Pentaquarks and Large Nc QCD:

- Motivation
- Large Nc and multiplets of exotics
- Large Nc and Chiral Soliton Models
- Large Nc and the heavy pentaquarks
  - Good news: heavy pentaquarks must exist in the combined large Nc and heavy quark limits
  - Bad news: the real world is rather far from this limit

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Why do large Nc?

- Only known practical nonperturbative approach is lattice QCD.
  - The question arises about how practical.
  - My old joke was that waiting for reliable lattice results was like being a character in a Samuel Beckett play:
  - Situation has clearly improved f reliable computation is (partic heavy quark sy
  - Compu f resonant state r, remain hard problem.

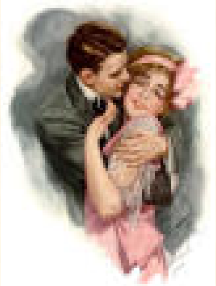
- Most theory about pentaquarks is based on modeling: *eg.* chiral solitons,quark models, QCD sum rules
- Generic Problem: the models are all to some extent *ad hoc*. Are predictions of models related in any real way to the predictions of QCD?

The **three great lies** of the model builders

The check is in the mail

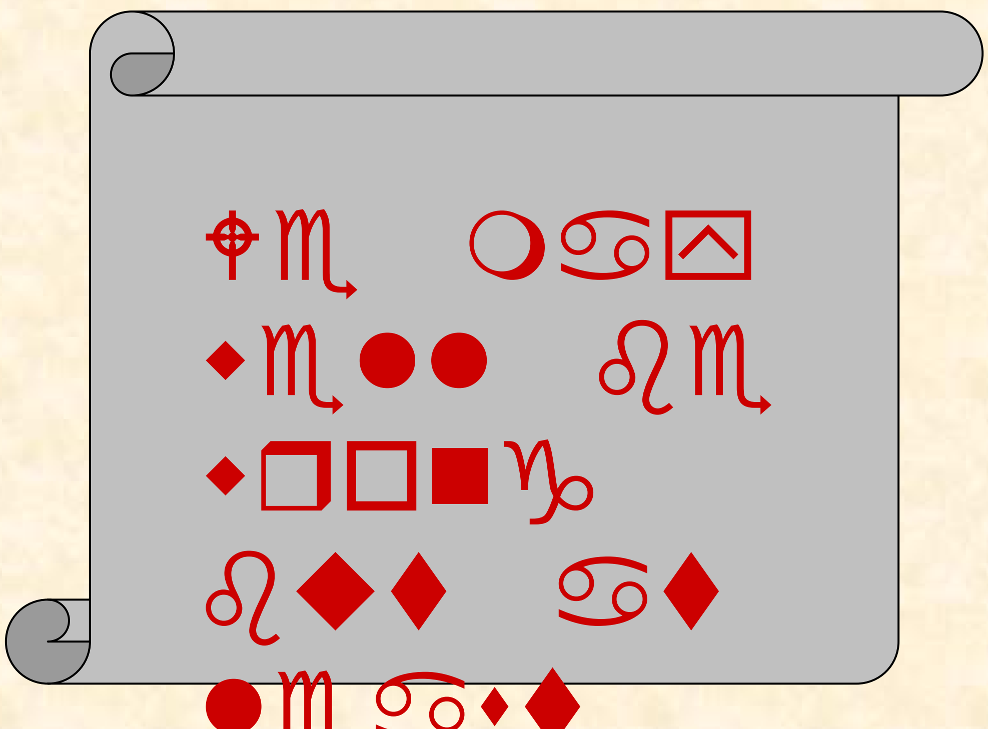
Of course, darling I will respect you in the morning

My model is based on QCD



Large Nc enables one to make model independent predicts---albeit about a fiction large Nc world. *If* 1/Nc corrections are small it is useful for the real world.

