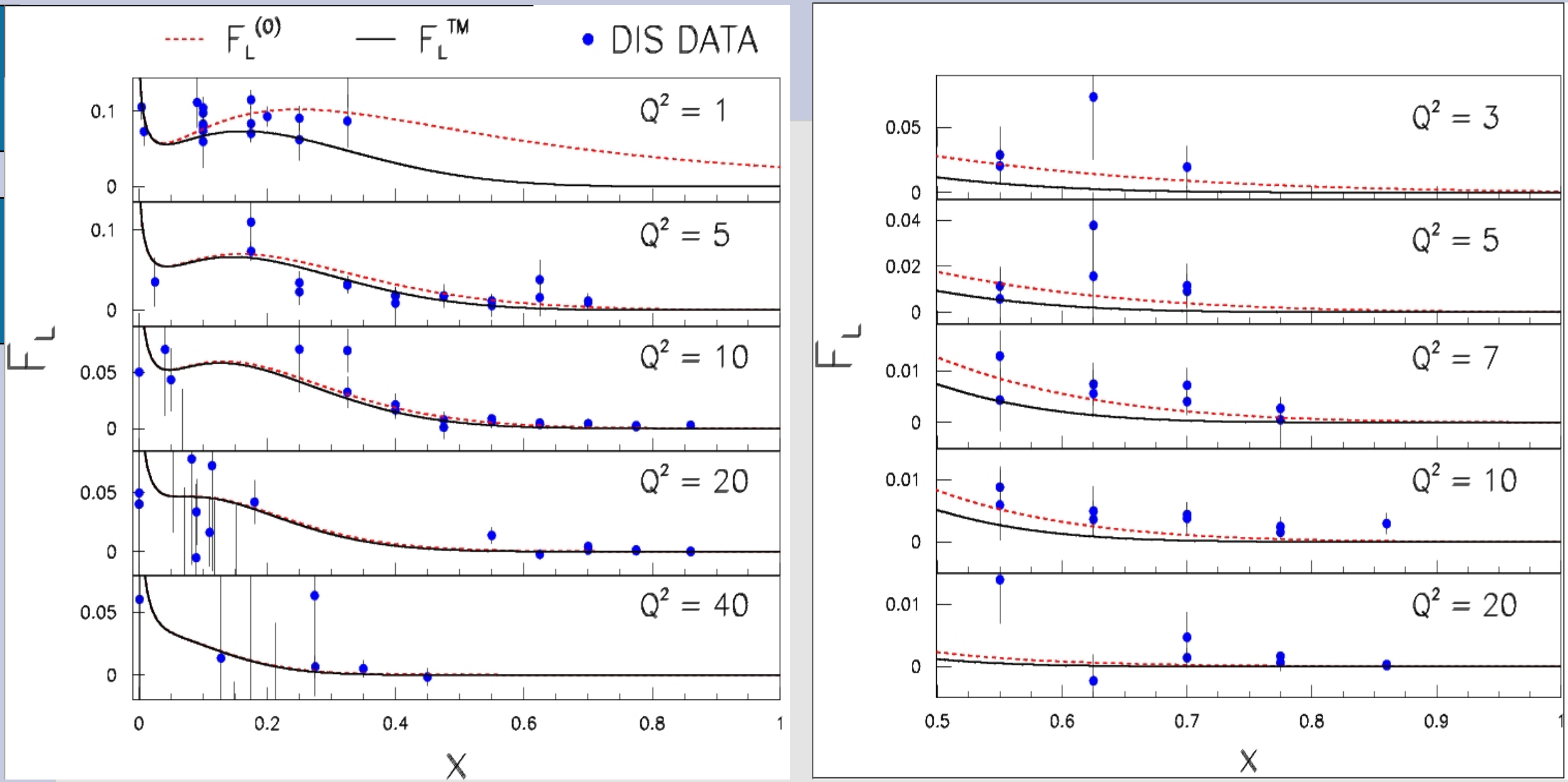
→ Global duality is assured if H-T are *canceling*. **DeRujula, Georgi, Politzer (1977)**

=> pQCD is *the* scaling curve

Note: doesn't tell us why this might be the case!

→ The determination of structure function moments allow us to study the transition of QCD from asymptotic to confinement scales.

F_L^p results from TMC fit
(MEC, J. Blumlein, H. Bottcher)



Can study → test pQCD evolution of extracted $F_{L,2}^{(o)}$

→ Further duality studies using as 'scaling' curve

- Quark-hadron duality is a non-trivial property of QCD

  → Soft-Hard Transition!

- Duality has been shown to hold in many observables thus far, including:

  1. All unpolarized structure functions (including Nuclei, see Donal Day's talk)
  2. Polarized structure functions (See Oscar Rondon's talk)
  3. Semi-inclusive

- Models are being confronted with new data, including ***free neutron***

- More experimental results are coming:

  1. *First* studies with neutrino scattering (MINERvA)!
     Unique information on F_3 and flavor sensitive probe.
  2. Higher Q^2 and x with Jlab upgrade.

