Open Questions in Parton-Hadron Duality
University of Virginia, March 13, 2015

Duality from QCD?

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Duality and QCD

- Operator product expansion

→ expand moments of structure functions in powers of $1/Q^2$

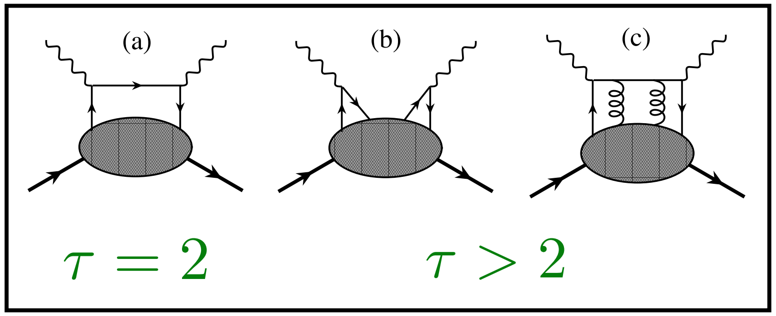
$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

de Rujula, Georgi, Politzer
Ann. Phys. ***103****, 315 (1975)*

Carlson & Mukhopadhyay
PRD ***41****, 2343 (1990)*



Duality and QCD

- Operator product expansion

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- If moment $\approx$ independent of Q^2

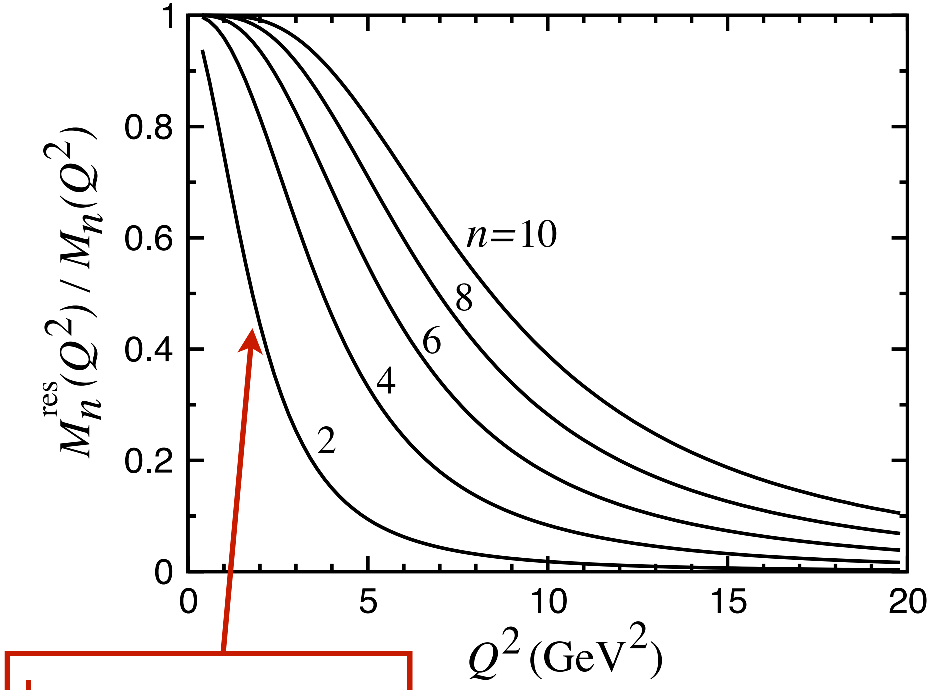
  → higher twist terms $A_n^{(\tau>2)}$ small

