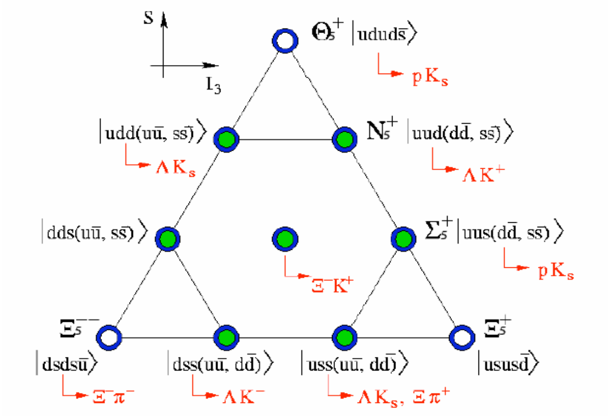
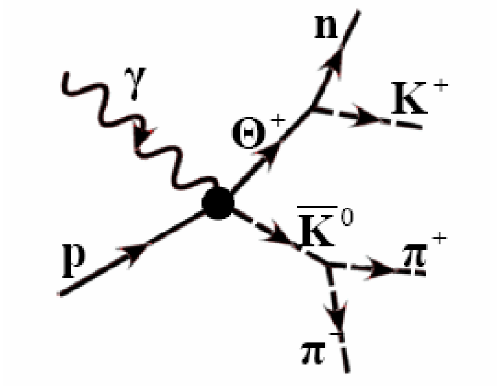




Variational Monte Carlo studies of pentaquark states

Status of $\Theta^+(1540)$ and non-relativistic constituent quark model calculations

(my name's not Estragon)

Thanks for discussions with: Jo Dudek (quark model/expt'l), Ross Young (lattice calculations), Bob Wiringa (VMC wave functions)

nucl-th/0507061, *PRL, In press*

Jefferson Lab

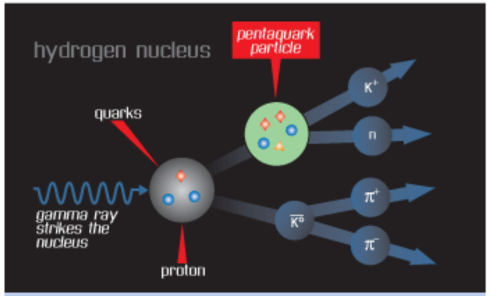
Overview

Organizing principle:

- Fix parameters of H_{NRCQM} from 3q,6q
- Narrow resonance approximation
- What is m_{5q}?

Outline

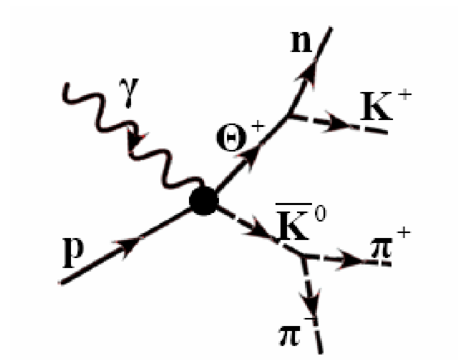
1. Pentaquark states
    - negative or positive parity
    - $J = \frac{1}{2}$
    - Isospin T=0
2. Define H
    - Non-relativistic constituent quark flux-tube model
3. Variational wave function & Monte Carlo
    - $|\Psi_V\rangle = \widehat{\mathcal{G}}|\Phi\rangle$
4. Results
    - $J^{\pi} = \frac{1}{2}^{\pm}$ Masses
    - diquark—diquark pair distributions



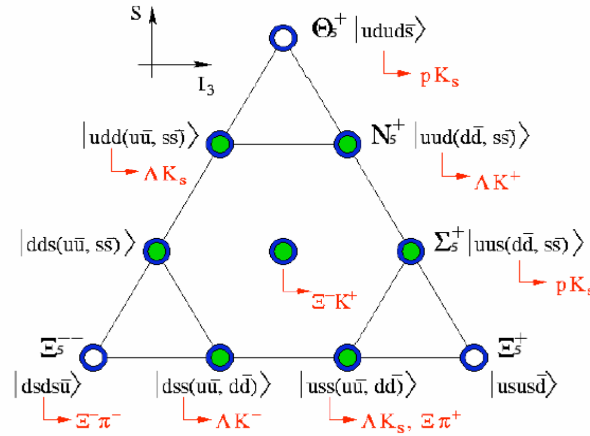
It was expected that a pentaquark may be produced when a gamma ray strikes a proton. However, the researchers saw no evidence of the pentaquark.

Pentaquark Debate Heats Up

New data from Jefferson Lab shows the θ^+ pentaquark doesn't appear in one place it was expected. The result contradicts earlier findings in this same region and adds to the controversy over whether research groups from around the world have caught a glimpse of a pentaquark, a particle built of five quarks.

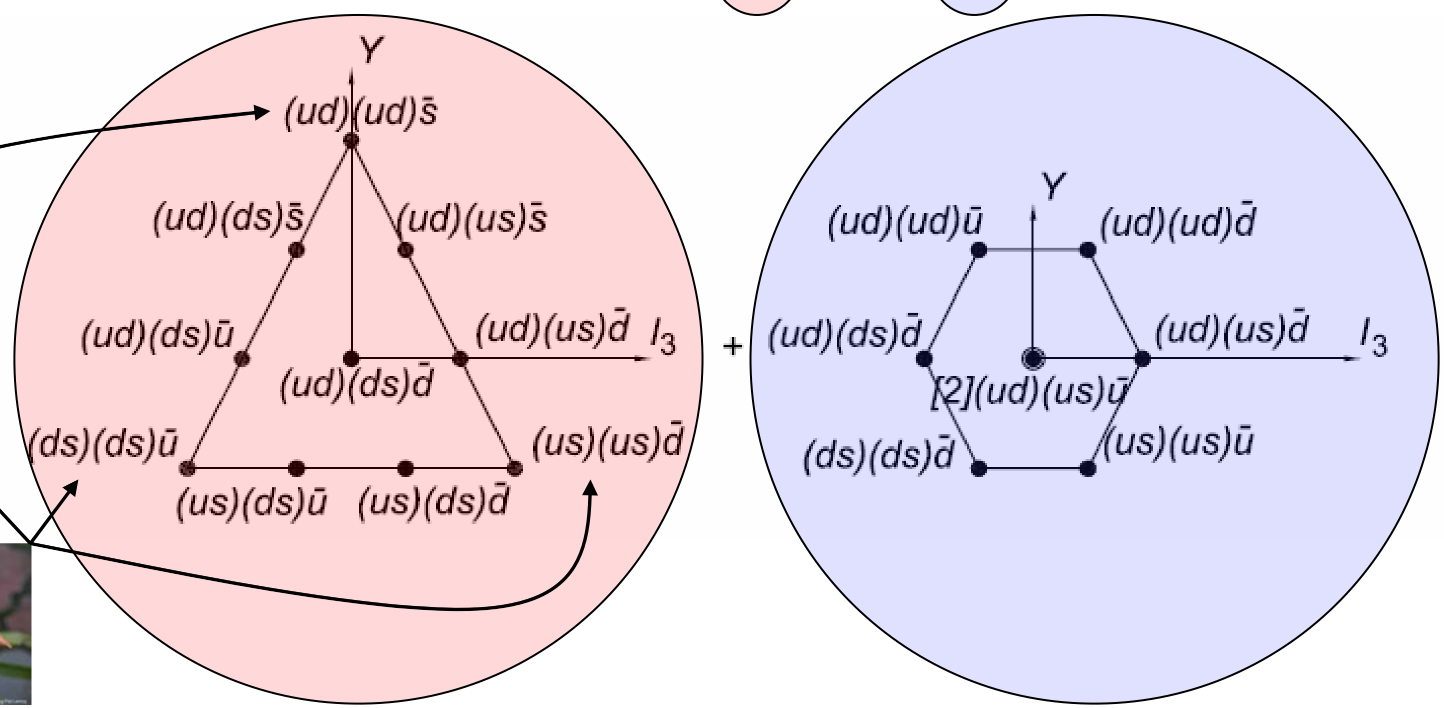


Pentaquark Taxonomy



- $|q_1 q_2 q_3 q_4 \bar{q}_5\rangle$ minimal quark content

- SU(3)$_F$:
  $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \overline{\mathbf{3}}$
  $= \mathbf{35} \oplus (3)\mathbf{27} \oplus (4)\mathbf{10} \oplus (2)\overline{\mathbf{10}} \oplus (8)\mathbf{8} \oplus (3)\mathbf{1}$



Flavor

Exotics

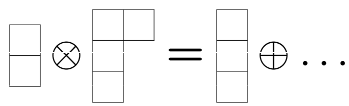
Θ^+("1540") Quantum numbers

- 4q totally antisymm: $P_{ij}|[4q]\bar{q}\rangle = -|[4q]\bar{q}\rangle$

  $m_q = 313\text{MeV};\ m_s = 510\text{MeV}$

- Antiquark coupling:

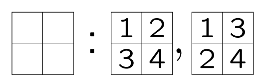
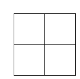
  $SU(3)_{\text{color}}$:

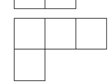
  

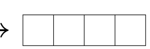
  $\bar{\mathbf{3}} \otimes \mathbf{3} = \mathbf{1} \oplus \cdots$

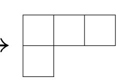
  Total Spin:

  $J_4 \otimes \frac{1}{2} = J_4 \pm \frac{1}{2} \rightarrow$ $\frac{1}{2}$ only

- [4q]:
  - Isospin: $T = 0 \leftrightarrow$ 
  - Spin/orbital: Spin : $S_4 = 0 \leftrightarrow$ 

    $S_4 = 1 \leftrightarrow$ 

    OAM : $L_4 = 0 \leftrightarrow$ 

    $L_4 = 1 \leftrightarrow$ 

    $\vec{J}_4 = \vec{L}_4 + \vec{S}_4$

    $L_4 : 0, 1$

    $S_4 : 0, 1, \not{2}$

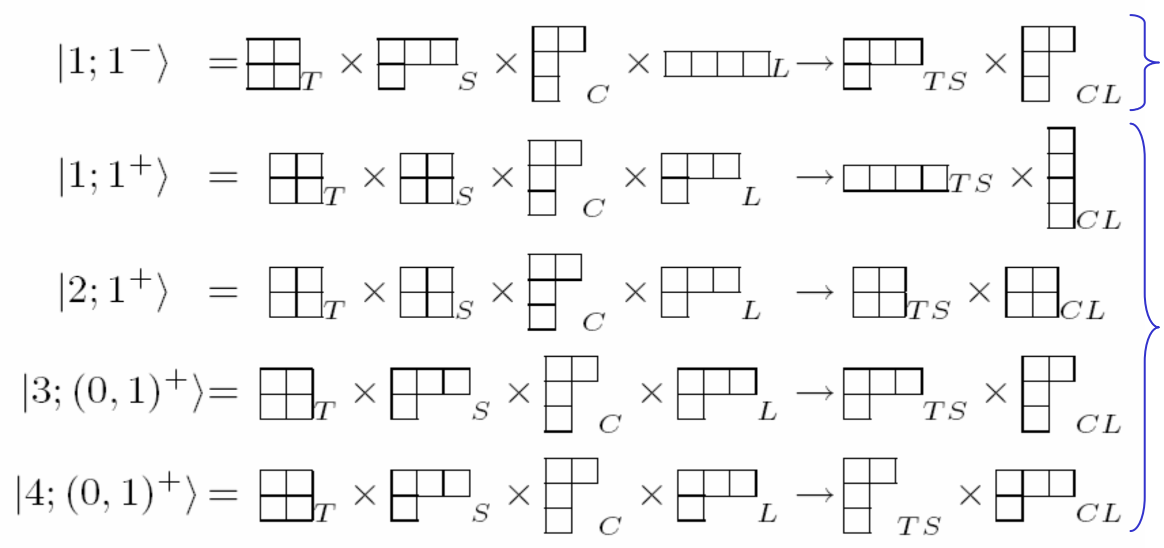
    $J_4 : 0, 1, \not{2}, \not{3}$
  - Total parity π=±1, either!

