# Duality in Inclusive Structure Functions: Present and Future

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# Quark-Hadron Duality

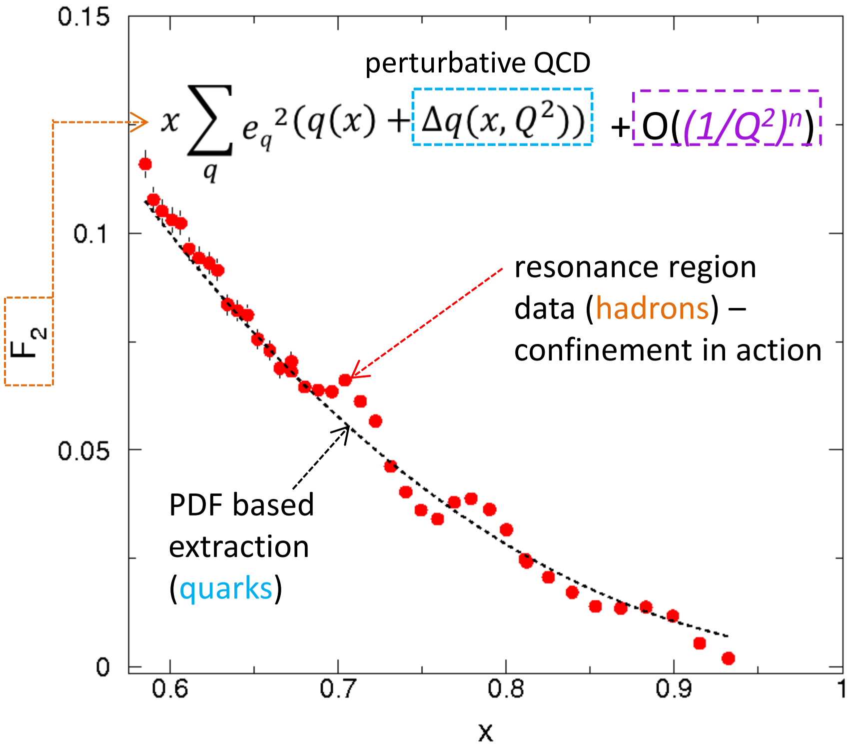
- What is Quark-hadron duality?

Quark-hadron duality = complementarity between quark and hadron descriptions of observables

$$\sum_{hadrons} = \sum_{quarks}$$

Description suitable for
low-energy regime
(confinament)

Description suitable for
high-energy regime
(asymptotic freedom)



*Resonance region data average to PDF based curve: $1/Q^{2n}$ corrections small or cancel on average*

→ We can use either set of **complete** basis states to describe physical phenomena

→ **In practice**, at finite energy we typically have access only to a **limited set of basis states**

→ **Even so**, quark-hadron duality shown to hold globally and locally in **many observables**

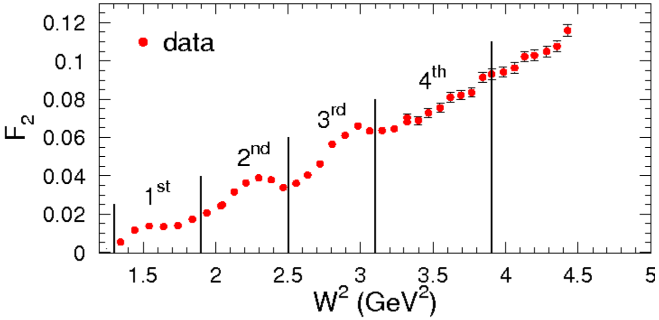
# Quark-Hadron Duality: Verification

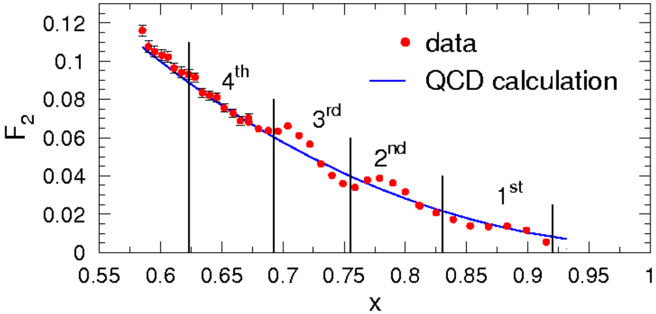
*Example: $F_2$ structure function*

❖ Define duality intervals

| **Region** | **1st** | **2nd** | **3rd** | **4th** | **DIS** | **global** |
|---|---|---|---|---|---|---|
| $W_{min}$ | 1.3 | 1.9 | 2.5 | 3.1 | 3.9 | 1.9 |
| $W_{max}$ | 1.9 | 2.5 | 3.1 | 3.9 | 4.5 | 4.5 |

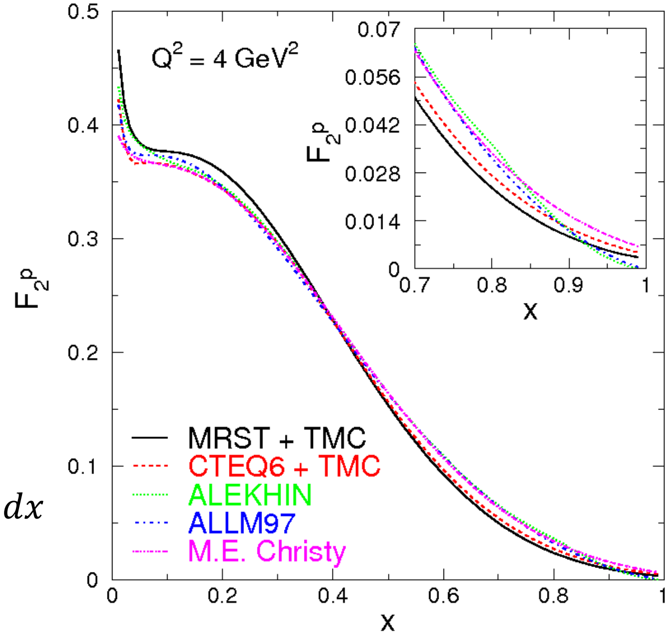
→ there is arbitrariness in defining the local W intervals; typically try to catch peaks and valleys within one interval





❖ Define *scaling curve*: PDF-based extraction constrained in the x regime where duality is verified

→ per fixed $Q^2$ the resonance region sits at highest x; scaling curve used for duality verification must be from 2nd generation PDF fits (Alekhin *et al*., CTEQ-JLab)



How well data average to the scaling curve?

❖ Calculate ratio: $\int_{x_{min}}^{x_{max}} F^{data}(x,Q^2)\,dx \Big/ \int_{x_{min}}^{x_{max}} F^{param.}(x,Q^2)dx$

# Duality: Inclusive Measurements

- **Unpolarized beam, unpolarized target**

**$\sigma_T$, $\sigma_L$ cross sections** – photo-absorption of T (helicity +/- 1) and/or L (helicity 0) $\gamma^*$

$$\frac{d^2\sigma}{d\Omega dE'} = \Gamma(\boldsymbol{\sigma_T}(x,Q^2) + \varepsilon\boldsymbol{\sigma_L}(x,Q^2)) = \Gamma\sigma_T(1+\varepsilon\boldsymbol{R})$$

always measured

sometimes measured

$\boxed{F_2(x,Q^2) \sim [\boldsymbol{\sigma_T} + \boldsymbol{\sigma_L}]}$ → *results in this talk*

$F_L(x,Q^2) \sim \boldsymbol{\sigma_L}(x,Q^2)$ → *results in Eric's talk*

$F_1(x,Q^2) \sim \boldsymbol{\sigma_T}(x,Q^2)$

- **Longitudinally polarized beam, longitudinally polarized target**

$$\frac{d\sigma}{d\Omega dE'} = \Gamma\left(\sigma_T + \varepsilon\sigma_L + hP_z\sqrt{1-\varepsilon^2}\sigma'_{TT}\right) \quad \sigma_T = \frac{1}{2}(\sigma_{1/2} + \sigma_{3/2}) \quad \sigma_{TT} = \frac{1}{2}(\sigma_{3/2} - \sigma_{1/2})$$

Helicity cross sections: **$\sigma_{1/2}$, $\sigma_{3/2}$**

→ correspond to the spin of $\gamma^*$ and proton anti-aligned (1/2) and aligned (3/2)

