Strangeness-changing scalar form factor from scattering data and CHPT

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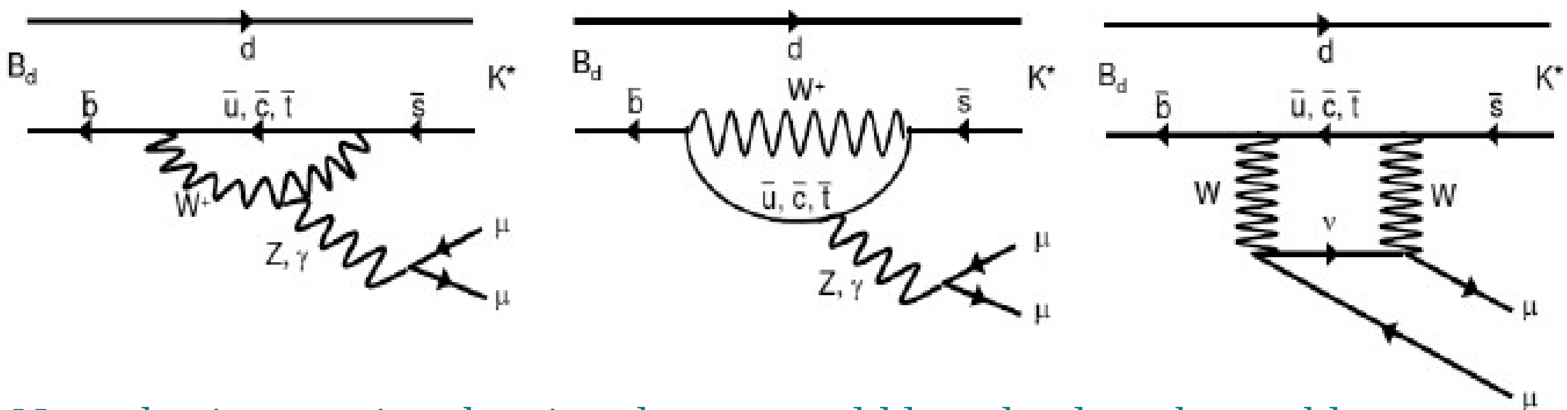
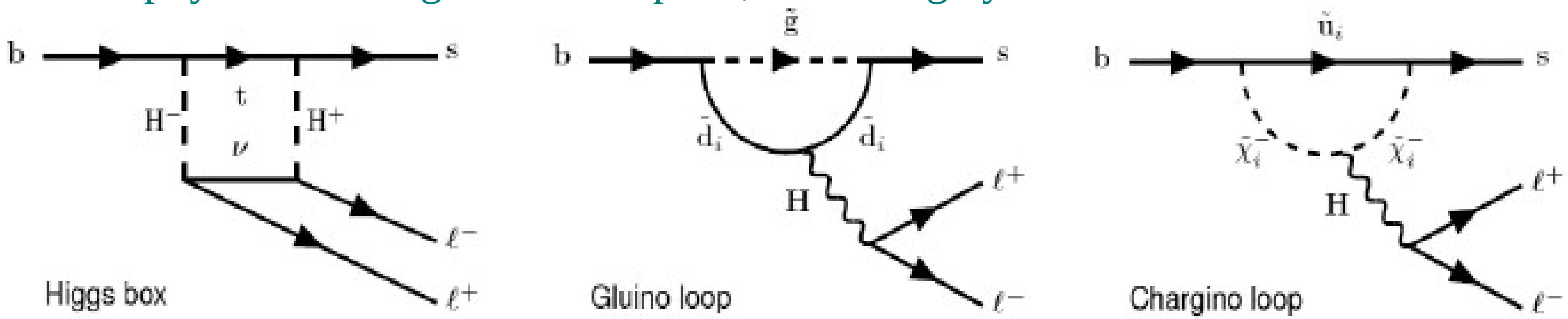
Motivation: $K^*l^+l^-$

- Within the SM, these processes proceed via loop diagrams like



- New physics entering the virtual parts, could largely alter observables



- Effective Hamiltonian: $\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} (C_i^{SM} + \Delta C_i^{NP}) \mathcal{O}_i$

Wilson coeffs.
(short-dist. interactions)

Operators
(long-dist. interactions)

Slide from **Wei Wang**

- V. Bernard, M. Oertel, E. Passemar and J. Stern, *Dispersive representation and shape of the K(l3) form factors: Robustness*, PRD 80 ('09)

  V. Bernard, *First determination of f+(0) |V_us| from a combined analysis of (tau → Kpi nu) tau decay and piK scattering with constraints from Kl3 decays*, JHEP 1406 (2014) 082

  V. Bernard and E. Passemar, *Chiral extrapolation of the strangeness changing $K\pi$ form factor*, *JHEP* **04** (2010) 001 [arXiv:0912.3792] [INSPIRE].

  V. Bernard and E. Passemar, *Matching chiral perturbation theory and the dispersive representation of the scalar $K\pi$ form-factor*, *Phys. Lett.* **B 661** (2008) 95 [arXiv:0711.3450] [INSPIRE].

  - 3-subtracted dispersion relation, subtraction constants from data ($K_{\ell 3}$ and $\tau \to K\pi\nu_\tau$)
  - Matching to CHPT to two loops

- S. Descotes-Genon and B. Moussallam, *The $K^{(*)0}(800)$ scalar resonance from Roy-Steiner representations of πK scattering*, *Eur. Phys. J.* **C 48** (2006) 553 [hep-ph/0607133] [INSPIRE].

- Z.-H. Guo, J. Oller and J. Ruiz de Elvira, *Chiral dynamics in form factors, spectral-function sum rules, meson-meson scattering and semi-local duality*, *Phys. Rev.* **D 86** (2012) 054006 [arXiv:1206.4163] [INSPIRE].

- M. Jamin, J.A. Oller and A. Pich, *Strangeness changing scalar form-factors*, *Nucl. Phys.* **B 622** (2002) 279 [hep-ph/0110193] [INSPIRE].

  M. Jamin, J. Oller and A. Pich, *Scalar $K\pi$ form factor and light quark masses*, *Phys. Rev.* **D 74** (2006) 074009 [hep-ph/0605095] [INSPIRE].