Uncorrelated [4q] states

- Classification $|\Phi\rangle = |n; J_4^{\pi}\rangle$

  symmetry index for given total parity

- Inner products on $\mathcal{S}_4$



$L = 0,\ \pi = -1$

$L = 1,\ \pi = +1$

- Overlaps
  - Jaffe-Wilczek $\langle [ud][ud]\bar{s}|1;1^+;\bar{s}\rangle = \frac{1}{6}$
  - Karliner-Lipkin $\langle [ud]\{ud\bar{s}\}|1;1^+;\bar{s}\rangle = 0;\ \langle [ud]\{ud\bar{s}\}|n>2;1^+;\bar{s}\rangle \neq 0$

Constituent quark model Hamiltonian

- Dynamical Hamiltonian model
  - non-relativistic kinetic energy

$$H_{CQM} = \sum_{i=1}^{N_q} \frac{\hbar^2}{2m_i}\nabla_i^2$$

  - many-body confining term
    - flux tube model

$$+V^{FT}(\mathbf{r}_1, \ldots, \mathbf{r}_{N_q})$$

  - two-body potential interactions
    - strong state dependence T=0,1 isospin
      S=0,1 spin
      C=**3***,**6** color
    - just about any configuration space potential can be handled
    - use one-gluon exchange + one-pion exchange

$$v_{ij}^{\pi} = v_{\sigma\tau}(r)\sigma_i \cdot \sigma_j \tau_i \cdot \tau_j + v_{t\tau}(r) S_{ij} \tau_i \cdot \tau_j$$

$$v_{ij}^{g} = [v_{cg}(r) + v_{\sigma g}(r)\sigma_i \cdot \sigma_j + v_{tg}(r) S_{ij} + v_{\ell s}(r)(\mathbf{L}\cdot\mathbf{S})_{ij}]\mathsf{T}_i \cdot \mathsf{T}_j$$

$$+\sum_{i<j=1}^{N_q} v_{ij}(\mathbf{r}_{ij})$$

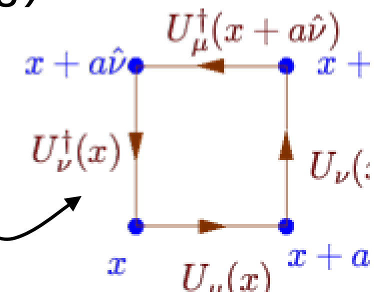
  - constant term
    - scales with number of quarks
      - flux tube ends
      - constant term

$$+N_q v_0$$

$$\sum_{i<j} \mathsf{T}_i \cdot \mathsf{T}_j \Phi = -\frac{1}{2}\left(\sum_i \mathsf{T}_i^2\right)\Phi = -\frac{8}{3} N_q \Phi$$

Confinement potential: flux tube model

- Motivated by lattice Hamiltonian (at strong coupling)
  - Kogut & Susskind

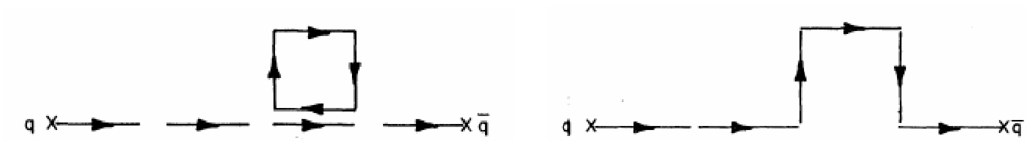


$$H_{KS} = \frac{g_s^2}{2a} \sum_{\alpha,links} \mathbf{E}_\alpha^2$$ color electric field

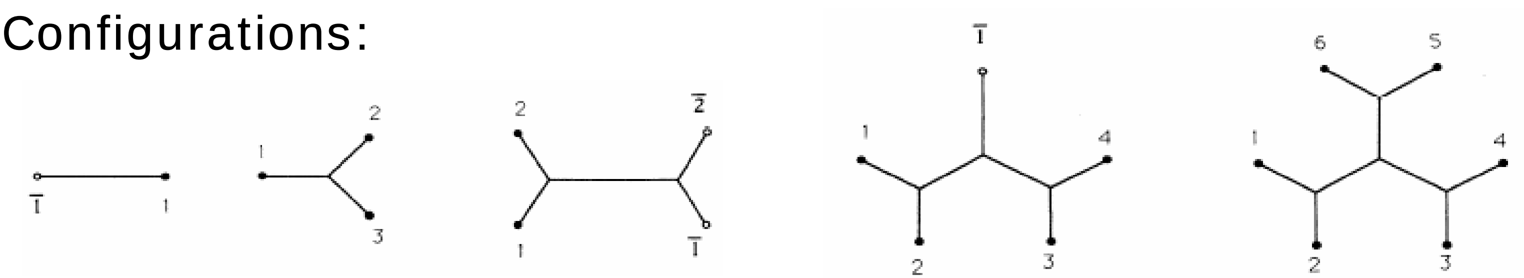
$$+ \frac{4}{a g_s^2} \sum_{\Box} Tr U_\Box$$ color magnetic field "plaquette"; Wilson loop

$$+ \frac{1}{a} \sum_{sites} \psi^\dagger(x) U_\mu(x) \psi(x+\mu) + m_0 \sum_{sites,\mu} (-1)^x \psi^\dagger(x)\psi(x)$$

  - Strong coupling: eigenstates for static quarks are definite paths of flux links; plaquettes induce configuration mixing



- V_{MB} = {string tension} $\times$ {flux tube length}
- Configurations:



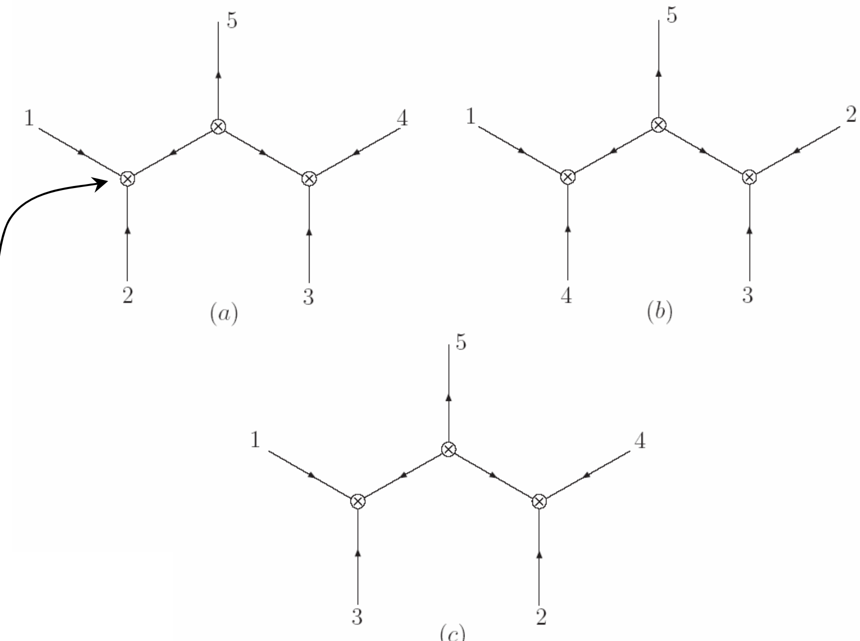
Many body confinement

Flux tube confinement

- -consistent with gauge invariance
- -Kogut & Susskind Hamiltonian
- -Lattice potential studies Suganuma, et. al.

$$V^{FT}(\mathbf{R}) = \sqrt{\sigma}\sum_{i=1}^{5} |\mathbf{r}_i - \mathbf{r}_{Y(i)}|$$

FT topologies



$(a)|[12][34]5\rangle$
$(b)|[14][23]5\rangle$
$(c)|[13][24]5\rangle$

color states

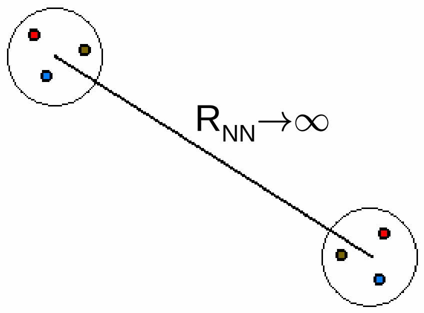
Many-body/2b decomposition

$$V^{FT}(\mathbf{R}) = \sqrt{\sigma}\sum_{i=1}^{5} |\mathbf{r}_i - \mathbf{r}_{Y(i)}| = \sum_{i<j} v^{FT}(r_{ij}) + V^{FT}_{MB}(\mathbf{R})$$

$$V^{FT}_{MB}(\mathbf{R}) = \sqrt{\sigma}\sum_{i=1}^{5} |\mathbf{r}_i - \mathbf{r}_{Y(i)}| - \frac{1}{n-1}\sqrt{\sigma}\sum_{i<j} r_{ij} \lesssim 15\%$$

Jefferson Lab

Model parameters



- Constituent quark mass
  - light quark mass $m_q = m_N/3$
    - fixed by non-relativistic form
  - strange quark mass $m_s = 550$ MeV
    - parameter
- string tension
  - N and Δ trajectories
  - E^2 vs. J for "stringlike" configurations $E^2 = 2\pi\sigma^{1/2} J$
  - $\sigma^{1/2} = 0.88$ GeV/fm
- perturbative gluon coupling constant
  - $m_\Delta - m_N \Rightarrow \alpha_s = 0.61$
- pion quark coupling – fit from elastic NN data
  - $R_{NN} \to \infty \Rightarrow f_{\pi\, qq} = 3 f_{\pi\, NN}/5$
- quark form factor
  - $F(\mathbf{q}^2) = \frac{\Lambda^2 - \mu^2}{\Lambda^2 - \mathbf{q}^2}$
  - $\Lambda = 5\ \mathrm{fm}^{-1}$; parameter
- constant term fitted to $m_N \Rightarrow v_0 \approx 130$ MeV (per flux tube end)

Solution of the Schrödinger equation

- Variational Monte Carlo

$$\langle H\rangle = \frac{\langle\Psi_V|H|\Psi_V\rangle}{\langle\Psi_V|\Psi_V\rangle} \qquad \frac{\delta\langle H\rangle}{\delta\Psi_V} = 0 \text{ S.T. } \langle\Psi_V|\Psi_V\rangle = \text{const.}$$

- How to write a *good* many-body wave function
  - Correlation operator $|\Psi_V\rangle = \hat{\mathcal{G}}|\Phi\rangle$
  - Ansätz $\hat{\mathcal{G}} \approx \mathcal{S}\prod_{i<j=1}^{N_q}\hat{F}_{ij}$
  - Two-body correlation

$$\hat{F}_{ij} = \sum_p f_p(r_{ij})\mathcal{O}_{ij}^p$$

$$\sum_{p=1}^{8} f_p(r_{ij})\mathcal{O}_{ij}^p|TSC\rangle = f_{TSC}(r_{ij})|TSC\rangle$$

$$\sum_{p=9}^{12} f_p(r_{ij})\mathcal{O}_{ij}^p S_{ij}|TSC\rangle = f_{tTC}(r_{ij})S_{ij}|TSC\rangle$$

p	$\mathcal{O}_{ij}^p$	Symb
1	1	c
2	$\tau_i\cdot\tau_j$	τ
3	$\sigma_i\cdot\sigma_j$	σ
4	$\sigma_i\cdot\sigma_j\tau_i\cdot\tau_j$	$\sigma\tau$
5	$\mathsf{T}_i\cdot\mathsf{T}_j$	g
6	$\tau_i\cdot\tau_j\mathsf{T}_i\cdot\mathsf{T}_j$	τg
7	$\sigma_i\cdot\sigma_j\mathsf{T}_i\cdot\mathsf{T}_j$	σg
8	$\sigma_i\cdot\sigma_j\tau_i\cdot\tau_j\mathsf{T}_i\cdot\mathsf{T}_j$	$\sigma\tau g$
9	S_{ij}	t
10	$S_{ij}\tau_i\cdot\tau_j$	$t\tau$
11	$S_{ij}\mathsf{T}_i\cdot\mathsf{T}_j$	tg
12	$S_{ij}\tau_i\cdot\tau_j\mathsf{T}_i\cdot\mathsf{T}_j$	$t\sigma g$