- Duality ⟷ suppression of higher twists

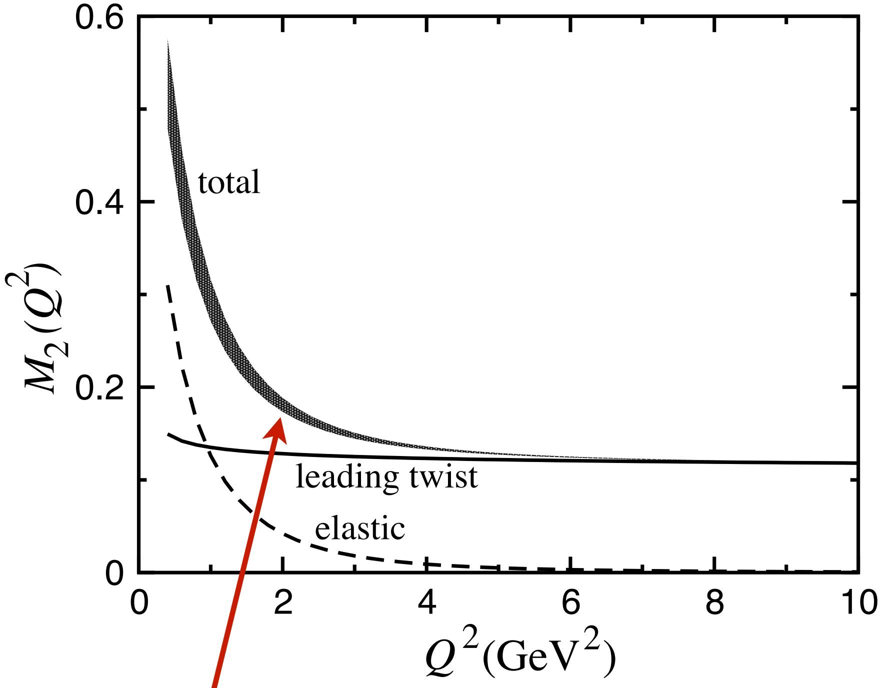
“global duality”

Duality and QCD

- Total higher twist is "small" at scales $Q^2 \sim \mathcal{O}(1\ \mathrm{GeV}^2)$
  - → on average, nonperturbative interactions between quarks and gluons are not dominant (at these scales)
  - → nontrivial interference between resonances



large resonance contributions at low Q^2



"small" higher twists at low Q^2

- Can we understand the resonance−scaling (parton) relation dynamically, at quark level?

  → is duality an accident?

- For simple (toy) quark model with spin-flavor symmetric wave function

  low energy

  → *coherent* scattering from quarks $d\sigma \sim \left(\sum_i e_i\right)^2$

  high energy

  → *incoherent* scattering from quarks $d\sigma \sim \sum_i e_i^2$

- For duality, these must be equal...

  → how can *square of a sum* become *sum of squares*?

Dynamical cancellations

- → $e.g.$ for toy model of two quarks bound in a harmonic oscillator potential, structure function given by

$$F(\nu, \mathbf{q}^2) \ \sim \ \sum_n \left|G_{0,n}(\mathbf{q}^2)\right|^2 \ \delta(E_n - E_0 - \nu)$$

- → charge operator $\Sigma_i \ e_i \exp(i\mathbf{q} \cdot \mathbf{r}_i)$ excites *even* partial waves with strength $\propto (e_1 + e_2)^2$
  odd partial waves with strength $\propto (e_1 - e_2)^2$

- → resulting structure function

$$F(\nu, \mathbf{q}^2) \ \sim \ \sum_n \left\{(e_1 + e_2)^2 \ G^2_{0,2n} + (e_1 - e_2)^2 \ G^2_{0,2n+1}\right\}$$

- → if states degenerate, cross terms $(\sim e_1 e_2)$ cancel when averaged over nearby even and odd parity states

*Close & Isgur, PLB **509**, 81 (2001)*

Dynamical cancellations

→ duality is realized by summing over at least one complete set of even and odd parity resonances

Close & Isgur, PLB ***509****, 81 (2001)*

→ in NR Quark Model, even & odd parity states generalize to **56** (L=0) and **70** (L=1) multiplets of spin-flavor SU(6)

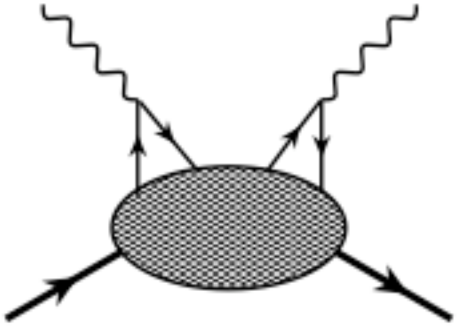
representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$

λ (ρ) $=$ (anti) symmetric component of ground state wave function

“local duality”

Close & WM, PRC ***68****, 035210 (2003)*
PRC ***79****, 055202 (2009)*

Accidental cancellations of charges?