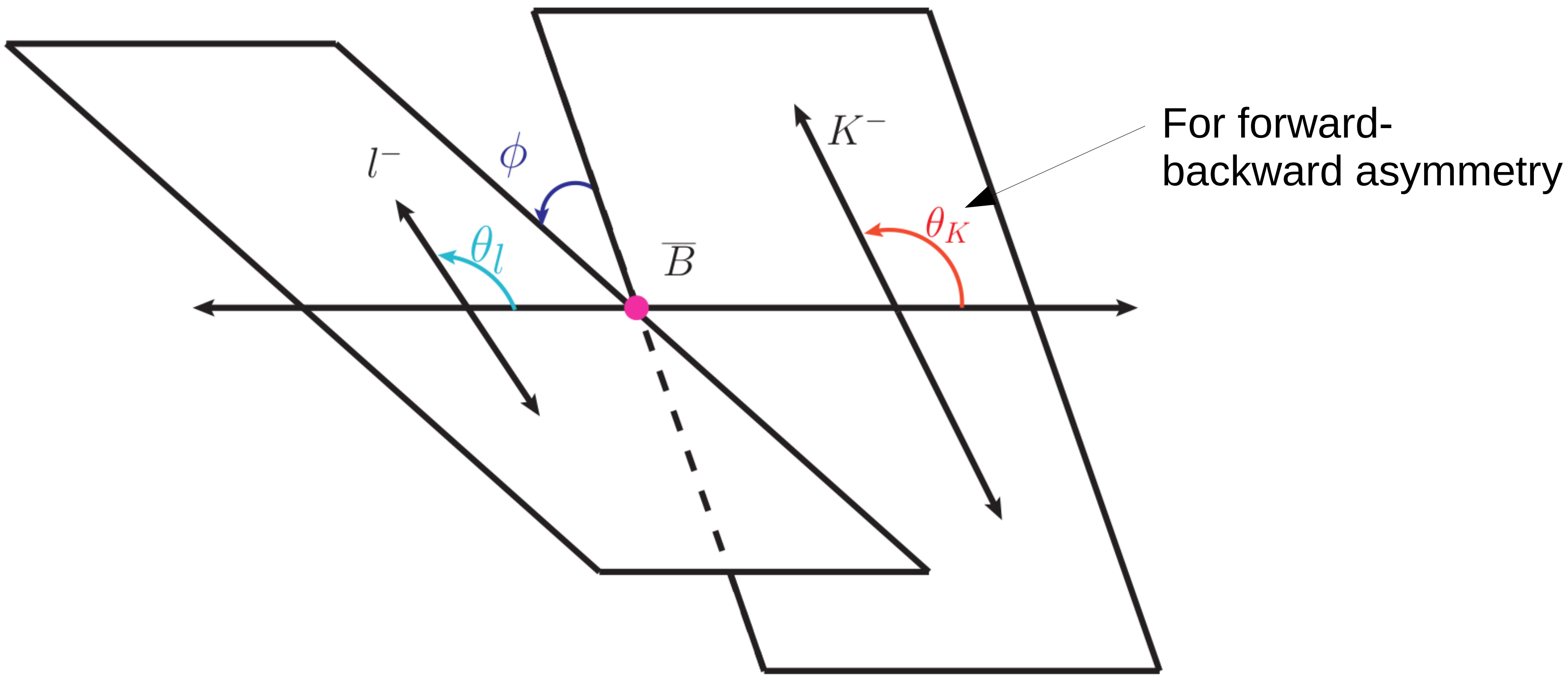
  - Up to three-channel, unsubtracted dispersion relations
  - Determination of strange quark mass

Incomplete list

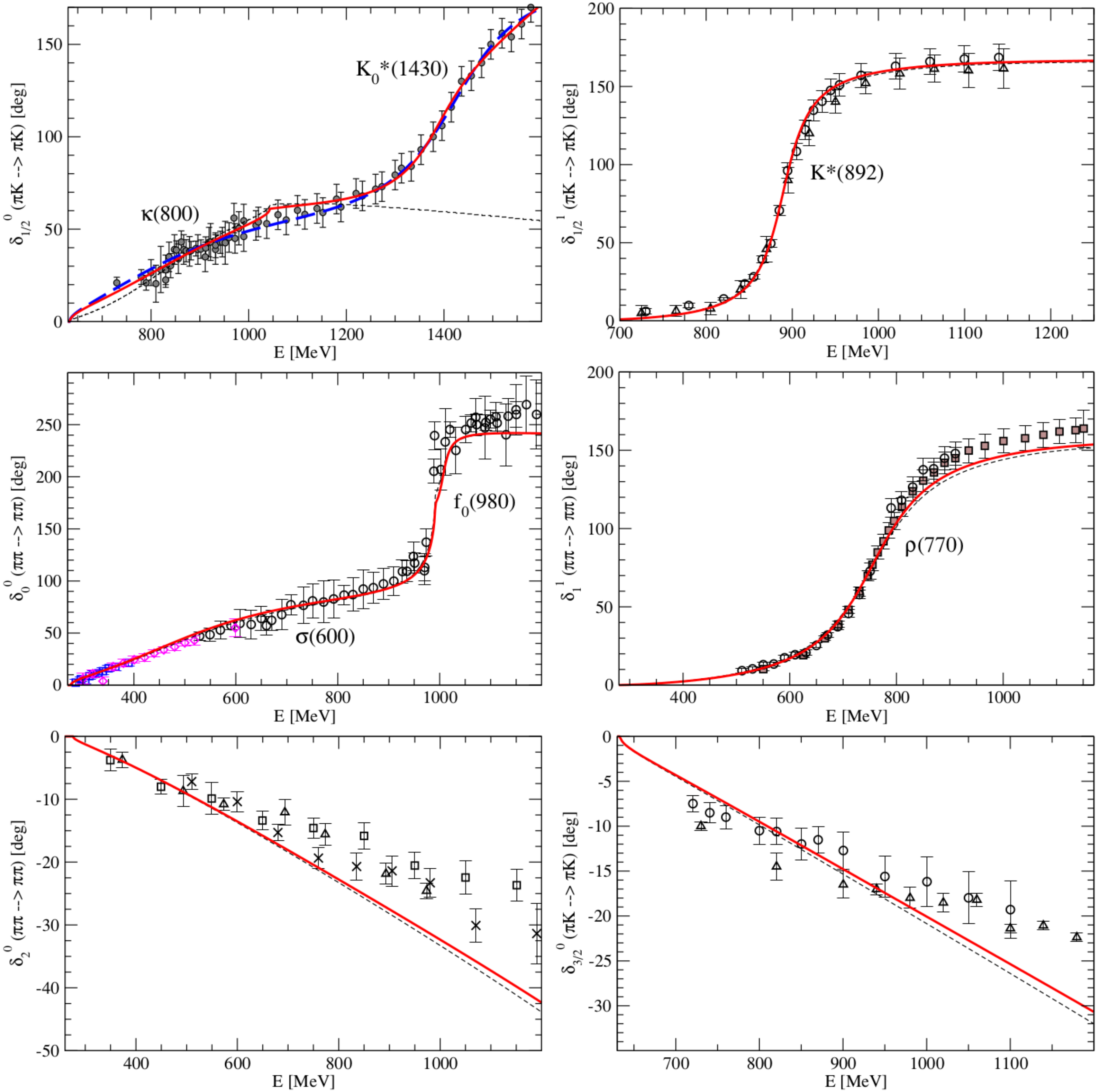
Reaction geometry

$$\overline{B} \rightarrow \overline{K}^*_J(\rightarrow K^-\pi^+)l^+l^-$$

- θ_l: angle of emission between K^{*0} and μ^- in di-lepton rest frame
- θ_{K^*}: angle of emission between K^{*0} and K^- in di-meson rest frame.
- ϕ: angle between the two planes
- q^2: dilepton invariant mass square