  - Solve two-body Schrodinger-like equation, eg.

$$\left\{-\frac{\hbar^2}{2\mu}\nabla_{ij}^2 + [v_{TSC}(r_{ij}) - \lambda_{TSC}(r_{ij})]\right\} f_{TSC}(r_{ij}) = 0$$

Results

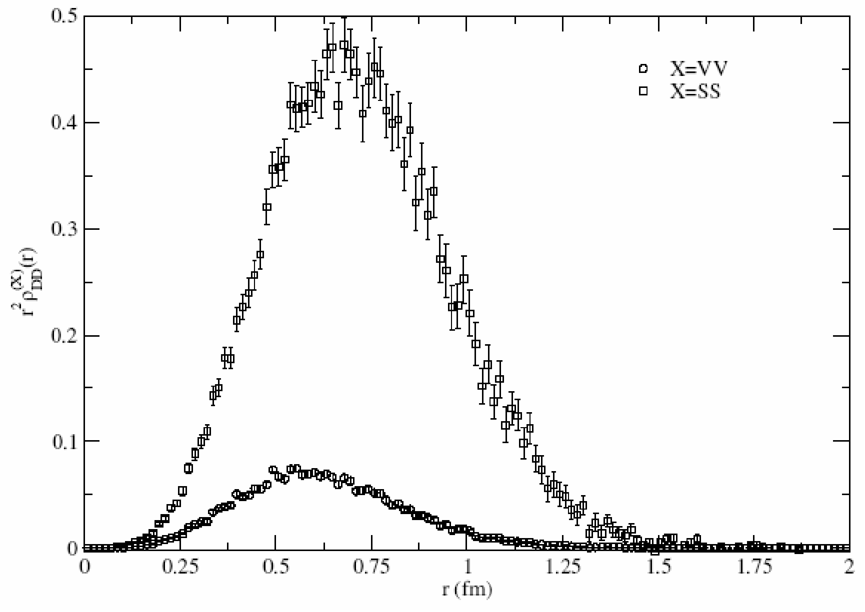
- Masses

$$M_{\theta+}(n;J_4^\pi) = 4m_q + m_s + \langle\hat{T}\rangle + \langle\hat{V}\rangle - V_0(4q\bar{q})$$

$n;J_4^\pi$	$1;1^-$	$1;1^+$	$2;1^+$	$3;0^+$	$3;1^+$	$4;0^+$	$4;1^+$
$M_{\theta+}$	2.22	2.50	2.57	2.75	2.81	2.83	2.88
$\langle\hat{T}\rangle$	1.68	2.13	2.02	2.03	2.00	1.92	1.90
$\langle\hat{V}\rangle$	0.92	0.74	0.93	1.10	1.19	1.29	1.36

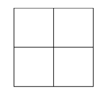
Notes:
1. Negative parity lightest
2. Positive parity
   - lower V
   - higher T

- Short range diquark structure



Notes:
1. Uncorrelated:
   - SS=VV=1/6
2. Correlated
   - SS+VV . 1/3
   - SS>>VV

Concluding remarks: pentaquark in NRCQM

- pentaquark masses & 2 GeV
  - Hamiltonian fit to 3q & 6q properties
  - solve 5q Schrodinger equation (almost) exactly
  - inconsistent with chiral quark soliton model & JW—correlated quark model
- negative parity < positive parity for J=1/2
  - potential is reduced for S : 
  - but not enough to compensate for kinetic energy increase due to orbital angular momentum
- short-range structure
  - significant JW component $\approx$ 1/3
  - other structures important
- existence at 1540 MeV?
  - Yes: what's been overlooked? χ symm? ...?
  - No: could it be elsewhere? why doesn't it exist?

Additional slides

$J^\pi = \frac{1}{2}^+$ pentaquark results

- Two types of pair correlation operators
  - lq-lq: $f_p^q(r_{ij}),\ p = \{c,\sigma,t,\ell s\} \otimes \{1,g\}, \{\sigma,t\} \otimes \{c,\tau\}$
  - lq-sq: $f_p^s(r_{ij}),\ p = \{c,\sigma\} \otimes \{c,g\}$

	CD		CND		LQSQ		T	
vgc=	-543.32+/-	0.46	-20.14+/-	0.12	-197.79+/-	0.20	-563.46+/-	0.46
vft=	1694.91+/-	2.15	0.00+/-	0.00	597.32+/-	0.90	1694.91+/-	2.15
vsg=	-160.62+/-	0.58	-78.79+/-	0.30	-7.70+/-	0.04	-239.41+/-	0.76
vtg=	-1.11+/-	0.00	-0.88+/-	0.00	0.02+/-	0.00	-1.98+/-	0.00
vlg=	1.96+/-	0.02	17.58+/-	0.08	3.92+/-	0.02	19.54+/-	0.10
vst=	-374.24+/-	1.34	0.00+/-	0.00	0.00+/-	0.00	-374.24+/-	1.34
vtt=	-11.19+/-	0.03	0.00+/-	0.00	0.00+/-	0.00	-11.19+/-	0.03
stS=	-410.02+/-	1.38	0.00+/-	0.00	0.00+/-	0.00	-410.02+/-	1.38
stL=	35.79+/-	0.05	0.00+/-	0.00	0.00+/-	0.00	35.79+/-	0.05
VMB=	236.53+/-	0.58	0.00+/-	0.00	0.00+/-	0.00	236.53+/-	0.58
<V>=	842.93+/-	4.65	-82.23+/-	0.31	395.76+/-	1.07	760.70+/-	4.80
<T>=	2125.77+/-	5.71	0.00+/-	0.00	0.00+/-	0.00	2125.77+/-	5.71
<H>=	2968.70+/-	1.42	-82.23+/-	0.31	395.76+/-	1.07	2886.47+/-	1.25
RqR=	0.53E+00+/-	.48E-03	0.00E+00+/-	NaN	0.00E+00+/-	NaN	0.53E+00+/-	.48E-03

```
%%%%
MZ+= 2500.97+/-   1.25
VFT= 1931.45+/-   2.51 v2f= 1694.91+/-   2.15 vmb=  236.53+/-   0.58
OGE= -785.32+/-   1.15
OPE= -385.43+/-   1.35
SKR=0.85E-03+/-.83E-03
SKI=-.11E-03+/-.60E-03
JW  =  1.0000+/- 0.0000
KL  =  1.0000+/- 0.0000
JW1=  0.1107+/- 0.0012
JW2=  0.8288+/- 0.0112
```

$$M(\Theta^+(\tfrac{1}{2}^+)) = 2.5\ \text{GeV}$$

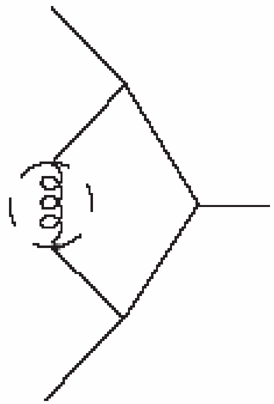
$\Theta^+(1540)$ is a WMD

(Weird Multiquark Demon)*

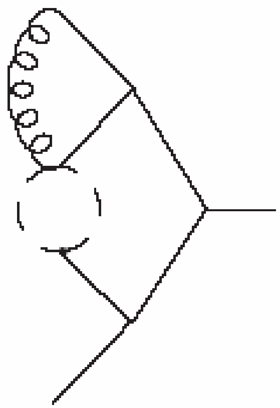
*F. Close

- After: Jaffe & Wilczek
- Remarkably narrow
  - $m_\Theta - (m_n + m_K) \approx +100$ MeV
    - COM threshold ~270 MeV ⇒ not-very-relativistic
  - short range interaction⇒only s- or p-wave likely
  - no annihilation (“exotic”)
- Potential scattering
  - s-wave resonance not possible
  - p-wave:
    - range of potential ~ 1 fm gives width ~ 10^2 MeV !!!
    - some other mechanism (dynamical) needed to get ~10 MeV

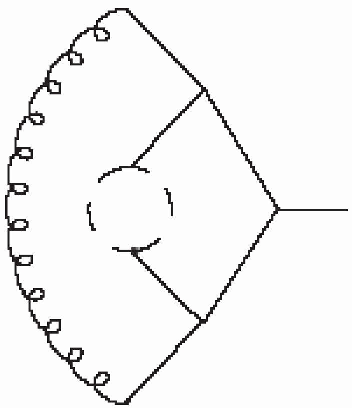
Flux tube exchange/CND matrix elements



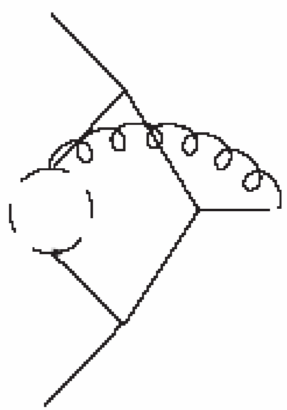
$\langle (12)(34)\bar{5}|\mathrm{T}_2 \cdot \mathrm{T}_4|(14)(32)\bar{5}\rangle$



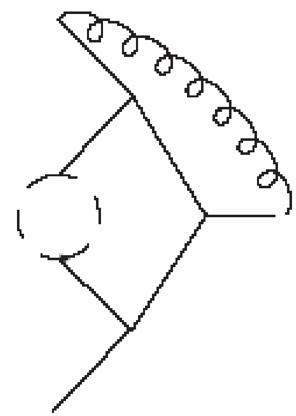
$\langle (12)(34)\bar{5}|\mathrm{T}_1 \cdot \mathrm{T}_2|(14)(32)\bar{5}\rangle$



$\langle (12)(34)\bar{5}|\mathrm{T}_1 \cdot \mathrm{T}_3|(14)(32)\bar{5}\rangle$



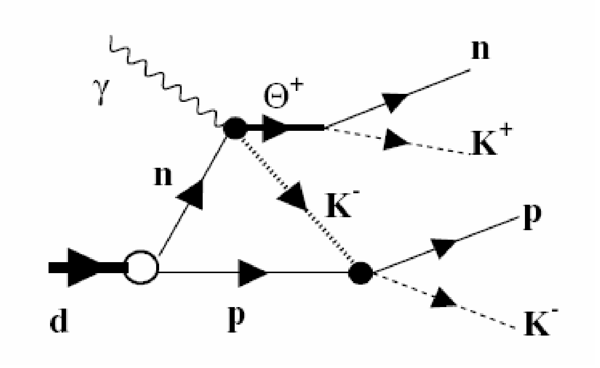
$\langle (12)(34)\bar{5}| - \mathrm{T}_2 \cdot \mathrm{T}_5^*|(14)(32)\bar{5}\rangle$

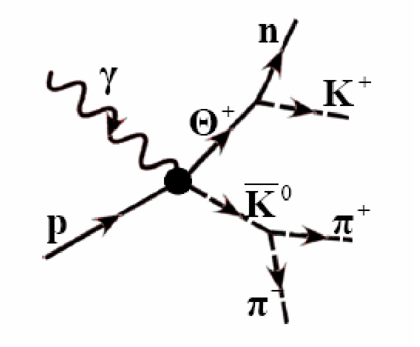


$\langle (12)(34)\bar{5}| - \mathrm{T}_1 \cdot \mathrm{T}_5^*|(14)(32)\bar{5}\rangle$

Production mechanisms

- photoproduction: $\gamma p/\gamma A \to \Theta^+ X$





- hadroproduction:
  - $pp/pA \to \Theta^+ X$
  - $K^+p/K^+A \to \Theta^+ X$
  - $AA \to \Theta^+ X$