<u>cat's ears diagram</u> (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i\neq j} e_i\, e_j \;\sim\; \Big(\sum_i e_i\Big)^2 - \sum_i e_i^2$$

coherent — incoherent

proton $$\mathrm{HT} \sim 1 - \left(2\times\frac{4}{9}+\frac{1}{9}\right) = 0\ !$$

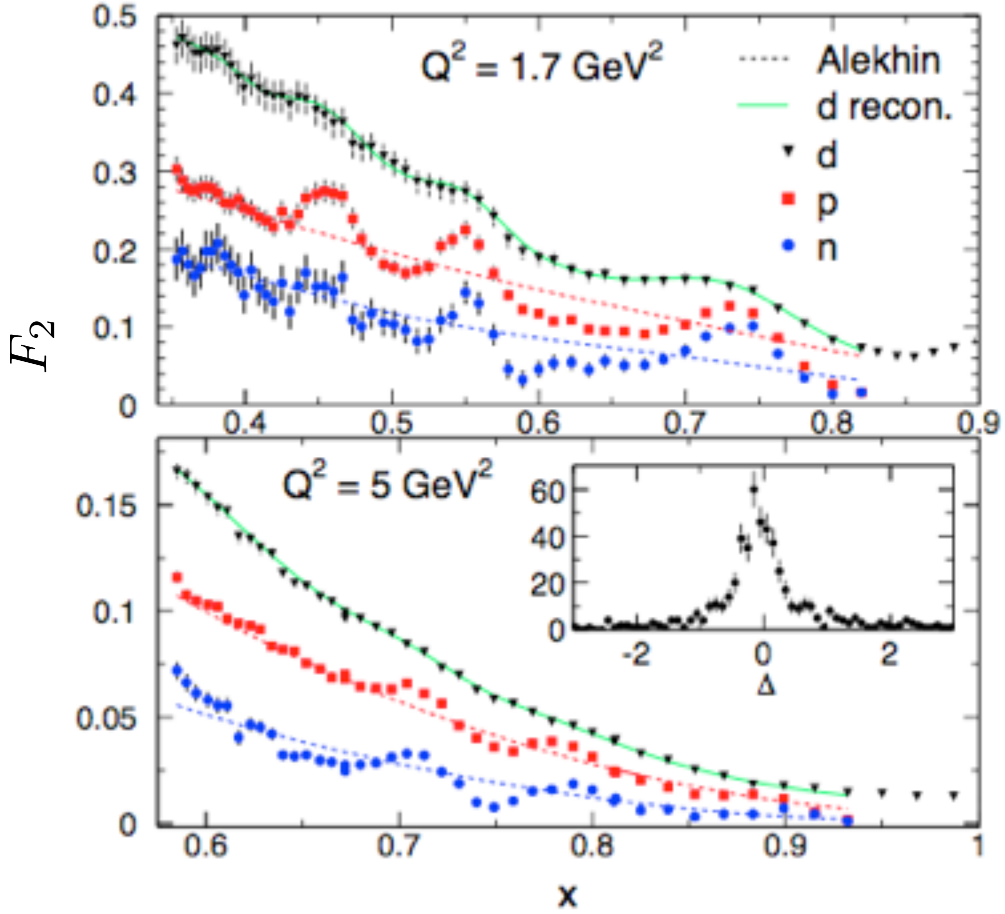
neutron $$\mathrm{HT} \sim 0 - \left(\frac{4}{9}+2\times\frac{1}{9}\right) \neq 0$$