Phase parametrization



Coupled-channel simple model from Unitarized CHPT [Oller, Oset, Pelaez, PRD (1999)]

Genuine s-channel resonance in kappa channel to model higher energies

Parametrization of πK phase shift and (model-dependent) prediction of ηK amplitude

Pole structure

			z_0 [MeV]	$a_{-1}(K\eta)$ $[M_\pi]$	$a_{-1}(K\pi)$ $[M_\pi]$
$\kappa(800)$	this work	(2-ch.)	$792-i\,279$		$-29-i\,57$
	this work	(1-ch.)	$715-i\,283$		$-45-i\,62$
	MD, Meißner ('12)	(χU)	$815-i\,226$		$-30-i\,57$
	Descotes-Genon, Moussallam ('06)	(Roy-S.)	$658-i\,279$		
$K_0^*(1430)$	this work	(2-ch.)	$1388-i\,71$	$-11-i\,5$	$11+i\,13$
	this work	(1-ch.)	$1425-i\,120$	0	$20+i\,39$
	Bugg ('10)	(phen.)	$1427-i\,135$		

Formalism

Match with NLO SU(3) CHPT form factor at s=0
[Gasser, Leutwyler NPB 250 ('85)]

Phase parametrization

$$F_{K\pi}(s) = F^{\chi}_{K\pi}(0) + (F^{\chi}_{K\pi})'(0)\, s + \frac{s^2}{\pi}\int_{s_{K\pi}}^{\infty} ds' \, \frac{F_{K\pi}(s')\,\sigma_{K\pi}(s')\,T^{*}_{K\pi,K\pi}(s')}{s'^2(s'-s-i\epsilon)}$$
$$+ \frac{s^2}{\pi}\int_{s_{K\eta}}^{\infty} ds' \, \frac{F_{K\eta}(s')\,\sigma_{K\eta}(s')\,T^{*}_{K\eta,K\pi}(s')}{s'^2(s'-s-i\epsilon)},$$

From UCHPT model

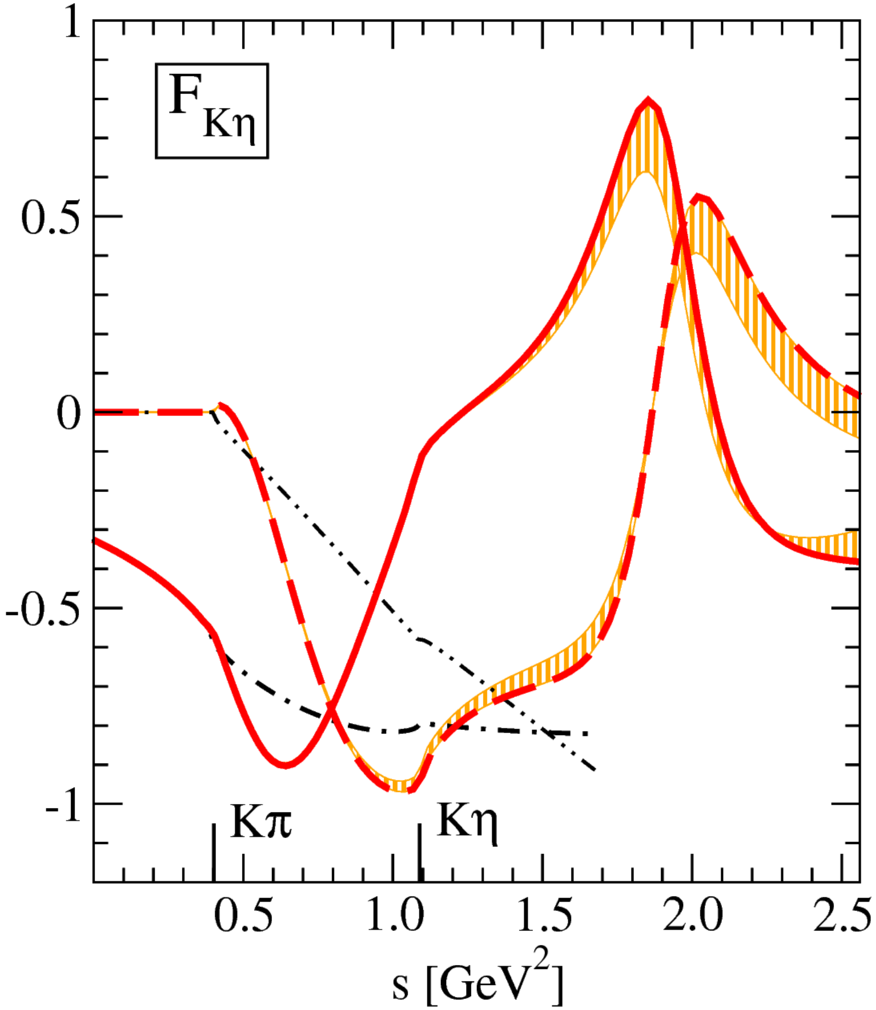
$$F_{K\eta}(s) = F^{\chi}_{K\eta}(0) + (F^{\chi}_{K\eta})'(0)\, s + \frac{s^2}{\pi}\int_{s_{K\pi}}^{\infty} ds' \, \frac{F_{K\pi}(s')\,\sigma_{K\pi}(s')\,T^{*}_{K\eta,K\pi}(s')}{s'^2(s'-s-i\epsilon)}$$
$$+ \frac{s^2}{\pi}\int_{s_{K\eta}}^{\infty} ds' \, \frac{F_{K\eta}(s')\,\sigma_{K\eta}(s')\,T^{*}_{K\eta,K\eta}(s')}{s'^2(s'-s-i\epsilon)}$$

