A Model That Realizes Veneziano Duality

- L. Jenkovszky, V. Magas, J.T. Londergan and A. Szczepaniak,
  Int'l Journ of Mod Physics **A27**, 1250517 (2012)

- Review, Veneziano dual model "resonance-Regge"
- Limitations of Veneziano model
  zero-width resonances, linear Regge trajectories
  violates unitarity, analyticity
- Unitary, analytic extensions of Veneziano picture
- A Toy Model to demonstrate extended Veneziano model
- examine Q^2, x dependence of our model

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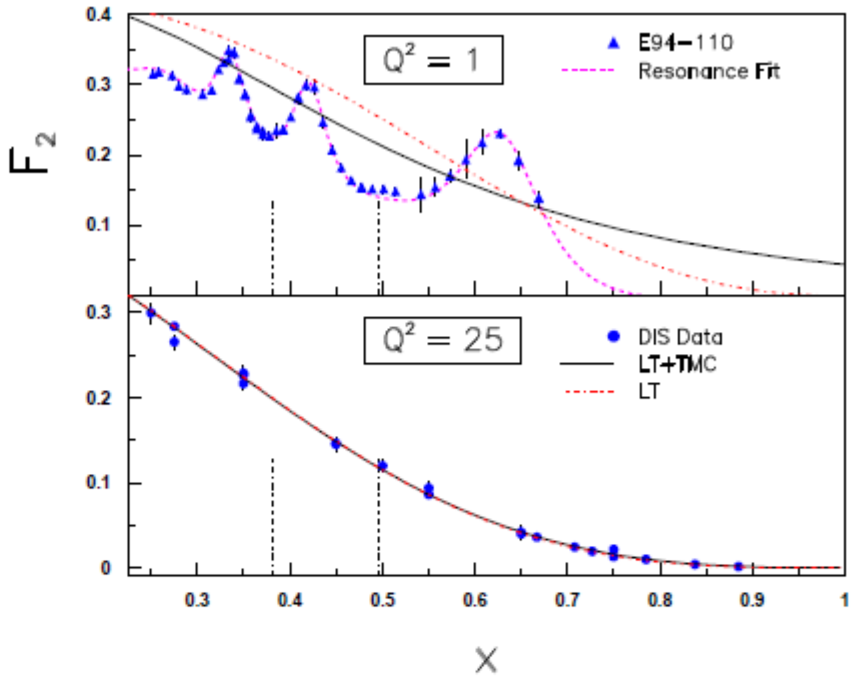
Bloom-Gilman Duality

Low Q^2: inelastic X-section shows series of hadron resonance peaks

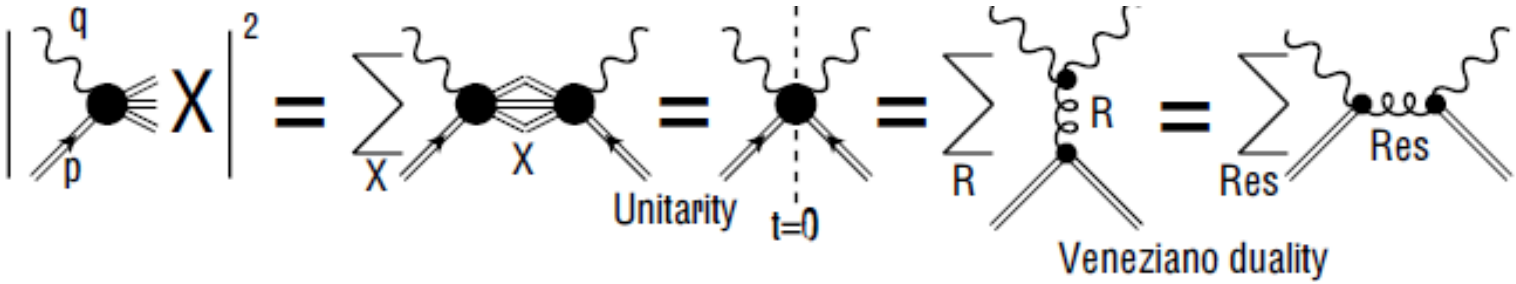
- **High Q^2**: X-section smooth, described by partonic processes



