hadspec

hadron spectrum collaboration

hadspec.org

meson-meson scattering from lattice QCD

Jozef Dudek

WILLIAM & MARY

CHARTERED 1693

Jefferson Lab

finite volume spectrum ⇔ scattering amplitudes

lattice QCD computes a spectrum in a periodic cube

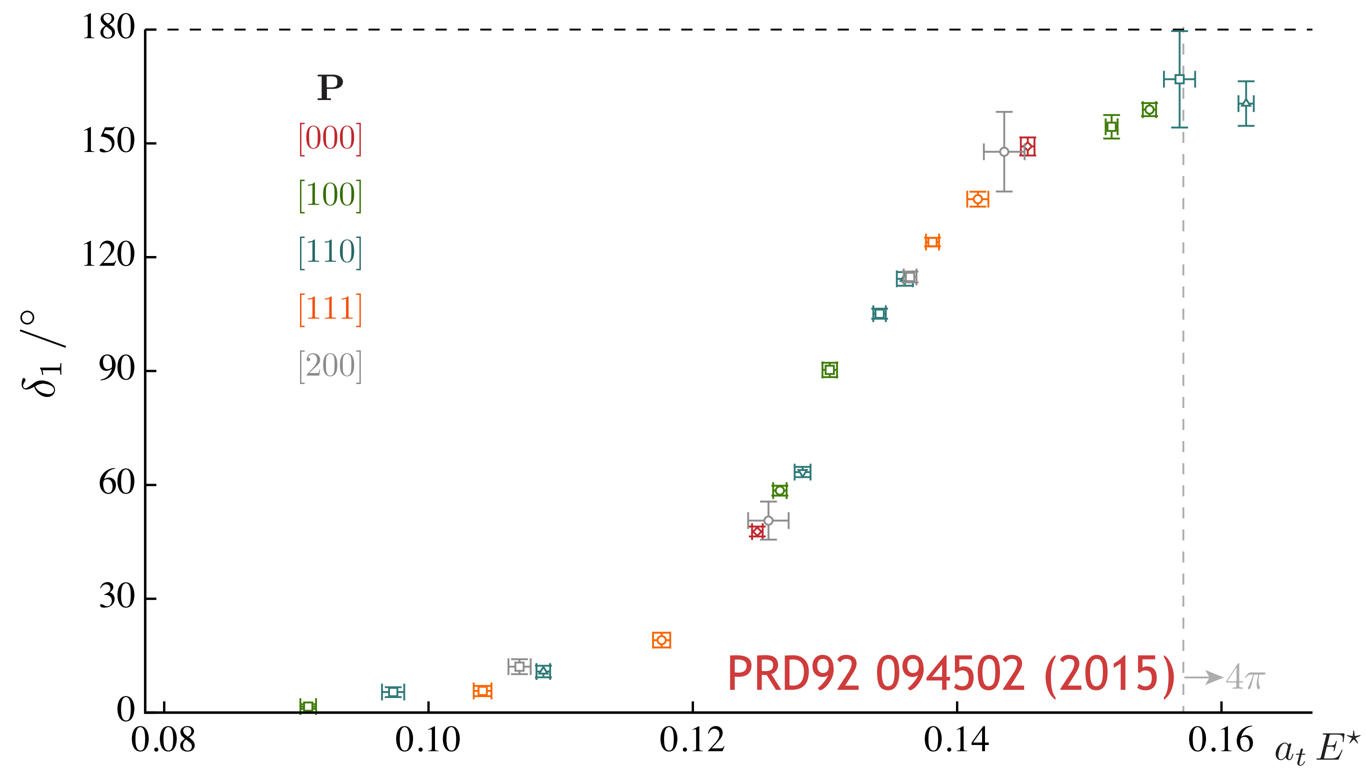
under **periodic boundary conditions:**

scattering continuum → discrete (volume dependent) spectrum

from the discrete spectrum, can determine scattering amplitudes

e.g. in the simplest elastic case — $E_n \rightarrow \delta_\ell(E_n)$

ππ I=1 *P*-WAVE — m_π~236 MeV



a review of the field: arXiv:1706.06223
(to appear in Rev.Mod.Phys)

hadron spectrum collaboration

has pioneered the extension to the **coupled-channel sector**

several studies of meson-meson systems with (initially) m_π~**391 MeV**

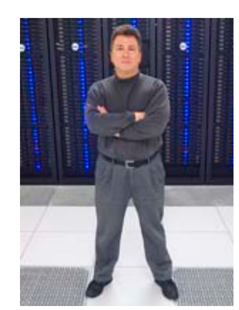
unique combination of ingredients:

- large operator basis
- multiple volumes
- moving frames
- varied parameterisations of energy dependence
- exploration of pole singularities

on the papers
I'll show today

D. Wilson

R. Edwards



R. Briceno

C. Thomas

$\pi\eta, K\bar{K}$ scattering & a_0 resonance

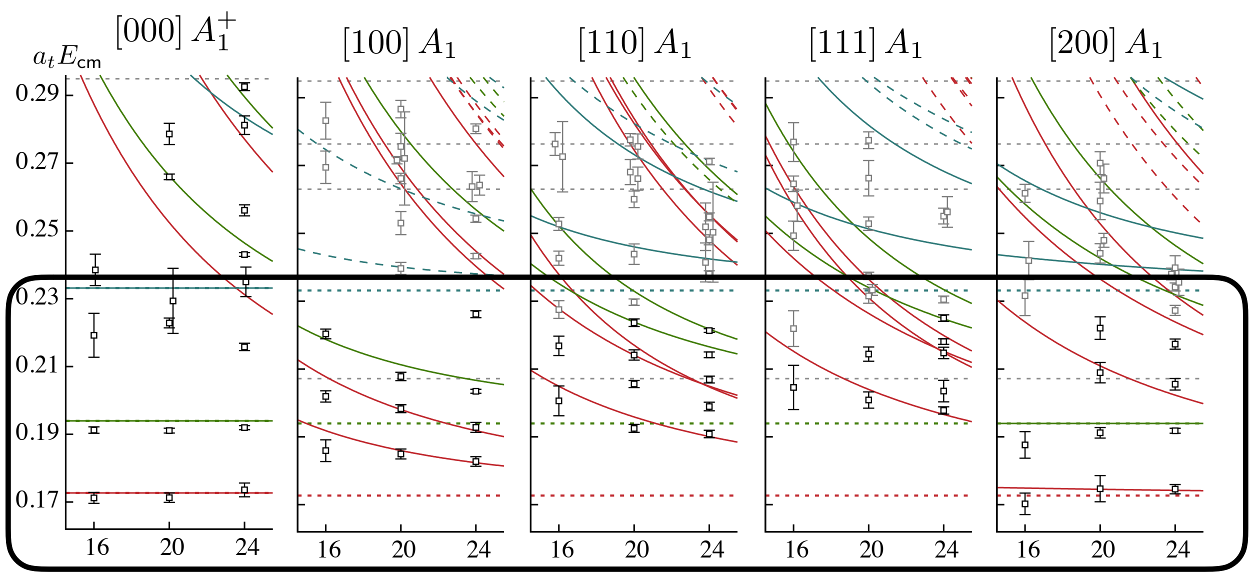
not easy to study $\pi\eta$ **scattering** experimentally (isolating η exchange not practical)

but well known there's an $a_0(980)$ resonance
at $K\bar{K}$ threshold decaying to $\pi\eta$

$\pi\eta, K\bar{K}$ scattering & a_0 resonance

PRD93 094506 (2016)

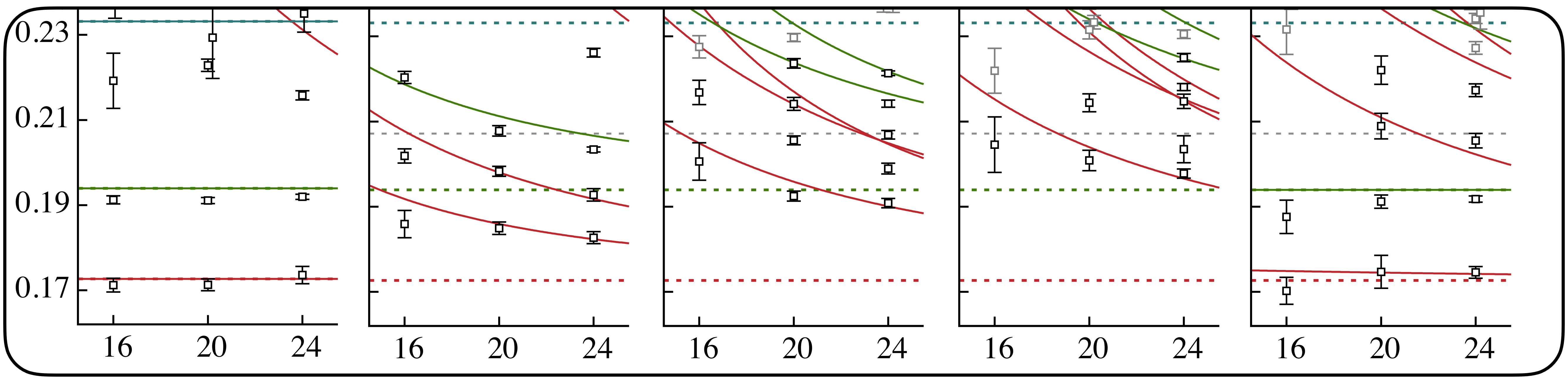
SPECTRA IN THREE VOLUMES



47 energy levels
below $\pi\eta'$ threshold

$\pi\eta, K\bar{K}$ scattering & a_0 resonance

PRD93 094506 (2016)

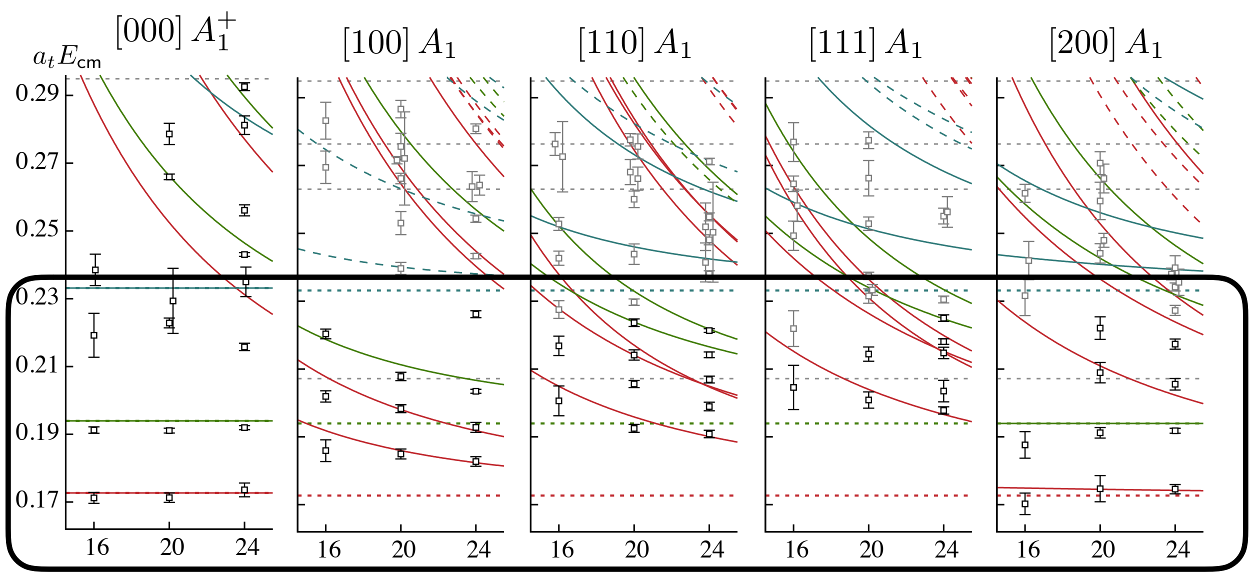


each point helps constrain scattering at that energy ...