The Beginning: Bloom-Gilman duality

- Inclusive e-P scattering.

-Resonance excitation at low W,Q^2

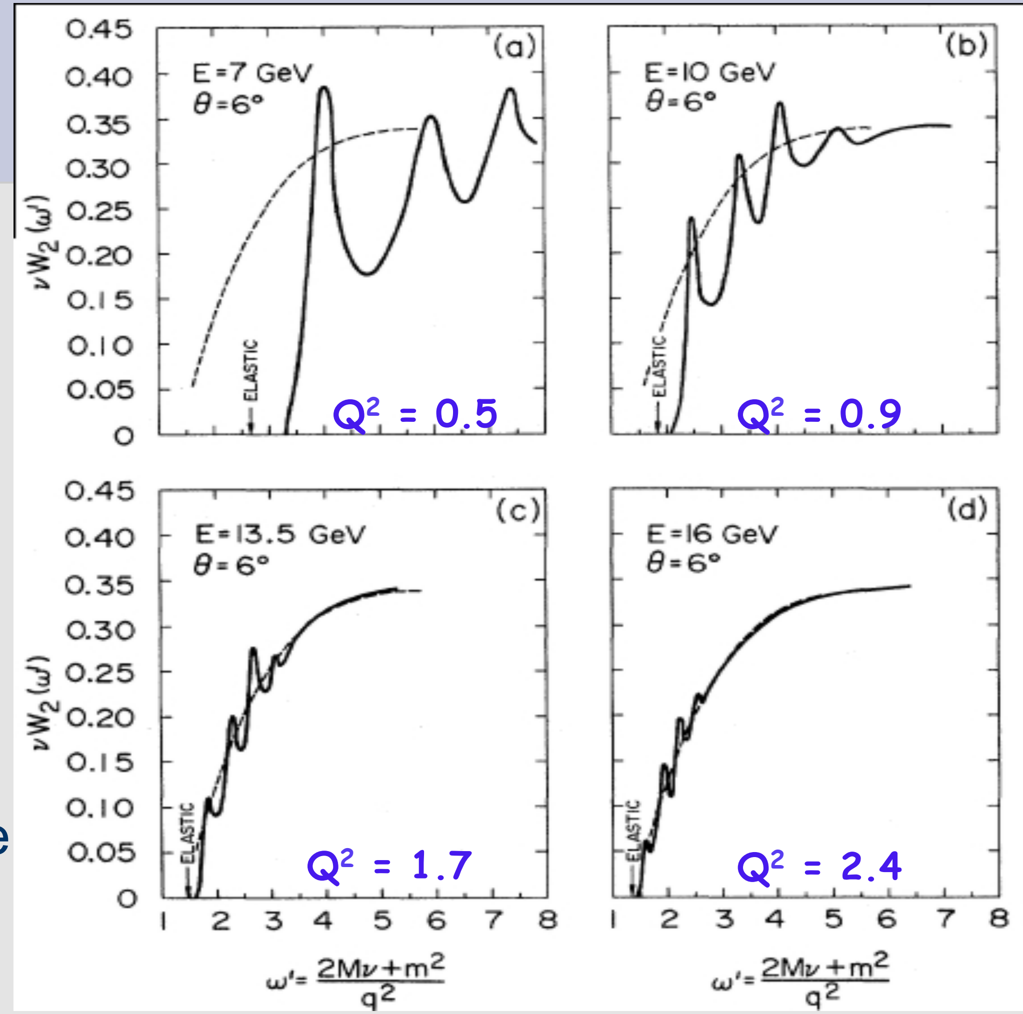
-Continuum at larger W,Q^2

- First observed by Bloom and Gilman at SLAC *prior* to the development of QCD.

  Phys.Rev.Lett.25:1140,1970.

- Noted that resonances oscillate around a 'scaling' curve at all Q^2.

- hadrons excitations follow the DIS scaling behavior.