- Bloom-Gilman or parton-hadron duality: smooth partonic X-section is average of X-section over resonance peaks

- Another kind of duality; Veneziano (resonance-Regge) duality

Veneziano (resonance-Regge) Duality



• **e.g. nucleon EM interactions:** can describe as sum over all resonant states (s-channel poles)

• alternatively, in limit of high s, small |t|, can describe as sum over t-channel singularities (Regge trajectories)

- Two descriptions give identical expansion of structure function
- Search for amplitudes that manifest this duality

The Veneziano Dual Amplitude:

G. Veneziano, Nuovo Cim **A57**, 190 (1968)

The Veneziano amplitude is given by

$$V(s,t) = \int_0^1 dz\, z^{-\alpha(s)}(1-z)^{-\alpha(t)} = \frac{\Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}$$

The Γ function has simple poles at all non-positive integers.
Thus the amplitude Veneziano amplitude can be decomposed as a series of resonance poles

$$V(s,t) = \sum_{n=1}^{\infty} \frac{C_n(t))}{n-\alpha(s)}\ , \qquad C_n(t) = \frac{\Gamma(n+\alpha(t)+1)}{n!\,\Gamma(\alpha(t)+1)}$$

Amplitude characterized by series of zero-width resonance poles;
"s-channel trajectory" $\alpha_s(s)$ determines location of resonances

Duality in the Veneziano Model:

Duality in the Veneziano model is demonstrated by taking the asymptotic form of the Γ function

$$V(s,t)_{|s|\to\infty} \to [-\alpha(s)]^{\alpha(t)}\,\Gamma(1-\alpha(t))$$

(using the Stirling formula for the Γ function). Therefore the Veneziano dual amplitude also exhibits Regge behavior

$$V(s,t) \sim s^{\alpha(t)}$$

The original Veneziano model contains a series of zero-width resonances, and corresponds to linear Regge trajectories.

Drawbacks of Original Veneziano Model:

Because the Veneziano model contains zero-width resonances, and perfectly linear Regge trajectories,

it violates both unitarity and analyticity.

Look for generalizations of the Veneziano formula that incorporate both finite resonance width and nonlinear (even complex) Regge trajectories.

From QCD we know that inclusive (e,p) reactions are most efficiently described in terms of the virtuality Q^2 of the exchanged photon.

Look for a dual amplitude of the form $D(s,t,Q^2)$. The amplitude needs to retain the dual features of poles in the s-channel and Regge behavior in the t-channel.

A Form for the Dual Amplitude:

A dual amplitude $D(s,t,Q^2)$ that satisfies the requirement of poles in the s-channel and Regge behavior in the t-channel can be written:

$$D(s,t,Q^2) = \int_0^1 dz \left(\frac{z}{g}\right)^{-\alpha_s(s')-\beta\left(Q^{2''}\right)-1} \left(\frac{1-z}{g}\right)^{-\alpha_t(t'')-\beta\left(Q^{2'}\right)-1}$$

Here α_s determines features of the s-channel resonances, α_t is a t-channel Regge trajectory, β is a smooth function of Q^2, and g is a constant. We use the notation $v' = v(1-z)$ and $v'' = vz$ where $v = [s,t,Q^2]$.

This function has been shown by Bugrij to have the pole structure, threshold singularities and spectral function behavior required by unitarity.

(not unique! Several alternative ways to address these issues)

Properties of the Dual Amplitude:

The dual amplitude D(s,t,Q^2) produces poles in the s-channel and Regge behavior in the t-channel.

s-channel poles in this model have the form

$$D_n(s,t,Q^2) = g^{n+1} \frac{C_n(t,Q^2)}{n - \alpha_s(s) - \beta(0)}$$

poles shifted by constant β(0)

In the limit |s| → ∞ and finite t the amplitude exhibits Regge behavior

$$D(s,t,Q^2) \sim s^{\alpha_t(t)+\beta(0)}\, g^{\beta\left(Q^2\right)} \ , \quad |s| \to \infty$$

explicit Q^2 dependence of dual amplitude

The Structure Function and the Dual Amplitude:

The structure function $F_2(x,Q^2)$ is related to the dual amplitude $D(s,t=0,Q^2)$ through the optical theorem.