$\pi\eta, K\bar{K}$ scattering & a_0 resonance

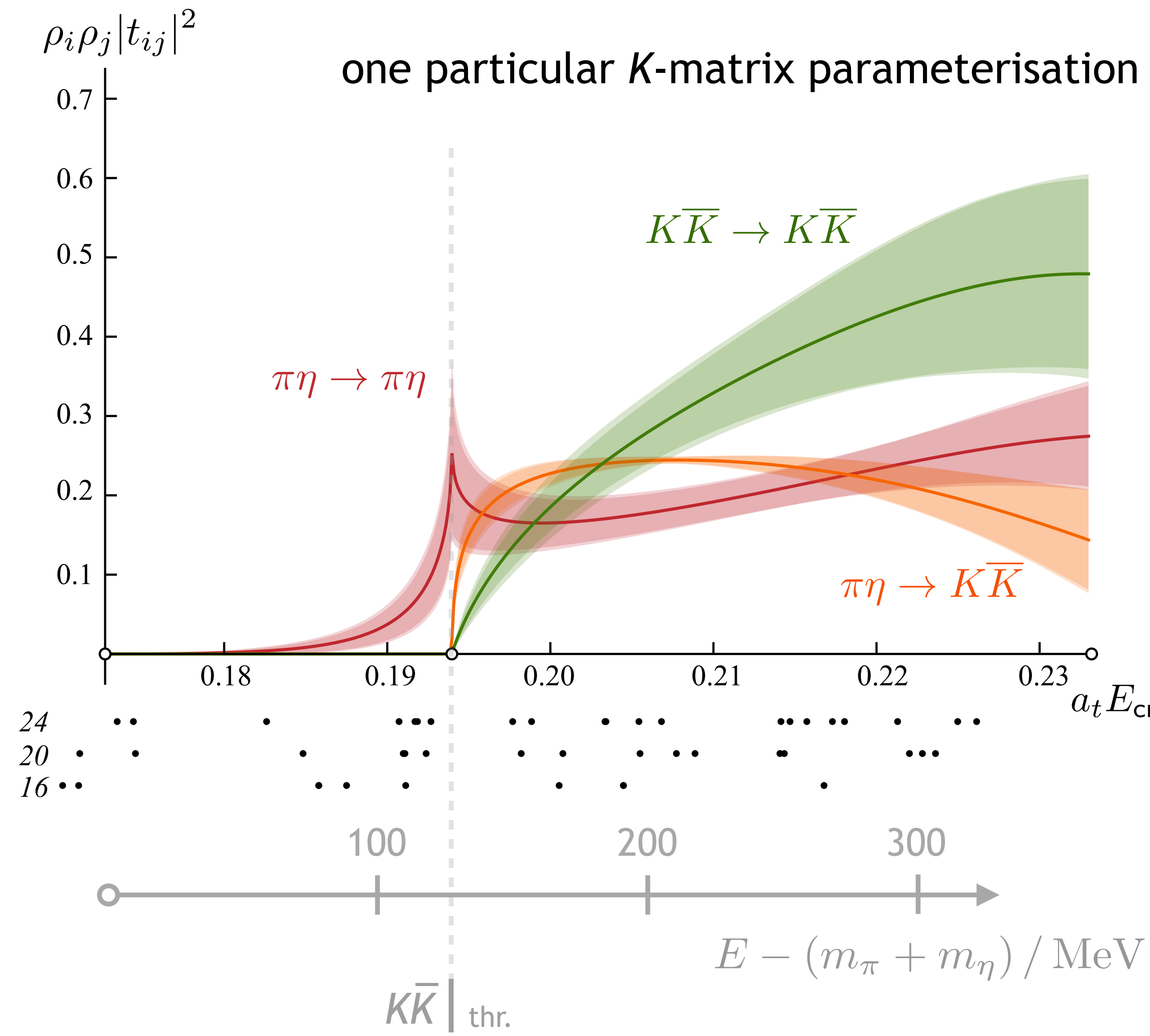
PRD93 094506 (2016)

SPECTRA IN THREE VOLUMES



47 energy levels
below $\pi\eta'$ threshold

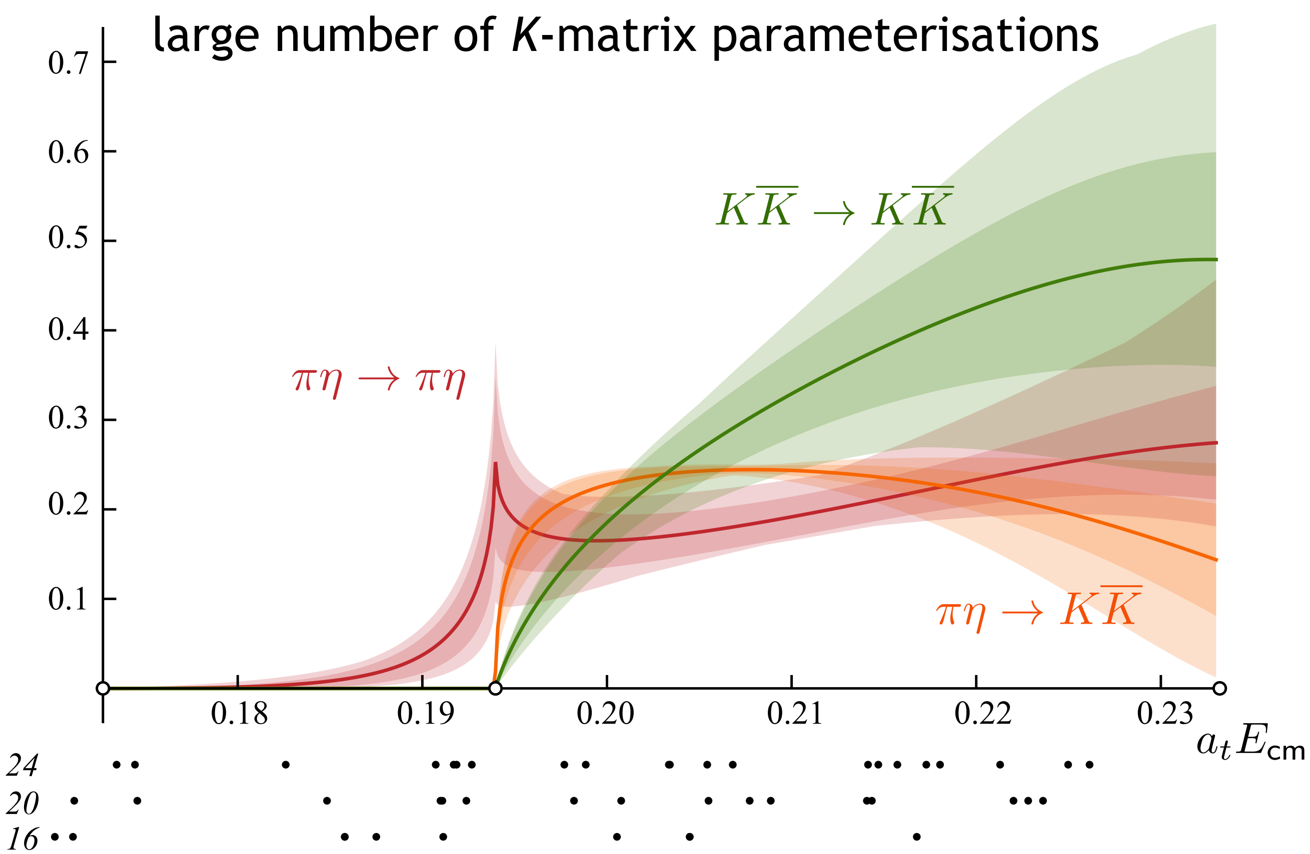
S-WAVE AMPLITUDES



$\pi\eta, K\bar{K}$ scattering & a_0 resonance

PRD93 094506 (2016)

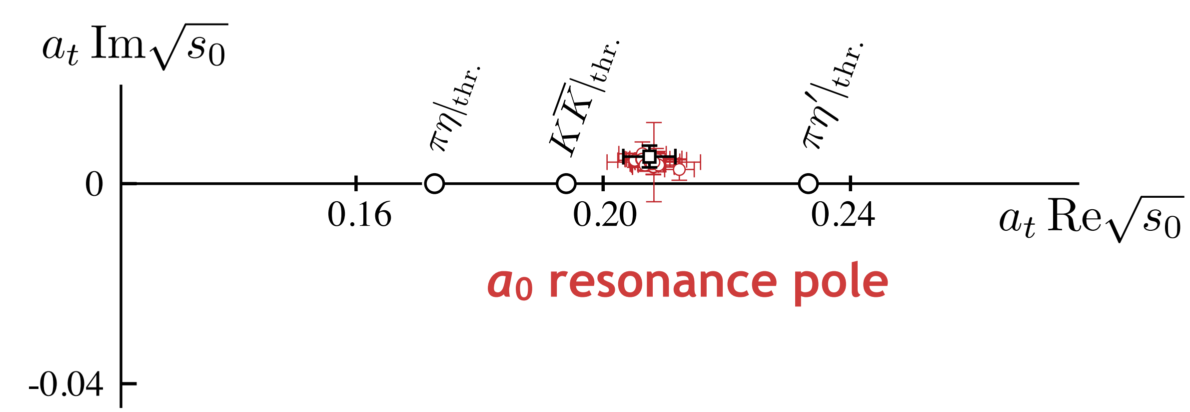
S-WAVE AMPLITUDES



RESONANCE POLE SINGULARITY

rapid energy variation at $K\bar{K}$ threshold proves to be due to a **pole singularity**