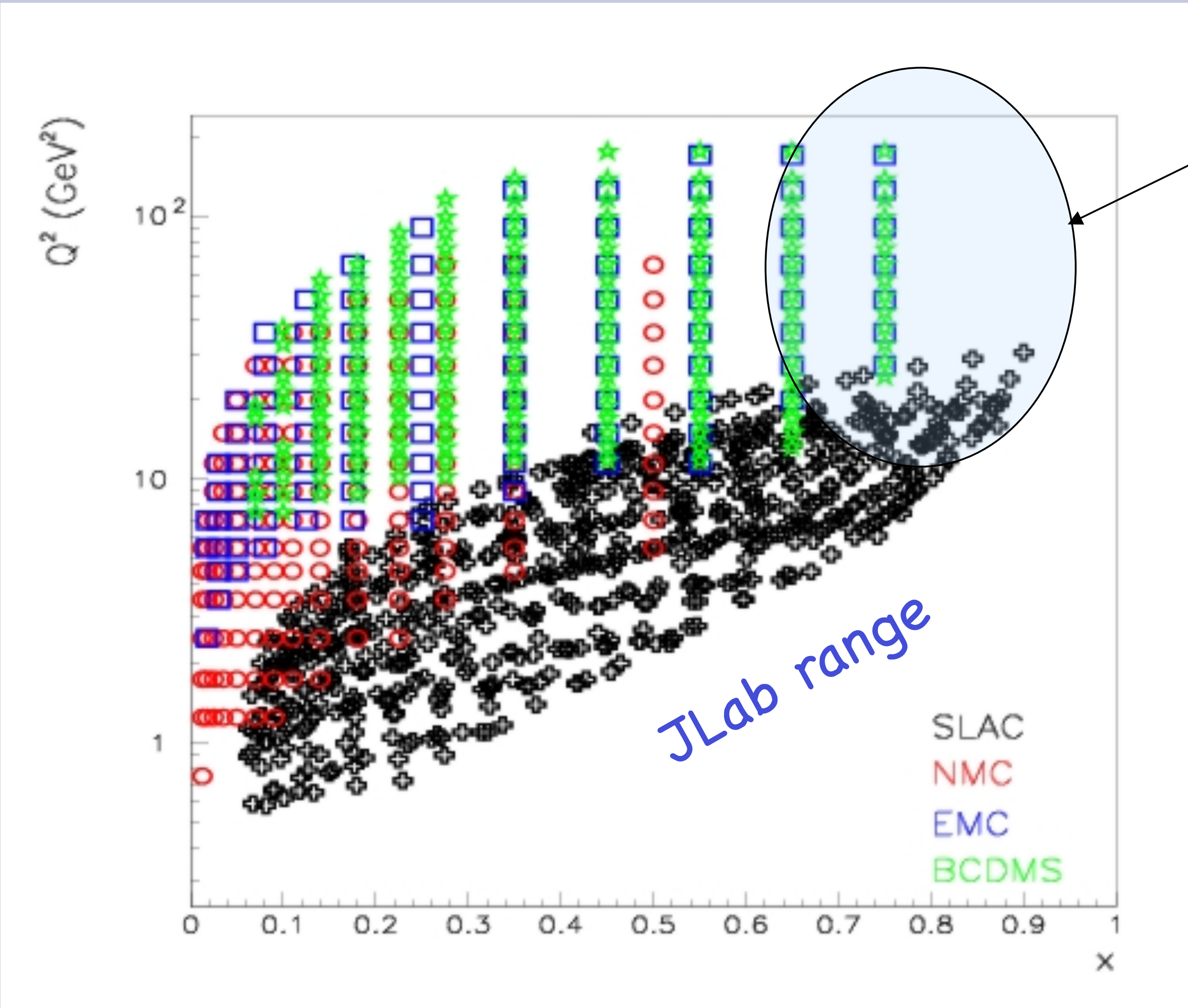
Bloom-Gilman Conclusions

- ✓ As Q^2 increased then resonances move toward [illegible]' =1 , ***each*** clearly following the smooth scaling-limit curve.

- ✓ The resonances are not a separate entity but are an intrinsic part of the scaling behavior.

- ✓ This connection between the behavior of resonances and scaling hints at a common origin in terms of a point-like substructure.

World data for charged lepton scattering from proton at high-*x*



Large uncertainties on F_2 here.

For DIS F_2, information on glue comes *only* from Q^2 evolution.