→ in the Bjorken limit proportional to the positive and negative helicity PDFs

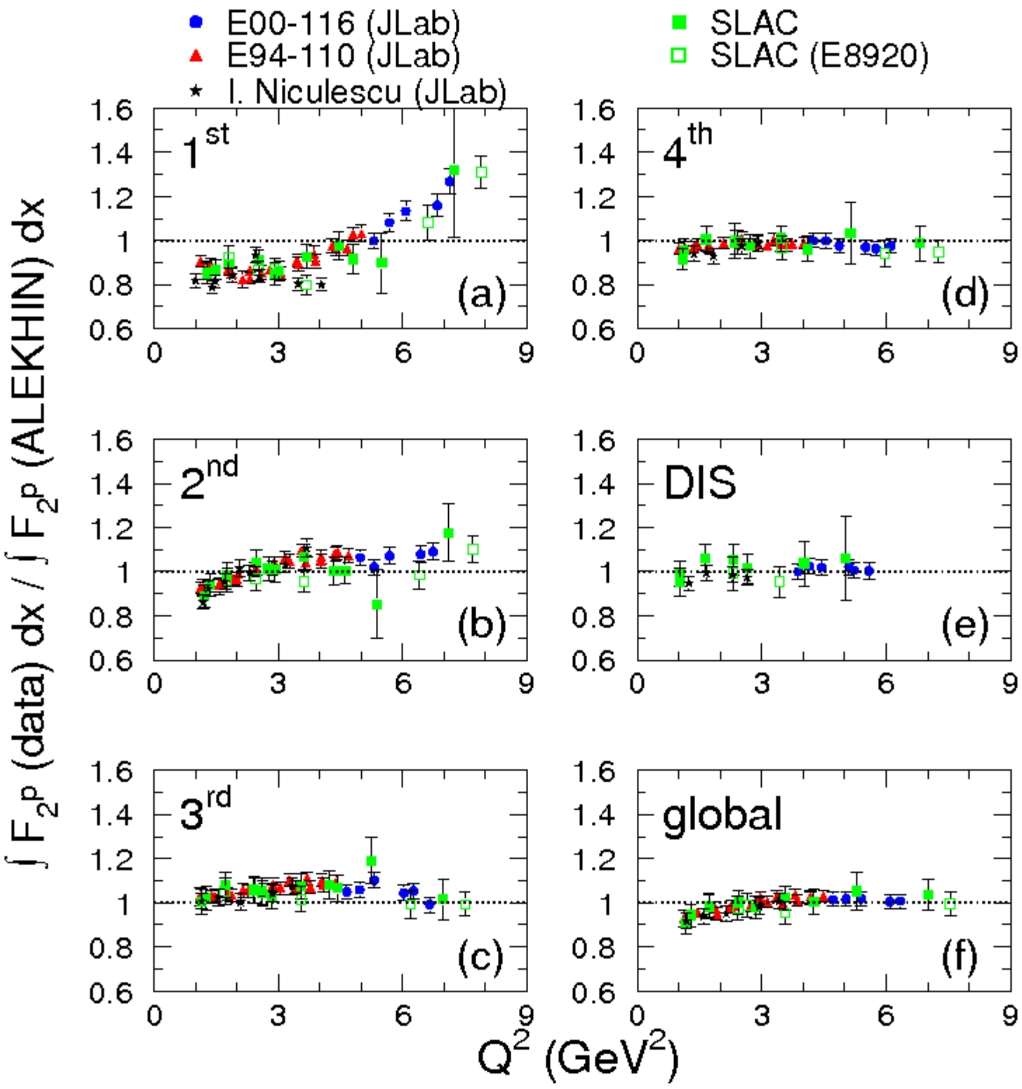
→ defined to be positive

Dimensionless Helicity Structure Functions: $\mathbf{H_{1/2}} = \frac{MK}{4\pi^2\alpha}\sigma_{1/2}$, $\mathbf{H_{3/2}} = \frac{MK}{4\pi^2\alpha}\sigma_{3/2}$

# Duality: $F_2$ Proton Structure Function

Verified!

- **Unpolarized beam (electron), unpolarized target (proton)**



Alekhin *et al*.: NNLO + HT + TM

→ Ratio within 10% globally

→ For 4th RES region and DIS, ratio very close to 1 for entire $Q^2$ range analyzed

→ For 2nd and 3rd regions ratio within 5-10 % for entire $Q^2$ range analyzed

1st : special case

→ models predict stronger violations of duality

→ calculation based on handbag diagram may break at such low W

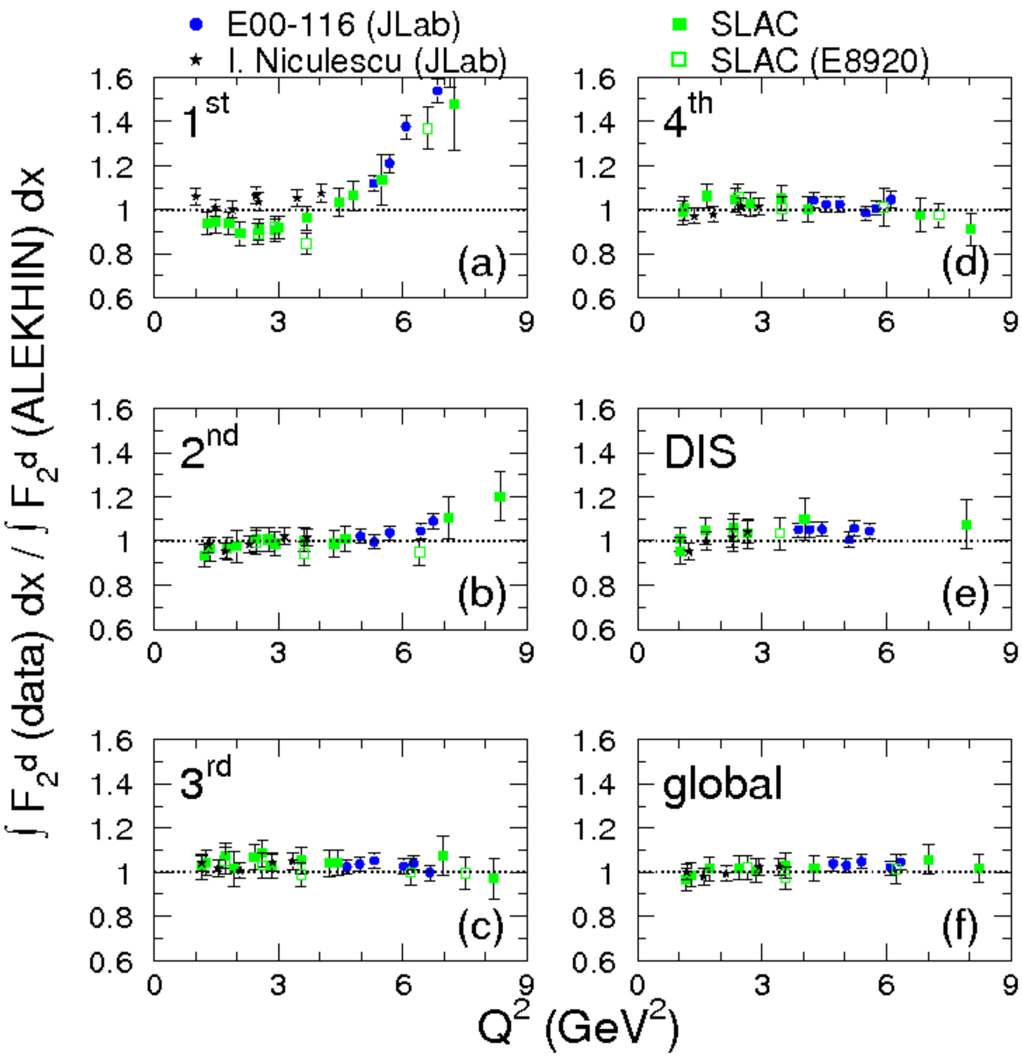
→ at the largest x, QCD fits poorly constrained -> difficult to test duality

S.P. Malace *et al*., Phys. Rev. C 80 035207 (2009)

# Duality: $F_2$ Deuteron Structure Function

Verified!