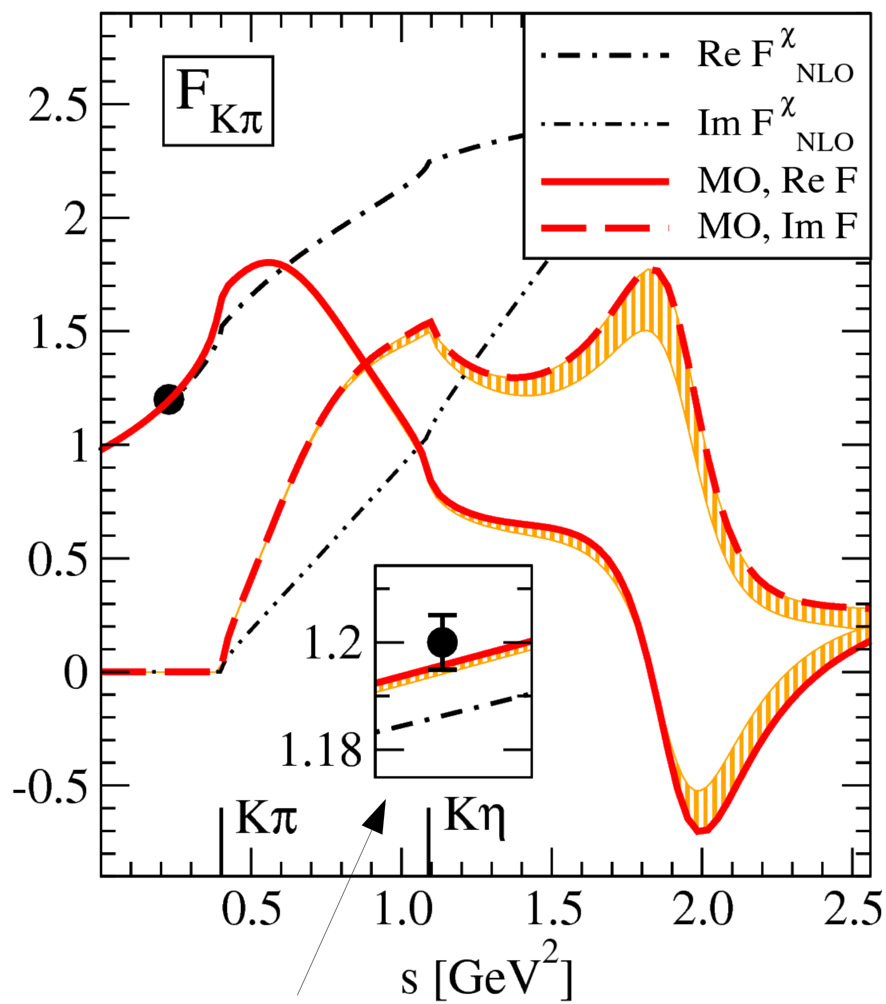
Direct inversion through discretization

$$\mathbf{F}^{\chi} = \left(F^{\chi}(0) + (F^{\chi})'(0)\, s_1, \ldots, F^{\chi}(0) + (F^{\chi})'(0)\, s_n\right)^T$$

$$\begin{pmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{F}^{\chi}_1 \\ \mathbf{F}^{\chi}_2 \end{pmatrix} + \begin{pmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{pmatrix}$$

Two-channel, twice subtracted DR

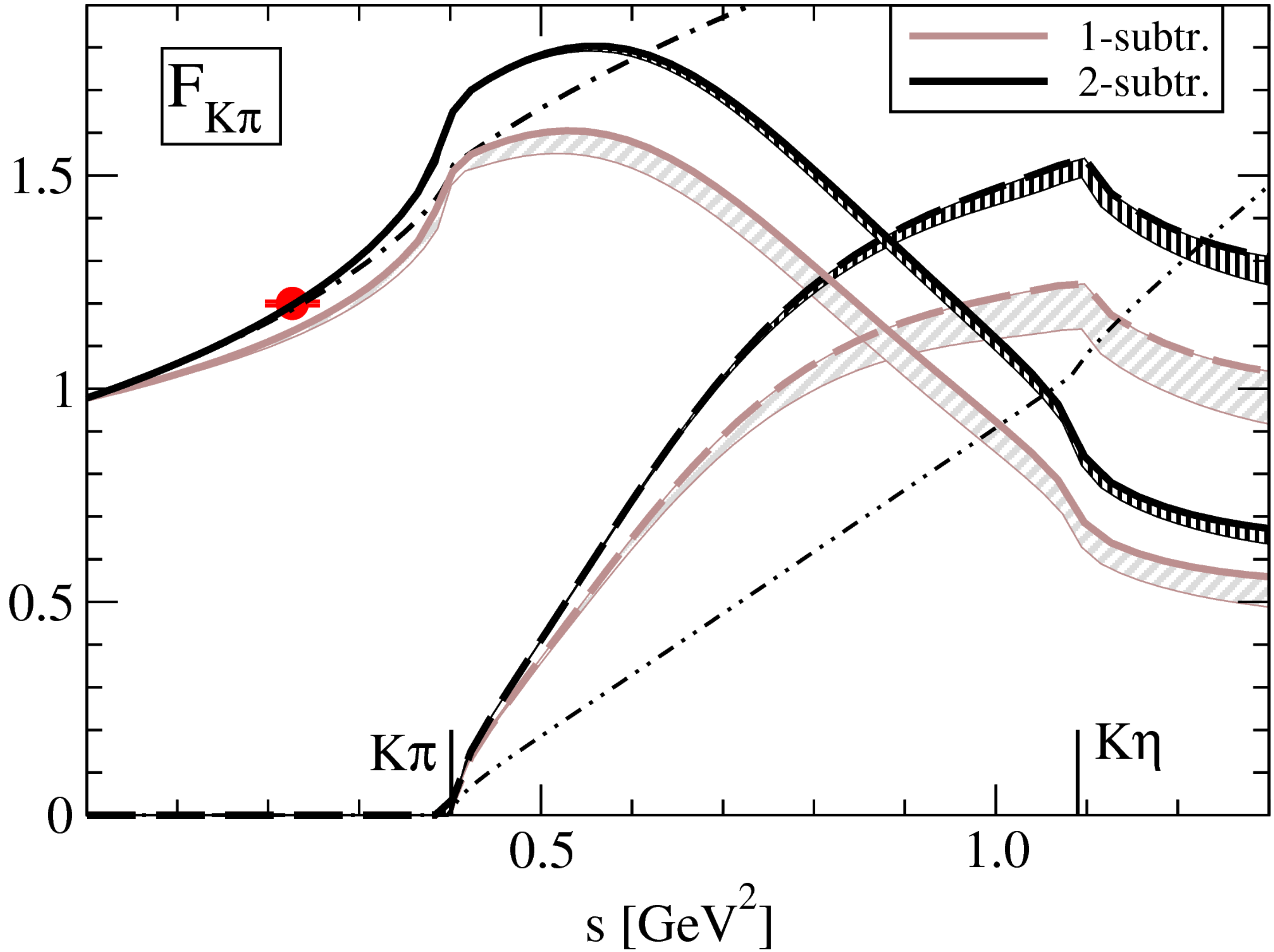




Callan-Treiman point

[Jamin et al., PRD 74 (2006)]