We choose a function $\beta(Q^2)$ that connects data between the large-Q^2 and small-Q^2 regions:

$$\beta(Q^2) = -1 - \frac{\alpha_t(0)}{\ln g} \ln\left(\frac{Q^2 + Q_0^2}{Q_0^2}\right)$$

This leads to asymptotic behavior of the structure function,

$$F_2(x,Q^2)\Big|_{x\to 0} \sim x^{1-\alpha_t(0)} \left(\frac{Q^2}{Q^2+Q_0^2}\right)^{\alpha_t(0)}$$

$$F_2(x,Q^2)\Big|_{x\to 1} \sim (1-x)^{2\alpha_t(0)\ln 2g/\ln g}$$

This choice of $\beta(Q^2)$ agrees with known behavior of F_2 at both large & small x; also Q^2 behavior consistent with scaling violations

A Toy Model to Demonstrate Duality:

For pedagogical purposes, choose a dual amplitude $D(s,t=0,Q^2)$ with only three resonance peaks, roughly equal in height.

We then construct the amplitude in all regions and examine the behavior of the dual amplitude and structure function.

In the resonance region,

$$D(s,t=0,Q^2) \sim \sum_{n=1}^{3} g^{n+1} \left(\frac{gQ_0^2}{Q^2+Q_0^2} \right)^{\alpha_t(0)} \frac{1}{n-\alpha_s(s)+1}$$

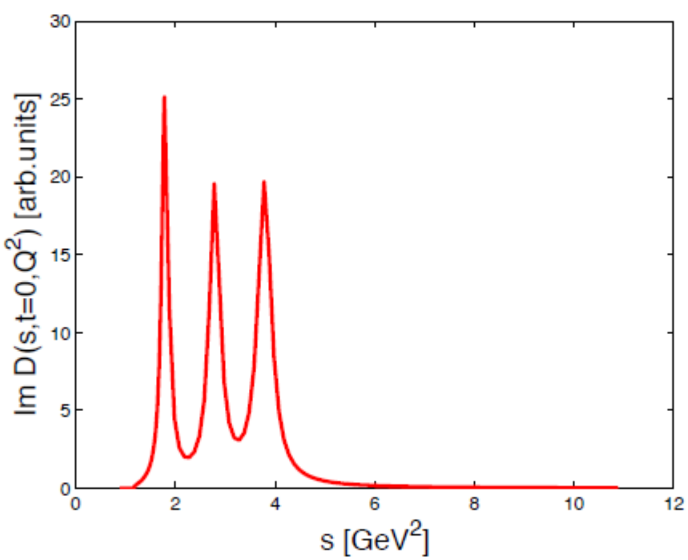
$$\alpha_s(s) = \alpha_s(0) + \alpha_s'(0)s + \gamma(\sqrt{s_0} - \sqrt{s_0 - s}) \ , \quad s_0 = (m+m_\pi)^2$$

Parameters $\alpha_s(0) = 0.1$, $\alpha_s'(0) = 1\ \text{GeV}^{-2}$, $\gamma = 0.1\ \text{GeV}^{-1}$
$\alpha_t(0) = 0.5$, $Q_0 = 1\ \text{GeV}$, $g = 1.5$