- **Unpolarized beam (electron), unpolarized target (deuteron)**



$F_2^d$(Alekhin) = $F_2^p$(Alekhin) * d/p (from empirical fit)

→ Ratio within 5-10% : globally, DIS, 4th, 3rd, 2nd

1st : special case

→ models predict stronger violations of duality

→ calculation based on handbag diagram may break at such low W

→ at the largest x, QCD fits poorly constrained -> difficult to test duality

→ d/p fit not well constrained at large x

S.P. Malace *et al.*, Phys. Rev. C 80 035207 (2009)

# Duality: $F_2$ Neutron Structure Function

- **Unpolarized beam (electron), unpolarized target (deuteron, proton)**

→ Impulse Approximation – virtual photon scatters incoherently from individual nucleons

$$F^D_2 = \widetilde{F^p}_2 + \widetilde{F^n}_2 + \boxed{\delta^{off}} F^D_2 \qquad \widetilde{F^{n,p}}_2 = \int_x^{M_D/M} dy \boxed{f(y,\gamma)} F^{n,p}_2\left(\frac{x}{y}\right)$$

off-shell correction

smearing function

→ $F_2^n$ via the additive extraction method: solve equation iteratively

$$f(y,\gamma) = \boxed{\mathrm{N}}\delta(y-1) + \boxed{\delta f(y,\gamma)} \rightarrow$$

finite width of smearing function

normalization of smearing function

$$\widetilde{F^n}_2(x) = NF^n_2(x) + \boxed{\int_x^{M_D/M} dy f(y,\gamma) F^n_2\left(\frac{x}{y}\right)} \rightarrow$$

perturbation

Initial guess for the neutron structure function

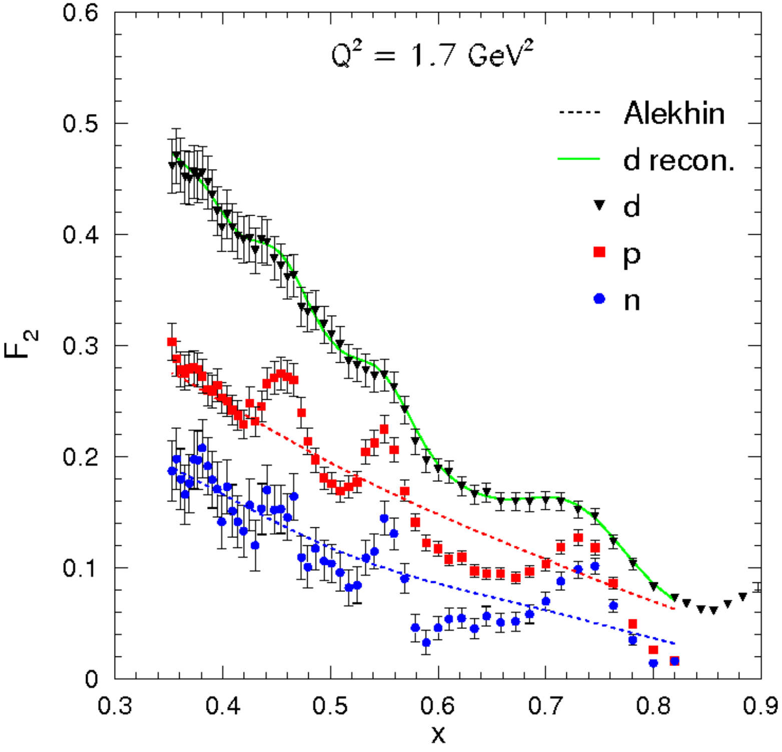
$$F^{n(1)}_2(x) = F^{n(0)}_2(x) + \frac{1}{N}\left[\widetilde{F^n}_2(x) - \int_x^{M_D/M} dy f(y,\gamma) \boxed{F^{n(0)}_2}\left(\frac{x}{y}\right)\right]$$

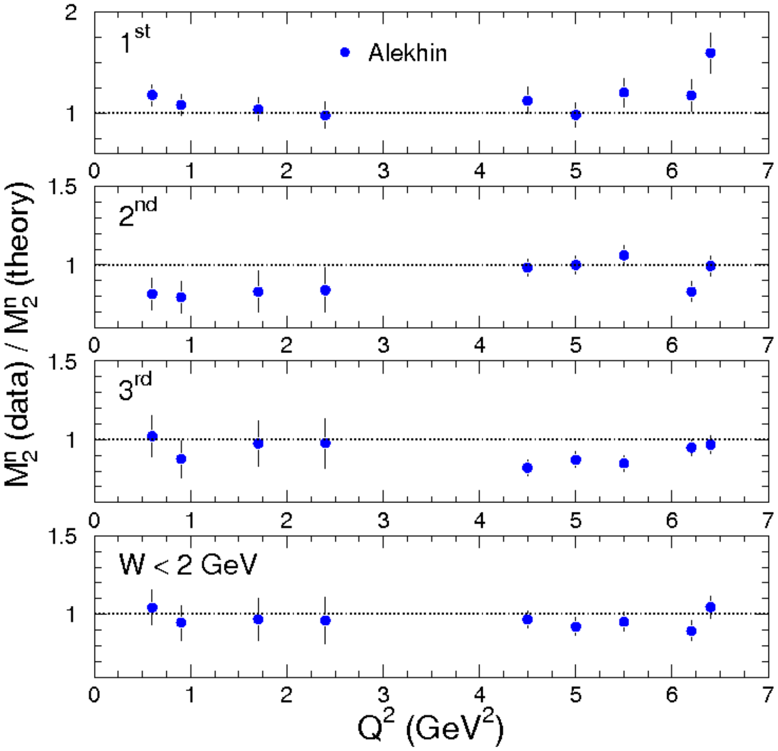
# Duality: $F_2$ Neutron Structure Function

Verified!

- **Unpolarized beam (electron), unpolarized target (deuteron, proton)**

→ $F_2^n$ extracted at fixed $Q^2$ from proton and deuteron data; the PDF-based $F_2^n$ from Alekhin *et al*. used as scaling curve for duality verification





→ Ratio within 10% globally and 15%-20% for 3rd, 2nd

S.P. Malace, Y. Kahn, W. Melnitchouk, C. Keppel, Phys. Rev. Lett. 104, 102001 (2010)