- leptoproduction:
  - $e^+e^- \to \Theta^+ X$
  - $eN/\nu N \to \Theta^+ X$

Experimental status (I) – Positive signal

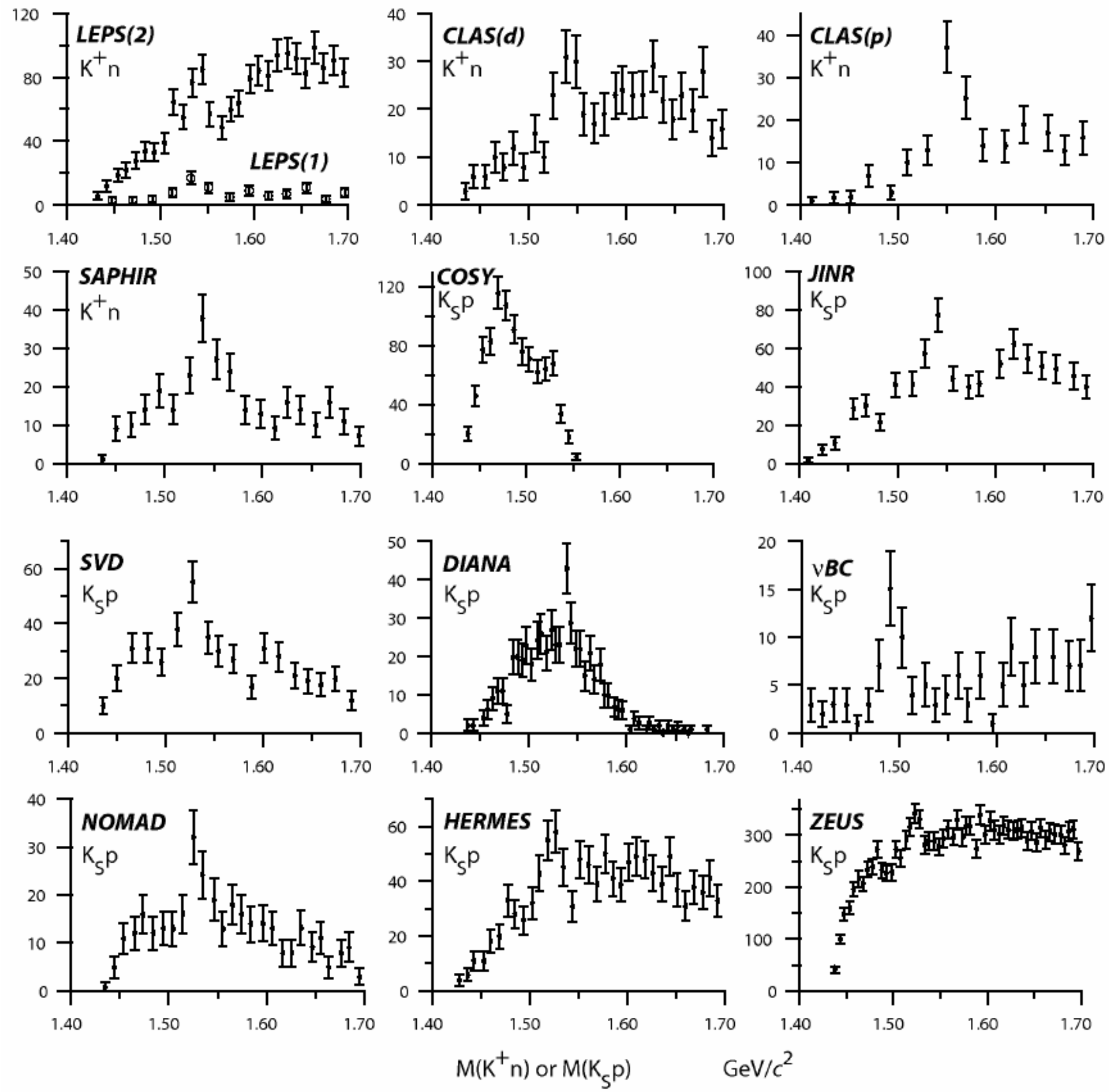
- Dzierba, Meyer, & Szczepaniak ~ 6 months ago hep-ex/0412077

	Experiment	Reaction	State	Mode	Reference
Photoproduction	LEPS(1)	$\gamma C_{12} \to K^+K^-X$	θ^+	K^+n	[4]
	LEPS(2)	$\gamma d \to K^+K^-X$	θ^+	K^+n	[5]
	CLAS(d)	$\gamma d \to K^+K^-(n)p$	θ^+	K^+n	[6]
	CLAS(p)	$\gamma p \to K^+\pi^-\pi^+(n)$	θ^+	K^+n	[7]
	SAPHIR	$\gamma p \to K^0_S K^+(n)$	θ^+	K^+n	[8]
Hadroproduction	COSY	$pp \to \Sigma^+ K^0_S p$	θ^+	$K^0_S p$	[9]
	JINR	$p(C_3H_8) \to K^0_S pX$	θ^+	$K^0_S p$	[10]
	SVD	$pA \to K^0_S pX$	θ^+	$K^0_S p$	[11]
	DIANA	$K^+Xe \to K^0_S p(Xe)'$	θ^+	$K^0_S p$	[12]
Leptoproduction	νBC	$\nu A \to K^0_S pX$	θ^+	$K^0_S p$	[13]
	NOMAD	$\nu A \to K^0_S pX$	θ^+	$K^0_S p$	[14]
	HERMES	quasi-real photoproduction	θ^+	$K^0_S p$	[15]
	ZEUS	$ep \to K^0_S pX$	θ^+	$K^0_S p$	[16]
	NA49	$pp \to \Xi\pi X$	Ξ_5	$\Xi\pi$	[17]
	H1	$ep \to (D^*p)X$	θ_c	D^*p	[18]

- Impressive array of experimental evidence – on the surface
  - statistical significance ~ 4—8 σ

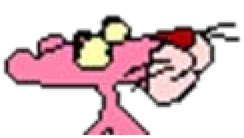
Experimental status (II) – positive signal – cross sections

- Sampling of experimental cross sections
  - counts vs. invariant mass recoiling against K^+n/K_Sp
- Error bars are statistical only
- Low statistics
  - 10 – 100 peak events



Experimental status (III) – Negative signal

- DMS (continued)
- Published and unpublished results
- generally high statistics
- fine resolution, 1—2 MeV
- benchmark resonances seen: $\phi(1020)$, $K^*(890)$, $\Lambda(1520)$, $\Xi(1320)$, $\Xi(1530)$



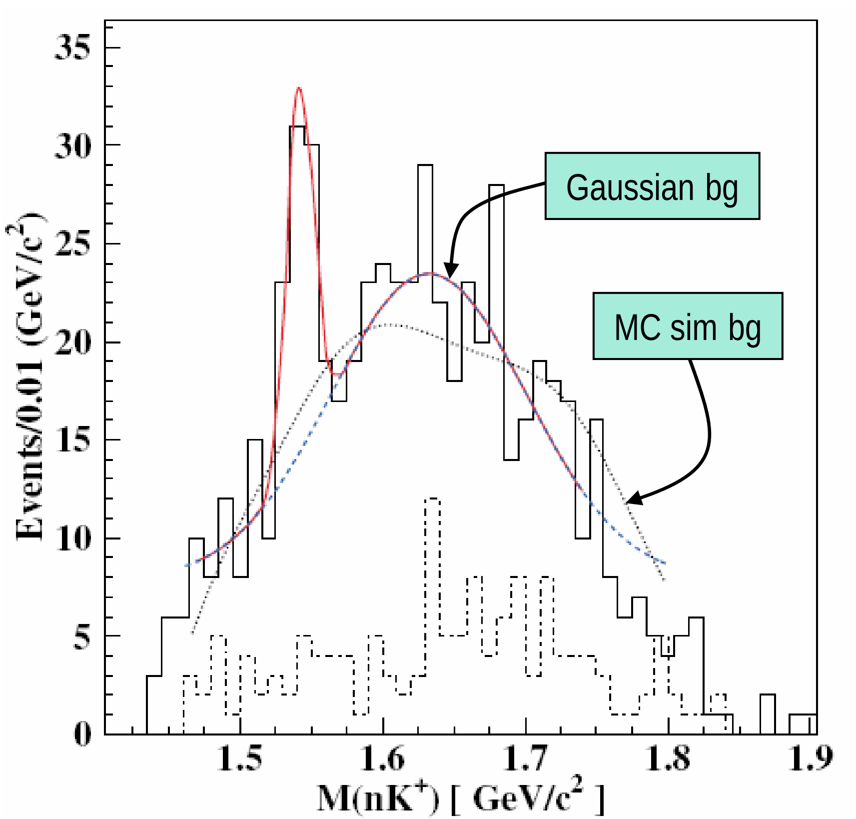
Experiment	Search Reaction	θ^+	Ξ_5	θ_c	Reference
ALEPH	Hadronic Z decays	$\Downarrow$	$\Downarrow$	$\Downarrow$	[19]
BaBar	$e^+e^- \to \Upsilon(4S)$	$\Downarrow$	$\Downarrow$	–	[20]
BELLE	$KN \to PX$	$\Downarrow$	–	$\Downarrow$	[21]
BES	$e^+e^- \to J/\psi(\psi(2S) \to \theta\bar{\theta}$	$\Downarrow$	–	$\Downarrow$	[22]
CDF	$p\bar{p} \to PX$	$\Downarrow$	$\Downarrow$	$\Downarrow$	[23]
COMPASS	$\mu^+(^6LiD) \to PX$	$\Downarrow$	$\Downarrow$	–	[24]
DELPHI	Hadronic Z decays	$\Downarrow$	–	–	[25]
E690	$pp \to PX$	$\Downarrow$	$\Downarrow$	–	[26]
FOCUS	$\gamma p \to PX$	$\Downarrow$	$\Downarrow$	$\Downarrow$	[27]
HERA-B	$pA \to PX$	$\Downarrow$	$\Downarrow$	–	[28]
HyperCP	$(\pi^+, K^+, p)Cu \to PX$	$\Downarrow$	–	–	[29]
LASS	$K^+p \to K^+n\pi^+$	$\Downarrow$	–	–	[30]
L3	$\gamma\gamma \to \theta\bar{\theta}$	$\Downarrow$	–	–	[25, 31]
PHENIX	$AuAu \to PX$	$\Downarrow$	–	–	[32]
SELEX	$(\pi, p, \Sigma)p \to PX$	$\Downarrow$	–	–	[33]
SPHINX	$pC(N) \to \theta^+C(N)$	$\Downarrow$	–	–	[34]
WA89	$\Sigma^-N \to PX$	–	$\Downarrow$	–	[36]
ZEUS	$ep \to PX$	$\Uparrow$	$\Downarrow$	$\Downarrow$	[16, 37, 38]

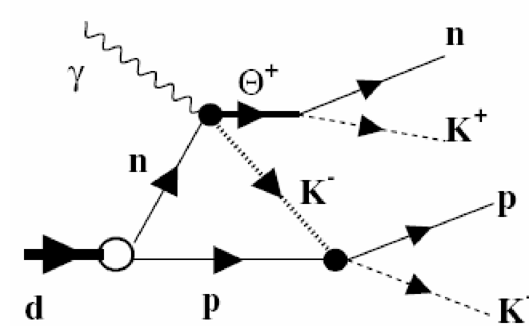
Experimental status (IV) -- Jefferson Lab g2

VOLUME 91, NUMBER 25 PHYSICAL REVIEW LETTERS week ending 19 DECEMBER 2003

Observation of an Exotic $S = +1$ Baryon in Exclusive Photoproduction from the Deuteron

S. Stepanyan,[1,28] K. Hicks,[2] D. S. Carman,[2] E. Pasyuk,[3] R. A. Schumacher,[4] E. S. Smith,[1] D. J. Tedeschi,[5] L. Todor,[4]

