The collective quantization of SU(3) solitons

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Exotic baryons, such the pentaquark, were predicted using rigid rotor quantization from SU(3) chiral soliton models by Praszalowicz and Diakonov, Petrov, and Polyakov (DPP).

- As has been argued previously (see Tom's talk) rigid rotor quantization is in fact not valid for $SU(3)$ solitons.

- In this talk, we will discuss how to collectively quantize $SU(3)$ solitons consistently in the large N_c limit.

Solitons - generalities

Suppose we have a classical model that admits static solitonic solutions.

- A solitonic solution can be interpreted as the first approximation to the ground state of the quantum theory, fluctuations around it are interpreted as corrections.

- But a solitonic solution breaks some classical symmetries, so we can get distinct solitonic solutions with the same energy.
  - The directions associated with symmetry breaking are energetically 'flat'.
  - A small fluctuation in an energetically flat direction is a zero mode.

Solitons - generalities

- We have to quantize the degrees of freedom associated with symmetry-breaking - that is, quantize the zero modes. These are also called the collective degrees of freedom.

- In extreme semi-classical regime (large N_c) the collective degrees of freedom decouple from other degrees of freedom, and can be quantized separately.

Zero modes

- Zero modes can be found by solving equation for small harmonic fluctuations of fields around the soliton.

$$\delta\ddot{\phi} = \mathbb{A}\delta\phi + \mathbb{B}\delta\dot{\phi}.$$

$$\Rightarrow -\omega^2\delta\phi = \mathbb{A}\delta\phi - i\omega\mathbb{B}\delta\phi.$$

- There are two kinds of zero modes: static and dynamic.

- If $B = 0$, they come in pairs:
  - $\delta\phi = \psi$ is a static zero mode if $\mathbb{A}\psi = 0$.
  - $\delta\phi = \psi t$ is corresponding dynamic zero mode.

Zero modes: $B \neq 0$

If $B \neq 0$, $\delta\phi = \psi$ with $A\psi$ is still a static zero mode. But the dynamic zero mode is more complicated (if it exists!):

$$\delta\phi(t) = \psi t + \Phi \ \text{ with } \ \mathbb{A}\Phi = -\mathbb{B}\psi$$

Φ exists if *either* $B\psi = 0$, or $B\psi$ is orthogonal to all the zero modes of A.

Collective quantization

- Once we have zero modes, we promote classical parameters corresponding to them to quantum variables - the collective variables, which can be quantized.

- This restores all the symmetries broken by the classical soliton.

Equation of motion for Skyrme model

The Skyrme model is the mother of all chiral soliton models, corresponds to QCD in the large N_c limit.

Equation of motion (EoM) for Skyrme model is

$$-\partial^\mu L_\mu - 2\frac{\epsilon^2}{f_\pi^2}\partial^\mu[L_\nu,[L_\mu,L^\nu]] + \underbrace{\frac{iN_c}{24\pi^2 f_\pi^2}\epsilon^{\alpha\beta\gamma\nu}L_\alpha L_\beta L_\gamma L_\nu}_{\mathrm{Witten-Wess-Zumino(WWZ)term}} = 0$$

$$L_\mu = U^\dagger \partial_\mu U, \quad U \in SU(3)$$

Note, WWZ term is first order in time $\Rightarrow \mathbb{B} \neq 0 \Rightarrow$ We need to be concerned about static/dynamic zero mode distinction.

$SU(3)$ **Hedgehog**

- The $SU(2)$ Skyrme model has a well known solitonic solution called a 'hedgehog Skyrmion'. In SU(3), we will take the soliton to be:

$$U_H = \left(\begin{array}{c|c} \overbrace{\exp i(\vec{\tau} \cdot \hat{r})F(r)}^{SU(2)\text{ hedgehog}} & 0 \\ \hline 0 & 1 \end{array} \right)$$

- Above embedding in $SU(3)$ is arbitrary $\Rightarrow$ statically rotated hedgehog $AU_HA^\dagger,\ A \in SU(3)/U(1)$ is solution of EoM.

- Since A can be specified via seven parameters, there are seven static zero modes.