*different way to access the neutron in Ioana's talk*

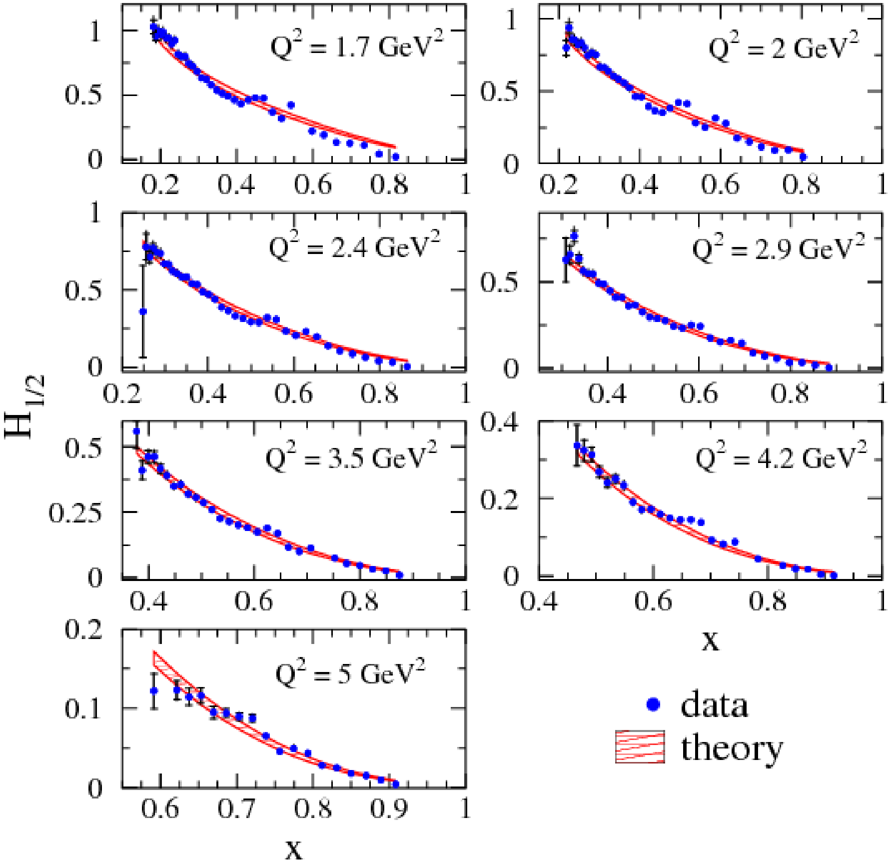
# Duality: Helicity Structure Functions

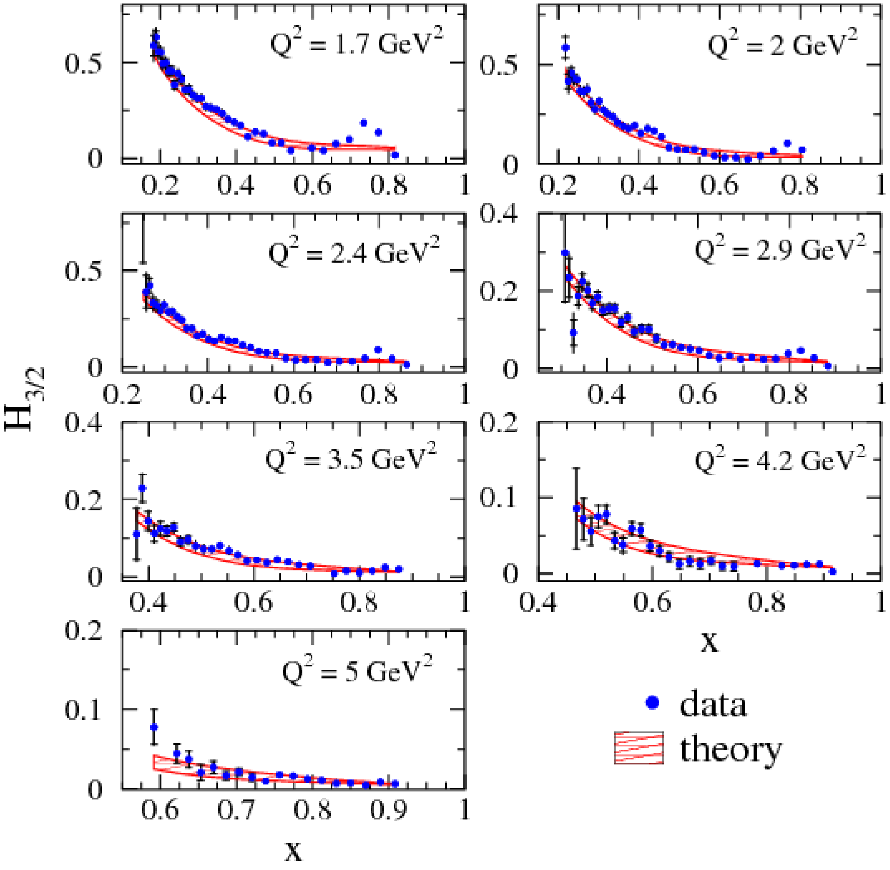
Verified!

- **Longitudinally polarized beam (electron), longitudinally polarized target ($NH_3$)**

$$H_{1/2} = F_1 + g_1 - \frac{Q^2}{\nu^2} g_2 \qquad H_{3/2} = F_1 - g_1 + \frac{Q^2}{\nu^2} g_2$$

$g_1$ data from CLAS E91-023





→ Ratio within 10% globally for $H_{1/2}$ and within 20% for $H_{3/2}$

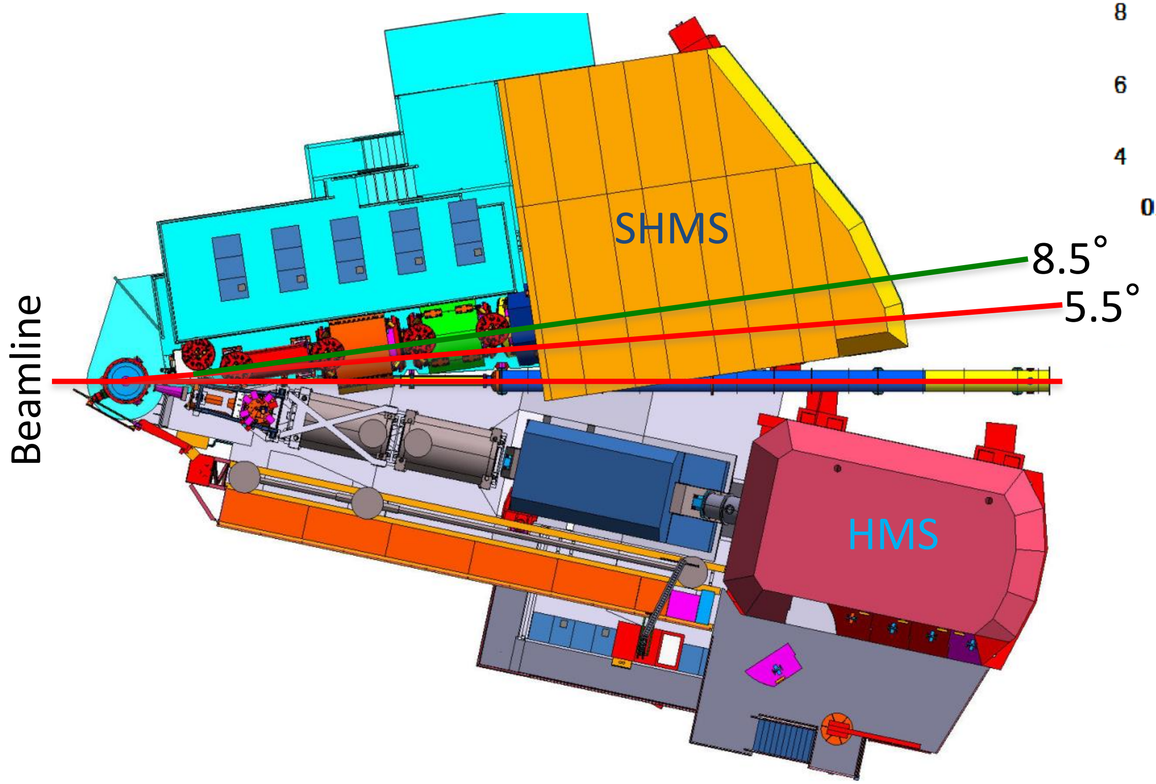
S.P. Malace, W. Melnitchouk, A. Psaker, Phys. Rev. C 83, 035203 (2011)

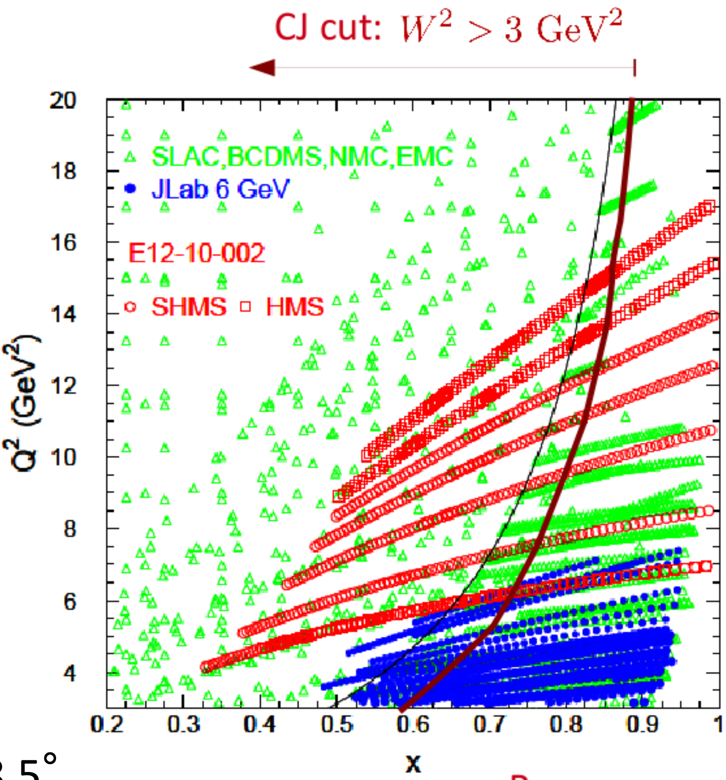
# Future: E12-10-002 in Hall C at JLab, 2016-2017

- **E12-10-002**: at Jefferson Lab in Hall C to measure cross sections and $F_2$ structure functions at large x and low to intermediate $Q^2$ on **proton and deuteron**