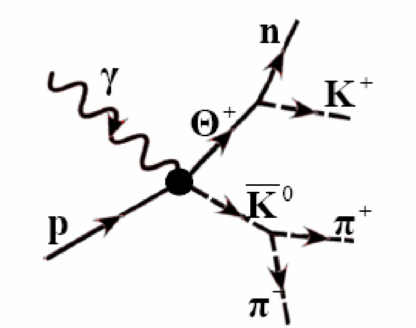


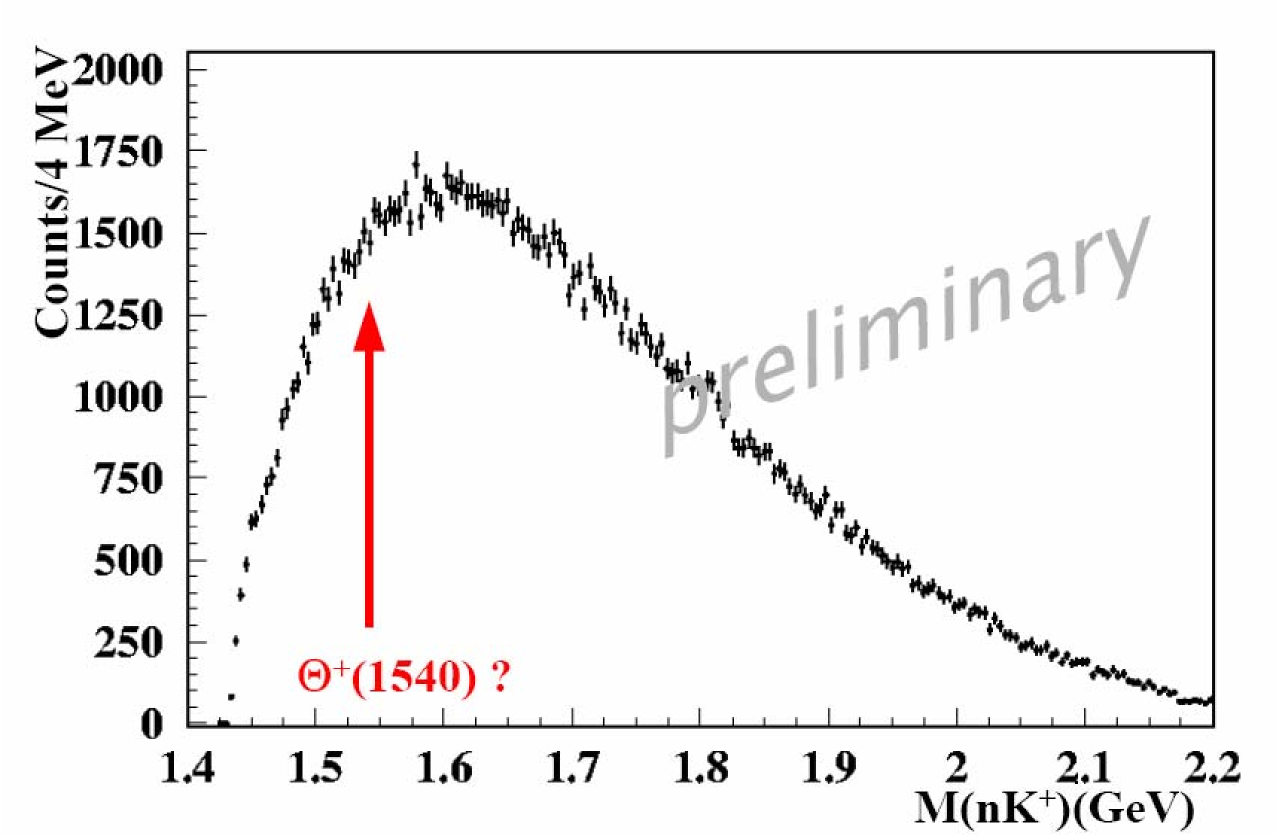


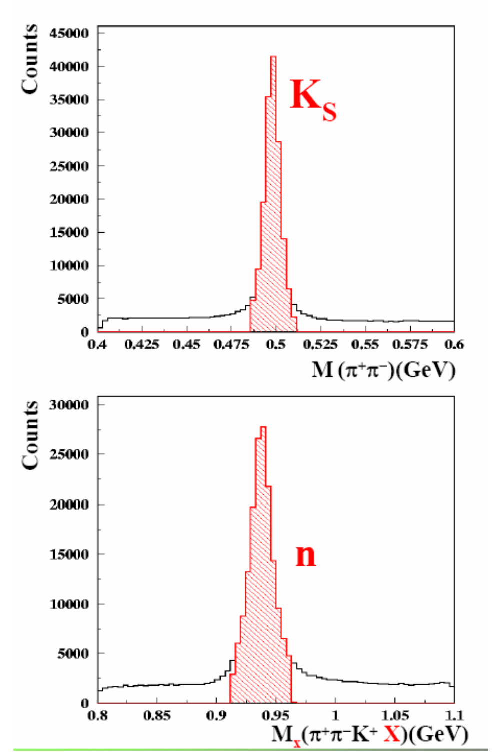
- 43 events above bg in peak
- bg cuts:
  - $\gamma p \rightarrow \phi X$
    $\rightarrow YX$: $M(K^+K^-) < 1.07$ GeV^2
  - $\gamma p \rightarrow \Lambda K^+$:
    $1.485 < M(pK^-) < 1.551$GeV^2
- $M_\Theta = 1542(5)$ MeV
- FWHM = 21 MeV

Experimental status (V) -- Jefferson Lab g11

- $\gamma p \rightarrow \Theta^+ \bar{K}^0, 1.6 < E_\gamma < 3.8$ GeV
- Exclusive process $\Theta^+ \rightarrow nK^+$
  $\bar{K}^0 \rightarrow K_S \rightarrow \pi^+\pi^-$
- High statistics – 120k events







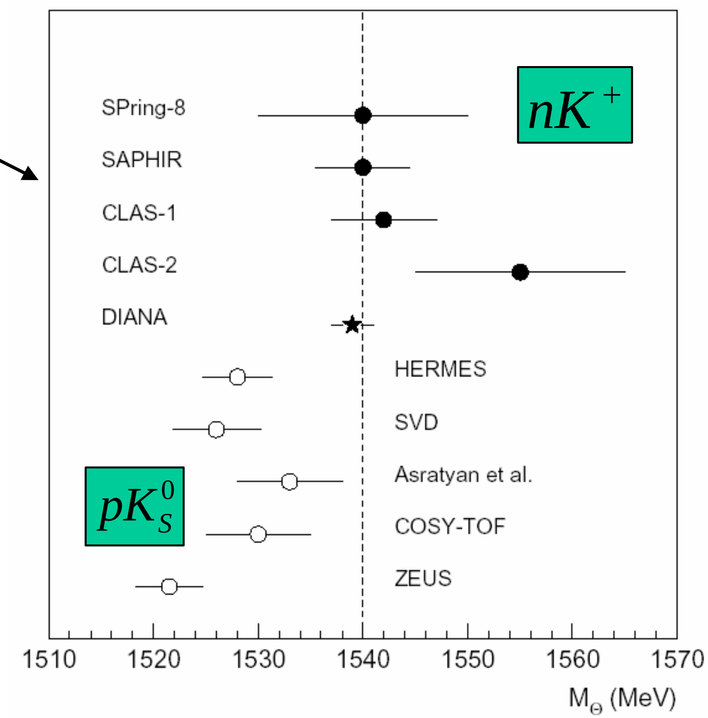
•g11 data courtesy R. diVita

Experimental analysis – possible problems

- Low statistics
  - statistical significance estimators: $s/\sqrt{b},\ s/\sqrt{s+b},\ s/\sqrt{s+2b}$
    - often quoted

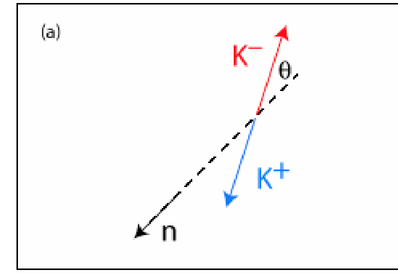
4→8 σ   2→6 σ

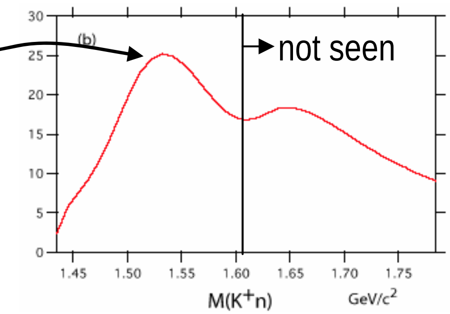
- Harsh angular cuts
  - capable of producing peaks
- Final state interaction effects
  - effects on cross section not studied
- Two masses?
  - 1530 MeV
  - 1540 MeV
- Low energy vs. high energy
  - production mechanism “exotic”?
  - fragmentation production
    - constituent counting rules Θ^+ high energy prod. suppressed
      Titov et.al. PRC,70,042202(R)
- If Θ^+ doesn’t exist, what implications on expt’l analysis?

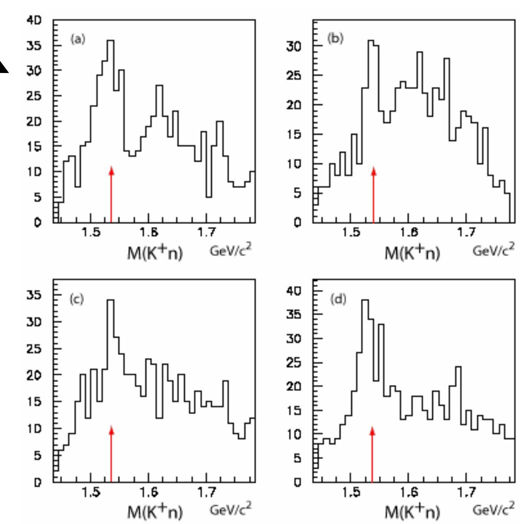


Alternative explanations

- Kinematic reflections Dzierba, Meyer, Szczepaniak hep-ex/0412077
  - a_2, f_2, ρ_3, ... $\rightarrow K^+K^-$ $\Rightarrow$ stucture in K^+n
    - decay angular distribution $\sim |Y_\ell^m(\theta)|^2$
    - $\theta \approx 0 \Rightarrow m_{K^+n}$ small; $\theta \approx \pi \Rightarrow m_{K^+n}$ large
    - amplitude
      $\sim$ t-channel Regge $\times$ decay ang. dist.
    - generated distributions ~ 20% have peak
    - model specifics wrong
      - π^0 Regge doesn't contribute
      - physics may be correct
        Oh, Nakayama, & Lee hep-ph/0412363; Hicks et.al. hep-ph/0411265







- Spurious peaks
  - $\Lambda(1115) \rightarrow \pi^- p$ ghost tracks
    - track "doubling" mimics Θ^+
  - kinematic cuts
    - peak only seen after angular cuts on K^+

(Quenched) Lattice QCD

after Lasscock, et.al. CSSM Lattice Collaboration hep-lat/0503008

- Interpolating functions [$\mp$ T=0,1]
  - color-singlet form

$$\chi_{NK} = \frac{1}{\sqrt{2}}(u^{Ta}C\gamma_5 d^b)[u^c(\bar{s}^e i\gamma_5 d^e) \mp (u \leftrightarrow d)]$$

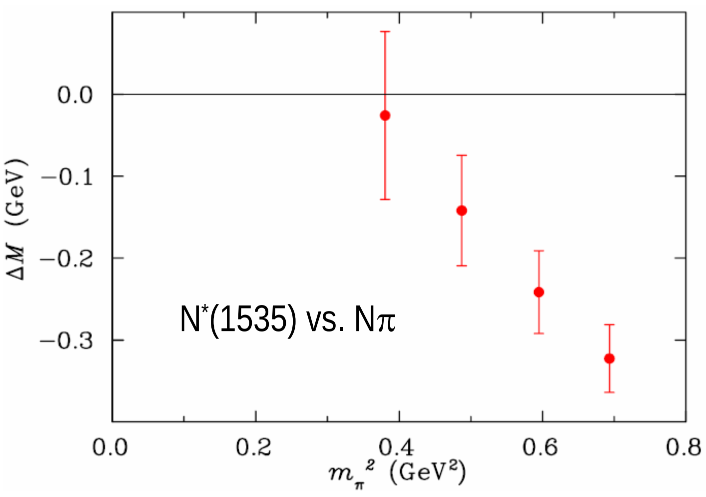
  - color-fused form

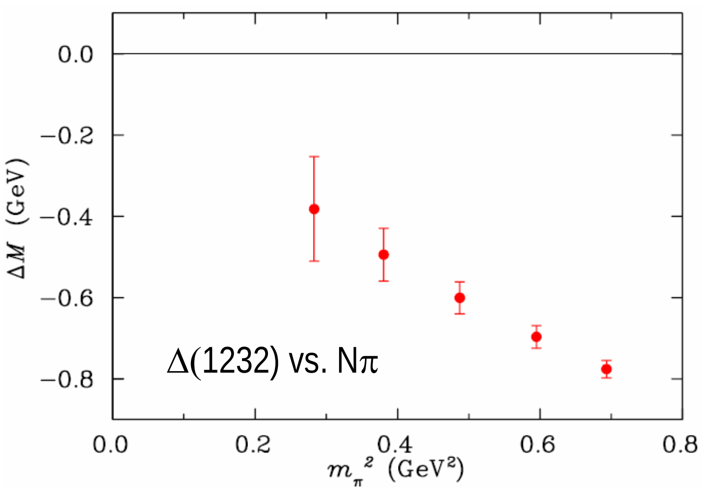
$$\chi_{\widetilde{NK}} = \frac{1}{\sqrt{2}}(u^{Ta}C\gamma_5 d^b)[u^e(\bar{s}^e i\gamma_5 d^c) \mp (u \leftrightarrow d)]$$