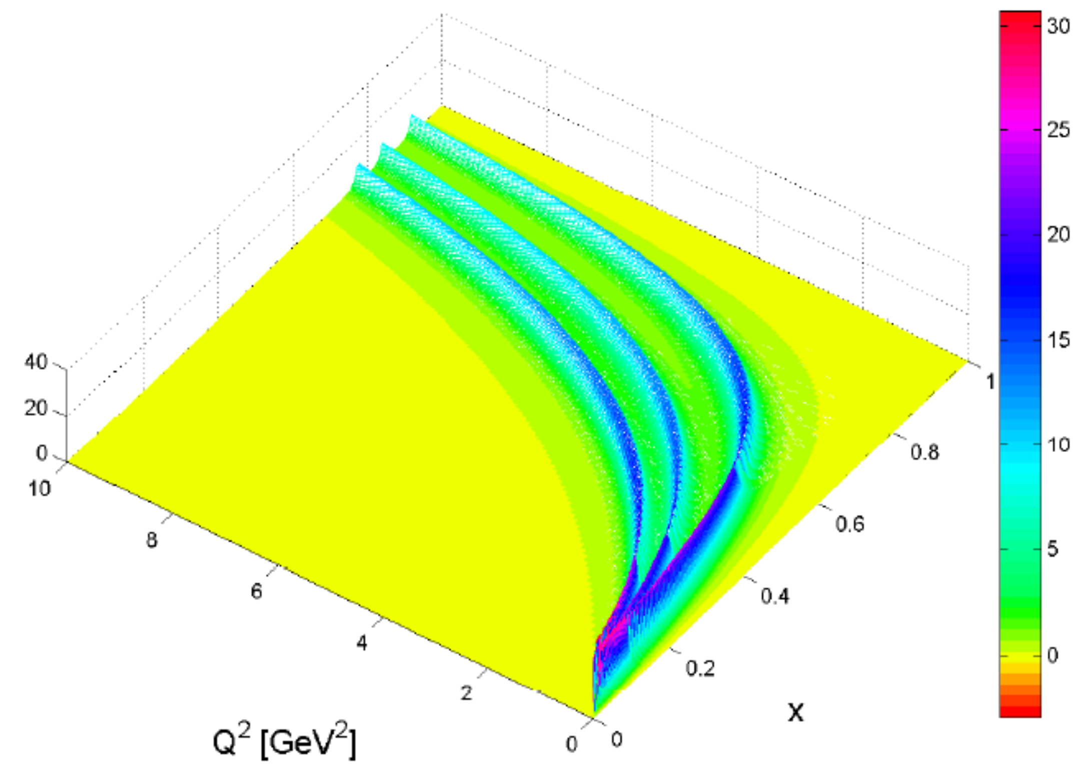
Dual Amplitude as a function of Q^2 and x

Plot imaginary part of the dual amplitude (arbitrary units).

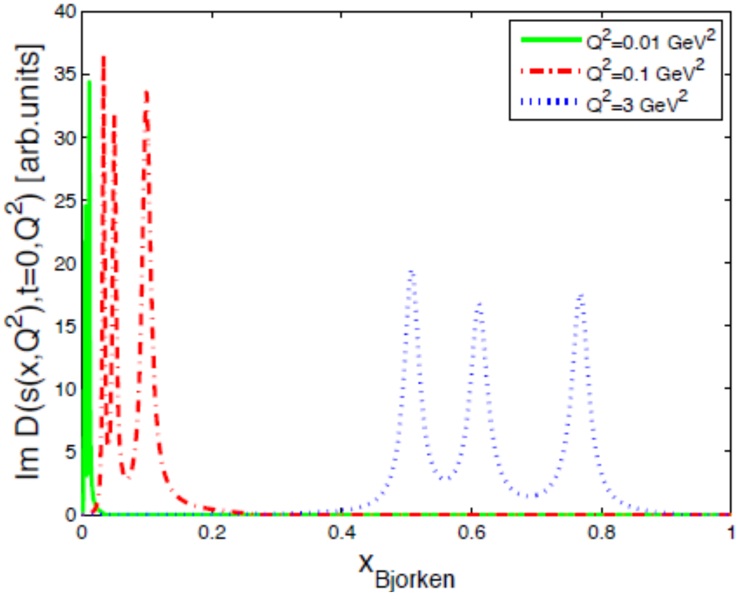
Imag. part of dual amplitude in the x-Q^2 plane.
Resonances are function of $s \sim Q^2(1-x)/x$.
As x decreases, 3 resonance peaks at large x move to lower Q^2. If more resonances were added, would merge into power-law (Regge-dominated) behavior at low x.