*S.P. Malace – contact and spokesperson*
*M.E. Christy, C. Keppel, I. Niculescu spokespeople*

→ Hall C Standard spectrometers: Super High Momentum Spectrometer (SHMS), High Momentum Spectrometer (HMS)





DIS region

⇣

Data will be included in the CTEQ-JLAB PDF global fits

Resonance region

⇣

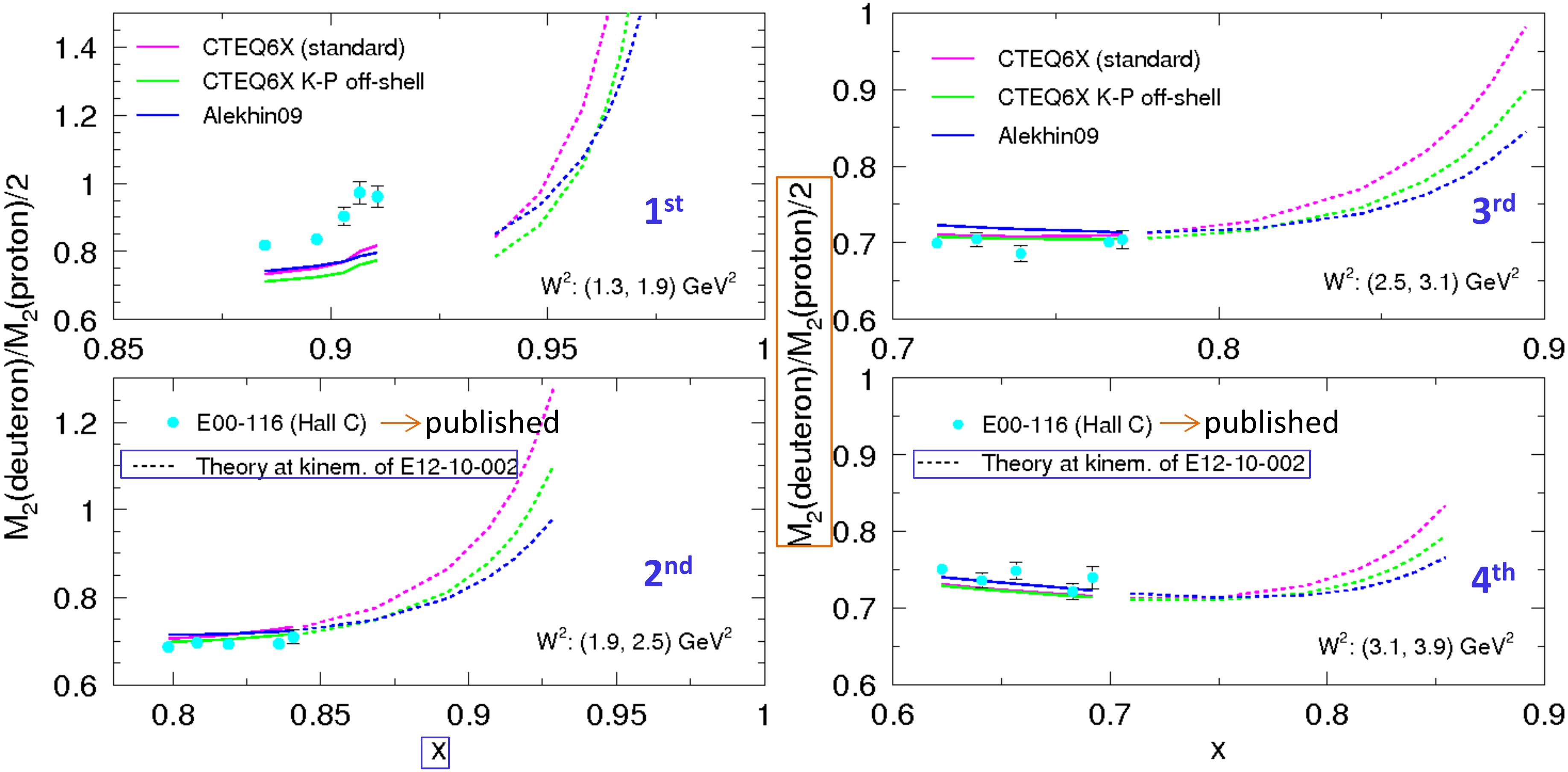
Data will be used to study confinement effects

*Quark-hadron duality*

# Future: E12-10-002 in Hall C at JLab, 2016-2017

- **E12-10-002**: Resonance Region coverage

*S.P. Malace – contact and spokesperson*
*M.E. Christy, C. Keppel, I. Niculescu spokespeople*

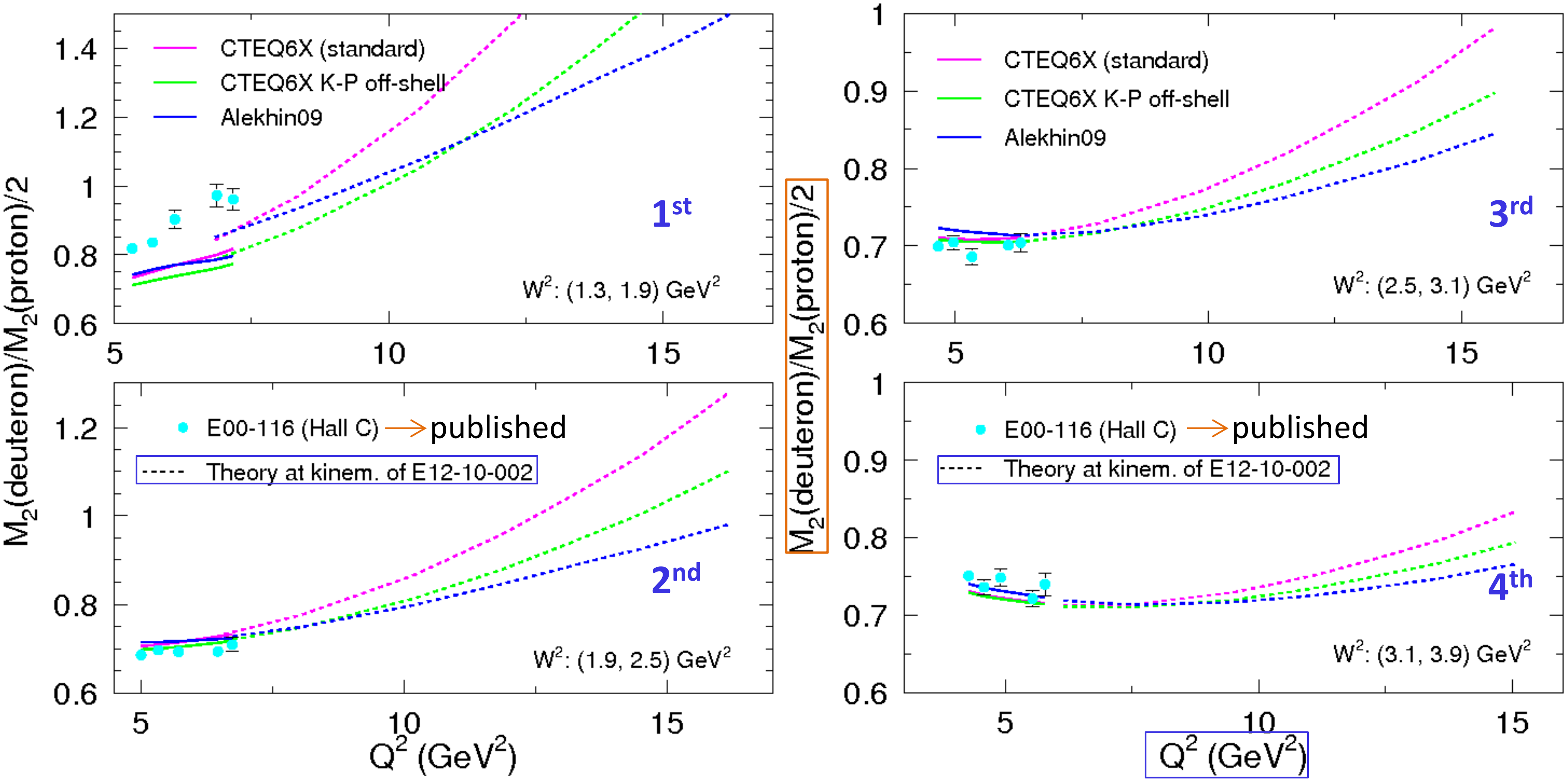


- **E12-10-002**: greatly extends the **x coverage** per resonance region

# Future: E12-10-002 in Hall C at JLab, 2016-2017

- **E12-10-002**: Resonance Region coverage

*S.P. Malace – contact and spokesperson*
*M.E. Christy, C. Keppel, I. Niculescu spokespeople*