  - Jaffe-Wilczek

$$\chi_{JW} = \frac{1}{2\sqrt{2}}\epsilon^{abc}(u^{Ta}C\gamma_5 d^b)[(u^{Tc}C\gamma_5 d^e)-(u^{Te}C\gamma_5 d^c)]C\bar{s}^{Te}$$

- Lattice baryon resonance signature
  - all N,Δ resonance states lie *below* sum of decay products





Lattice signature

- Look for q5 state with mass smaller than N+K
- Isoscalar
- Positive and negative parity

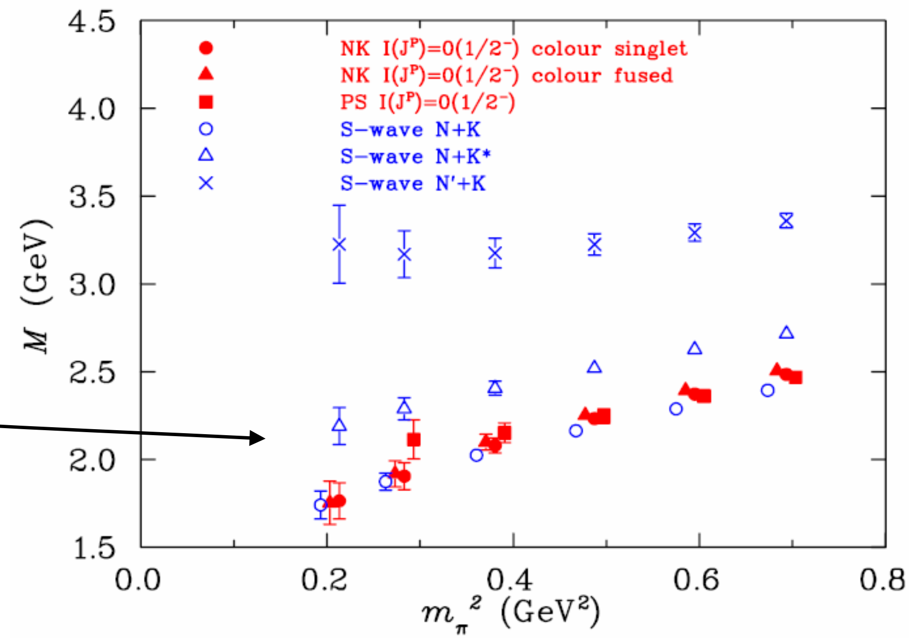


FIG. 6: Masses of the $I(J^P) = 0(\frac{1}{2}^-)$

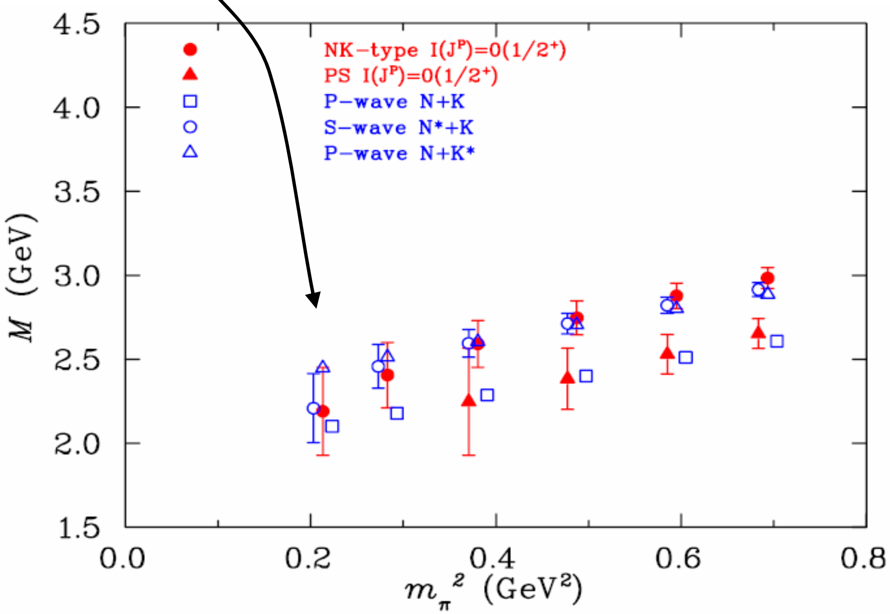


FIG. 20: Masses of the $I(J^P) = 0(\frac{1}{2}^+)$

- Standard lattice resonance signature not observed for any of the spin—1/2 pentaquark states
- Negative parity mass < positive parity (for m_π large)

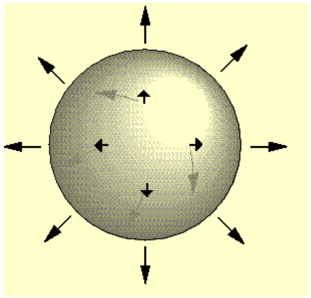
Chiral quark soliton model

- Baryons are solitons of the pion (chiral) field
  - Hedgehog

$$\pi^a(\mathbf{r}) = \frac{x^a}{r} P(r)$$

  - T=J

$$E_{rot} = \frac{J(J+1)}{2I}$$



- Successes
  - Gell-Mann—Okubo relations

$$2(m_N + m_\Xi) = 3m_\Lambda + m_\Sigma$$

$$m_\Delta - m_{\Sigma^*} = m_{\Sigma^*} - m_{\Xi^*} = m_{\Xi^*} - m_{\Omega^-}$$

within 8 and 10

  - Guadagnini formula

$$8(m_{\Xi^*} - m_N) + 3m_\Sigma = 11m_\Lambda + 8m_\Sigma$$

between 8 and 10

- Anti-decuplet
  - Fix zero by fitting $N^{*1/2-}(1710)$
  - Capstick, Page, and Roberts hep-ph/0307019
    - point out that N(1710) identification is arbitrary, using N(1440) $\Rightarrow$ Θ stable

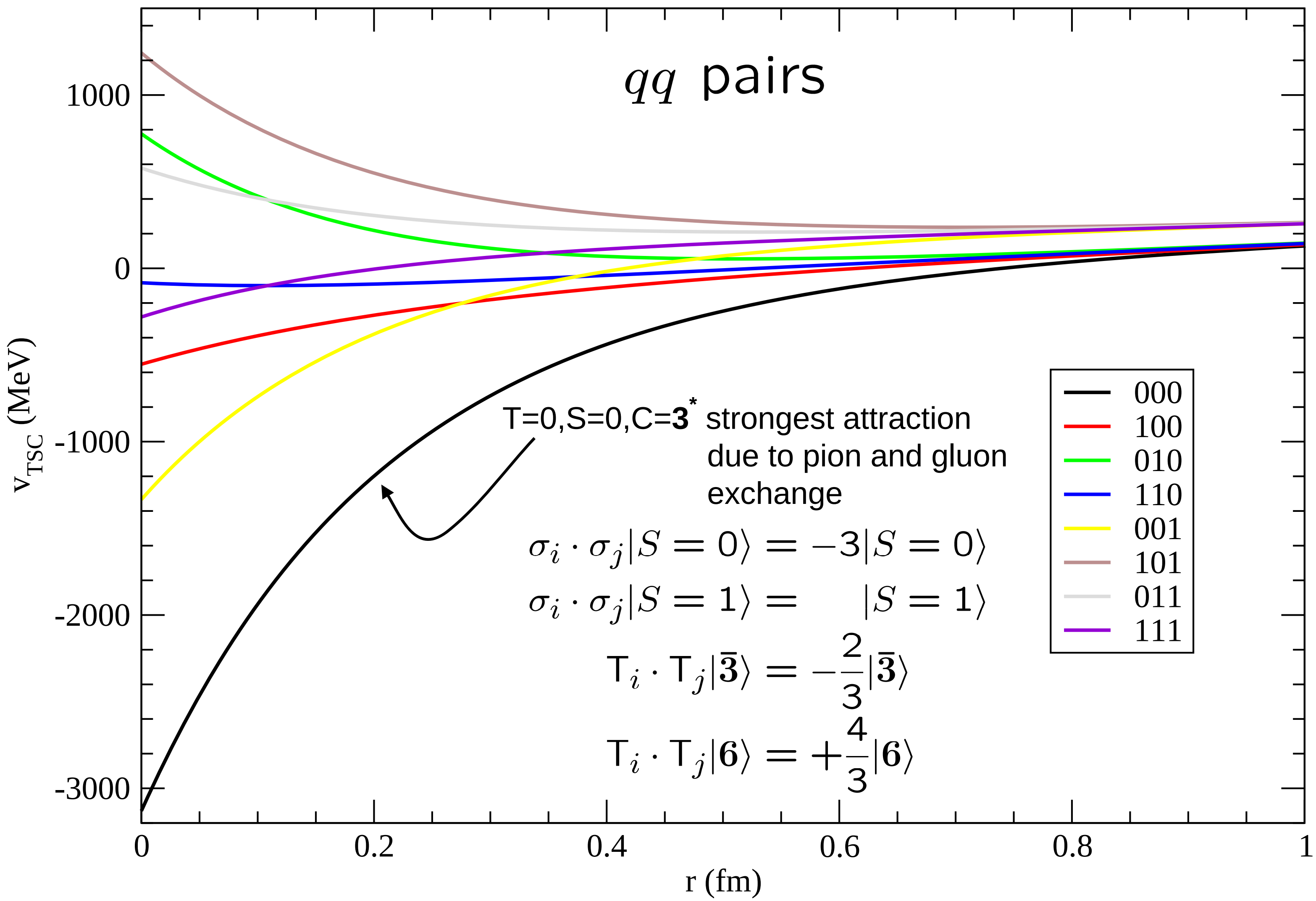
Model calculations
(in no particular order)

- Diakonov, Petrov, & Polyakov (χ-Quark Soliton Model)
- Jaffe & Wilczek (Constituent Quark Model (CQM) – diquark)
- Carlson et. al. (CQM-uncorrelated)
- Close & Dudek (CQM-variational)
- Karliner & Lipkin (CQM-uncorrelated)
- Praszalowicz (Skyrme)
- Huang, Zhang, Yu, & Zou (χSU(3))
- Hosaka (χ bag model)
- Zhu (QCD sum rules)
- Stancu (CQM-variational)
- Kanada-En'yo et.al. (CQM-AMD)
- Oh, Nakayama, & Lee (CQM+phen.)
- Hiyama (CQM-variational)
- Takeuchi & Shimizu (CQM-variational)
- Capstick (CQM)
- Gerasyuta & Kochkin (Relativistic CQM)

why do another?
-QMC methods exact
-anti-symmetrized WF
-fully correlated WF
-any/all TSL structures
-investigate SR (^{2}Q?)