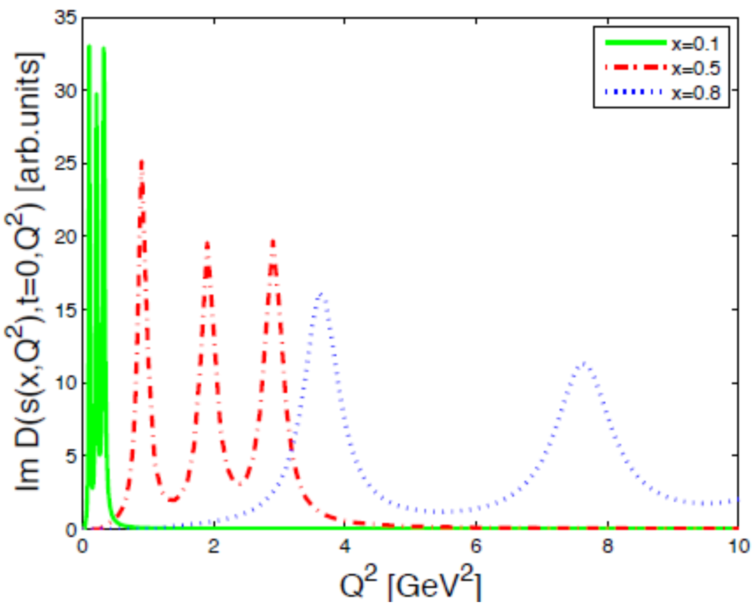


Plot vs s shows 3 resonance peaks



Dual Amplitude – slices in Q^2 and x

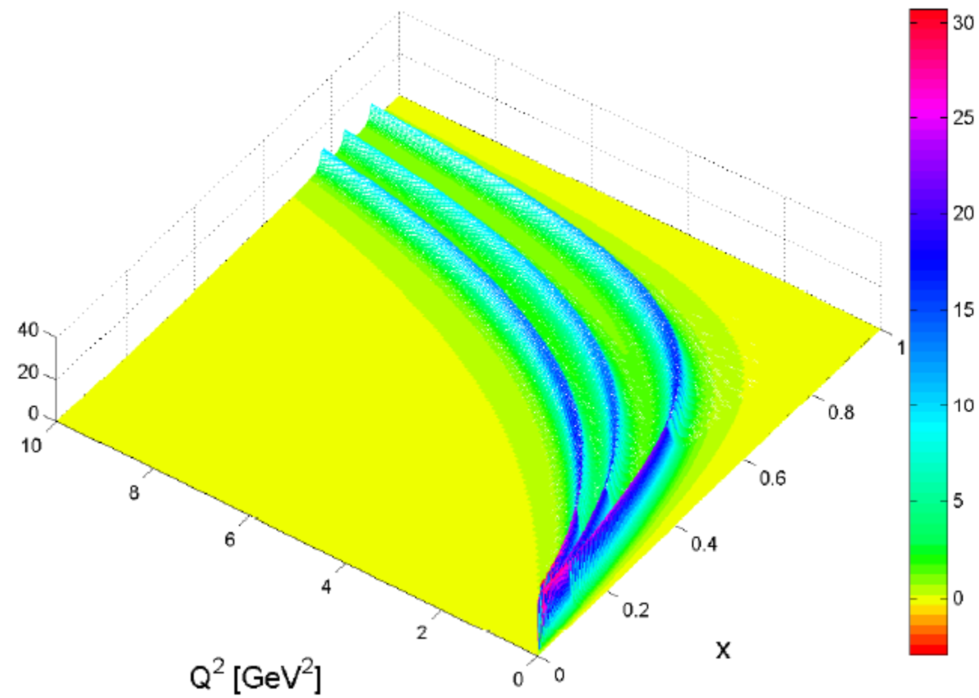




Top: imag part of dual amplitude vs. x for various Q^2.