→ here duality in proton is a coincidence

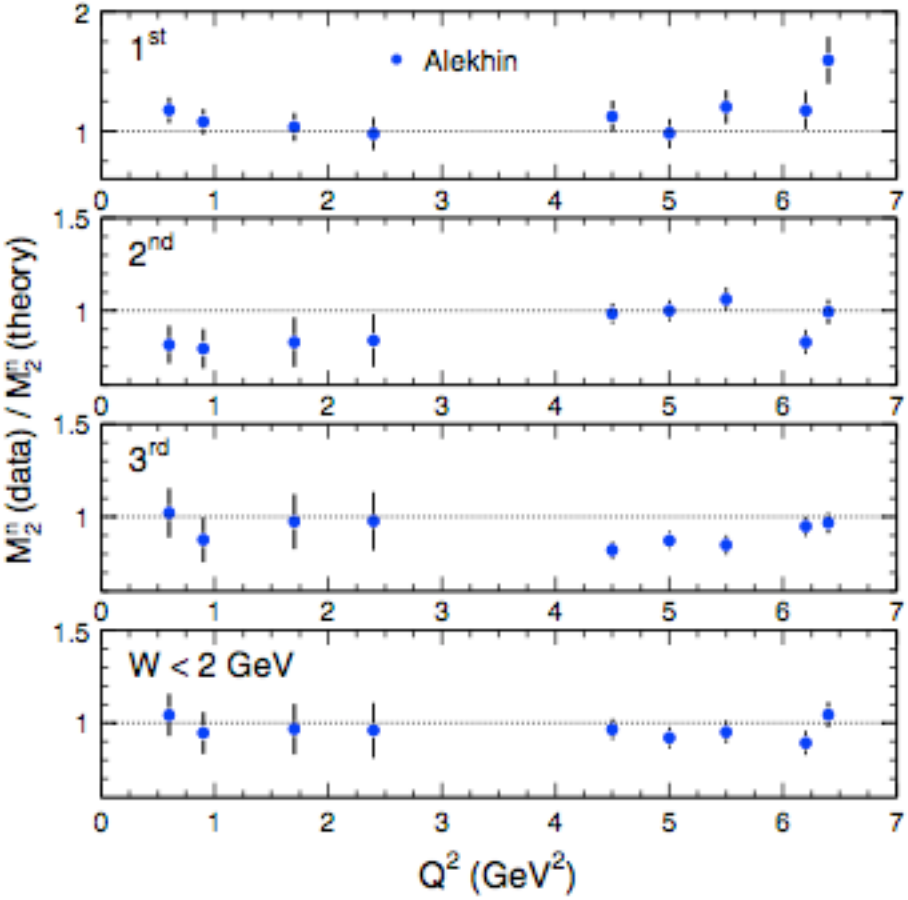
→ should *not* hold for neutron!

S. Brodsky (2000)

Neutron: the smoking gun



Malace et al., PRL ***104****, 102001 (2010)*



duality violations < 10%

→ duality is *not* accidental, but a general feature of resonance−scaling transition!

How to build up a scaling structure function from $\gamma^* N N^*$ transitions?

- Earliest attempts predate QCD

→ $e.g.$ harmonic oscillator spectrum $M_n^2 = (n+1)\Lambda^2$
including states with spin = 1/2, ..., n+1/2
(n even: I = 1/2, n odd: I = 3/2)

*Domokos et al., PRD **3**, 1184 (1971)*

→ at large Q^2 magnetic coupling dominates

$$G_n(Q^2) = \frac{\mu_n}{(1+Q^2 r^2/M_n^2)^2} \qquad r^2 \approx 1.41$$

→ in Bjorken limit, $\sum_n \longrightarrow \int dz\,, \quad z \equiv M_n^2/Q^2$

$$F_2 \sim (\omega' - 1)^{1/2}(\mu_{1/2}^2 + \mu_{3/2}^2)\int_0^\infty dz\, \frac{z^{3/2}(1+r^2/z)^{-4}}{z+1-\omega'+\Gamma_0^2 z^2}$$

→ scaling function of $\omega' = \omega + M^2/Q^2 \qquad (\omega = 1/x)$

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Domokos et al., PRD ***3****, 1184 (1971)*

→ in $\Gamma_n \to 0$ limit

$$F_2 \sim (\mu_{1/2}^2 + \mu_{3/2}^2)\frac{(\omega' - 1)^3}{(\omega' - 1 + r^2)^4}$$

cf. Drell-Yan-West relation

$$G(Q^2) \sim \left(\frac{1}{Q^2}\right)^m \quad \Longleftrightarrow \quad F_2(x) \sim (1-x)^{2m-1}$$

→ similar behavior found in many other models

Einhorn, PRD ***14****, 3451 (1976)* ('t Hooft model)
Greenberg, PRD ***47****, 331 (1993)* (NR scalar quarks in HO potential)
Pace, Salme, Lev, PRC ***57****, 2655 (1995)* (relativistic HO with spin)
Isgur et al., PRD ***64****, 054005 (2001)* (transition to scaling)
....

How to build up a scaling structure function from $\gamma^* N N^*$ transitions?