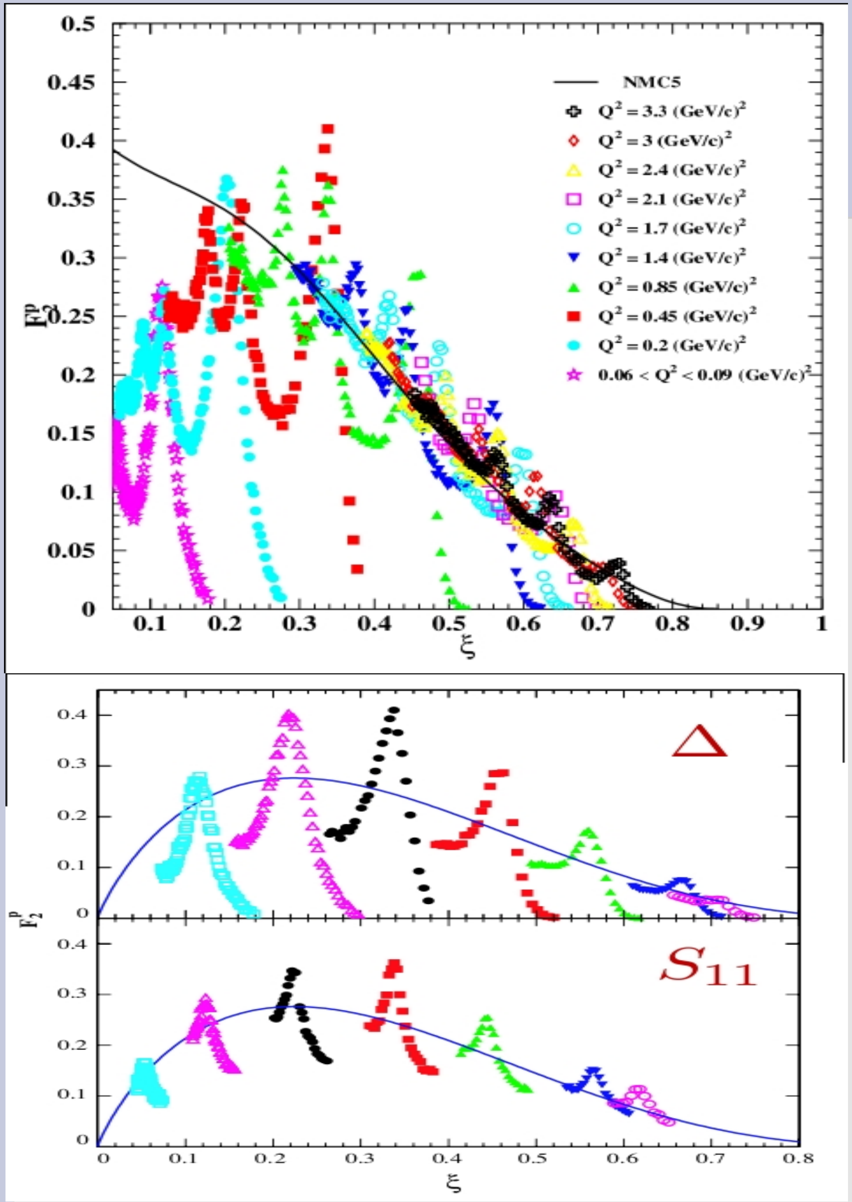
=> Use F_L to provide direct sensitivity to gluon distribution.

PDF fits should be to reduced cross sections:

$$\boxtimes_T + \boxtimes\boxtimes_L \propto 2xF_1 + \boxtimes$$

First Hall C F_2 data

(I. Niculescu et.al, Phys. Rev. Lett. 85, 1186)



→ Confirmed Bloom-Gilman observation in spectacular fashion.

→ Observed that data trace out a *valence-like* curve when $Q^2 < 0.5$

→ *local* duality is observed.

but additional contributions at finite Q^2, e.g.

Kinematic 'Target Mass' Corrections':

Fractional nucleon momentum carried by the struck quark away from Bjorken limit

$$\xi = 2x/(1+r)$$ With $$r = 1 + \nu^2/Q^2 = \sqrt{1 + \frac{4M^2x^2}{Q^2}}$$

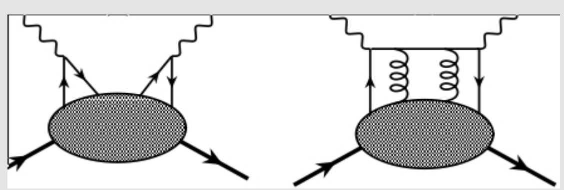
Note that [illegible] → [illegible] x [illegible] for $Q^2 \rightarrow \infty$ (or $M \rightarrow 0$) at fixed x

$$F_2^{TM}(x,Q^2) = \frac{x^2}{r^3}\frac{F_2^{(0)}(\xi,Q^2)}{\xi^2} + 6\frac{M^2}{Q^2}\frac{x^3}{r^4}\int_\xi^1 dx' \frac{F_2^{(0)}(x',Q^2)}{x'^2} + 12\frac{M^4}{Q^4}\frac{x^4}{r^5}\int_\xi^1 dx' \int_{x'}^1 dx'' \frac{F_2^{(0)}(x'',Q^2)}{x''^2}$$

'Massless' limit

Higher Twist contributions (H-T)':

Quark-Quark correlations: eg. gluon exchange between struck and spectator quarks.



→ **Older PDFs not enough strength at large *x***

=> *looks* like larger duality violations (20-30%).

→ **Not as much a failure of duality, but unconstrained PDFs at large *x***

→ **New efforts to relax kinematic constraints and include TMCs and HTs in PDF fits result in much smaller duality violations observed (< 10%, except at Δ(1232)).**

=> telling us that *on average* resonance region H-T are the same as the DIS.