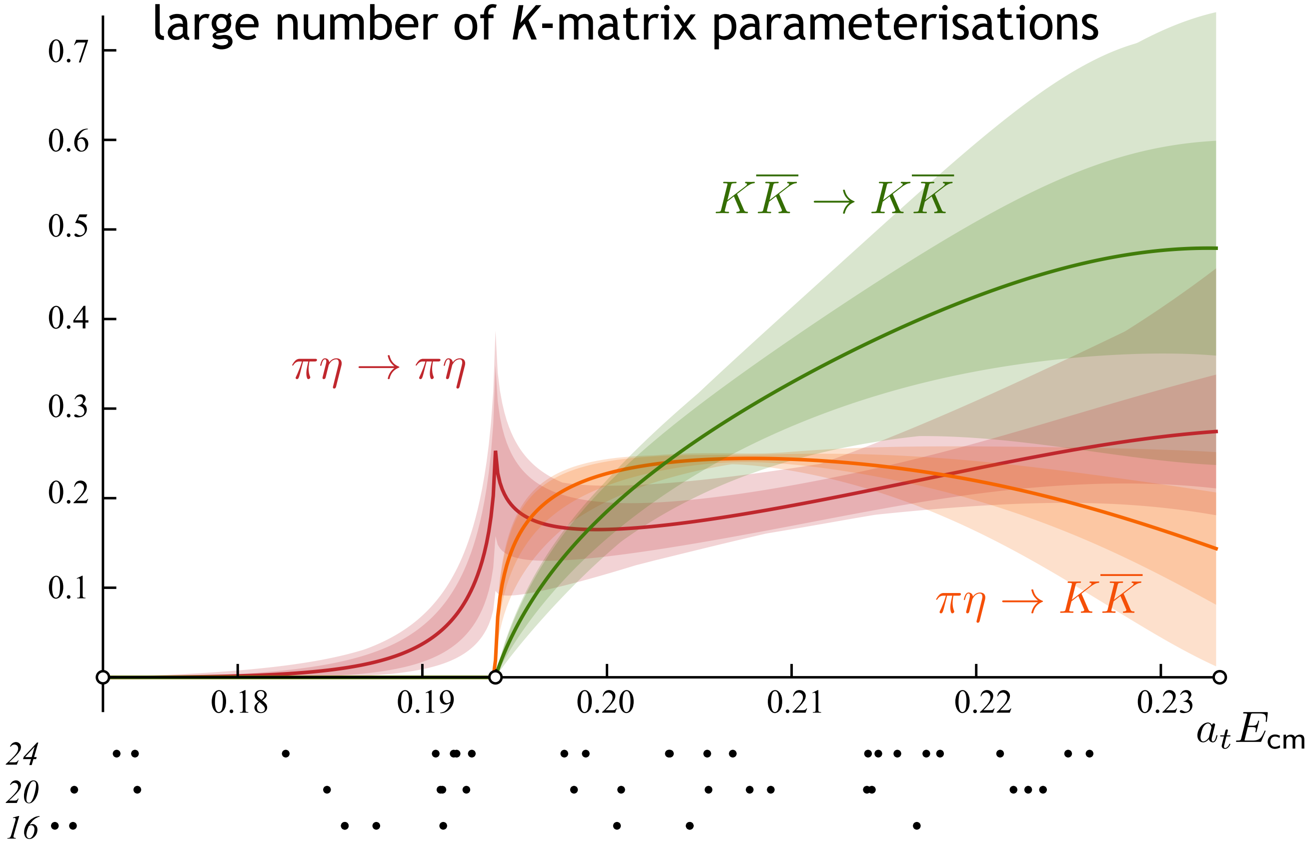
$$t_{ij}(s) \sim \frac{c_i\, c_j}{s_0 - s}$$

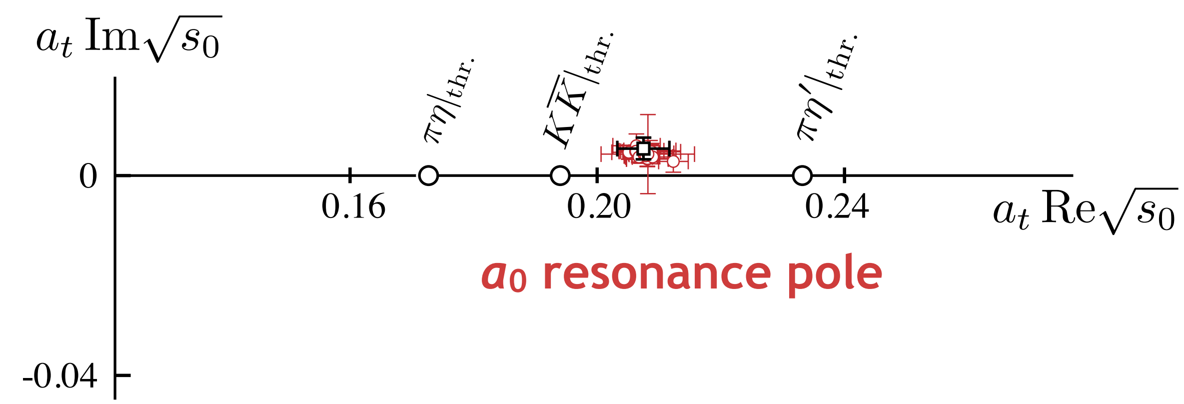


$\pi\eta, K\bar{K}$ scattering & a_0 resonance

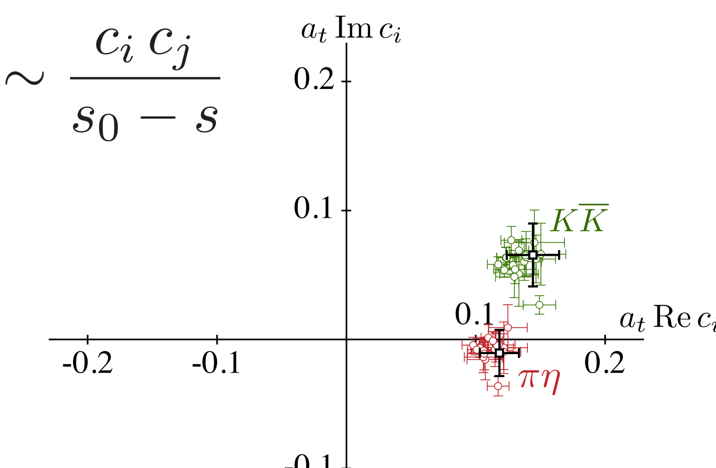
PRD93 094506 (2016)