- More recent phenomenological analyses at finite Q^2

→ additional constraints from threshold behavior at $\mathbf{q} \to 0$ and asymptotic behavior at $Q^2 \to \infty$

Davidovsky & Struminsky,
Phys. Atom. Nucl. ***66****, 1328 (2003)*

$$\left(1+\frac{\nu^2}{Q^2}\right) F_2^R = M\nu \ \left[|G_+^R|^2 + 2|G_0^R|^2 + |G_-^R|^2\right] \ \delta(W^2 - M_R^2)$$

→ 21 isospin-1/2 & 3/2 resonances (with mass < 2 GeV)

$$\left|G_\pm^R(Q^2)\right|^2 = \left|G_\pm^R(0)\right|^2 \left(\frac{|\vec{q}|}{|\vec{q}|_0}\,\frac{\Lambda'^2}{Q^2+\Lambda'^2}\right)^{\gamma_1} \left(\frac{\Lambda^2}{Q^2+\Lambda^2}\right)^{m_\pm}$$

$$m_{+,0,-} = 3, 4, 5$$

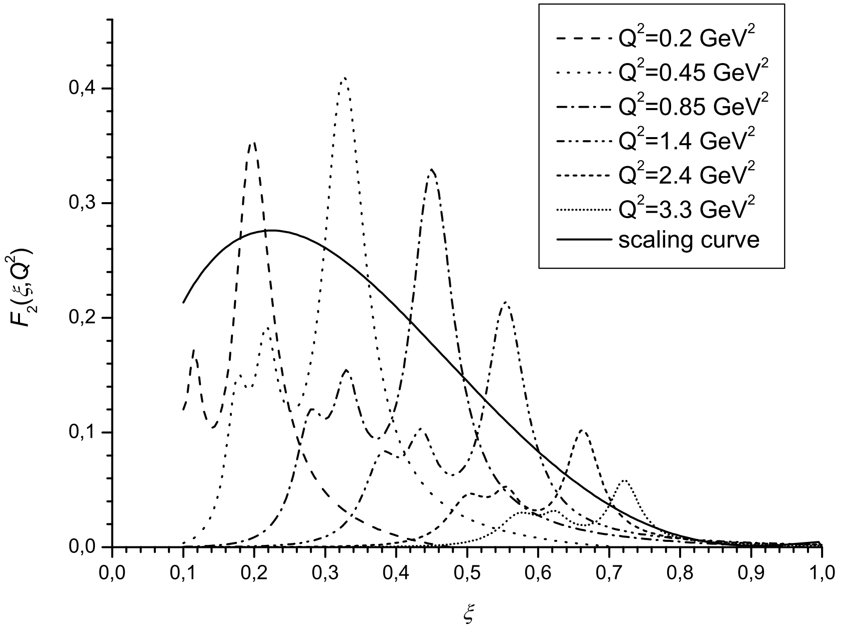
$$\left|G_0^R(Q^2)\right|^2 = C^2 \left(\frac{Q^2}{Q^2+\Lambda''^2}\right)^{2a} \frac{q_0^2}{|\vec{q}|^2} \left(\frac{|\vec{q}|}{|\vec{q}|_0}\,\frac{\Lambda'^2}{Q^2+\Lambda'^2}\right)^{\gamma_2} \left(\frac{\Lambda^2}{Q^2+\Lambda^2}\right)^{m_0}$$

→ in $x \to 1$ limit,

$$F_2(x) \sim (1-x)^{m_+}$$

How to build up a scaling structure function from $\gamma^* N N^*$ transitions?

- More recent phenomenological analyses at finite Q^2



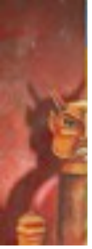
Davidovsky & Struminsky,
Phys. Atom. Nucl. ***66****, 1328 (2003)*

$\longrightarrow$ valence-like structure of dual function suggests "two-component duality":

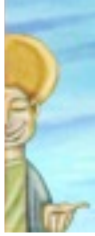
- valence (Reggeon exchange) dual to resonances $F_2^{(\text{val})} \sim x^{0.5}$
- sea (Pomeron exchange) dual to background $F_2^{(\text{sea})} \sim x^{-0.08}$

$\longrightarrow$ *T. Londergan*

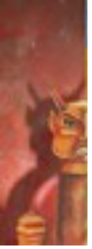
Freund, PRL ***20****, 235 (1968)*
Harari, PRL ***20****, 1395 (1969)*



Open questions



- Is there a QCD-based understanding of local duality?
  - → quark models give insights into emergence of "scaling" behavior from resonances
  - → large-N_c ? HQET?
- Role of nonresonant background in "resonance" cancellations?
  - → mostly unexplored territory
- Definitions of duality
  - → which moments (C-N, Nachtmann)?
  - → which structure functions (resonance region *vs*. LT, or total low-W *vs*. high-W)?