- **E12-10-002**: greatly extends the **Q$^2$ coverage** per resonance region

# Future: E12-14-002 in Hall C at JLab, 2019-2020?

*S.P. Malace - Spokesperson and contact*
*E. Christy, D. Gaskell, C. Keppel, P. Solvignon spokespeople*

- **E12-14-002** at Jefferson Lab plans to extract *in a* ***model independent*** *fashion via the Rosenbluth technique*:

  → $R_p$, $R_D - R_p$
  → $R_A - R_D$ for C, Cu, Au
  → $F_1$, $F_L$, $F_2$ for H, D, C, Cu, Au

  x: 0.1 - 0.6 ; $Q^2$: 1 - 5 $GeV^2$

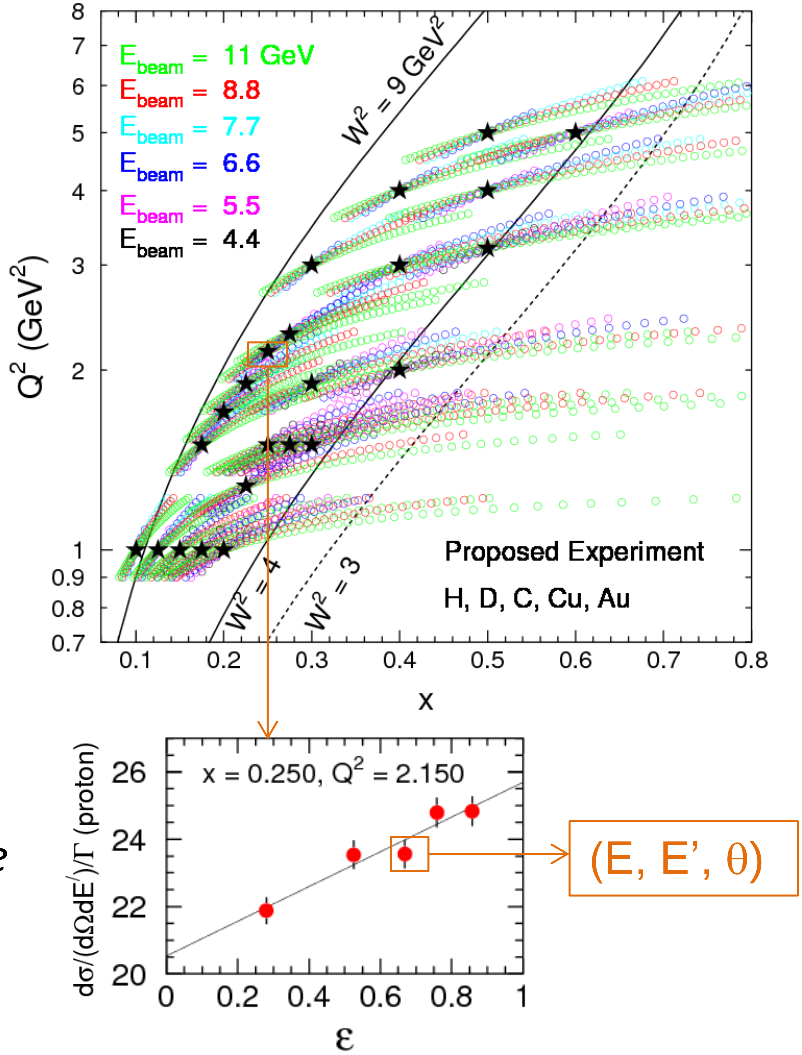
- Each central L/T extraction (black stars) :

  → Hall C spectrometers, SHMS and HMS
  → up to 6 beam energies
  → D, Cu at all kinematics shown; H, C, Au at select kinematics

- Statistical goal: 0.2 – 0.5% (depending on the target) in a $W^2$ bin of 0.1 $GeV^2$

→ *Resonance Region covered within the acceptance of the spectrometers*

→ *Allows for duality studies on separated structure functions on proton and nuclei*



# Open Questions

→ I showed studies of quark-hadron duality in the proton, deuteron and neutron $F_2$ structure functions as well as in the helicity structure functions $H_{1/2}$ and $H_{3/2}$

The procedure to verify how well do resonance region data average to "scaling curves" is rather simple:

→ We define local and global resonance regions using W as parameter
→ We generate the "scaling curve" at the exact same kinematics as the data (discrete points from models, fits)
→ We apply the same integration procedure to data and generated "scaling curve"
→ The ratio of integrals from data and scaling curves will then ONLY be a measure of how well the data average to the scaling curve

- There is arbitrariness in defining the local W intervals
  → how should we quantify this arbitrariness?

- What is a "reasonable" scaling curve?
  → Since we study quark-hadron duality, my first choice would be a PDF-based scaling curve
  → Second generation PDF fits better constrained at large x are ideal for these studies; make sure that the "scaling curve" of choice is well constrained in the kinematic region of interest

# Open Questions

→ I showed studies of quark-hadron duality in the proton, deuteron and neutron $F_2$ structure functions as well as in the helicity structure functions $H_{1/2}$ and $H_{3/2}$

→ Resonance region data will be available from upcoming JLab experiments: **E12-10-002** (and **E12-14-002**) and studies of duality verification will be extended to larger x and $Q^2$

- How do we move past the "verification of duality" point?
- How do we make the observation of duality practically useful?

In the context of PDF fits:
→ can we now use the PDF fits framework (CTEQ-JLab) to understand how duality arises?
→ can we develop a robust procedure to yield duality averaged data for use in PDF fits, for example?
→ based on the applicability of QCD calculations at low values of W, which resonance region data would should we include?
- criterion proposed by Alberto Accardi: separation between target jet and current jet

*Special thanks to Simonetta for organizing this workshop*

# Backup

# Duality: Helicity Structure Functions

- ❖ **Longitudinally polarized beam (electron), longitudinally polarized target ($NH_3$)**
- ❖ Choice of F1, g1, g2 $$H_{1/2} = F_1 + g_1 - \frac{Q^2}{\nu^2} g_2 \qquad H_{3/2} = F_1 - g_1 + \frac{Q^2}{\nu^2} g_2$$
  - "Data" -> $g_1$ from E91-023 (CLAS) K.V. Dharmawardane *et al.*, Phys. Lett. B 641, 11 (2006)

    -> $F_1$ from Christy-Bosted fit E.M. Christy *et al.*, Phys. Rev. C 81, 055213 (2010)

    -> $g_2$ from Simula *et al.* S. Simula *et al.*, Phys. Rev. D 65, 034017 (2002)

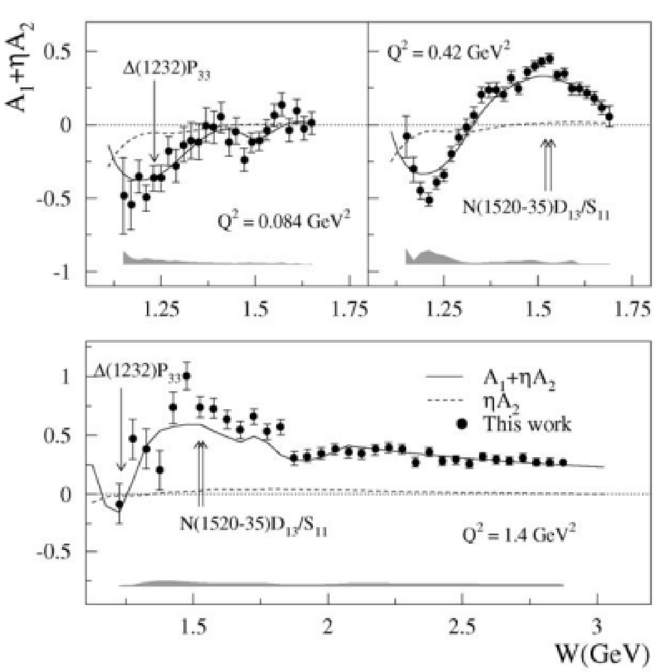


Fig. 1. Results for the asymmetry $A_\parallel/D = A_1 + \eta A_2$ on the proton versus final-state invariant mass $W$, for three bins in $Q^2$. Arrows indicate the masses of several resonances. The first two panels show data obtained with 1.6 GeV beam energy, while the last panel comes from the 5.7 GeV data. The solid line close to the data points is the result for $A_\parallel/D$ of our parametrization of previous world data. The dashed line close to zero is the estimated contribution from the unmeasured asymmetry $A_2$ to $A_\parallel/D$. Bands at the bottom of all figures indicate systematic errors.