S-WAVE AMPLITUDES





$$t_{ij}(s) \sim \frac{c_i\, c_j}{s_0 - s}$$



$$\left|\frac{c_{K\bar{K}}}{c_{\pi\eta}}\right| = 1.3(4)$$

@ m_π~391 MeV

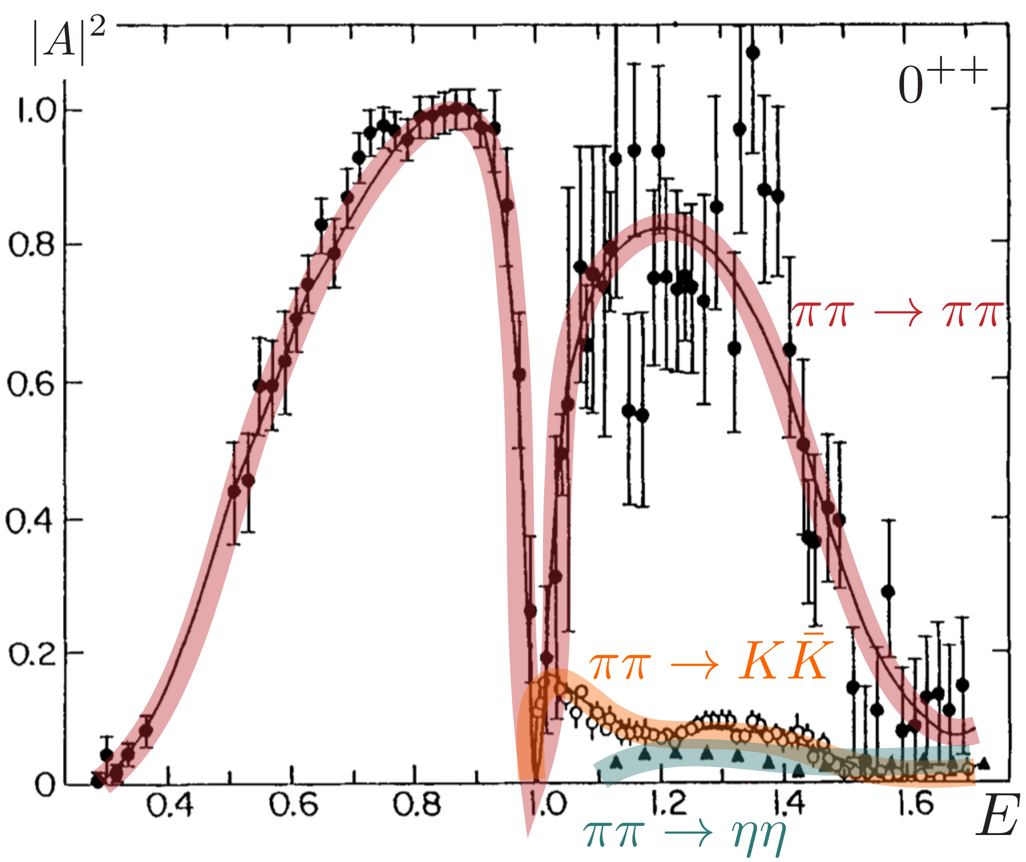
a_0('980') resonance

$$\sqrt{s_0} = 1177(27) \pm \tfrac{i}{2} 49(33)\ \mathrm{MeV}$$

$$2\,m_K = 1098\,\mathrm{MeV}$$

$\pi\pi, K\bar{K}, \eta\eta$ scattering & f_0, f_2 resonances

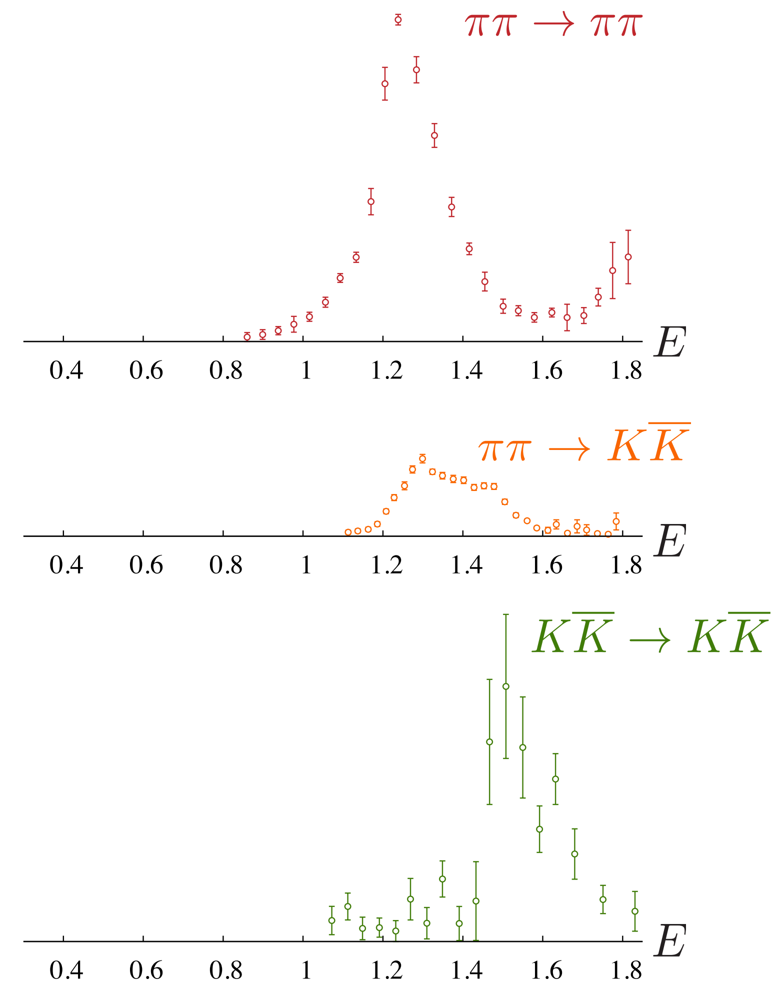
S-WAVE AMPLITUDES

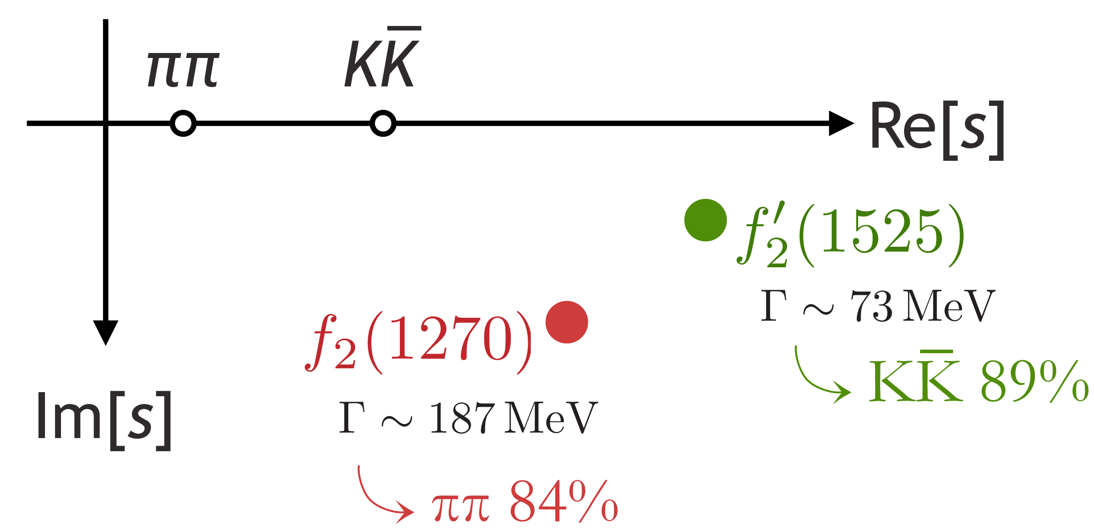


due to broad $\sigma = f_0$('500')
and narrow $f_0(980)$

constrained dispersive
approaches pin down the σ

D-WAVE AMPLITUDES

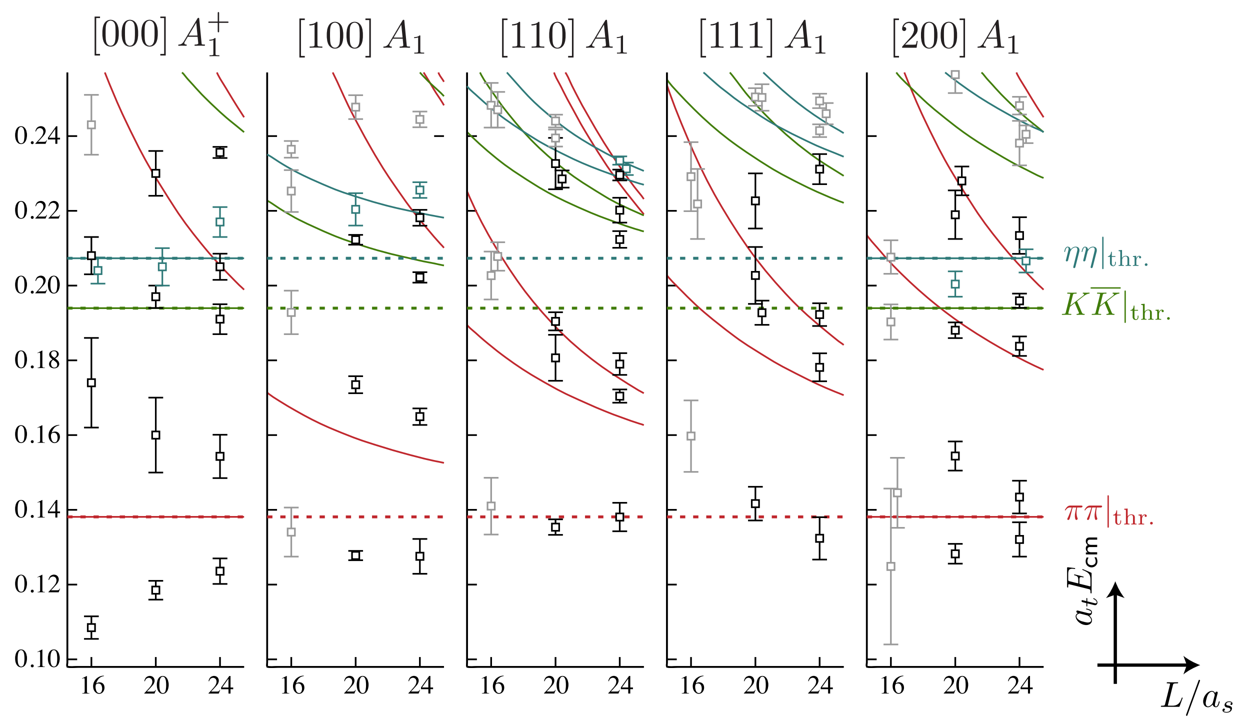




$\pi\pi, K\overline{K}, \eta\eta$ scattering & f_0, f_2 resonances

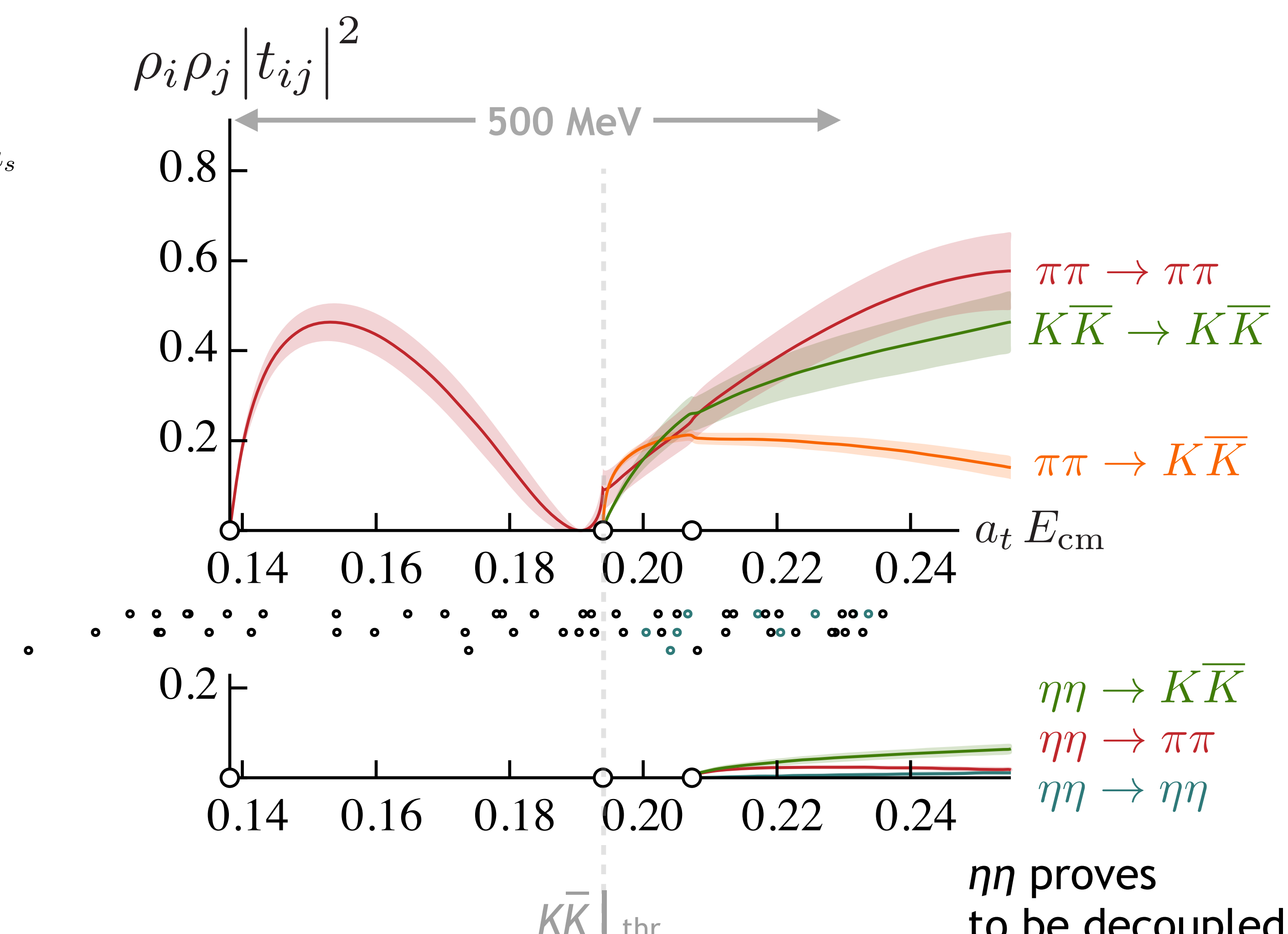
arXiv:1708.06667