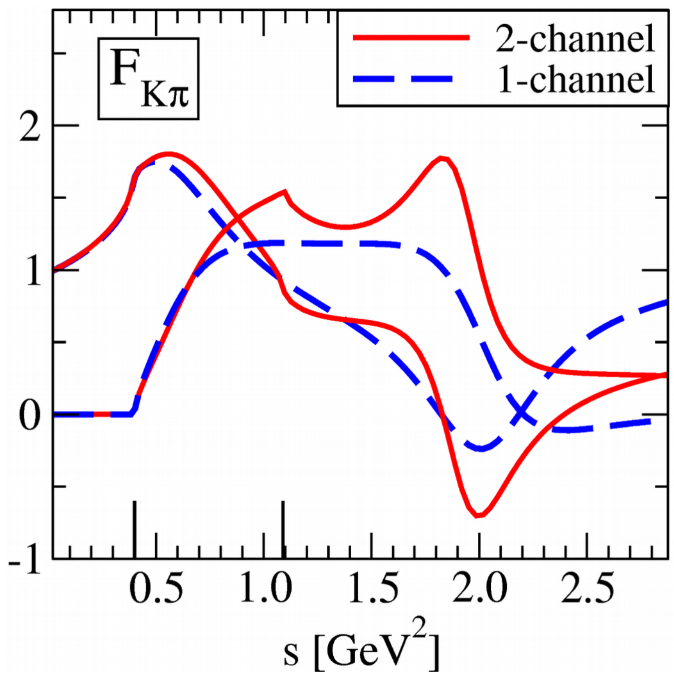
Compare to once subtracted DR

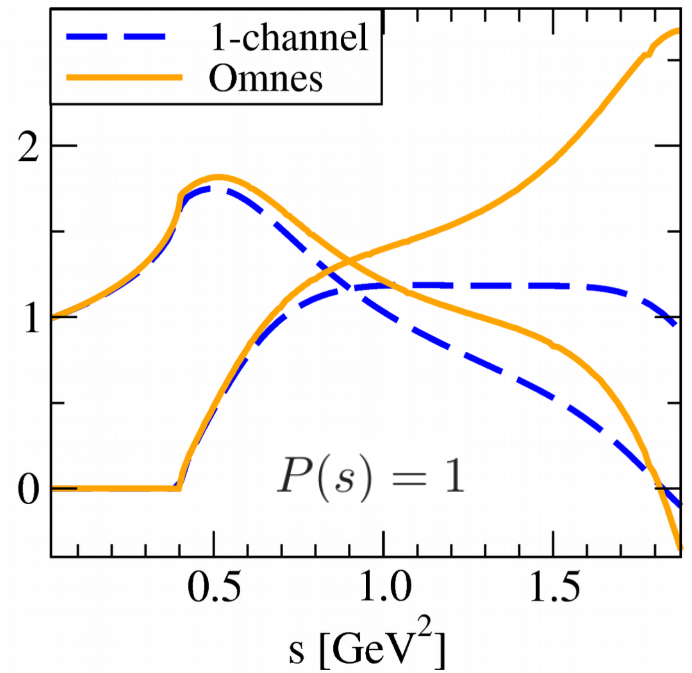


- Slope of *CHPT* form factor at *s=0* no longer matched
- More influence from high-energy input

Other comparisons

- 1-channel, twice subtracted
- Omnès function (no zeros)





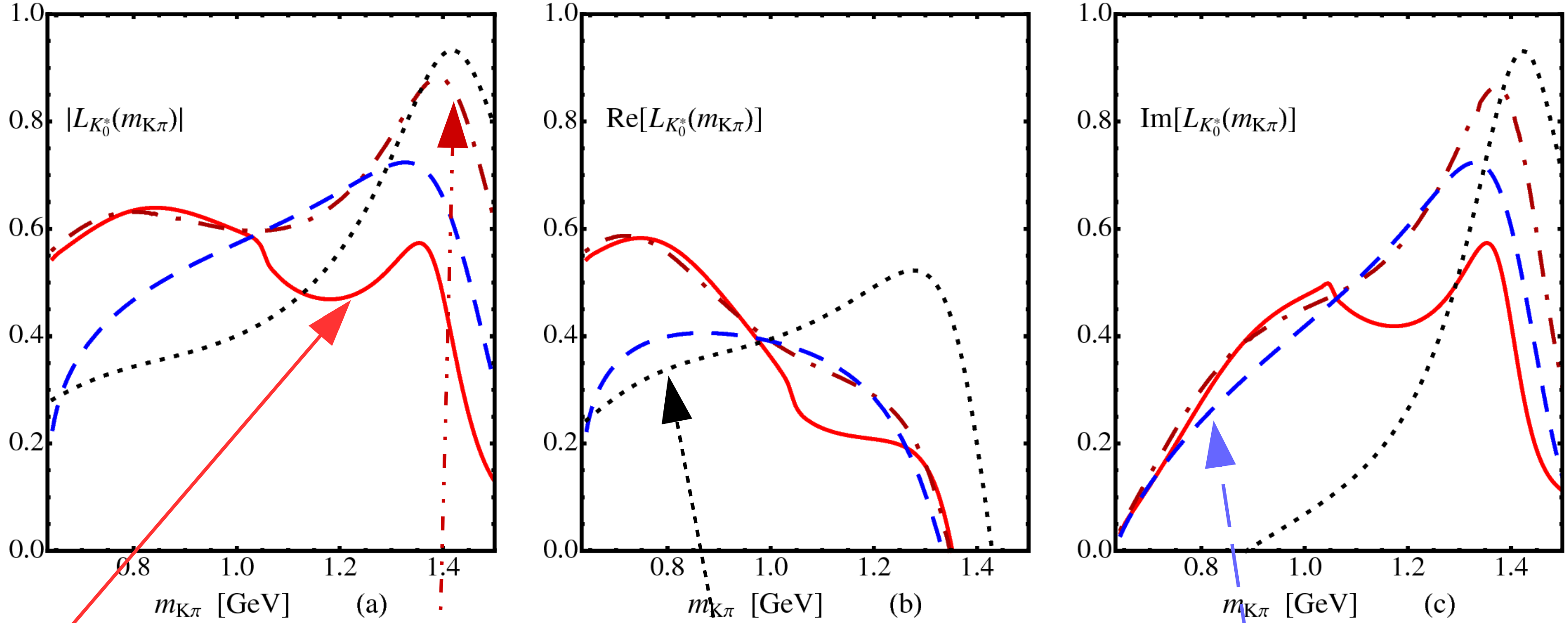
direct inversion

$$F_{K\pi}(s) = F^{\chi}_{K\pi}(0) + (F^{\chi}_{K\pi})'(0)\, s + \frac{s^2}{\pi} \int_{s_{K\pi}}^{\infty} ds' \, \frac{F_{K\pi}(s')\, \sigma_{K\pi}(s')\, T^{*}_{K\pi,K\pi}(s')}{s'^2(s' - s - i\epsilon)}$$

Omnès

$$F_{K\pi}(s) = P(s)\, F^{\chi}_{K\pi}(0) \, \exp\left[s\, \frac{(F^{\chi}_{K\pi})'(0)}{F^{\chi}_{K\pi}(0)} + \frac{s^2}{\pi} \int_{s_{K\pi}}^{\infty} \frac{ds'}{s'^2} \, \frac{\delta(s')}{s' - s} \right]$$

Lineshapes



2-channel,
2-subtracted

Omnes

LASS
[PRD 72 (‘05)]

Bečirević, Tayduganov
[NPB 868 (‘13)]