CTEQ6x

S. Alekhin, J. Blumlein, S. Klein, S. Moch, Phys. Rev. D 81, 014032 (2010).

Accardi, E.C, Keppel, Melnitchouk, Monaghan, Morfín, Owens, Phys. Rev. D 81, 034016 (2010).

Can we use duality data to constrain large *x* parton distributions?

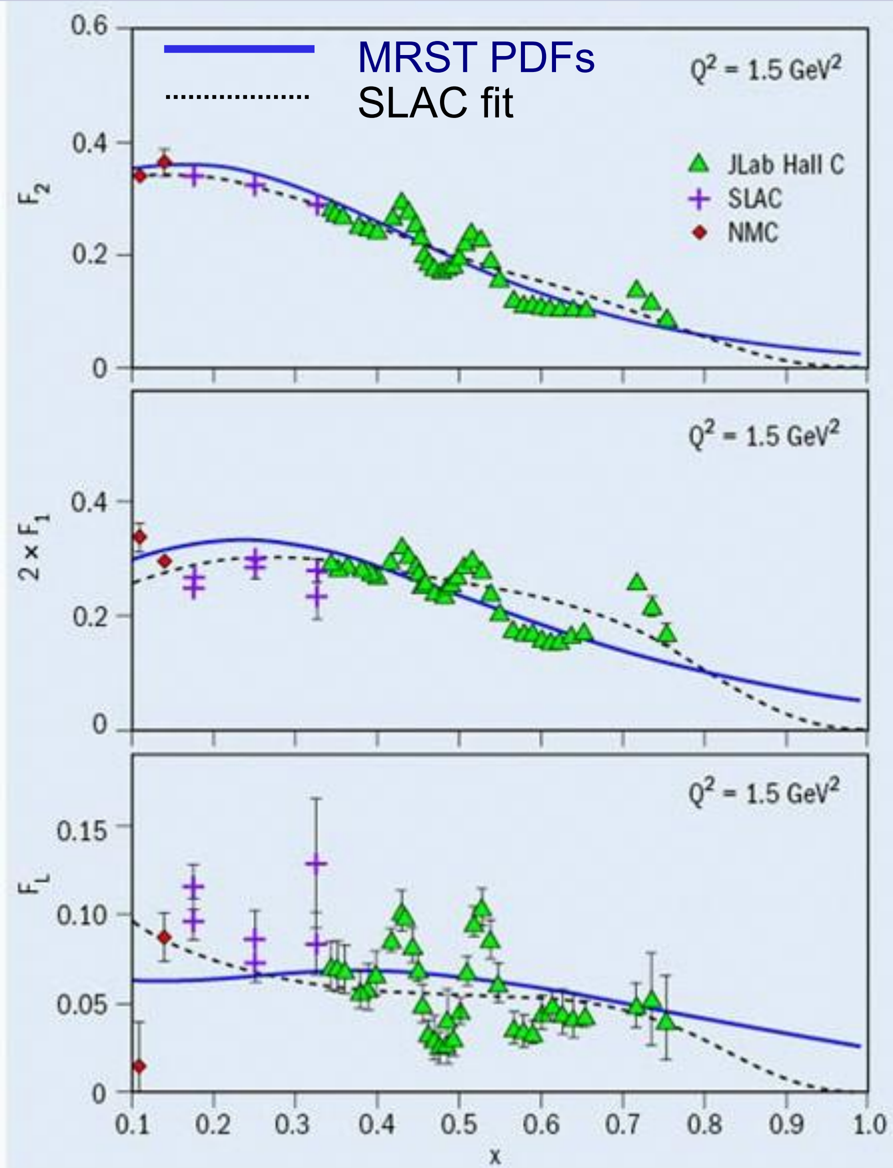
Perhaps... must test if duality averaged data can be fit consistently with higher W data when including TM / H-T.

In principle this is no different then how H-T is handled in the fits to scattering data with relaxed kinematics.

Important to constrain standard model physics as much as possible for cleanest interpretation of new physics at LHC and Tevatron.

Since uncertainties on large *x* PDFs at small Q^2 evolve to smaller *x* at large Q^2.

Later duality observed in separated F_1 and F_L



Observed now in separated transverse (F_1) and longitudinal (F_L) structure functions.

