On the Existence of Heavy Pentaquarks:

The large N_c and the Heavy Quark Limits and Beyond

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Introduction

- We want to determine if pentaquarks could exist in the real world.
- We have beautiful model-independent result that in the combined heavy quark, large N_c limits, pentaquarks must exist. (See T.D.Cohen's talk)
- Question: How close is the real world to this limit?
- Can't solve QCD, therefore we must consider some model with "realistic" parameters (one pion exchange potential).
  - In combine limit, (attractive) OPEP must bind
- We found that the real world is not near this limit for the case of pentaquarks.

Heavy Pentaquarks

- Consider pentaquarks with c or b antiquarks

- With very large mass → heavy quark symmetry arises
  - Spin of heavy quark is irrelevant
  - Mass degeneracy between D(B) and D*(B*)

- Combine vector and pseudoscalar into an H-field

$$H \equiv \frac{(1+ \forall)}{2}[P_1^{\text{¤}\circ\, 1} - P^{\circ}{}_5]$$

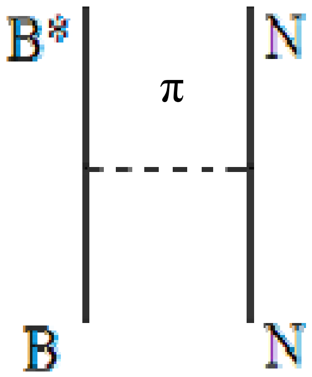
Large N_c and Heavy Quark Limits

- Heavy Pentaquarks not only can exist, but must exist! (see talk by T. D. Cohen)
- Emergent symmetry $SU(4) \times O(8) \times SU(2)$
- Any potential with a region of attraction guarantees the existence (in the limit)!
- Is the real world close enough to this limit, so that this qualitative result holds?

Therefore…

- Can we cook up a realistic potential with some attractive region?
- Of course, the One Pion Exchange Potential (OPEP)
- Thus if a OPEP, with realistic parameters, binds a pentaquark, irrespective of the short distance physics, then pentaquarks must exist in the real world!

One Pion Exchange Potential



$$\mathcal{L}_{N N ¼} = -\frac{g_A}{f_{¼}\, p\overline{2}} \bar{N}^{¹} ¿^{a} °_{º} °_5 N \, @^{º} ¼^{a}$$

$$\mathcal{L}_{int} = -\frac{g_H}{f_{¼}\, p\overline{2}} \mathrm{Tr}\overline{H} ¿^{a} °_{¹} °_5 H \, @^{¹} ¼^{a}$$

$$V_{¼}(\vec{r}) = (I^2 - I_N^2 - I_H^2)[S_{12} V_T(r) + (K^2 - S_N^2 - S_l^2) V_c(r)]$$

$$V_c(r) = \frac{g_A g_H}{2¼ f_{¼}^2} \frac{e^{i\ r}}{3r} \qquad V_T(r) = \frac{g_A g_H}{2¼ f_{¼}^2} \frac{e^{i\ r}}{6r} \; | \; \frac{3}{r^2} + \frac{3}{r} + 1$$

Short Distance Physics

- The OPEP was cutoff ~ 1 fm
- Short distance physics modeled as either constant or quadratic
- Some parameters were determined by fitting to a form which for nucleons yields correct deuterium binding (to make sure potential wasn't too stupid)
- Varied depth, width, and coupling constant to determine robustness of results

B/N Bound State Results

Channel J	S	P	I	A +	A −	B +	B −	C +	C −	D +	D −
$\frac{1}{2}$	$\frac{1}{2}$	−	0	1.30	1.35	3.89	1.92, 3.62	139.38, 142.14	–	14.49, 16.01	15.46, 16.15
			1	–	–	0.35	0.27	–	139.38, 140.76	15.32, 15.60	15.04, 15.46
$\frac{1}{2}$	$\frac{1}{2}$	+	0	–	–	–	–	14.9, 32.39	4, 19.32, 46.5	–	–
			1	–	–	–	–	12.72, 18.22, 26.91	9.45	–	–
$\frac{1}{2}$	$\frac{3}{2}$	−	0	1.30	1.31	3.89	3.67	140.76	140.76	15.87	15.32
			1	–	–	–	0.26	140.76	140.76	15.04	15.32
$\frac{1}{2}$	$\frac{3}{2}$	+	0	–	–	–	–	32.15	3.35, 45.95	–	–
			1	–	–	–	–	12.12, 27.19	8.36, 22.08	–	–
$\frac{3}{2}$	$\frac{1}{2}$	−	0	1.42	1.31	3.89	3.67	140.76	140.76	15.87	15.32
			1	–	–	–	0.26	140.76	140.76	15.04	15.32
$\frac{3}{2}$	$\frac{1}{2}$	+	0	–	–	–	–	15.32, 18.49, 32.43	4.65	–	–
			1	–	–	–	–	12.80	17.25, 17.66, 22.91	–	–
$\frac{3}{2}$	$\frac{3}{2}$	−	0	1.42	1.25	3.89	3.67	140.76	140.76	15.87	15.32
			1	–	–	–	0.20	140.76	140.76	15.04	15.32
$\frac{3}{2}$	$\frac{3}{2}$	+	0	–	–	–	–	18.22, 32.29	–	–	–
			1	–	–	–	–	4.18, 23.18	–	–	–