SPECTRA IN THREE VOLUMES



57 energy levels

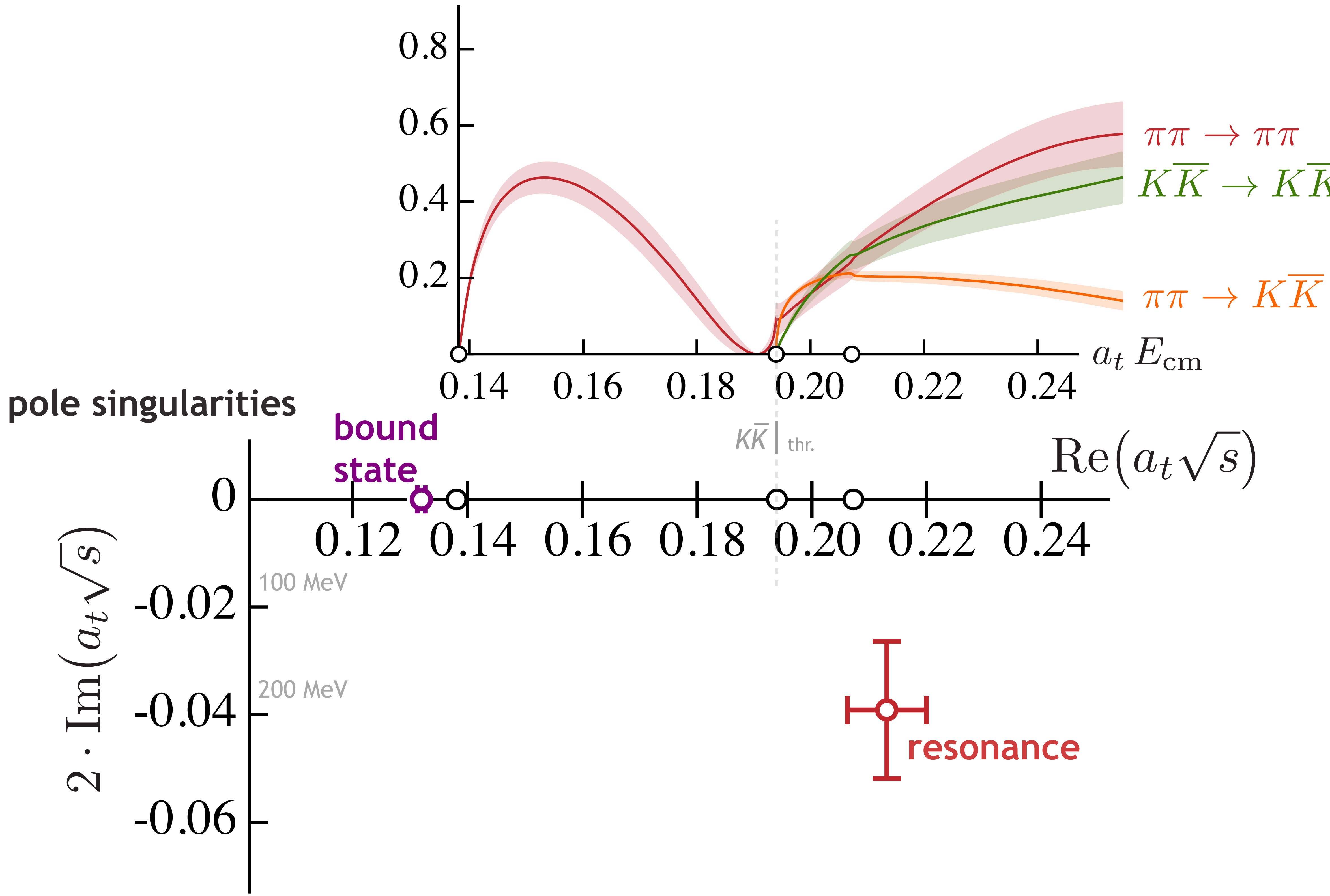
ISOSPIN-0 S-WAVE AMPLITUDES



$\eta\eta$ proves to be decoupled

$\pi\pi, K\bar{K}, \eta\eta$ scattering & f_0, f_2 resonances

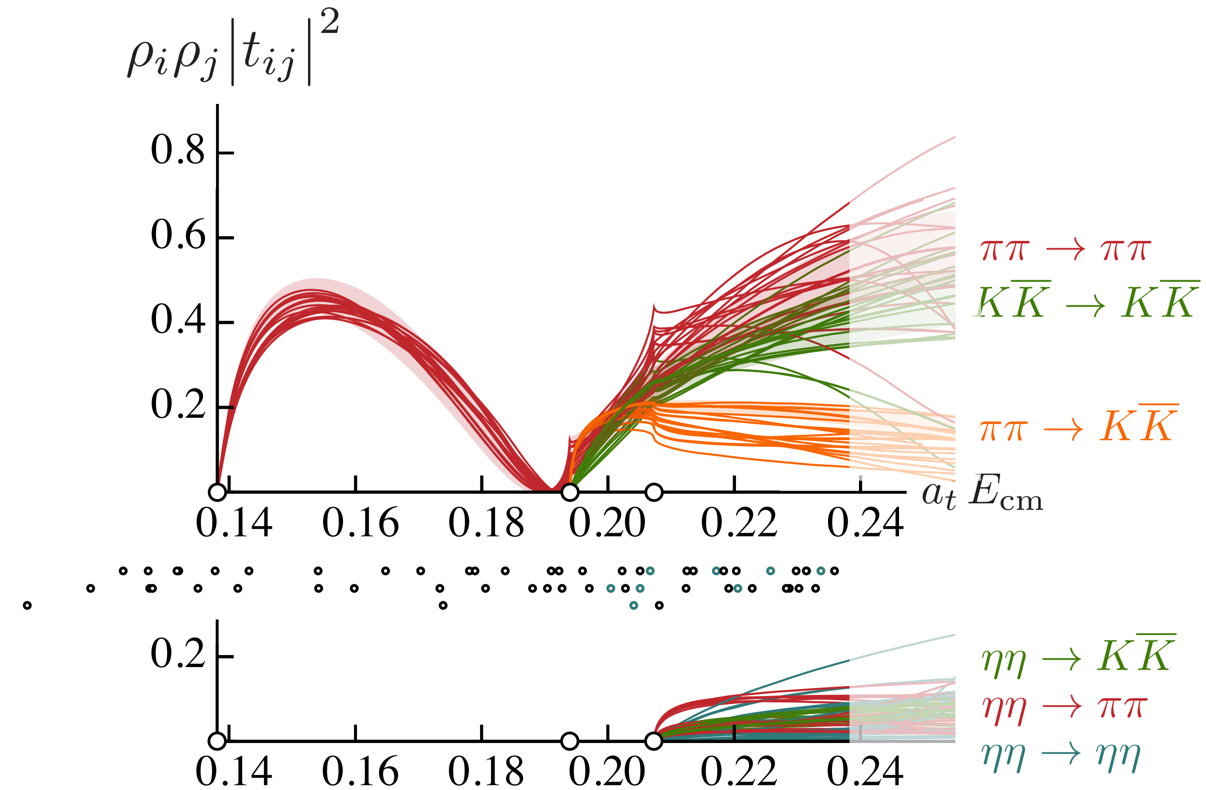
arXiv:1708.06667

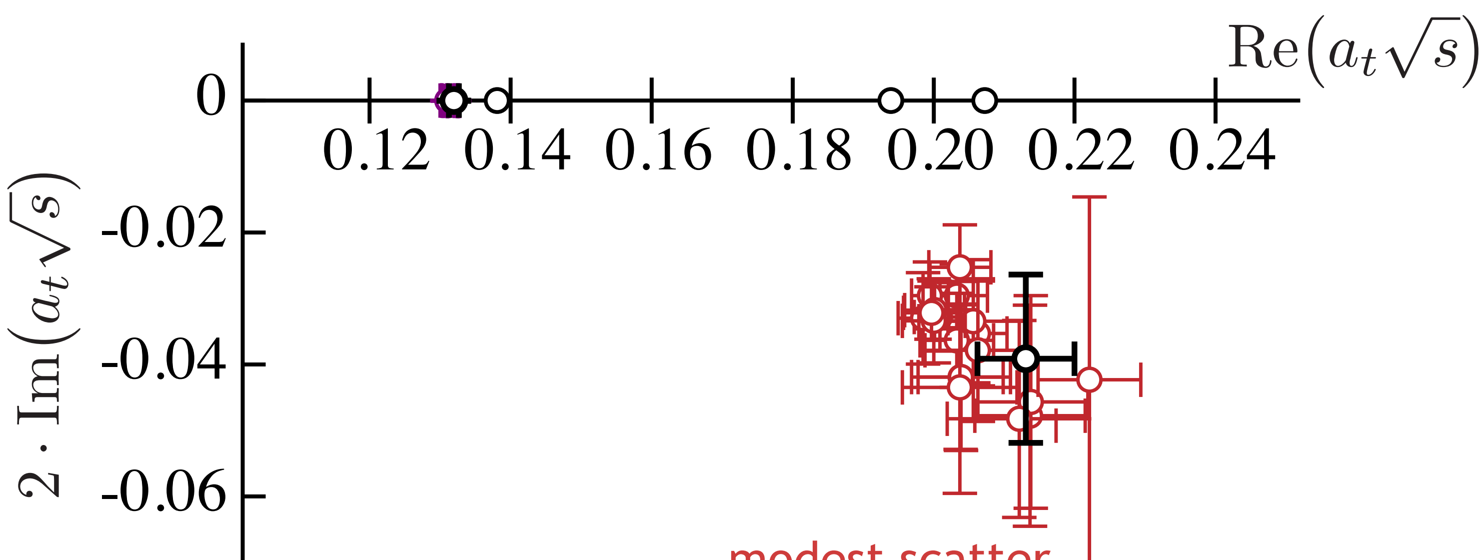


$\pi\pi, K\bar{K}, \eta\eta$ scattering & f_0, f_2 resonances

arXiv:1708.06667

variation under change in parameterization





@ m_π~391 MeV

stable σ bound-state

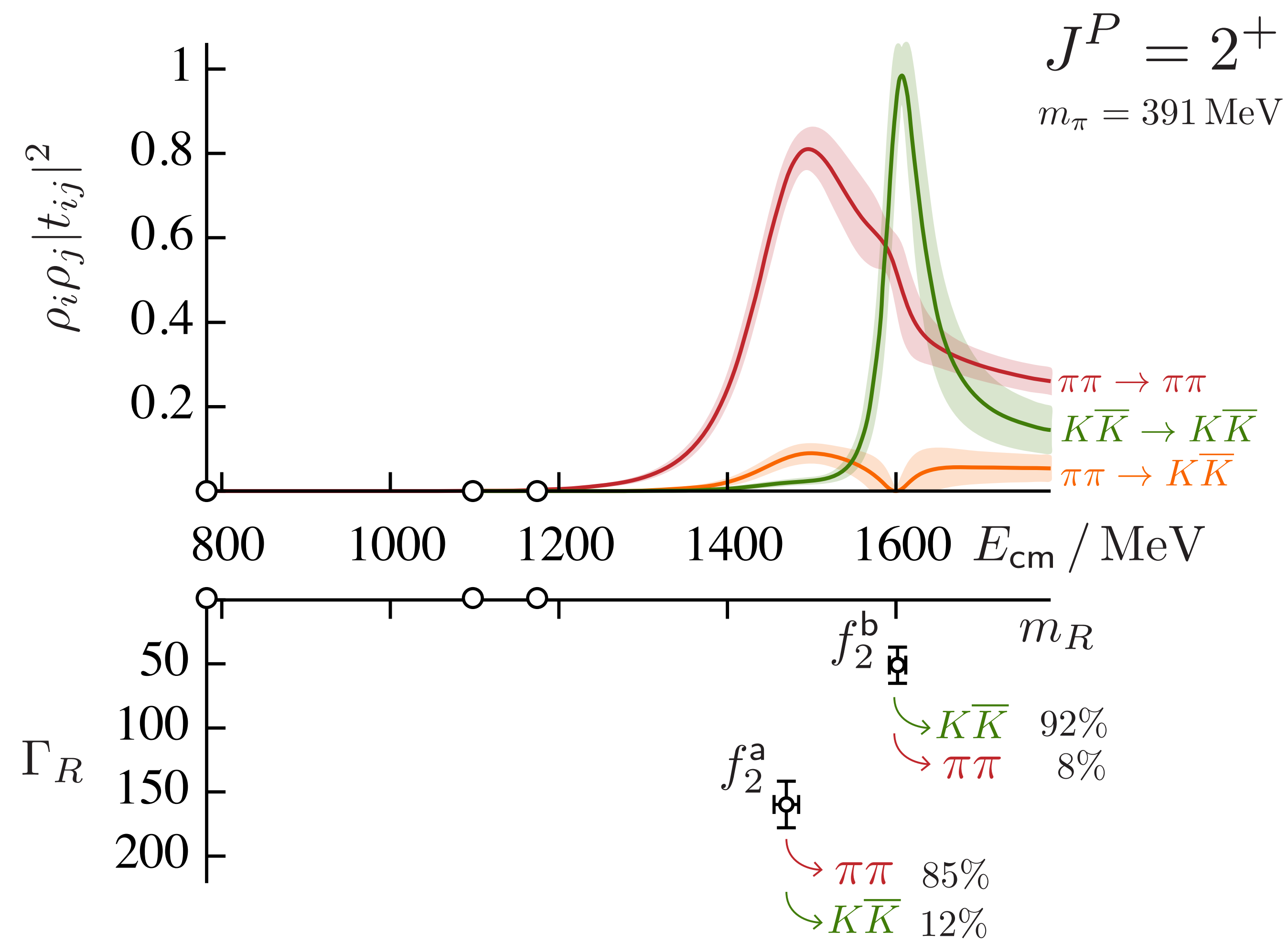
binding energy ~ 40 MeV

f0('980')-like resonance

$$\sqrt{s_0} = 1166(45) \pm \tfrac{i}{2} 181(68)\ \mathrm{MeV}$$

$\pi\pi, K\bar{K}, \eta\eta$ scattering & f_0, f_2 resonances

arXiv:1708.06667



two ‘OZI’-like resonances