A Motto for 1/Nc practitioners



On Pentaquark Multiplets

- Large Nc does not predict the existence of pentatquarks.
- If a pentaquark does exist however it requires that other nearly degenerate pentaquarks also exist. TDC&RF Lebed, PLB 578,(2004) 150; A. Manohar& E. Jenkins JHEP 0406 (2004) 039.
- This is a consequence of the general spin-flavor contracted SU(2 N_f) symmetry which emerges for ground band baryons at large Nc.

Contracted SU($2N_f$) Symmetry

- Derivable by “large Nc consistency rules” in a model independent way. Gervais & Sakita (1984), Dashen & Manohar (1993)
- Identical results seen in Skyrme model and other chiral solitons.
- Large Nc quark model also yields this emergent symmetry.
- We will only impose 2-flavor symmetry in this analysis.

- One key result: I=J rule all leading order matrix elements have quantum numbers with I=J. Operators violating this suppressed by factor of $\left(\frac{1}{N_c}\right)^{|I-J|}$

(Kaplan& Manohar 1998; I=J rule seen in Skyrme model by Mattis & colloborators in the late 1980's)

- Leading order nucleon operators are either *scalar-isoscalar* or *vector-isovector ;* *scalar-isoscalar* and *vector-isovector* operators are *1/Nc* suppressed.

Use to study resonant states

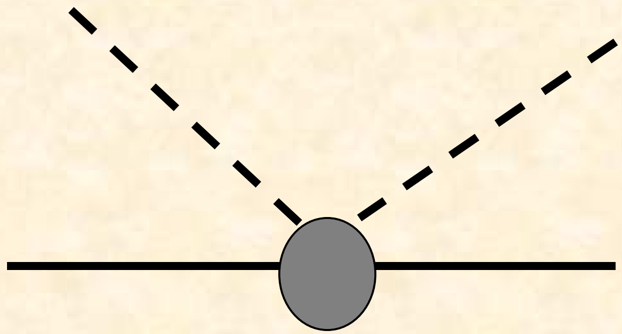
- Key idea: Focus on physical observables (such as meson nucleon scattering).
- Scattering observables are just operators acting on nucleons. TDC&RF.Lebed (2003)
- Logic is directly applicable to pentaquarks.Conservative scheme only rely isospin symmetry but not SU(3). (Suggestion out there by Jaffe&Wilczek that SU(3) may be badly broken in cases of ideal mixing)

Focus on K-N scattering

- Label scattering amplitudes by $S_{LL'I_sJ_s}$

L (L’) initial (final) L for K;

I_s (J_s) total isospin (angular momentum of state



I=J for nucleon operator; t channel for scattering

- Most general amplitude does not have $I_t = J_t$ but large Nc QCD does. Fewer amplitudes at large Nc than in general: large Nc QCD requires relations among amplitudes. (Modulo 1/Nc corrections)
- Express large Nc amplitude in terms of most general amplitude with $I_t = J_t$ (requires recoupling) and then use 6-J coefficient identities:

$$S_{LL'IJ} = \sum_k 2(2k+1)$$

$$\times \begin{Bmatrix} k & I & J \\ \frac{1}{2} & L' & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} k & I & J \\ \frac{1}{2} & L & \frac{1}{2} \end{Bmatrix} S_{kLL'}$$

- Note the same “reduced” amplitude contributes to many physical channels.
- A resonance is a pole in the scattering amplitude (at a complex energy).
- If there is a pole in a physical amplitude there must be a pole in some reduced amplitude---which implies a pole in another physical channel with the same mass and width.

$$S_{LL'IJ} = \sum_{k} 2(2k+1) \times \begin{Bmatrix} k & I & J \\ \frac{1}{2} & L' & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} k & I & J \\ \frac{1}{2} & L & \frac{1}{2} \end{Bmatrix} S_{kLL'}$$