Bottom: dual amplitude vs. Q^2 for various x.

poles in $s \sim Q^2(1-x)/x$

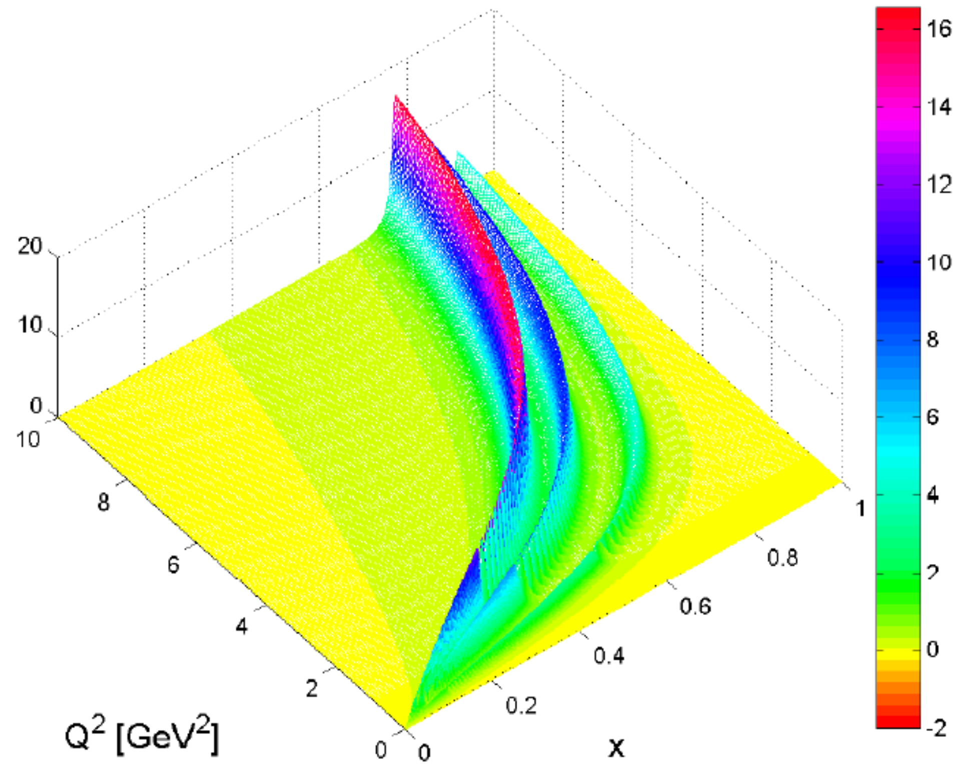


Structure function F_2 vs. Q^2 and x

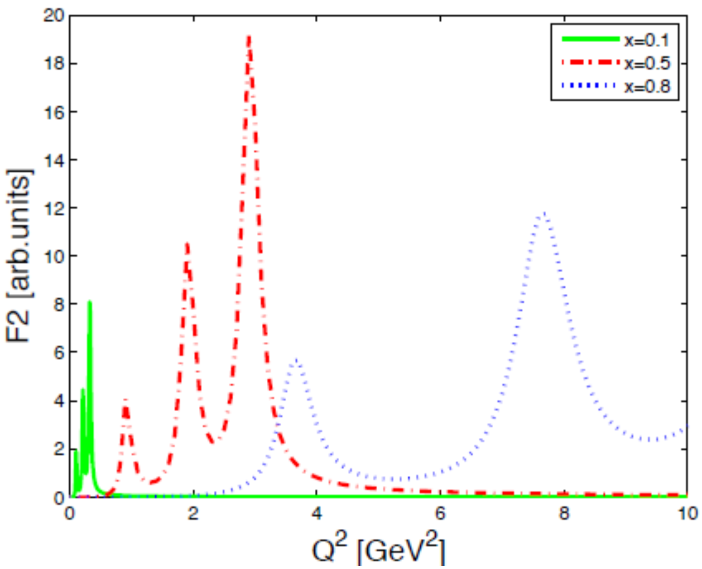
Behavior of the structure function F_2 in the x-Q^2 plane, showing the behavior of the structure function with only 3 s-channel resonances.