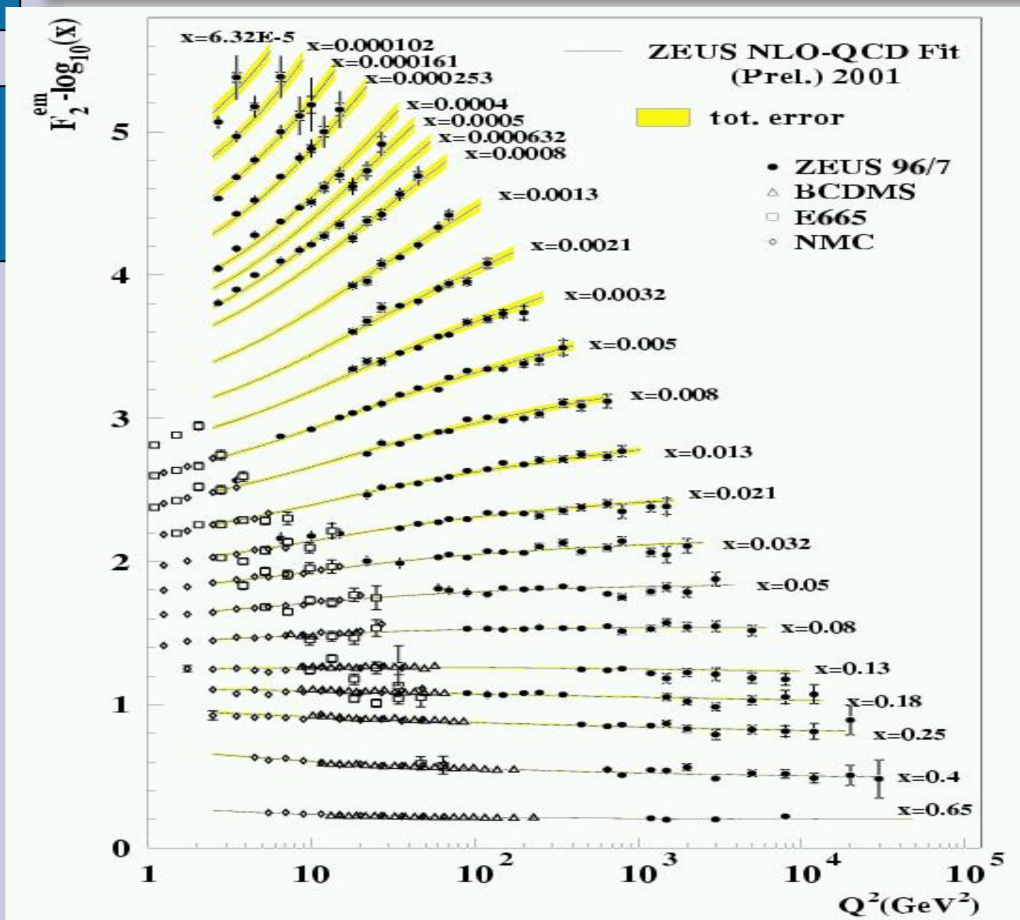
Fascinating link between hadron and quark phenomenology- challenges our understanding of strong interaction dynamics.

"The successful application of duality to extract known quantities suggests that it should also be possible to use it to extract quantities that are otherwise kinematically inaccessible."
(CERN Courier, December 2004)

Tool to access large *x* regime?

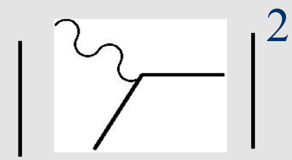
Separation of scale => Q^2 dependence of DIS structure functions governed by perturbative QCD

Scaling in F_2 measured to high precision over many orders of magnitude in *x* and Q^2,

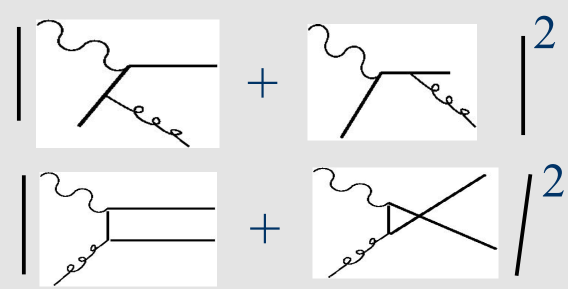


Single quark scattering (leading twist)