$f_2^{\mathsf{a}} \sim u\bar{u} + d\bar{d}$ $\qquad$ $f_2^{\mathsf{b}} \sim s\bar{s}$

c.f. $f_2(1270)$ $\qquad$ $f_2'(1525)$

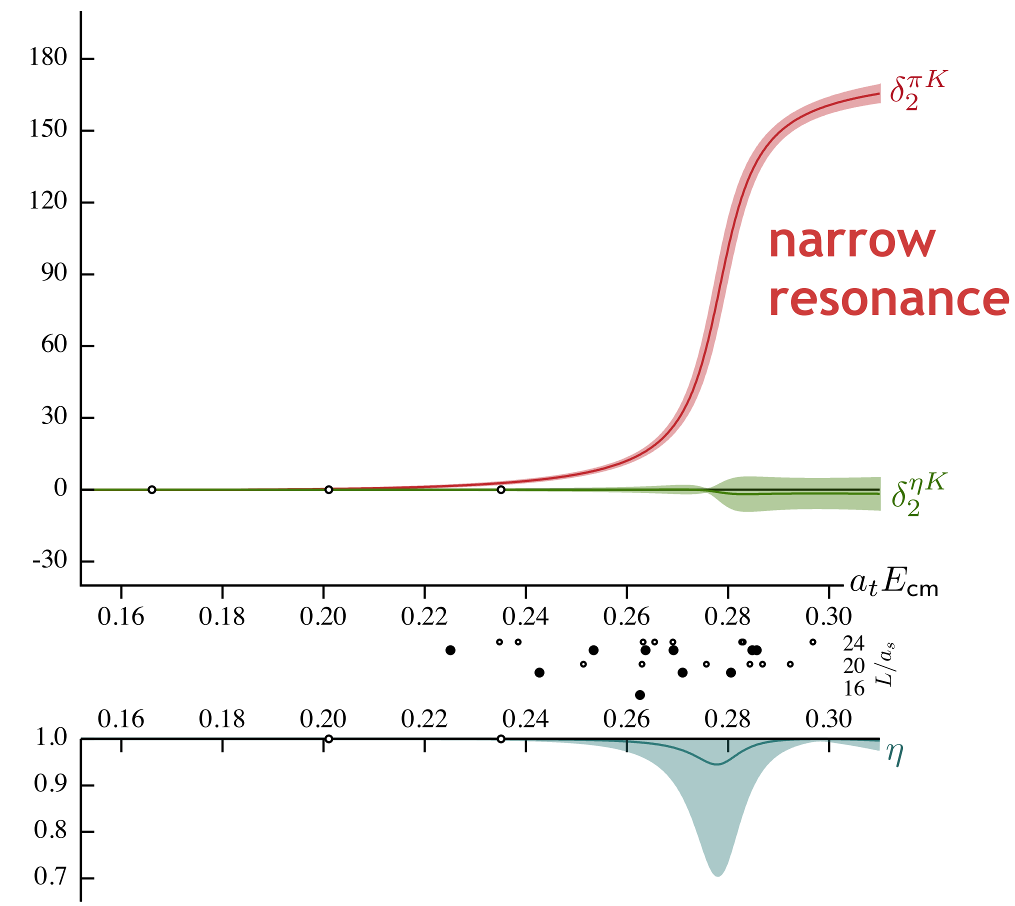
role of three (and four) hadron channels not considered
… but reasonable (?) arguments they may be suppressed

$\pi K, \eta K$ scattering

the first coupled-channel calculation in lattice QCD (in 2014)

P-wave contains <u>stable</u> vector meson K^* at threshold *[fluke of the quark mass chosen]*

D-WAVE $\pi K / \eta K$ SCATTERING

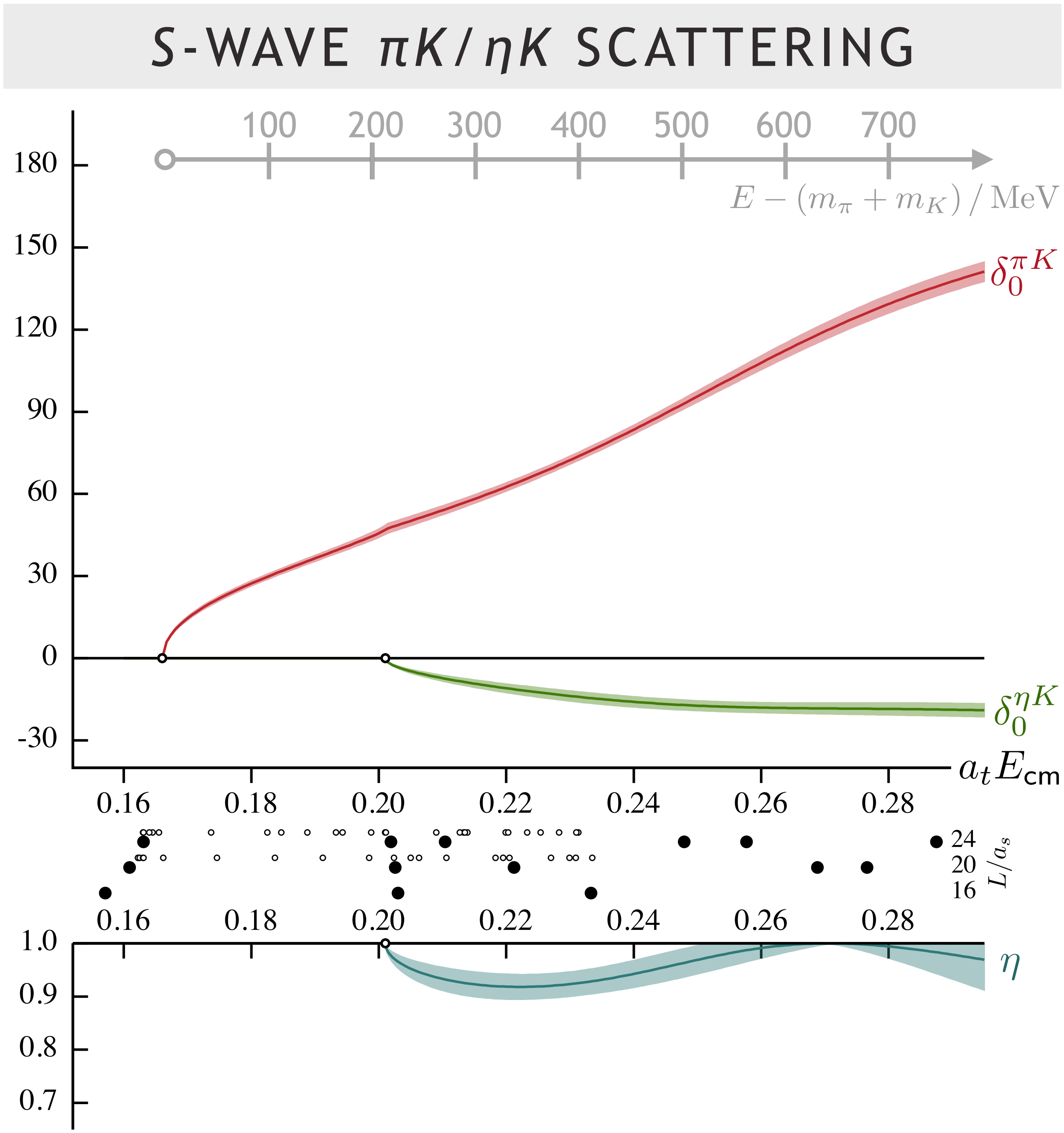


$$\mathbf{S} = \begin{bmatrix} \eta\, e^{2i\delta_{\pi K}} & i\sqrt{1-\eta^2}\, e^{i(\delta_{\pi K}+\delta_{\eta K})} \\ i\sqrt{1-\eta^2}\, e^{i(\delta_{\pi K}+\delta_{\eta K})} & \eta\, e^{2i\delta_{\eta K}} \end{bmatrix}$$