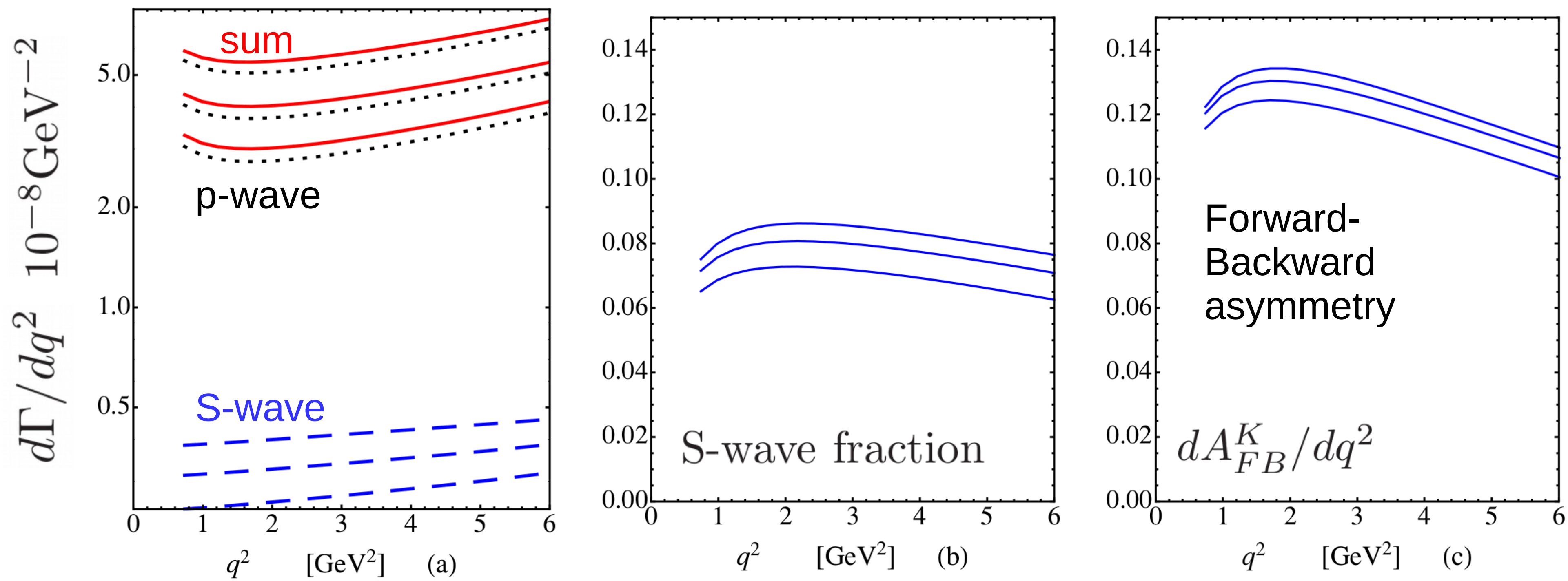
Predicted observables

$$\bar{B}^0 \to K^-\pi^+ l^+ l^-$$

Integral over invariant mass:

$$\frac{d\Gamma_S}{dq^2} \equiv \int_{(m_{K^*}-\delta m)^2}^{(m_{K^*}+\delta m)^2} dm_{K\pi}^2 \frac{d^2\Gamma_S}{dq^2 dm_{K\pi}^2}$$

$m_{K^*} \equiv m_{K^*(892)}$ and $\delta_m = 100\,\mathrm{MeV}$



Comparison to experiment

(Predictions from 2013 compared to LHCb measurement 2015, JHEP 1611 (2016) 047)

Measurements of the S-wave fraction in $B^0 \to K^+\pi^-\mu^+\mu^-$ decays and the $B^0 \to K^*(892)^0\mu^+\mu^-$ differential branching fraction

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The LHCb collaboration

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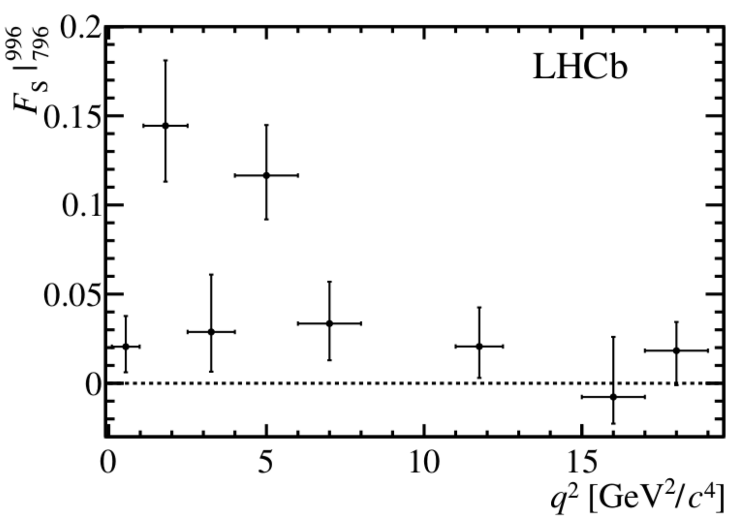
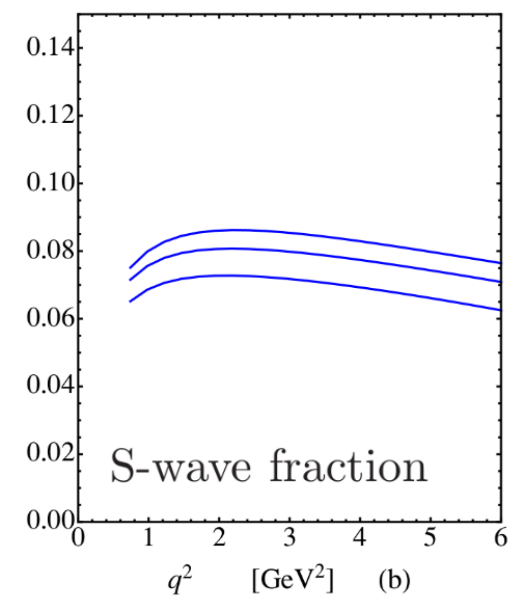
$$F_S = 0.101 \pm 0.017(\text{stat}) \pm 0.009(\text{syst})$$

Binning: $796 < m_{K\pi} < 996\ \text{MeV}/c^2$

$$1.0 < q^2 < 6.0\,\text{GeV}^2/c^4$$



$\bar{B}^0 \to K^-\pi^+l^+l^-$

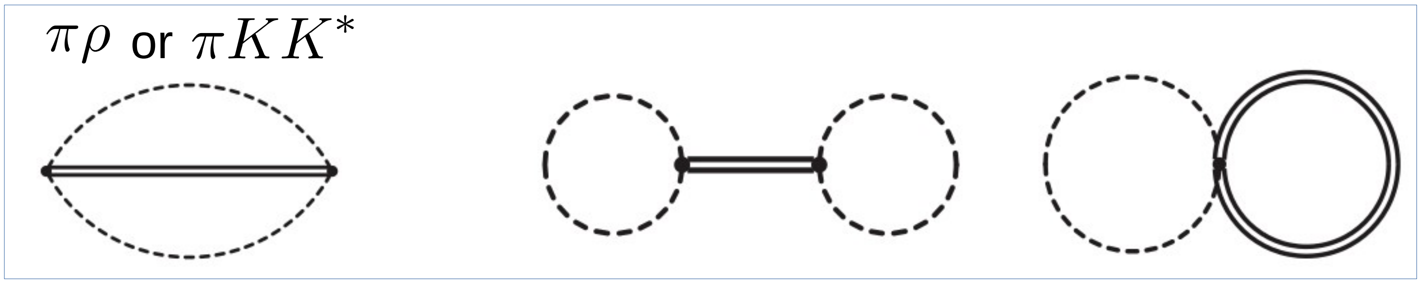


And now to something entirely different:

Interacting pions and kaons for the hadron gas

M. D. and V. Koch, *Charge fluctuations and electric mass in a hot meson gas*, PRC (2007)

See also: J.R. Pelaez et al, PRD66 (‘02), Gomez Nicola, Ruiz de Elvira (‘13),...