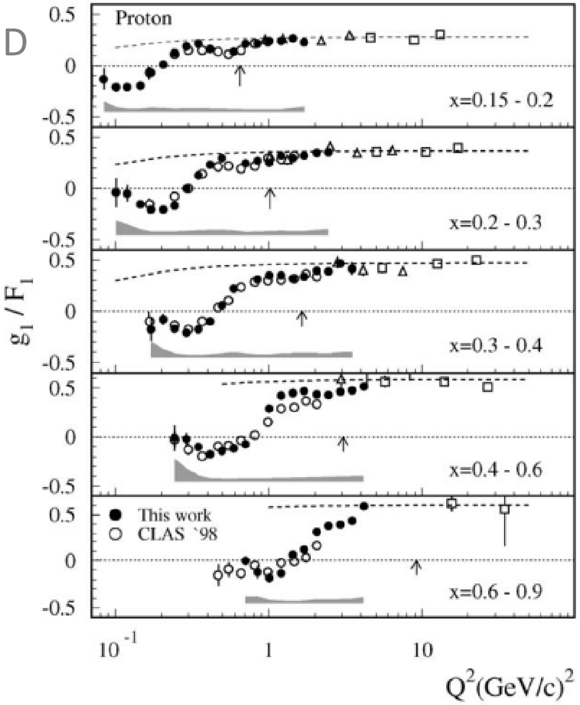


Fig. 2. Measured ratio $g_1/F_1$ as a function of momentum transfer squared $Q^2$ for several bins in $x$ for the proton. A few data points from SLAC experiments E143 [6] (open triangles) and E155 [8] (open squares) are also shown for comparison, as well as data from the first run with CLAS [12,13] (open circles). The dashed line represents our parametrization of the world data in the DIS region [8]. Arrows indicate the conventional limit of the resonance region at $W = 2$ GeV.

# Duality: Helicity Structure Functions

- **Longitudinally polarized beam (electron), longitudinally polarized target ($NH_3$)**

- Choice of F1, g1, g2

$$H_{1/2} = F_1 + g_1 - \frac{Q^2}{\nu^2} g_2 \qquad H_{3/2} = F_1 - g_1 + \frac{Q^2}{\nu^2} g_2$$

  - “Theory”

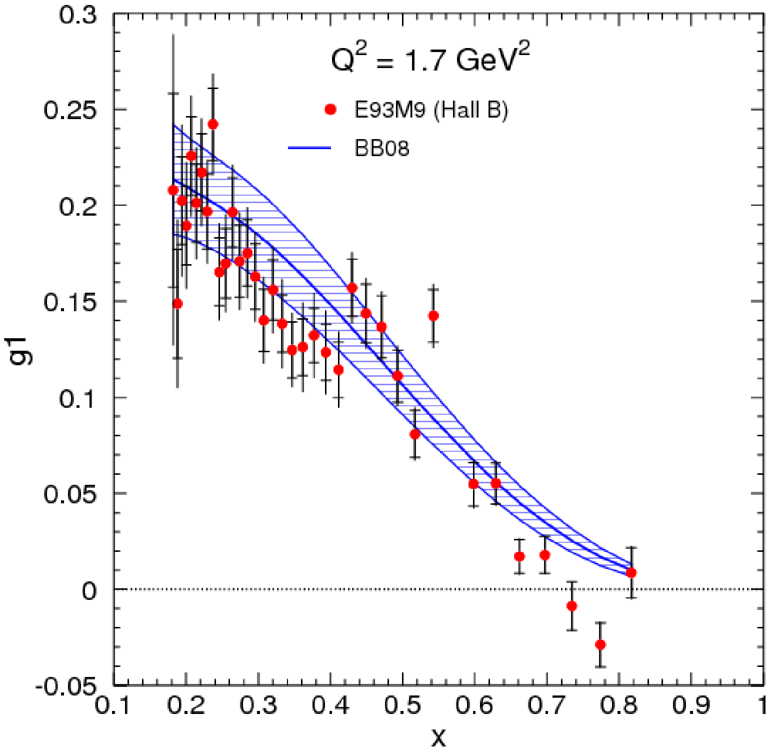
    -> g1 from Blümlein and Böttcher

    Nucl. Phys. B 841, 205 (2010)

    -> F1 from ABKM

    Phys. Rev. D 81, 014032 (2010)