$$F_2(x,Q^2) = x \, \Sigma \, e_q^{\,2} \, \mathbf{q}(x,Q^2)$$

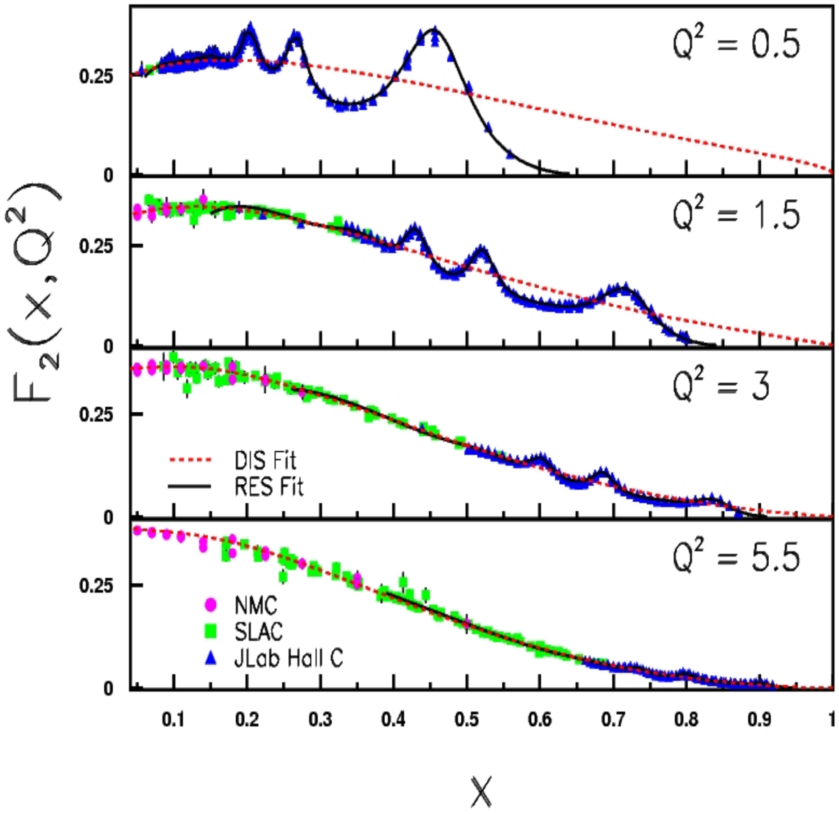


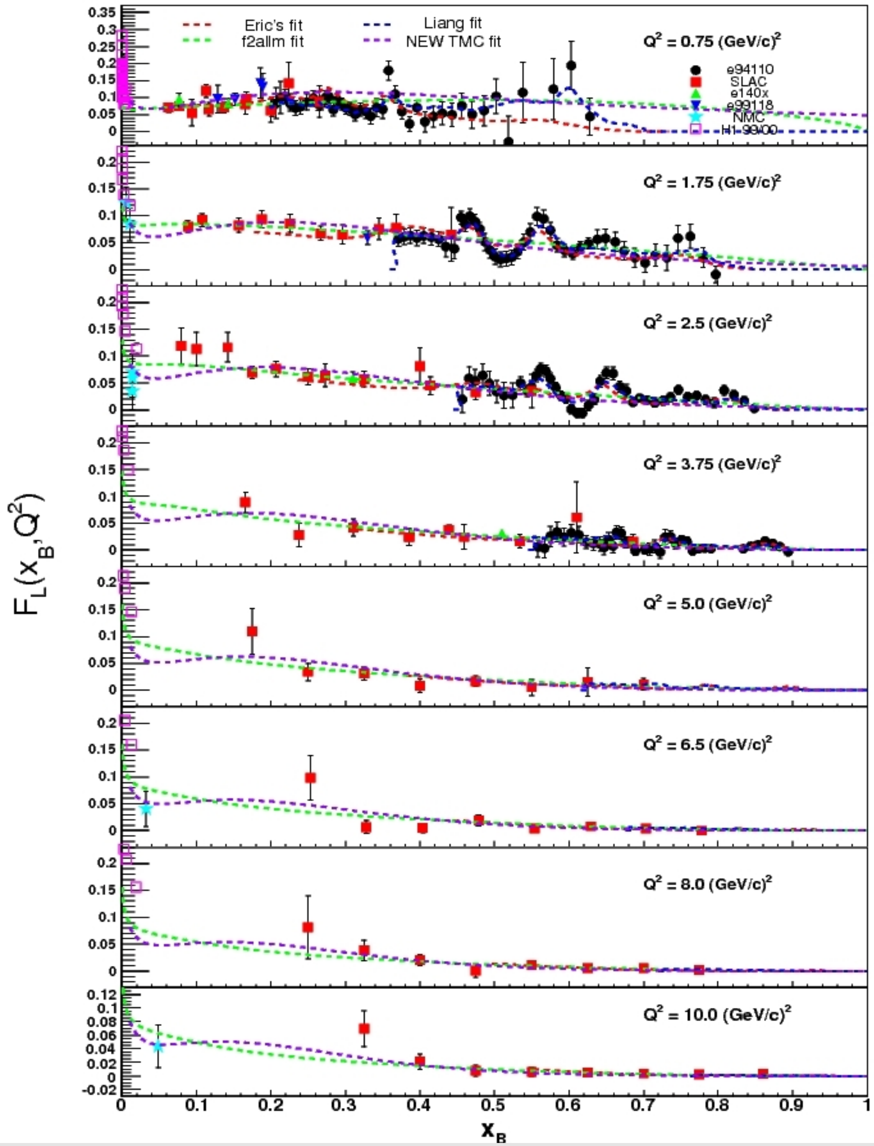
Where the $\mathbf{q}(x,Q^2)$ evolve via pQCD.
Order $\alpha_s(Q^2)$ corrections



Status of unpolarized proton

DIS fit – 'F2ALLM' H.Abramowicz and A.Levy, hep-ph/9712415
Res fit - E.C. and P.E. Bosted, PRC 81,055213





→ Duality observed in ALL unpolarized structure functions

Are the CN moments of data what should be compared to pQCD?