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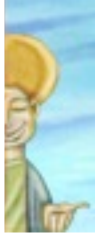
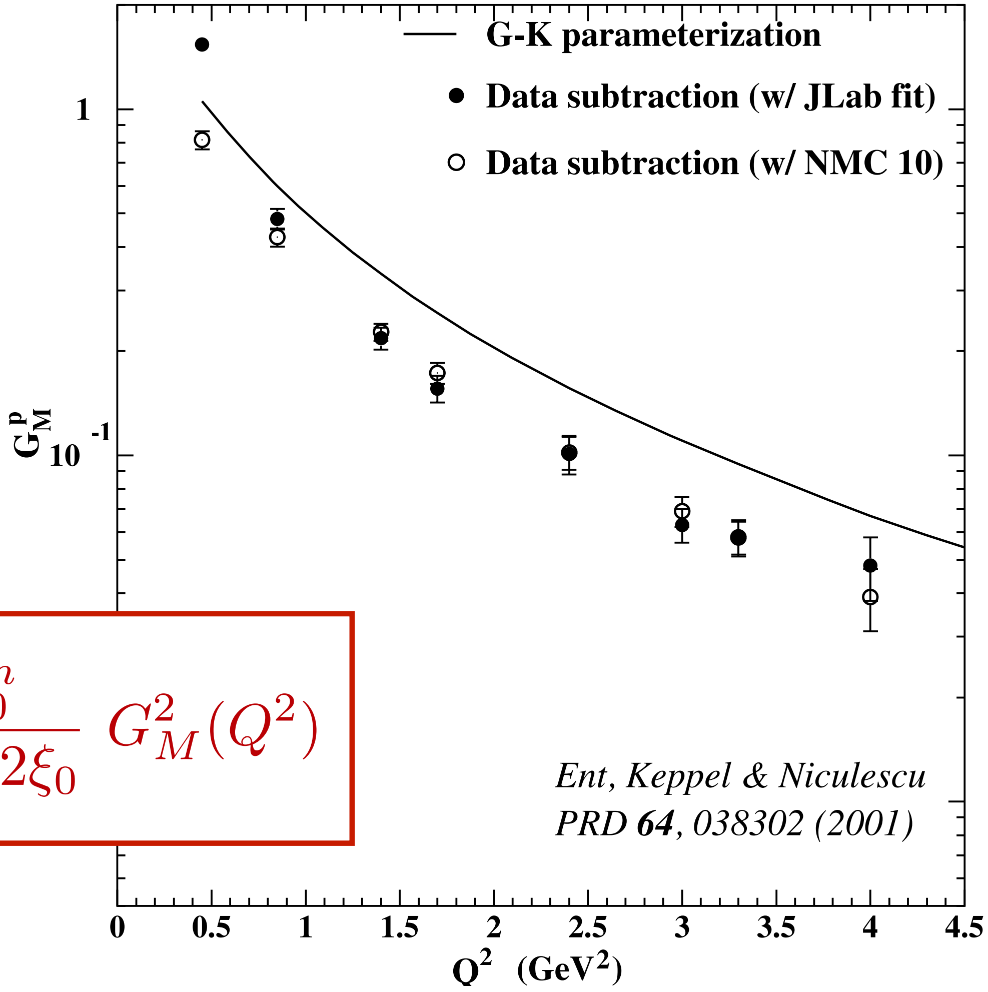


- Why does “local elastic duality” work *at all*?

$$F_1^{\rm el} = M\tau\, G_M^2\, \delta\left(\nu - \frac{Q^2}{2M}\right)$$

$$\boxed{\int_{\xi_{\rm th}}^{1} d\xi\; \xi^{n-2}\; F_1(\xi, Q^2) = \frac{\xi_0^n}{4-2\xi_0}\; G_M^2(Q^2)}$$



Ent, Keppel & Niculescu
*PRD **64**, 038302 (2001)*