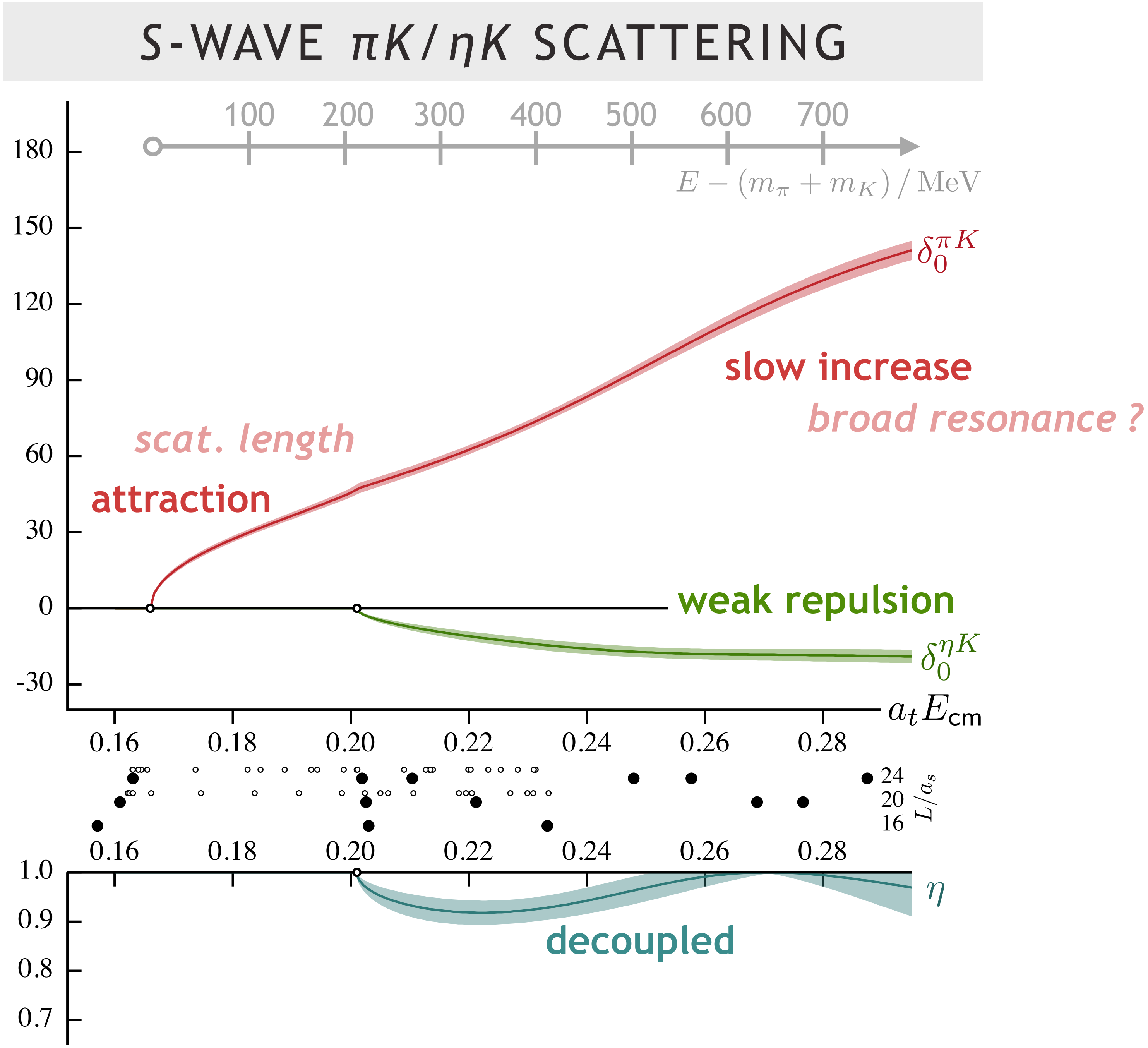
two phase-shifts, inelasticity

but role of $\pi\pi K$ not considered!

$\pi K, \eta K$ scattering



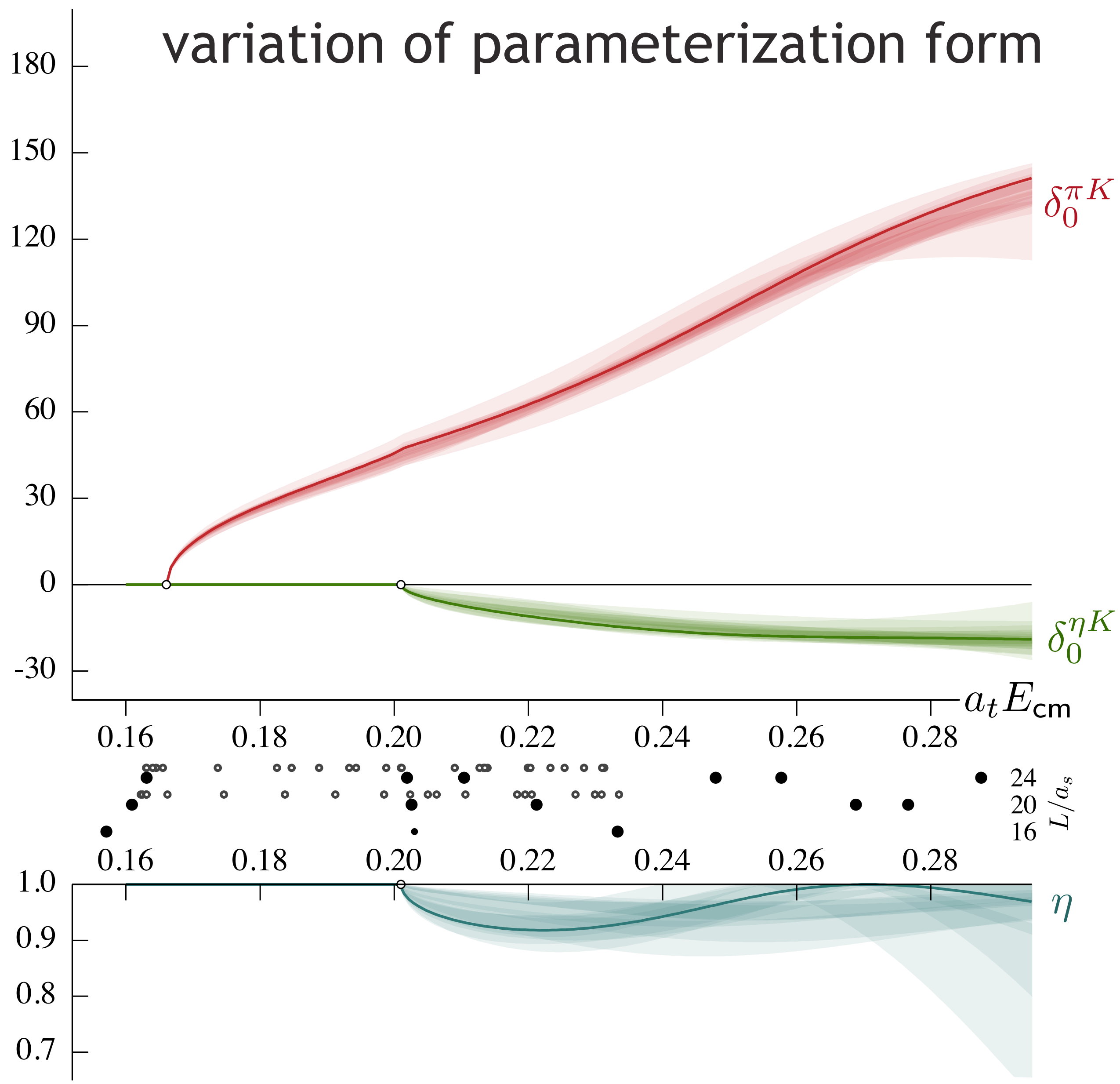
$\pi K, \eta K$ scattering



$\pi K, \eta K$ scattering

PRL113 182001 (2014)
PRD91 054008 (2015)