Dynamic modes in Skyrme model

Take static hedgehog configuration, and slowly rotate it (time-dependently, with frequency $\omega \sim 1/N_c$).

- If linearized EoM is satisfied, then we have the $B\psi = 0$ situation $\Rightarrow$ dynamical zero mode.
- Otherwise, linearized EoM is violated by $\omega B\psi$.
  - If $\omega B\psi$ is orthogonal to all static zero modes of A $\Rightarrow$ dynamic zero mode.
  - If above is false, no dynamic zero mode exists!

Computing dynamic zero modes

Explicit computation shows:

- $B = 0$ for rotations generated by $\lambda_1, \lambda_2, \lambda_3$ – inside u-d subspace.
- $B \neq 0$ and $B\psi$ is not orthogonal to static zero modes for $\lambda_4, \lambda_5, \lambda_6, \lambda_7$ due to WWZ.
- Therefore dynamical rotations out of the u-d subspace do not correspond to dynamical zero modes - they are not collective.
- There are a total of only three dynamical zero modes (not seven as in rigid rotor) $\Rightarrow$ Total of $10 = 7 + 3$ zero modes, not 14 as in rigid rotor approach.

$B\psi$ coming from WWZ term

Rotating by ω in λ_i direction, EoM is violated by:

$$\begin{gathered}
\Gamma_{wwz}(\lambda_{1,2,3}) = 0 \\
\Gamma_{wwz}(\lambda_4) = \omega(a\lambda_4 - b\lambda_5 + c\lambda_6 - d\lambda_7) \\
\Gamma_{wwz}(\lambda_5) = \omega(b\lambda_4 + a\lambda_5 + d\lambda_6 + c\lambda_7) \\
\Gamma_{wwz}(\lambda_6) = \omega(c\lambda_4 + d\lambda_5 - a\lambda_6 - b\lambda_7) \\
\Gamma_{wwz}(\lambda_7) = \omega(-d\lambda_4 + c\lambda_5 + b\lambda_6 + -a\lambda_7),
\end{gathered}$$

where

$$\begin{aligned}
a &= \tfrac{6i}{r^2}\cos\theta\sin^3(F(r))F'(r) & b &= \tfrac{48i}{r^2}\cos^2\left(\tfrac{F(r)}{2}\right)\sin^4\left(\tfrac{F(r)}{2}\right)F'(r) \\
c &= \tfrac{6i}{r^2}\cos\phi\sin\theta\sin^3(F(r))F'(r) & d &= \tfrac{6i}{r^2}\sin\phi\sin\theta\sin^3(F(r))F'(r).
\end{aligned}$$

Quantization at large N_c

- Collective coordinate ansatz is

$$Ae^{i(\vec{\tau}\cdot\vec{\omega})t}U_H e^{-i(\vec{\tau}\cdot\vec{\omega})t}A^\dagger, \ \ \text{with } A \in SU(3)/U(1), \ \vec{\tau} = \{\lambda_1, \lambda_2, \lambda_3\}.$$

- Quantizing the 10 zero modes now gives the spectrum of the model.

- Need to specify $SU(3)$ reps states can lie in, characterized by (p, q).

- To get (p, q), start with dimension of rep $d = 2Y_{max} + 1$, $Y = \frac{N_c B}{3} + S$.

- We show that $Y_{max} = \frac{N_c}{3}$ at large N_c independently of rotations $\Rightarrow$ $S > 0$ states can not arise as collective excitations.

Quantization at large N_c

We use straightforward group theory arguments to characterize the allowed collective states:

J=I	p	Collective State
$\frac{1}{2}$	1	p,n . . .
$\frac{3}{2}$	3	Δ's . . .
$\frac{5}{2}$	5	Large N_c artifact

Table 1: Allowable Representations

- Notice no exotic states appear as collective excitations!
- These results are identical to model-independent result of Dashen, Jenkins & Manohar, and agree with the results of large N_c quark model.

Conclusions

- We have quantized the collective degrees of freedom in $SU(3)$ Skyrme model.
- Showed that there are only ten (10) such degrees of freedom, not fourteen as has assumed in the past.
- Found that in quantizing the collective degrees of freedom in the large N_c limit no exotic baryon states arise as collective excitations.