We see the behavior of our dual model in both large and small-x regions and how it changes with Q^2.

poles in $s \sim Q^2(1-x)/x$



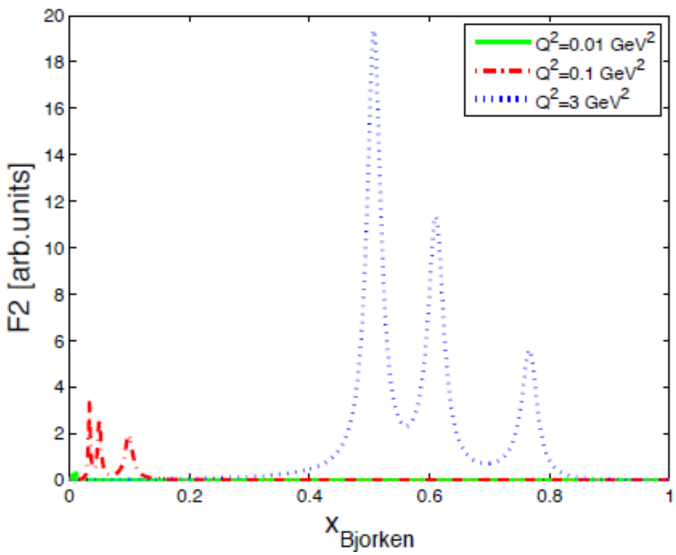
Slices showing F_2 behavior vs. Q^2 and x

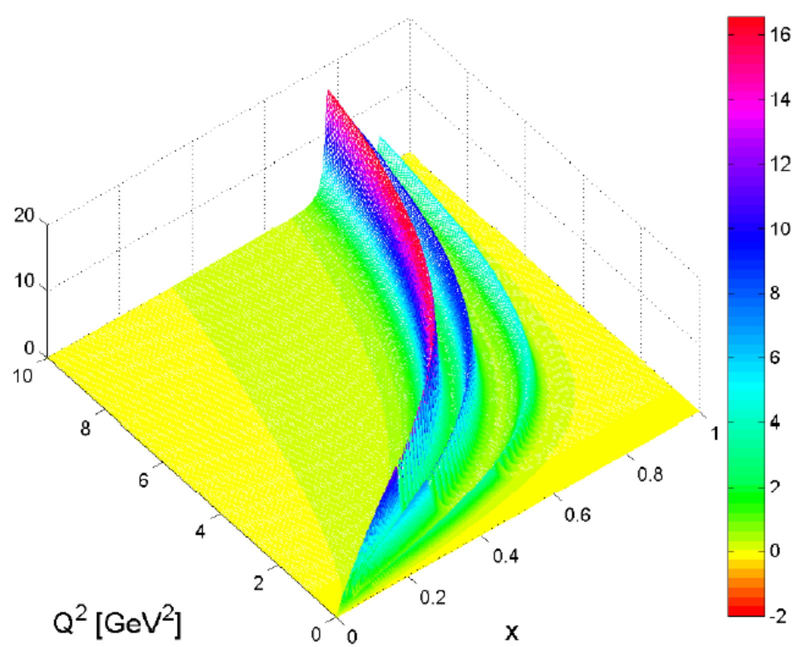


Top: structure function F_2 vs. Q^2 for various values of x.

Bottom: F_2 vs. x for various Q^2 values. With increasing Q^2, addition of resonances leads to smooth large-x behavior (similar to Bloom-Gilman scaling).

poles in s ~ $Q^2(1-x)/x$





Conclusions:

✓ Original Veneziano model – amplitude producing s-channel resonances also shows Regge behavior

✓ Veneziano model: zero-width resonances, linear Regge trajectories → violated unitarity, analyticity

✓ Modifications of Veneziano: retain resonance-Regge connection but insure unitarity, analytic dual amplitudes

✓ Include explicit Q^2 dependence, with guidance from observed scaling violations

- Toy Model:
- include only 3 resonances
- construct dual amplitude, F_2 structure function
- observe how resonances move in x-Q^2 plane, intuit how duality will arise in this model.
- this model will produce something qualitatively similar to Bloom-Gilman duality (at large x and Q^2, individual resonance peaks will merge into a smooth F_2 structure function).

Conclusions (cont'd):

- Future work:
- extend model to incorporate a reasonable set of s-channel e-N resonances.
- obtain quantitative Q^2 dependence of structure function?
- incorporate spin and isospin dependence of e-N scattering.
- can this model simultaneously produce resonance-Regge and Bloom-Gilman duality? Qualitative: **yes**. Quantitative test: **??**

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