    -> g2 from Wandzura-Wilczek relation with g1 from BB08

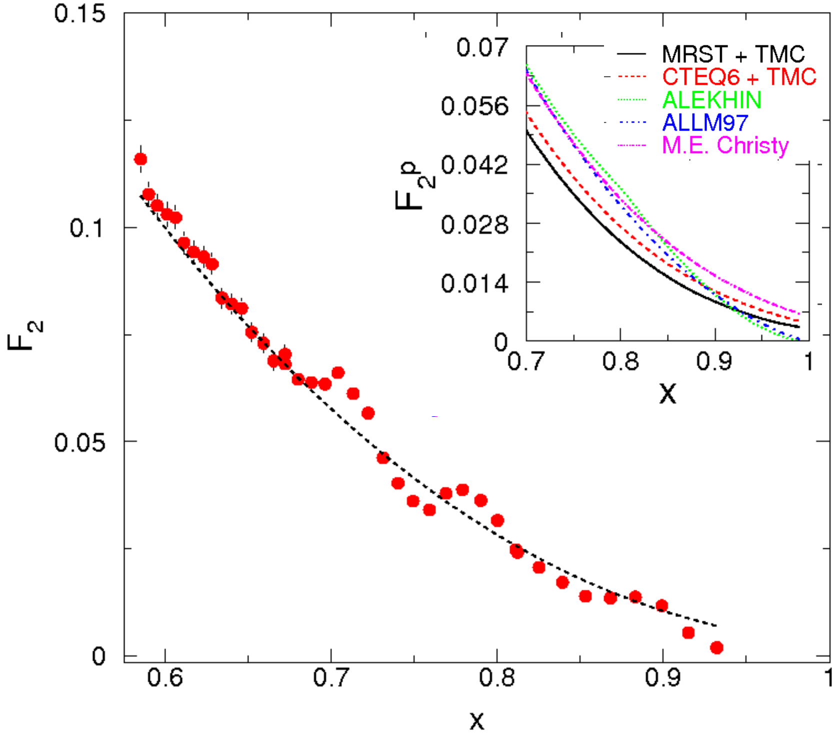


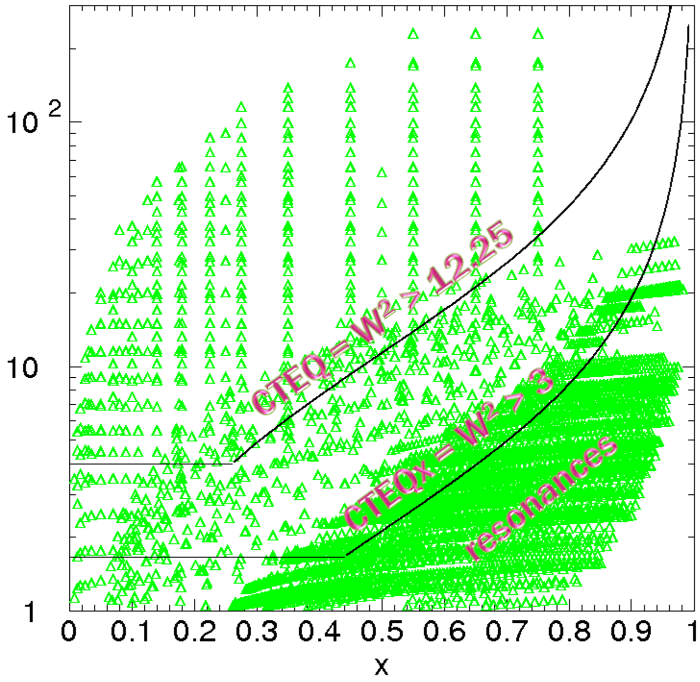
# Duality: Scaling Curves

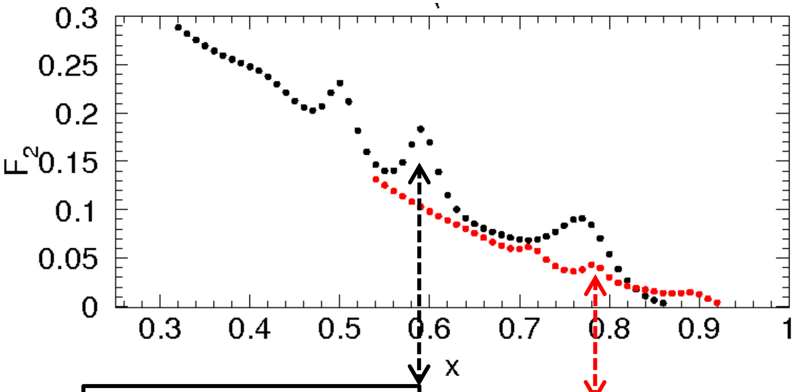
Presently $W^2>3$ GeV$^2$ data cover up to x ~ 0.85

Extending to larger x at finite $Q^2$

encounter the resonance region







**$2^{nd}$ resonance region: $Q^2$ = 2 GeV$^2$**

**$2^{nd}$ resonance region: $Q^2$ = 5 GeV$^2$**

Lack of well constrained scaling curves for x > 0.85 hinders verification of duality at largest x

# Duality: $F_2$ Proton Structure Function

Comparison: data to CTEQ6 (PDF fits with $W^2 > 12.25\ GeV^2$)

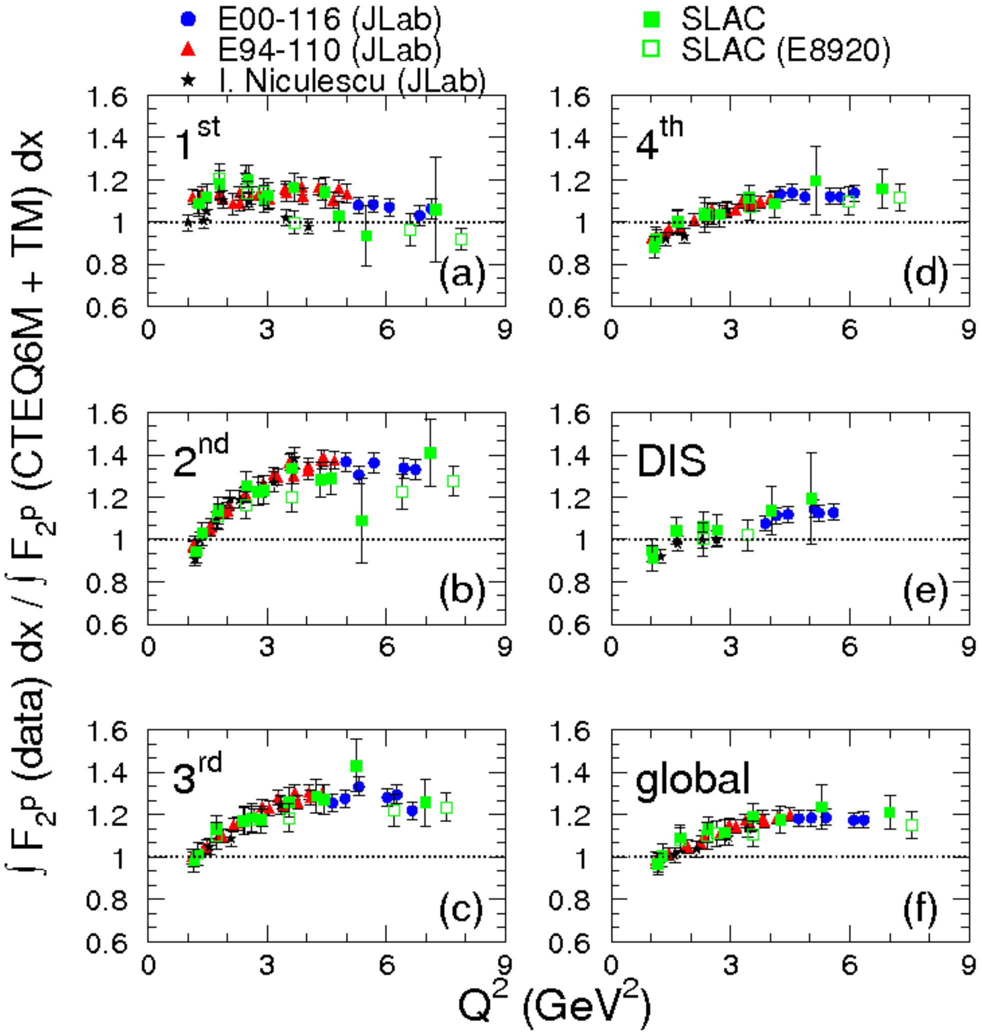
Ratio ~ 1 at $Q^2$ ~ 1.5 $GeV^2$ then rises with increasing $Q^2$ and reaches a plateau at ~ 4 $GeV^2$; above 4 $GeV^2$ $Q^2$ dependence saturates

⇩

Not failure of pQCD to describe the $Q^2$ evolution but paucity in the strength of PDFs at large x

Ratio becomes constant at different value for each RES region

Possibly related to unconstrained PDFs strength at large x



S. Malace *et al.*,Phys. Rev. C 80, 035207 2009

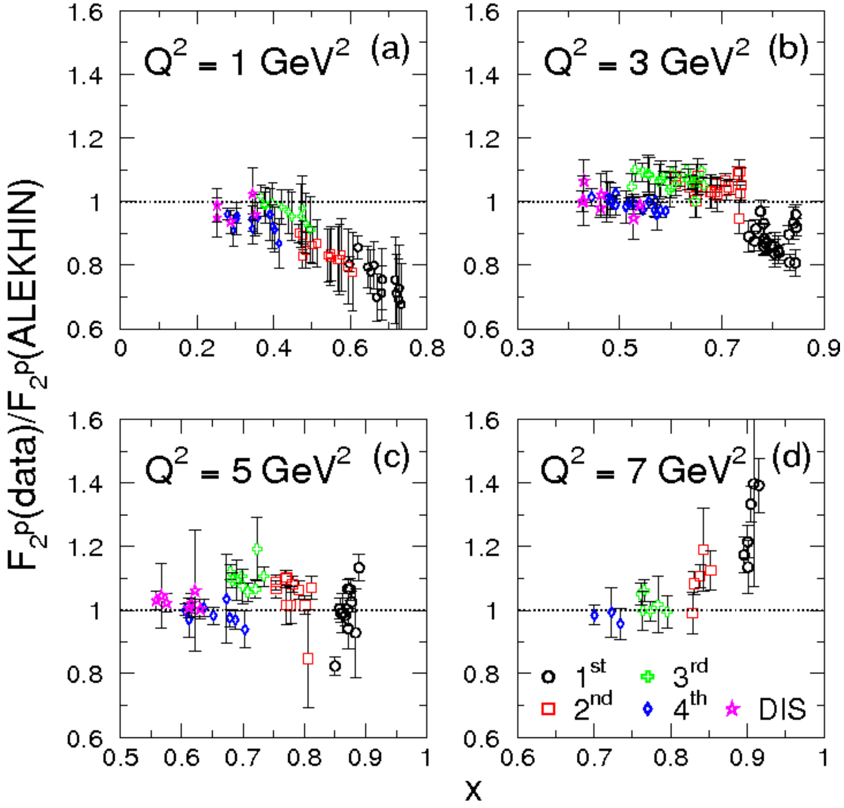
# Duality: $F_2$ Proton Structure Function

## ALEKHIN

Good description at $Q^2$= 3,5 $GeV^2$ (except for largest x regime: 1st RES)

$Q^2$= 7 $GeV^2$ : largest x (ALEKHIN least constrained) => growing discrepancy

$Q^2$= 1 $GeV^2$ : discrepancy as x grows ⬄ reached limits of applicability



Better description of data by ALEKHIN than CTEQ6

## CTEQ6

Fails to describe x dependence of data