Central potential

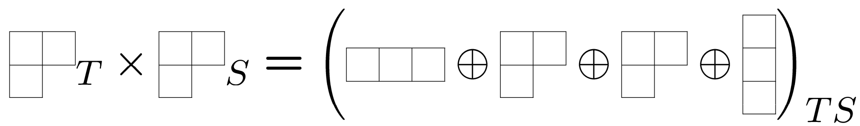
v_{TSC} vs. r



Single hadron wave function

- S- and P-wave N & Δ states
  - uncorrelated |qqq⟩ states
    - color singlet ⇒ $T_i \cdot T_j$ = -2/3 ∀ i,j
    - spin—isospin

$$\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{3}{2}^{S} \oplus \frac{1}{2}^{\rho} \oplus \frac{1}{2}^{\lambda}$$



$N^{\frac{1}{2}+}(939)$	$\chi_\lambda(m_S)\chi_\lambda(m_T) + \chi_\rho(m_S)\chi_\rho(m_T)$
$\Delta^{\frac{3}{2}+}(1232)$	$\chi_S(m_S)\chi_S(m_T)$
$N^{\frac{1}{2}-}(1535)$ $N^{\frac{3}{2}-}(1520)$	$\chi_\rho(m_T)\left[\phi_\lambda(m_L)\chi_\rho(m_S) + \phi_\rho(m_L)\chi_\lambda(m_S)\right]$ $+\chi_\lambda(m_T)\left[-\phi_\lambda(m_L)\chi_\lambda(m_S) + \phi_\rho(m_L)\chi_\rho(m_S)\right]$
$\Delta^{\frac{1}{2}-}(1620)$ $\Delta^{\frac{3}{2}-}(1700)$	$\chi_S(m_T)\left[\phi_\lambda(m_L)\chi_\lambda(m_S) + \phi_\rho(m_L)\chi_\rho(m_S)\right]$
$N^{\frac{1}{2}-}(1650)$ $N^{\frac{3}{2}-}(1700)$ $N^{\frac{5}{2}-}(1675)$	$\chi_S(m_S)\left[\phi_\lambda(m_L)\chi_\lambda(m_T) + \phi_\rho(m_L)\chi_\rho(m_T)\right]$

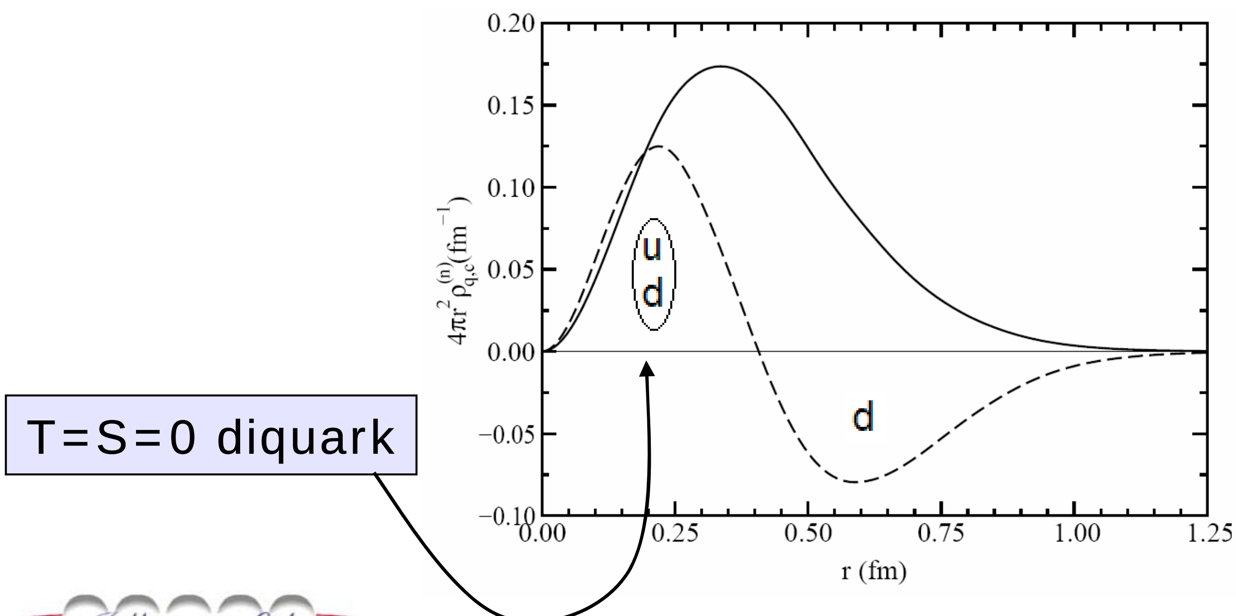
- Apply correlation operator: $|\Psi_V\rangle = \hat{\mathcal{G}}|\Phi\rangle$

Single hadron results

- s- & p-wave spectra

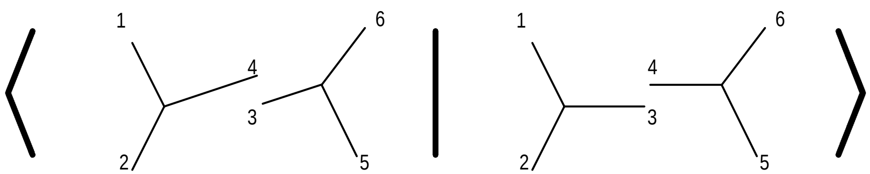
	$N\frac{1}{2}^+$	$\Delta\frac{3}{2}^+$	$N\frac{1}{2}^-$	$N\frac{3}{2}^-$	$\Delta\frac{1}{2}^-$	$\Delta\frac{3}{2}^-$	$N\frac{1}{2}^-$	$N\frac{3}{2}^-$	$N\frac{5}{2}^-$
Exp.	939(0)	1232(2)	1535(13)	1520(8)	1620(30)	1700(50)	1650(20)	1700(50)	1675(8)
H	939(1)	1232(1)	1581(1)	1571(1)	1703(1)	1702(1)	1603(1)	1672(1)	1734(1)
T	1007(5)	767(3)	1063(3)	1024(4)	965(3)	949(3)	1021(3)	940(3)	904(3)
V	308(5)	840(3)	894(3)	923(4)	1114(2)	1129(3)	958(3)	1108(3)	1206(2)

- Neutron quark & charge densities



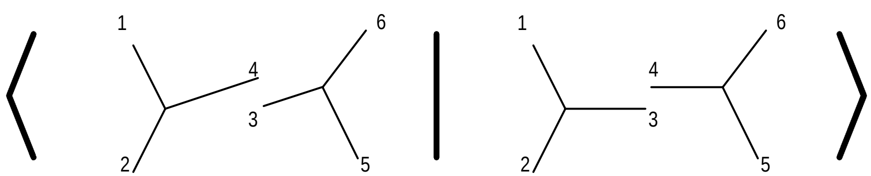
Flux tube dynamics

- Strong coupling: $g_s \to \infty$
  - different quark configurations have orthogonal flux tube configurations



$$= 0$$

- Finite coupling:
  - γ_{FT} – flux tube overlap parameter

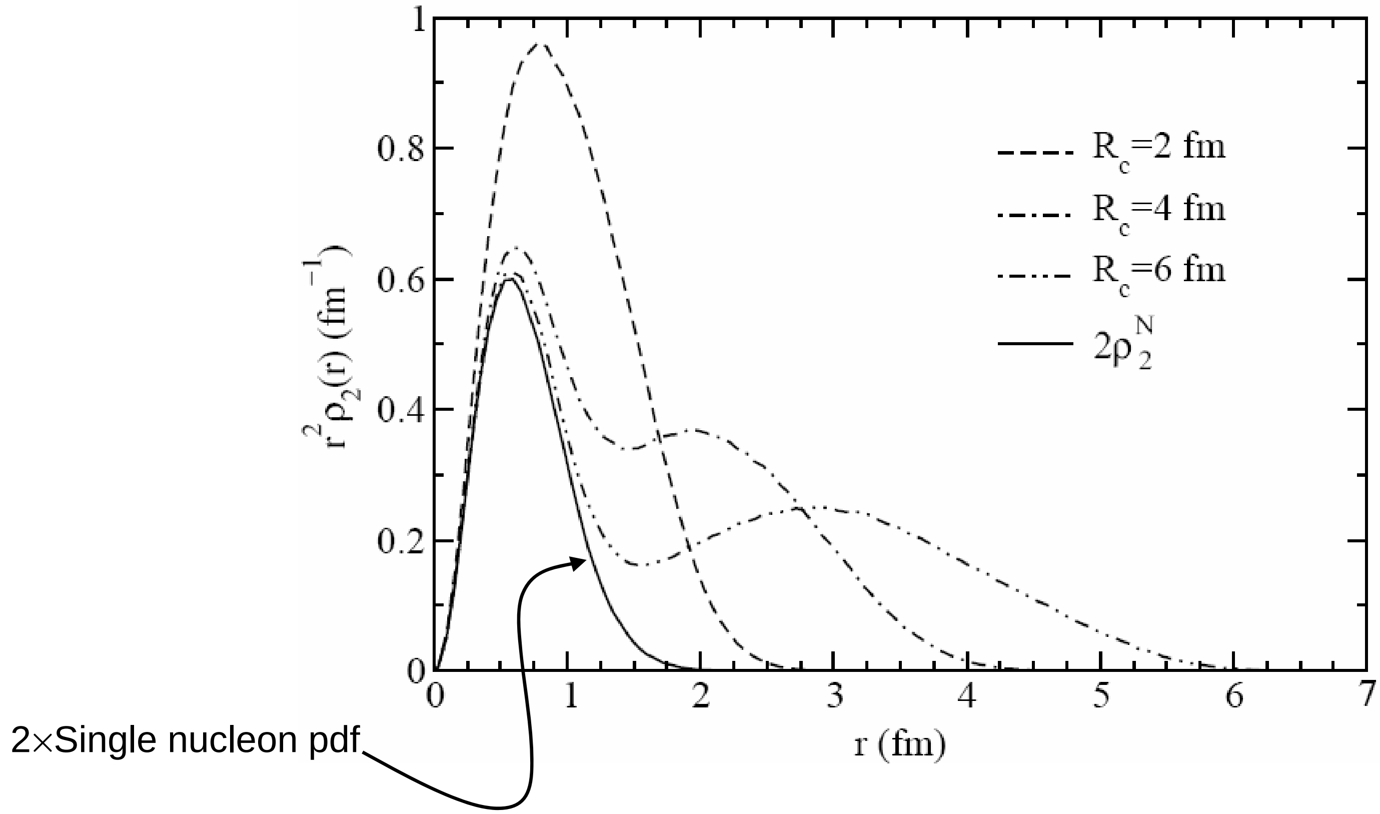


$$= e^{-\gamma_{FT}^2 r_{34}^2}$$

  - More complicated quark position dependence possible
    - eg. Green, Micheal, Paton NPA 554, 701 (1993)

Quark pair distribution function

$$\rho_2(r) = \frac{\int d\mathbf{R}\Psi_V^\dagger(\mathbf{R}) \sum_{q<q'} \delta^{(3)}\left(\mathbf{r} - (\mathbf{r}_q - \mathbf{r}_{q'})\right) \Psi_V(\mathbf{R})}{\int d\mathbf{R}\Psi_V^\dagger(\mathbf{R})\Psi_V(\mathbf{R})}$$



Deuteron properties

- Add $V_{2\pi}$-type interaction

$$v^S = c^S \sum_{q<q'\leq 6} \tilde{T}^2_\mu(r_{qq'})$$

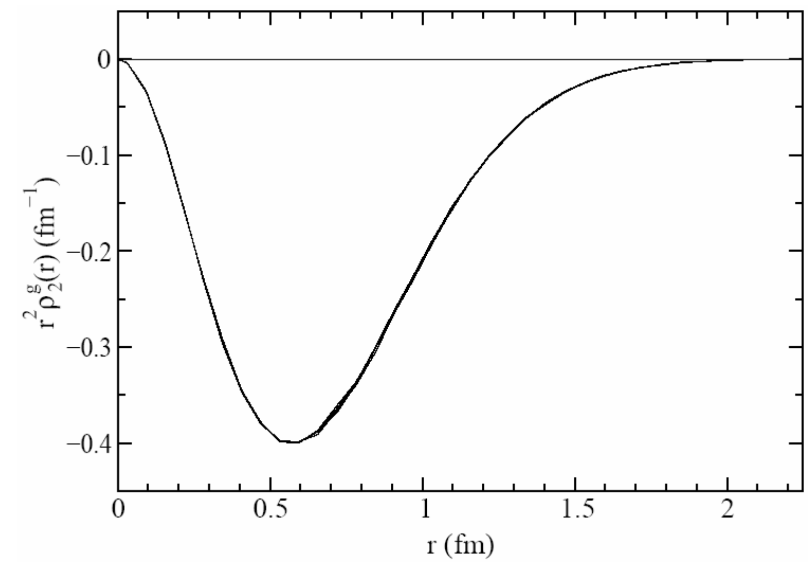
$$\tilde{T}_\mu = T_\mu - \frac{\Lambda_S^3}{\mu^3} T_{\Lambda_S} - \frac{1}{2}\frac{\Lambda_S}{\mu}\left(\frac{\Lambda_S^2}{\mu^2} - 1\right)(\Lambda_S r + 1) Y_{\Lambda_S}$$