- *Eg.* Suppose θ^+ has *I=0 and* $J^p=(1/2)^+$ and is created in p-wave scattering (initial and final state).
- Six-J coefficient: k=1/2 is only possibility: $S_{1/2\ 1\ 1}$ has pole at resonance position.
- $S_{1/2\ 1\ 1}$ also contributes to channel with L=L'=1; I=0, J=3/2

- *Ergo: at large Nc there is resonant state with I=0,* $J^p=(3/2)^+$ *at the same mass and width as orginal* θ^+ *.*

- 1/Nc correction shifts mass "slightly" (M_{Δ}-M_N).
- Width could have larger shift due to phase space effects but coupling constant will be the same as the θ^+ (+ 1/Nc corrections)
- **Model Independent Prediction of large Nc QCD.**
  - ***Similar analysis yields same qualitative conclusion regardless of quantum #s.***
  - ***A full three flavor analysis was recently completed by Rich Lebed & TDC***

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$$(\text{“}\overline{10}\text{”},\ \tfrac{1}{2}^+),\ (\text{“}27\text{”},\ \tfrac{1}{2}^+),\ (\text{“}27\text{”},\ \tfrac{3}{2}^+),\ (\text{“}35\text{”},\ \tfrac{3}{2}^+)$$

Large Nc and The Collective Quantization of chiral solitons

- Pentaquarks predicted via chiral soliton models using rigid rotor quantization
  - Basic idea of this is to quantize the zero modes of the model which separate out at large Nc. Assumed to be exact at large Nc
- Technical approach: **assume** the rigid rotor modes decouple. Works for SU(2) (ANW) why not SU(3)?

- Large Nc analysis of this approach gives obvious inconsistencies for exotics:

$M_{\theta+}-M_N \sim N_c^0$ *Not collective; does not decouple from vibrational d.of.*

Contrast with nonexotixcs $M_\Delta - M_N \sim N_c^{-1}$

Widths: $\Gamma_{\theta+} \sim N_c^0$ *but direct collective quantization simply gives a mass.*

Contrast with nonexotixcs $\Gamma_\Delta = 0$ *at large Nc.*

States do not arise as collective states in model independent large Nc analysis (Dashen-Manohar “large Nc conistency” approach)

All other collective properties are exactly captured (at large Nc)in this approach.

- What’s wrong?

  Witten-Wess-Zumino term gives interaction linear in time. This implies that some static zero modes do not have dynamical partners.

  Analog of charged particle in magnetic field

- Pentaquarks do not exist as purely collective excitations in these models
- For correct semiclassical collective quantization see Aleksey Cherman’s talk.
- Does not necessarily mean models cannot have pentaquarks---but they are non-collective and must be computed using Callan-Klebanov approach.

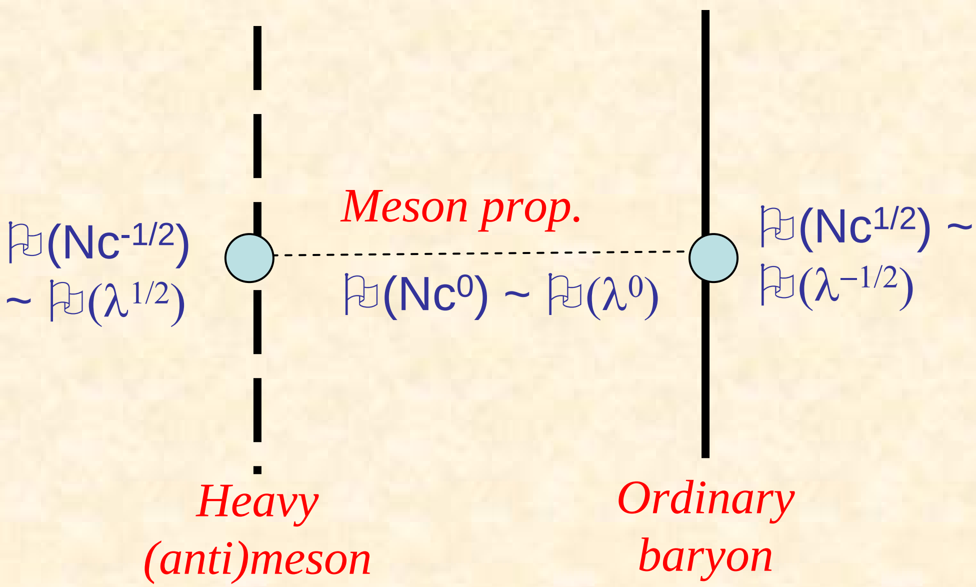
On the Existence of heavy pentaquarks in large Nc & heavy quark limits