Interaction corrections to *log* Z_μ

OR: $$\log Z = \log Z_0 + \sum_{i_1, i_2} z_1^{i_1}\, z_2^{i_2}\, b(i_1, i_2)$$

Virial expansion $B_2^{(\pi\pi),\,\mathrm{Boltz}}(\mu = 0)$

Used, e.g., in: Venugopalan, Prakash, NPA 546 (‘92)

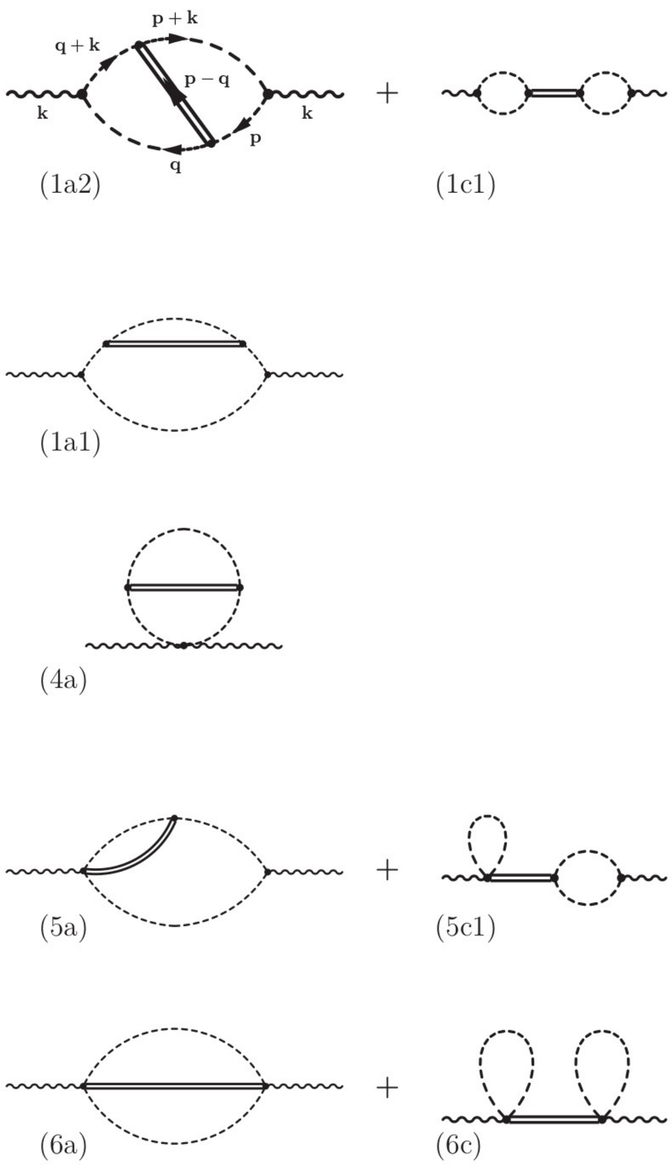
$$= \frac{1}{2\pi^3 \beta} \int_{2m_\pi}^{\infty} dE\, E^2\, K_2(\beta E) \sum_{\ell, I} (2I+1)(2\ell+1)\, \frac{\partial \delta_\ell^I(E)}{\partial E}$$

“Density” expansion

based on [Dashed, Ma, Bernstein, (‘69)]

$$B_2^{(\pi\pi),\mathrm{Bose}}(\mu) = \frac{\beta}{4\pi^3} \int_{2m_\pi}^{\infty} dE \int_{-1}^{1} dx \int_0^{\infty} dk\, \frac{E\, k^2}{\sqrt{E^2 + k^2}} \Big[\delta_0^2(E)(n[\omega_1 + \mu] n[\omega_2 + \mu] + n[\omega_1 - \mu] n[\omega_2 - \mu])$$
$$+ \delta_0^2(E)(n[\omega_1 + \mu] n[\omega_2] + n[\omega_1 - \mu] n[\omega_2]) + 3\, \delta_1^1(E)(n[\omega_1 + \mu] n[\omega_2] + n[\omega_1 - \mu] n[\omega_2])$$
$$+ \delta_0^2(E)\left(\frac{1}{3} n[\omega_1 + \mu] n[\omega_2 - \mu] + \frac{2}{3} n[\omega_1] n[\omega_2]\right) + 3\, \delta_1^1(E)\, n[\omega_1 + \mu] n[\omega_2 - \mu]$$
$$+ \delta_0^0(E)\left(\frac{2}{3} n[\omega_1 + \mu] n[\omega_2 - \mu] + \frac{1}{3} n[\omega_1] n[\omega_2]\right) \Big].$$

Straightforward calculation



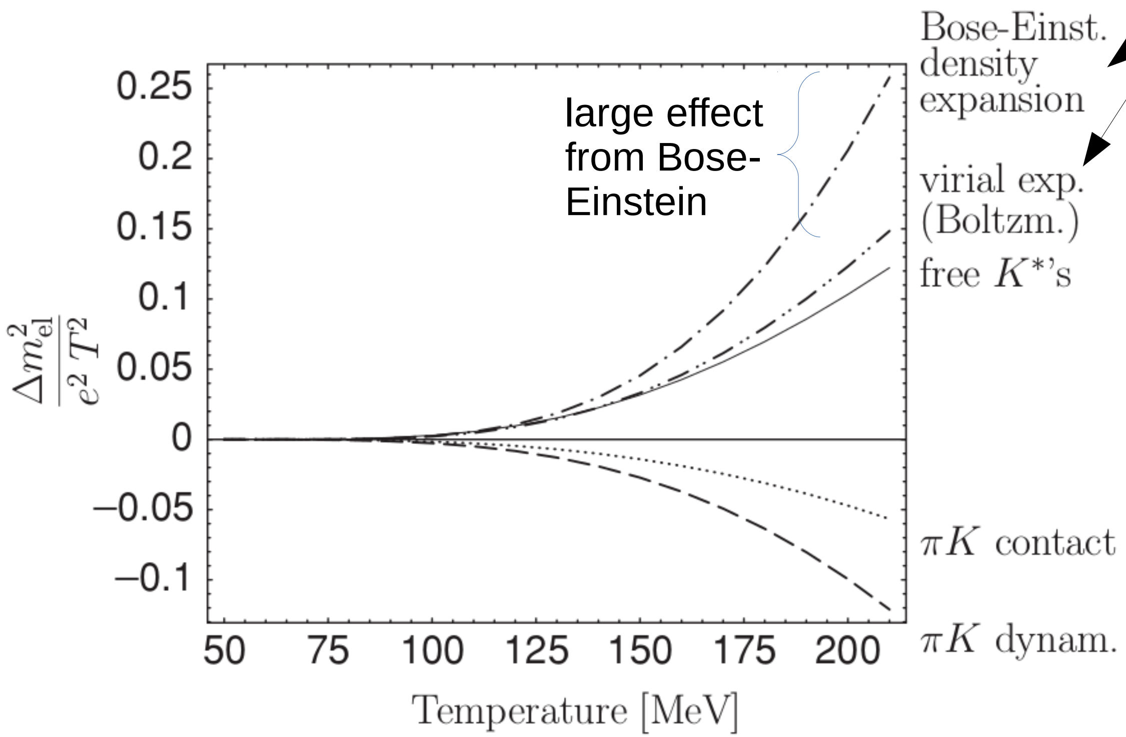
Charge fluctuations:

$$\langle \delta Q^2 \rangle = VT\ \Pi_{00}(\omega = 0, \mathbf{k} \to 0) = VTm_{\text{el}}^2$$

$$\Pi_{\mu\nu}(\omega, \mathbf{k}) = i \int dt d^3x e^{-i(\omega t - \mathbf{kx})} \langle J_\mu(\mathbf{x}, t) J_\nu(0) \rangle$$

J_μ: EM current-current correlation

Interacting πK: Influence on electric mass



Input:

$\delta_0^{1/2}, \delta_1^{1/2}$, and $\delta_2^{1/2}, \ \delta_0^{3/2}$

Interactions lead to larger changes than in pi-pi case

FIG. 13. Corrections to the electric mass or CF, $\langle \delta Q^2 \rangle / (e^2 V T^3)$ for πK interaction. The density and virial expansions are from Eqs. (F4) and (F6), respectively. The loop expansions "πK dynamical" and "πK contact" are from Eqs. (F1) and (F2) and Eq. (F3), respectively. The solid line shows the electric mass of a gas of free $\kappa(800)$, $K^*(892)$, $K_0^*(1430)$, and $K_2^*(1430)$ mesons.

Conclusions

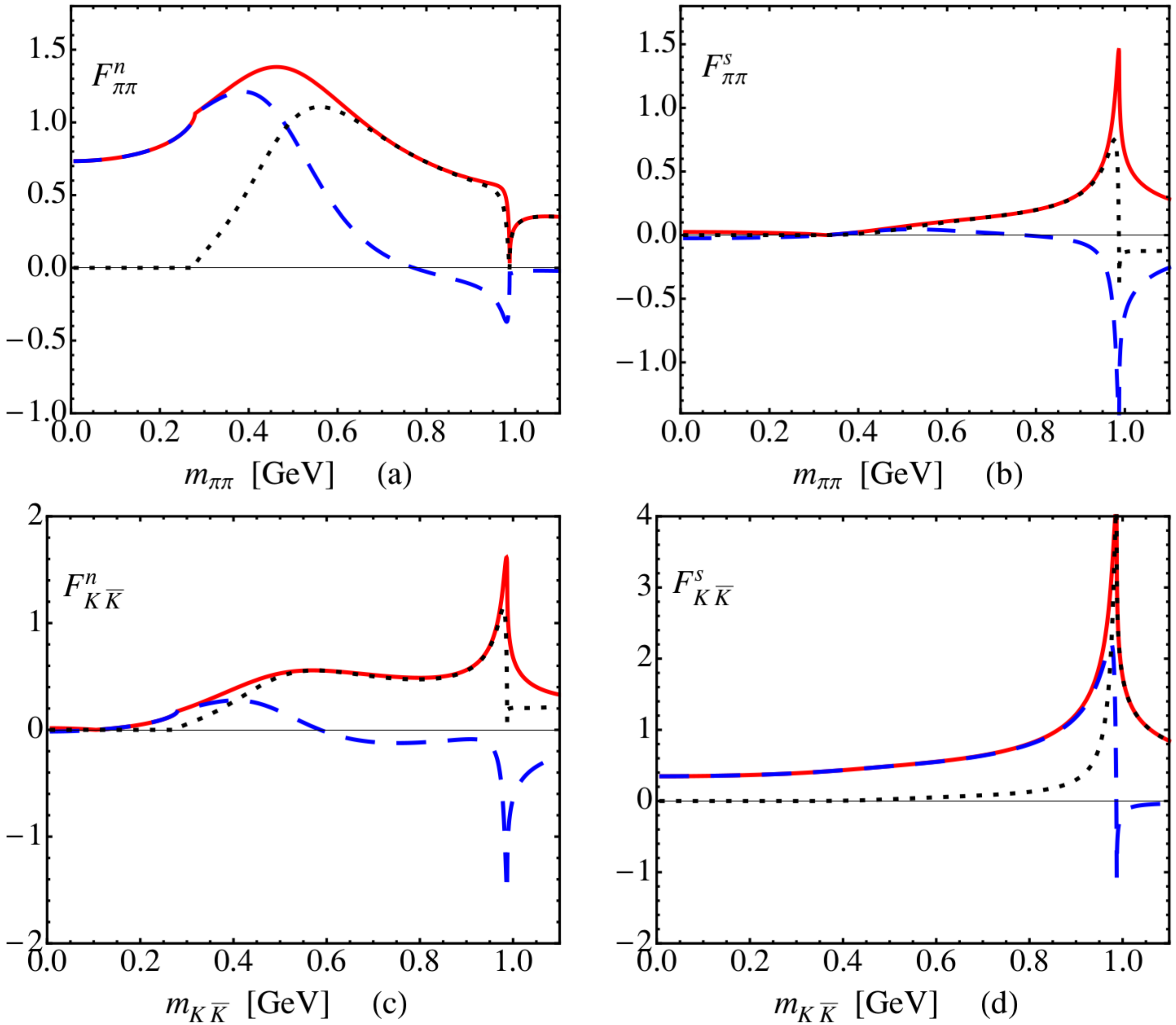
- Evaluation of the scalar, strangeness-changing form factor
- Twice-subtracted dispersion relation & matching to NLO CHPT
- Systematic influence from the ηK checked through a two-channel formulation (influence is generally small)
- Recent LHCb measurements are in agreement with predicted s-wave/p-wave ratio (~0.1)

  (although within large uncertainties)
- Better partial waves needed to improve precision

Spare slides

Pion-pion and K$\bar{K}$ form factors

(Calculated with UCHpT, not dispersion relations)



Predicted observables for $K\bar{K}$

$$\bar{B}_s^0 \to K^+K^-l^+l^-$$