Column A: constant potential V_0 = -62.79 MeV and r_0 = 1 fm

Column B: quadratic potential

Column C: constant potential V_0 = -276 MeV and r_0 = 1 fm

Column D: constant potential V_0 = -62.79 MeV and r_0 = 1.5 fm

D/N Bound State Results

Channel J S P	I	A +	A −	B +	B −
$\frac{1}{2}$ $\frac{1}{2}$ −	0	113.99, 110.4	–	7.36, 9.00	8.45, 9.27
	1	–	114.82, 115.78	8.40, 8.79	8.16, 8.63
$\frac{1}{2}$ $\frac{1}{2}$ +	0	2.91	16	–	–
	1	–	–	–	–
$\frac{1}{2}$ $\frac{3}{2}$ −	0	117.3	116.2	9.00	8.45
	1	115.23	115.23	8.45	8.45
$\frac{1}{2}$ $\frac{3}{2}$ +	0	2.10	15.87	–	–
	1	–	–	–	–
$\frac{3}{2}$ $\frac{1}{2}$ −	0	117.3	116.20	9.00	8.45
	1	115.37	115.78	8.45	8.45
$\frac{3}{2}$ $\frac{1}{2}$ +	0	2.91	–	–	–
	1	–	–	–	–
$\frac{3}{2}$ $\frac{3}{2}$ −	0	117.3	116.20	9.00	8.45
	1	115.09	115.09	8.45	8.45
$\frac{3}{2}$ $\frac{3}{2}$ +	0	2.53	–	–	–
	1	–	–	–	–

Column A: constant potential V_0 = -276 MeV and r_0 = 1 fm

Column B: constant potential V_0 = -62.79 MeV and r_0 = 1.5 fm

Summary of Results

- When short distance physics (SDP) could bind deuterium, only states with negative parity could bind the B meson.
- Deeper and wider potentials supported more and stronger bound states.
- B mesons bound better than D mesons
- When the SDP was turned off, neither B nor D mesons bound

⇨ **Short distance physics plays a crucial role in the existence of heavy pentaquarks!!**

Conclusion

- Existence of heavy pentaquarks depends greatly on the short distance physics
- The real world is not near the combined large N_c and heavy quark limits
- More understanding of short distance physics is needed to deduce the existence of heavy pentaquarks

$$V_{¼}(x) = ½ \begin{array}{ll} (I^2 - I_N^2 - I_H^2)[S_{12}V_T(r) + (K^2 - S_N^2 - S_H^2)V_c(r)] & r > r_0 \\ V_1(r) \text{ or } V_2(r) & r < r_0 \end{array}$$

$$V_c(r) = \frac{g_A g_H}{2¼f_{¼}^2} \frac{e^{i\, m_{¼} r}}{3r} m_{¼}^2$$

$$V_T(r) = \frac{g_A g_H}{2¼f_{¼}^2} \frac{e^{i\, m_{¼} r}}{6r} µ \frac{3}{m_{¼}^2 r^2} + \frac{3}{m_{¼} r} + 1 ¶ m_{¼}^2$$

$$V_1(r) = V_0 \; (V_0 = -62{:}79 \text{ MeV or } -276 \text{ MeV})$$

$$V_2(r) = -252{:}659 \frac{\text{MeV}}{\text{fm}^2} r^2 + 541{:}321 \frac{\text{MeV}}{\text{fm}} r - 309{:}822 \text{ MeV}$$

(1)

Motivation

- Let’s consider the binding of a heavy meson and a nucleon
- Any attractive potential between these will yield bound pentaquarks
- At large distances, there must be a one pion exchange potential (OPEP) which is attractive.