R_{cavity}

$c^S (MeV)$	Λ_s (fm^{-1})	2 fm	4 fm	6 fm
0		8.8(4)	−1.7(4)	−1.3(4)
−0.0077	5	−14.2(4)	−6.5(4)	−2.6(4)
−0.67	2	−14.2(4)	−11.0(4)	−5.1(4)
δE_{emp}		−14.2	−10.5	−6.0

- Distortion of the nucleon?
  - let $\gamma_{FT} \to \infty$
    - $\langle \mathcal{P}' | \mathsf{T}_i \cdot \mathsf{T}_j | \mathcal{P} \rangle = -\frac{2}{3}\delta_{\mathcal{P}'\mathcal{P}}$

    $\Rightarrow$ given q$\leftrightarrow$N
  - finite γ_{FT}
    - $\langle \mathcal{P}' | \mathsf{T}_i \cdot \mathsf{T}_j | \mathcal{P} \rangle \neq 0,\ \mathcal{P}' \neq \mathcal{P}$

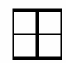
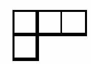
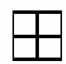
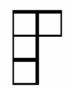
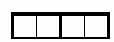
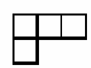


$$\rho_2^g(r) = \frac{\int d\mathbf{R}\Psi_V^\dagger(\mathbf{R}) \sum_{q<q'} \mathsf{T}_q \cdot \mathsf{T}_{q'} \delta^{(3)}\left(\mathbf{r} - (\mathbf{r}_q - \mathbf{r}_{q'})\right) \Psi_V(\mathbf{R})}{\int d\mathbf{R}\Psi_V^\dagger(\mathbf{R})\Psi_V(\mathbf{R})}$$

Pentaquark states -- $|uudd\bar{s}\rangle$

- Treat pentaquark as a bound state
  - narrow resonance approximation
- Variational wave function $|\Psi_5\rangle = \mathcal{S} \prod_{i<j} \hat{F}_{ij} |\Phi_5\rangle$
- Uncorrelated states

$$|\Phi_5\rangle = \frac{1}{\sqrt{3}} \sum_{c=R,G,B} \left[|(TSCL)_{[1^4]}; S_z, c, M_L\rangle \otimes |\bar{s}; s_z, m_\ell, \bar{c}\rangle \right]_{J,J_z}$$

	$(1^{r_1}, 2^{r_2}, 3^{r_3}, \ldots)$	Young	Yamanouchi
$S = 0$	(2^2)		$[2211], [2121]$
$S = 1$	$(3, 1)$		$[2111], [1211], [1121]$
$T = 0$	(2^2)		$[2211], [2121]$
$C = 3$	$(2, 1^2)$		$[3211], [3121], [1321]$
$L = 0$	(4)		$[1111]$
$L = 1$	$(3, 1)$		$[2111], [1211], [1121]$

- Take appropriate inner products of these to obtain a state which is totally anti-symmetric w.r.t. the four light quarks

Negative parity pentaquark

- states of 4q
  - T=0
  - S=0,1,~~2~~
  - C=**3** * anti-quark ∈ **3** *
  - parity even ⇒ L=0

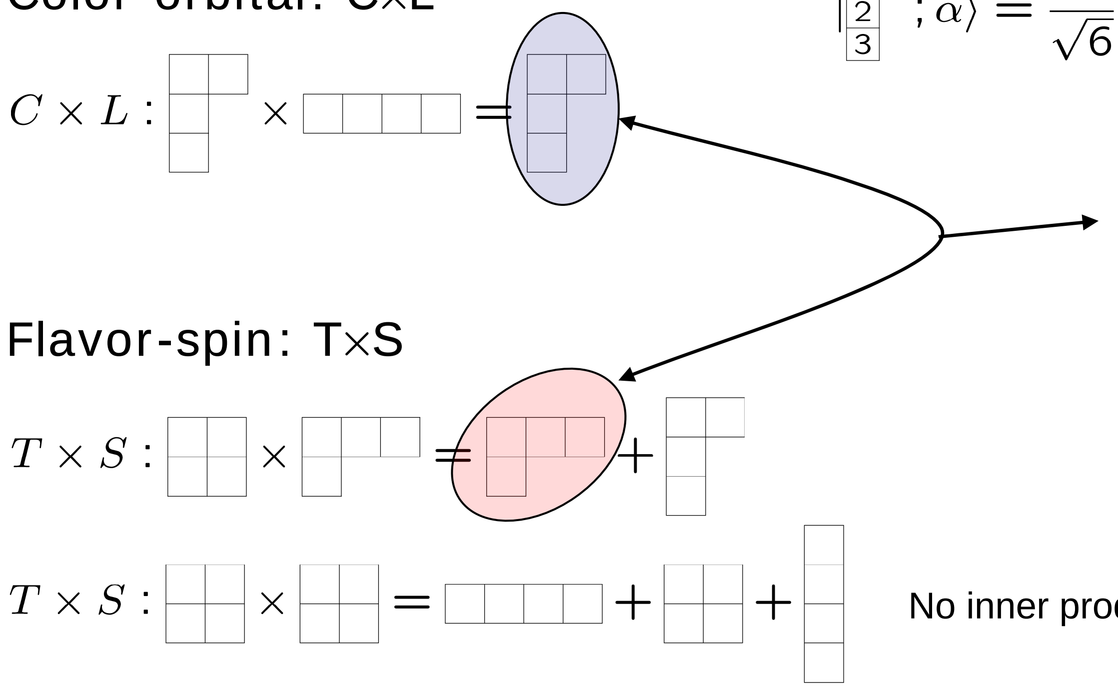
$$\left|\begin{smallmatrix}1&2\\3&\\4&\end{smallmatrix};\alpha\right\rangle = \frac{1}{2\sqrt{2}}(c_1^\alpha c_2^\beta + c_2^\alpha c_1^\beta)\epsilon_{\beta\gamma\delta}c_3^\gamma c_4^\delta$$

$$\left|\begin{smallmatrix}1&3\\2&\\4&\end{smallmatrix};\alpha\right\rangle = \frac{1}{4\sqrt{3}}c_1^\gamma c_2^\delta \epsilon_{\gamma\delta\beta}(3\, c_3^\alpha c_4^\beta - c_3^\beta c_4^\alpha)$$

$$\left|\begin{smallmatrix}1&4\\2&\\3&\end{smallmatrix};\alpha\right\rangle = \frac{1}{\sqrt{6}}\epsilon_{\beta\gamma\delta}c_1^\beta c_2^\gamma c_3^\delta c_4^\alpha$$

- Color-orbital: C×L

- Flavor-spin: T×S



yields singlet

No inner product with CL yields (1)[4]

Negative parity pentaquark results

- Two types of pair correlation operators
  - lq-lq: $f_p^q(r_{ij}),\ p = \{c, \sigma, t, \ell s\} \otimes \{1, g\}, \{\sigma, t\} \otimes \{c, \tau\}$
  - lq-sq: $f_p^s(r_{ij}),\ p = \{c, \sigma\} \otimes \{c, g\}$

Z01+1540

	CD		CND		LQSQ		T	
vgc=	-571.08+/-	1.09	-12.93+/-	0.13	-213.91+/-	0.49	-584.01+/-	1.08
vft=	1486.79+/-	4.81	0.00+/-	0.00	526.57+/-	2.04	1486.79+/-	4.81
vsg=	-40.61+/-	0.36	-182.92+/-	1.15	-51.85+/-	0.39	-223.53+/-	1.40
vtg=	-4.71+/-	0.02	-7.31+/-	0.04	-0.42+/-	0.01	-12.02+/-	0.05
vlg=	-4.50+/-	0.03	1.62+/-	0.02	-0.87+/-	0.01	-2.89+/-	0.02
vst=	-211.54+/-	1.69	0.00+/-	0.00	0.00+/-	0.00	-211.54+/-	1.69
vtt=	-37.72+/-	0.15	0.00+/-	0.00	0.00+/-	0.00	-37.72+/-	0.15
stS=	-228.58+/-	1.73	0.00+/-	0.00	0.00+/-	0.00	-228.58+/-	1.73
stL=	17.04+/-	0.05	0.00+/-	0.00	0.00+/-	0.00	17.04+/-	0.05
VMB=	288.76+/-	1.03	0.00+/-	0.00	0.00+/-	0.00	288.76+/-	1.03
<V>=	905.39+/-	8.50	-201.55+/-	1.18	259.52+/-	2.86	703.85+/-	9.58
<T>=	1958.01+/-	12.70	0.00+/-	0.00	0.00+/-	0.00	1958.01+/-	12.70
<H>=	2863.40+/-	4.26	-201.55+/-	1.18	259.52+/-	2.86	2661.85+/-	3.28
RqR=	0.47E+00+/-	.11E-02	0.00E+00+/-	NaN	0.00E+00+/-	NaN	0.47E+00+/-	.11E-02

%%%%

VFT= 1775.55+/- 5.79 v2f= 1486.79+/- 4.81 vmb= 288.76+/- 1.03

OGE= -822.45+/- 2.48

OPE= -249.26+/- 1.80

SKR=-.19E-03+/-.23E-03

SKI=0.14E-04+/-.32E-04

$M(\Theta^+(\frac{1}{2}^-)) = 2010$ GeV

Jefferson Lab

Comparisons

- with uncorrelated variational
  - M(Θ) decreased by ~40 MeV
    - compare with $M(\Theta)-(m_K+m_n)\approx 100$ MeV

	Correlated		Uncorrelated	
vgc=	-584.01+/-	1.08	-547.91+/-	1.13
vft=	1486.79+/-	4.81	1586.89+/-	5.24
vsg=	-223.53+/-	1.40	-167.14+/-	1.15
vtg=	-12.02+/-	0.05	0.00+/-	0.00
vlg=	-2.89+/-	0.02	0.00+/-	0.00
vst=	-211.54+/-	1.69	-125.34+/-	1.05
vtt=	-37.72+/-	0.15	0.00+/-	0.00
stS=	-228.58+/-	1.73	-139.43+/-	1.10
stL=	17.04+/-	0.05	14.09+/-	0.05
VMB=	288.76+/-	1.03	305.02+/-	1.15
<V>=	703.85+/-	9.58	1051.52+/-	9.40
<T>=	1958.01+/-	12.70	1650.85+/-	11.07
<H>=	2661.85+/-	3.28	2702.38+/-	2.54

- Carlson, et. al.
  - uncorrelated negative parity state; bag model wave functions

$$\Delta M = -C_\chi \langle \Theta(\tfrac{1}{2}^-)| \sum_{i<j} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j |\Theta(\tfrac{1}{2}^-)\rangle \approx -300 \text{ MeV}$$

C_χ: $m_\Delta - m_N$

Pending results

- Positive parity mass: higher or lower??
  - lattice: $m_- < m_+$
  - CQM: $m_- > m_+$
    - (increased symmetry of state compensates for L=1)
- Widths
  - Flux tube model predict narrow states?
- Overlap of fully antisymmetrized states with favorite flavors:
  - Jaffe-Wilczek diquark
  - nK molecular
  - etc.
- Short range structures
  - diquark size, shape, distributions

TO BE CONTINUED…

(but not for much longer)

Frank Close's observation

Arndt
Buccella
Carlson
Dyakanov
Ellis
Faber
Giannini
Huang
Inoue
Jaffe
Karliner
Lipkin
Maltman
Nussinov
Oh
Polyakov
Qiang
Rosner
Stech
Trilling
U
Veneziano
Wilczek
Xiang
Yang
Zhu

If Theta doesn't exist,
then these (and many other theorists)
should be congratulated on their creativity