In pQCD

This is not true for finite M^2/Q^2 due to TMCs. However, *Nachtmann (1973* found a way to project out the massless limit contribution via

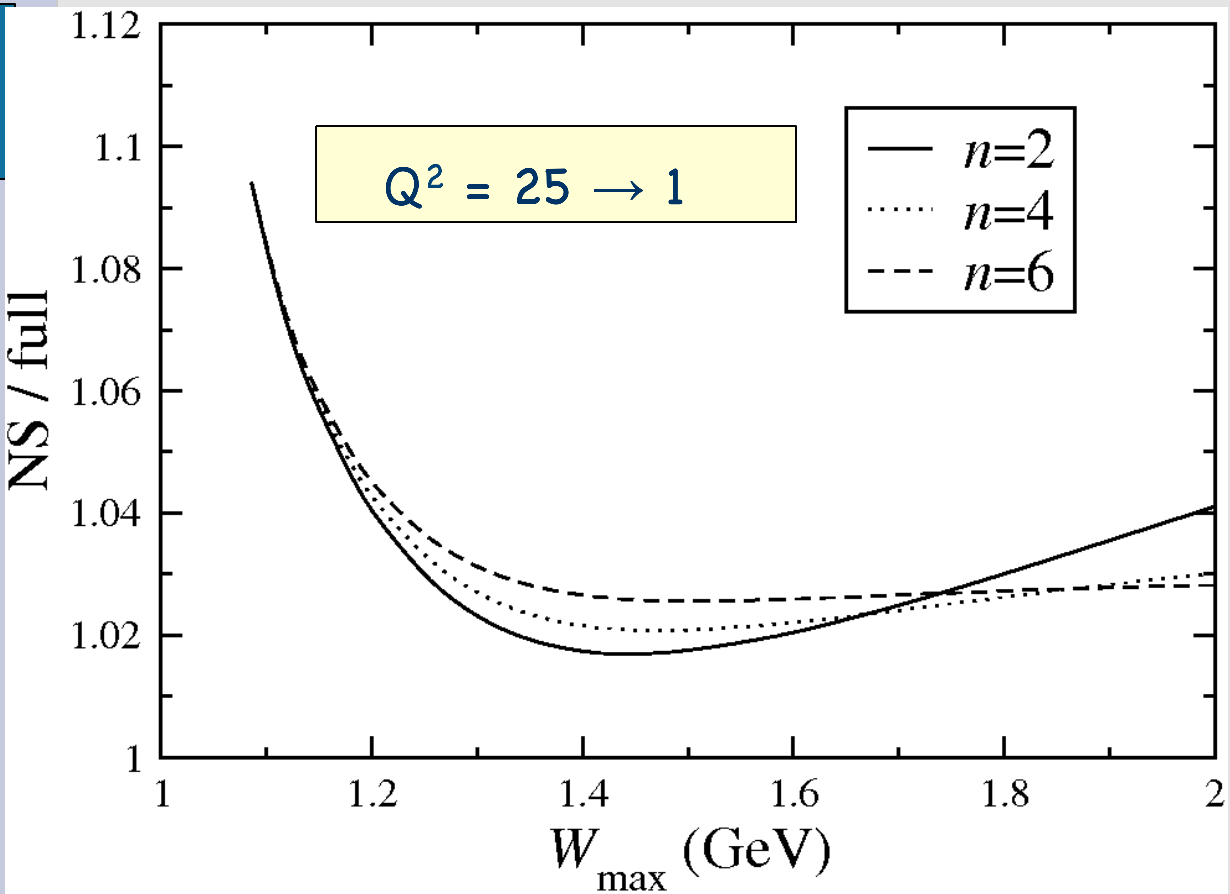
$$M_L^{(n)}(Q^2) = \int_0^1 dx\, \frac{\xi^{n+1}}{x^3} \Big\{ F_L(x,Q^2) + \frac{4M^2x^2}{Q^2}\frac{(n+1)\xi/x - 2(n+2)}{(n+2)(n+3)} F_2(x,Q^2) \Big\} \qquad (1)$$

→ **Here F_2, F_L are the *experimental* structure functions.**

→ **Nachtmann moment effectively removes the TM contributions.**

First check Non-Singlet vs full evolution.

Evolve F_2 from MRST PDFs from Q^2 = 25 to 1 GeV^2 using both **N-S** and full (**N-S + Singlet**).



Largest difference for n=2 ***moments***

~4% effect

Higher order (higher n) moments dominated by larger *x* (smaller W) regime

Recall - high W corresponds to low x - glue increasingly more important. Becomes dominant uncertainty.