- So if a OPEP with realistic parameters binds a pentaquark irrespective of the short distance physics, then pentaquark must exist in the real world!!

B Meson Results

Channel			I	A		B		C		D	
J	S	P		+	–	+	–	+	–	+	–
$\frac{1}{2}$	$\frac{1}{2}$	–	0	1.30	1.35	3.89	1.92, 3.62	139.38, 142.14	{	14.49, 16.01	15.46, 16.15
			1	{	{	0.35	0.27	{	139.38, 140.76	15.32, 15.60	15.04, 15.46
$\frac{1}{2}$	$\frac{1}{2}$	+	0	{	{	{	{	14.9, 32.39	4, 19.32, 46.5	{	{
			1	{	{	{	{	12.72, 18.22, 26.91	9.45	{	{
$\frac{1}{2}$	$\frac{3}{2}$	–	0	1.30	1.31	3.89	3.67	140.76	140.76	15.87	15.32
			1	{	{	{	0.26	140.76	140.76	15.04	15.32
$\frac{1}{2}$	$\frac{3}{2}$	+	0	{	{	{	{	32.15	3.35, 45.95	{	{
			1	{	{	{	{	12.12, 27.19	8.36, 22.08	{	{
$\frac{3}{2}$	$\frac{1}{2}$	–	0	1.42	1.31	3.89	3.67	140.76	140.76	15.87	15.32
			1	{	{	{	0.26	140.76	140.76	15.04	15.32
$\frac{3}{2}$	$\frac{1}{2}$	+	0	{	{	{	{	15.32, 18.49, 32.43	4.65	{	{
			1	{	{	{	{	12.80	17.25, 17.66, 22.91	{	{
$\frac{3}{2}$	$\frac{3}{2}$	–	0	1.42	1.25	3.89	3.67	140.76	140.76	15.87	15.32
			1	{	{	{	0.20	140.76	140.76	15.04	15.32
$\frac{3}{2}$	$\frac{3}{2}$	+	0	{	{	{	{	18.22, 32.29	{	{	{
			1	{	{	{	{	4.18, 23.18	{	{	{

Column A: constant potential V_0 = -62.79 MeV and r_0 = 1 fm

Column B: quadratic potential

Column C: constant potential V_0 = -276 MeV and r_0 = 1 fm

Column D: constant potential V_0 = -62.79 MeV and r_0 = 1.5 fm

D Meson Results

Channel			I	A		B	
J	S	P		+	–	+	–
$\frac{1}{2}$	$\frac{1}{2}$	–	0	113.99, 110.4	{	7.36, 9.00	8.45, 9.27
			1	{	114.82, 115.78	8.40, 8.79	8.16, 8.63
$\frac{1}{2}$	$\frac{1}{2}$	+	0	2.91	16	{	{
			1	{	{	{	{
$\frac{1}{2}$	$\frac{3}{2}$	–	0	117.3	116.2	9.00	8.45
			1	115.23	115.23	8.45	8.45
$\frac{1}{2}$	$\frac{3}{2}$	+	0	2.10	15.87	{	{
			1	{	{	{	{
$\frac{3}{2}$	$\frac{1}{2}$	–	0	117.3	116.20	9.00	8.45
			1	115.37	115.78	8.45	8.45
$\frac{3}{2}$	$\frac{1}{2}$	+	0	2.91	{	{	{
			1	{	{	{	{
$\frac{3}{2}$	$\frac{3}{2}$	–	0	117.3	116.20	9.00	8.45
			1	115.09	115.09	8.45	8.45
$\frac{3}{2}$	$\frac{3}{2}$	+	0	2.53	{	{	{
			1	{	{	{	{

Column A: constant potential V_0 = -276 MeV and r_0 = 1 fm

Column B: constant potential V_0 = -62.79 MeV and r_0 = 1.5 fm

Example of Results I

- Consider the state J=1/2, S=1/2, P= +1, I = 0
- Consider the potential with constant inner region and depth of -276 MeV and the coupling constants have same sign.

Bound states exist at14.9 MeV and 32.39 MeV below threshold.

Example of Results II

- Consider the state J=1/2, S=1/2, P= +1, I = 0
- Consider the potential with constant inner region and depth of -62.79 MeV and the coupling constants have same sign.

No Bound State exists!!