- General argument is based on effective field theory Hilbert space states in QCD with
  - Baryon number equal to unity; Heavy quark number of -1 (One net anti-heavy quark)
  - Energies less than the $M_N + M_H + m_\pi$ (H is heavy meson). Work below threshold for three-body final states. “Integrate out” (“project out”) all three hadron final states; theory is nonlocal.
  - QCD is this regime is necessarily completely equivalent to a (nonrelativistic) two-body quantum theory.

- Power Counting (λ as a common prameter; modeled on counting for nonexotic heavy baryons at large Nc C.K. Chow &TDC PRL 84 (2000) 5474; NPA 688 (2001) 842)
  - $\lambda \sim 1/\mathrm{Nc}$; $\lambda \sim \Lambda/m_h$ (Λ is typical hadronic scale)
  - Expansion in $\lambda^{1/2}$ (marginal as a quantitive description of Nc=3 world)
  - At threshold relative p: $p=(2\mu m_\pi)^{1/2}$;

    $$\mu = M_N M_H (M_N + M_H) \sim 1/\lambda; \quad p \sim \lambda^{-1/2}$$
  - Nonlocality at length scale $1/p \sim \lambda^{1/2}$.
  - Typical velocity $p/\mu \sim \lambda^{1/2}$.

- Effective theory assumes the form of a non-relativistic Schrödinger equation with a local potential at leading order.
  - nonlocalities & relativistic corrections $\sim\lambda^{1/2}$
  - Potential has strength and range of $\mathcal{O}(\lambda^0)$
    - Check via Witten approach of quark line counting
    - Easily seen in meson-exchange picture:



- In general a Schrödinger equation of the form $$\left(-\lambda\frac{\nabla^2}{2\bar{\mu}}+V(r)\right)\psi(\vec{r})=E\psi(\vec{r}) \quad \text{where } \bar{\mu}=\mu\lambda$$ has (many) bound states of both parities with many spins as λ gets large provded there exists at least some region where *V(r)<0.*
- We know that for large *r*, *V(r)* , is well described by a OPEP potential.
  - OPEP is necessarily attractive in some channels (and repulsive in others) depending on the relative sign of the heavy quark coupling constant to pions and the nucleons.

- Large Nc QCD **must** have (strong interaction) **stable** pentaquarks in the combined large Nc and heavy quark limits.
  - Pentaquarks of both parities and many angular momenta must exist.
- Nearly degenerate multiplets of heavy pentaquarks exist as the limit is approached.
  - Fall into representations of contracted SO(8)xSU(4)xSU(2)

Collective vibrations | ***Spin-flavor for light quarks*** | ***Heavy quark spin***

- Proof of principle QCD can have exotic baryons.
- What about the real world? Does this argument strongly suggest that stable heavy pentaquarks exist for Nc=3?
- General argument of this type also suggests that many deeply bound 2-nucleon states should exist but in practice we have one barely bound state---the deuteron.

Caveat Emptor

- Modeling suggests real world is, alas far from the ideal limit. (See P. Hohler’s talk).

Summary

- Large Nc provides a model independent approach to the problem
- If pentaquarks exist they must form multiplets at large Nc
- Initial chiral soliton model calculations of pentaquarks based on quantization which is unjustifed at large Nc
- Combined large Nc and heavy quark limits requires heavy pentaquarks