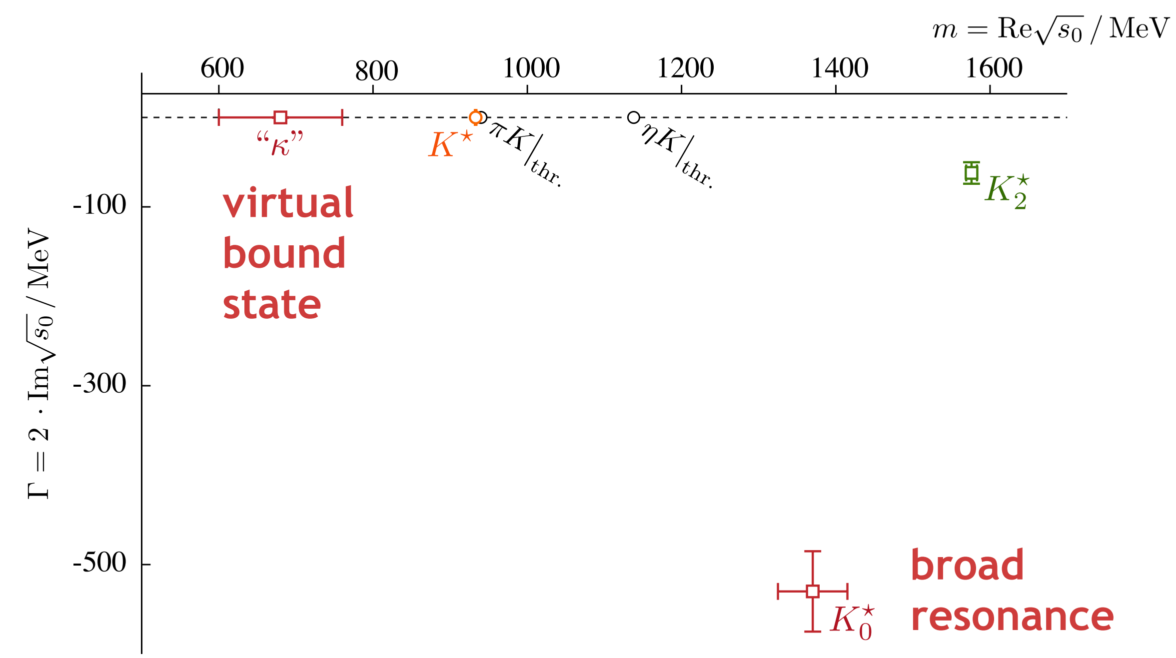
S-WAVE $\pi K / \eta K$ SCATTERING



$$k_{\text{cm}}^{2\ell+1} \cdot \cot \delta_\ell \to a_\ell^{-1}$$

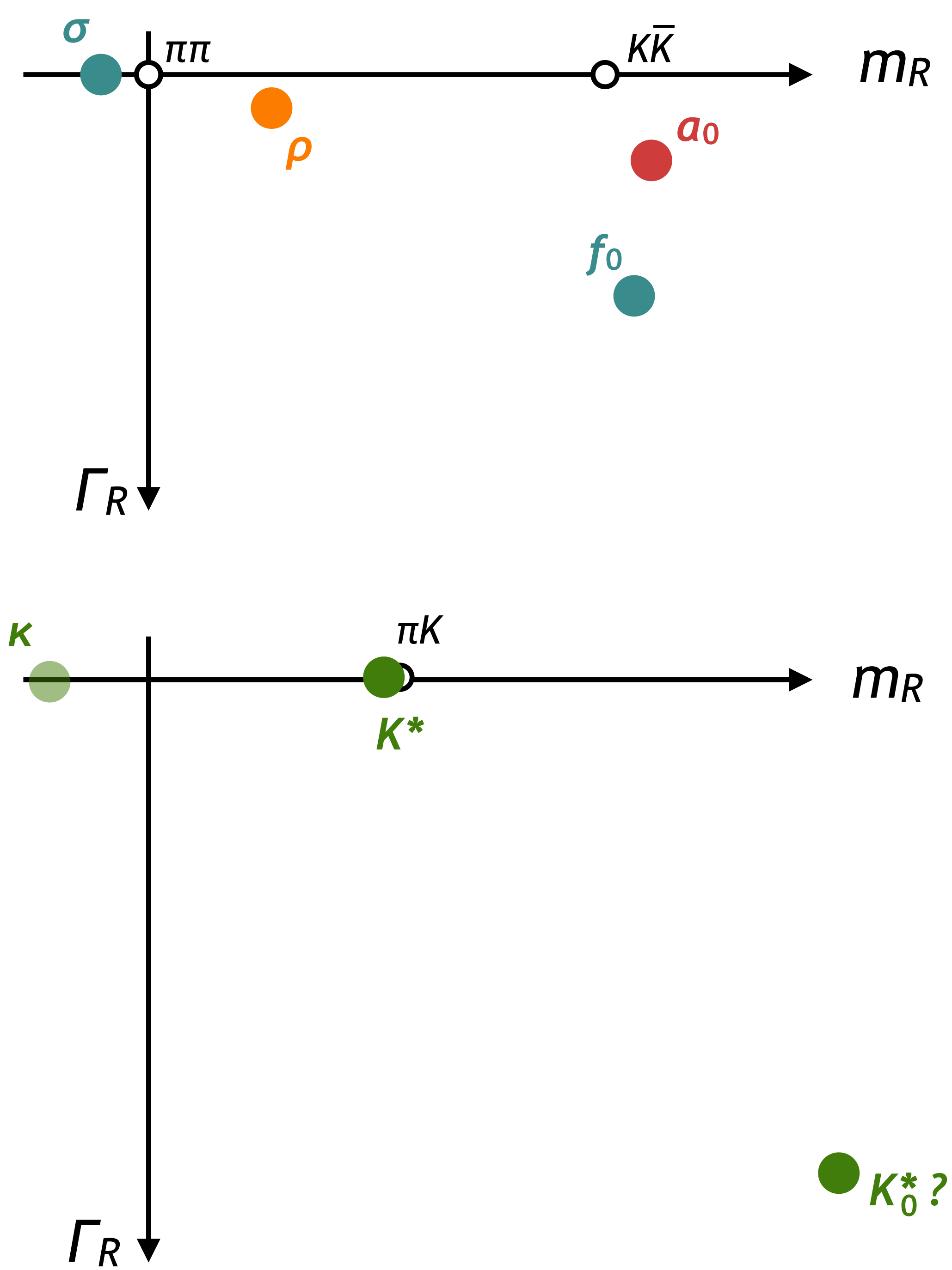
$$m_\pi \cdot a_{\ell=0} = 1.10(10)$$

POLE SINGULARITIES



low-lying meson spectrum

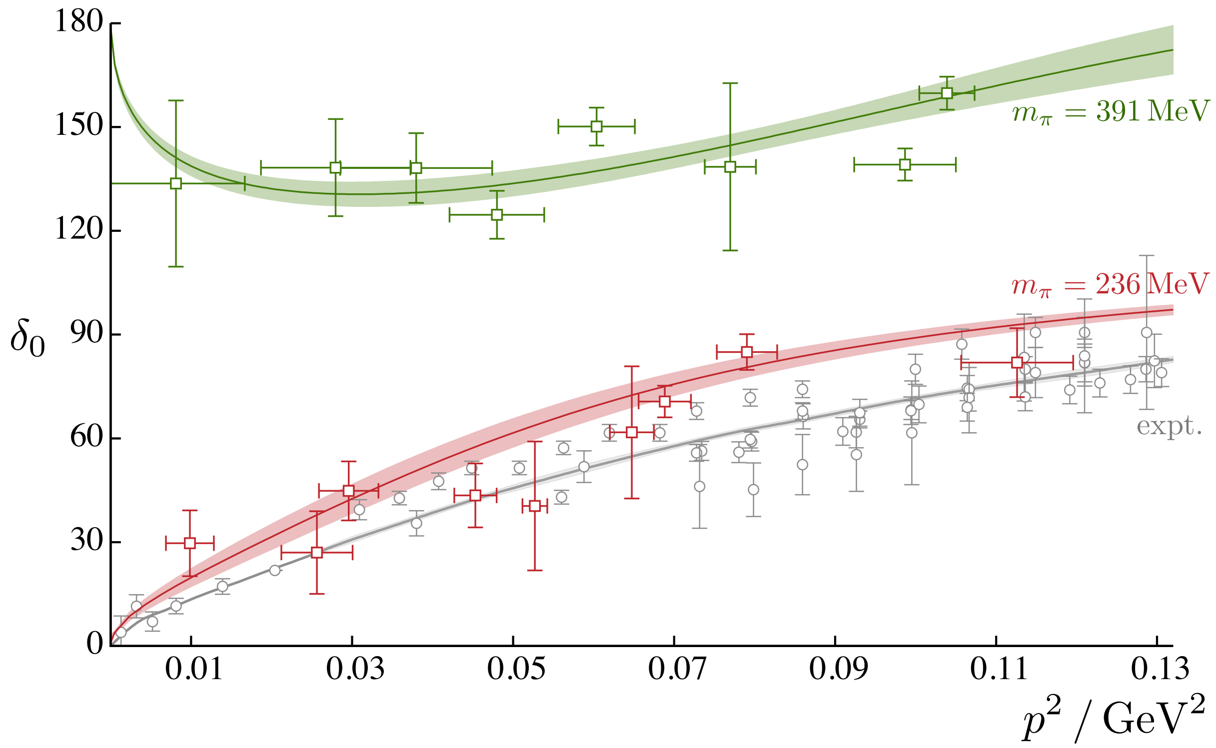
m_π~391 MeV

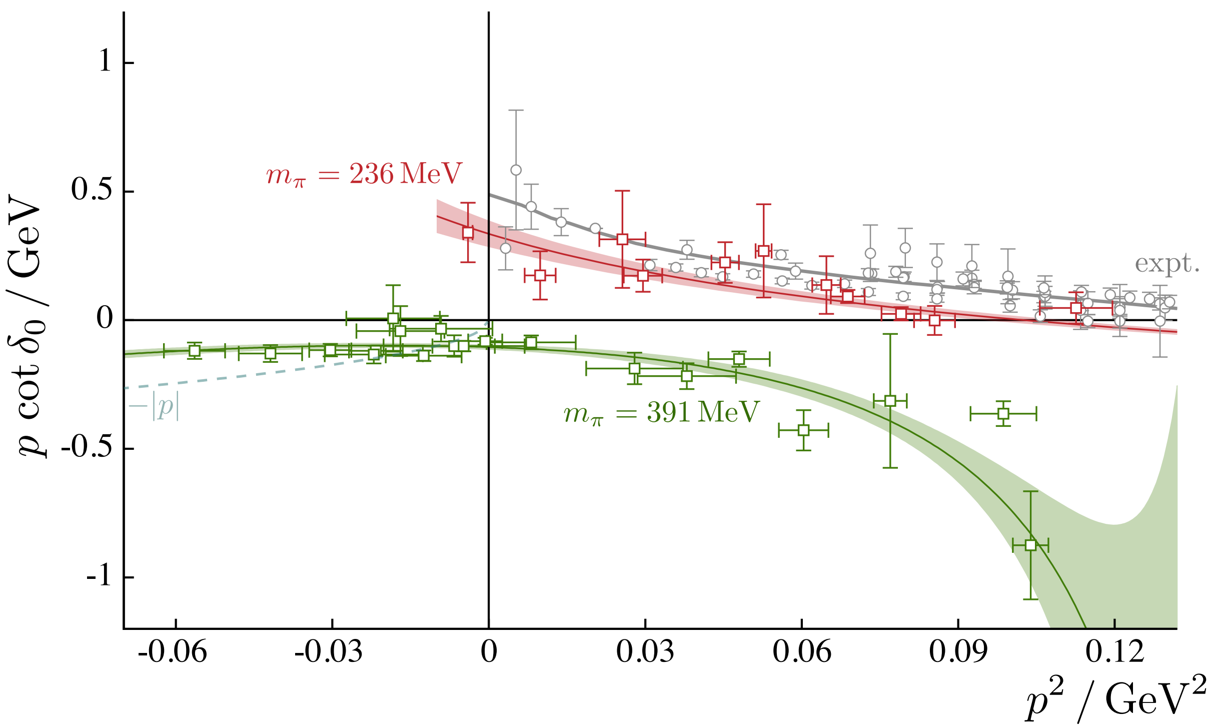


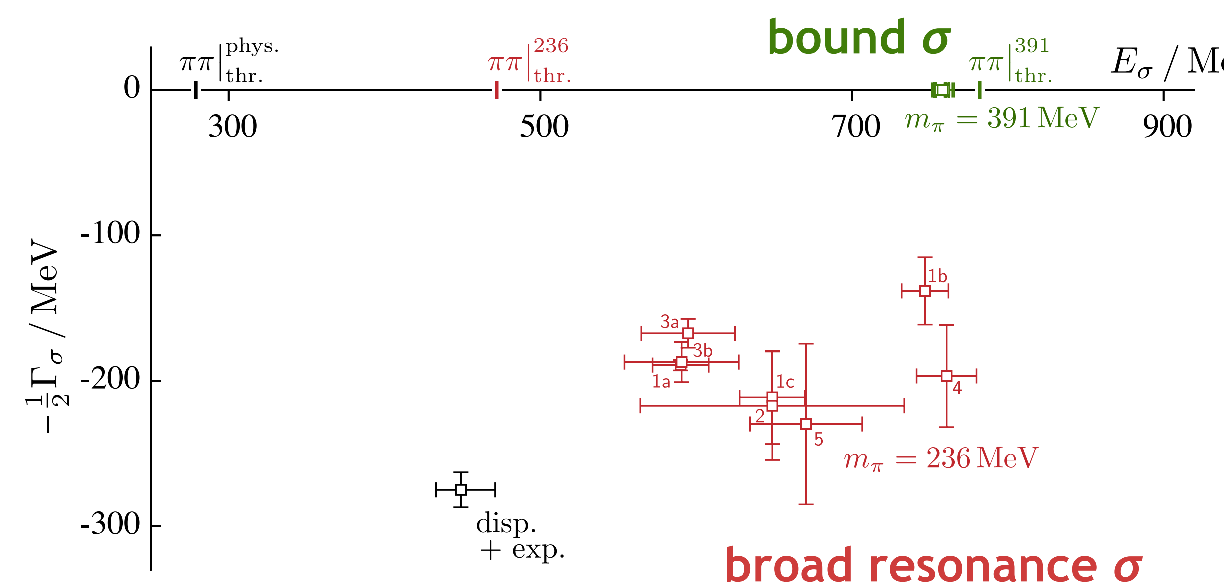
quark mass evolution of σ

m_π ~ 391 MeV → 236 MeV

PRL118 022002 (2017)







state of play

what do we learn from these heavy *u*,*d* quark masses ?

demonstration of methodology

hints of ‘evolution’ of resonances *mild* for $a_0(980)/f_0(980)$, **drastic** for σ, κ ?

[but as UχPT ?]

but can’t directly compare with experimental data yet

are there advantages to the lattice approach ?

‘stable pion target’ — no need to extrapolate to the pion-exchange pole in ***t***

[is the beam momentum precise enough to achieve this?]

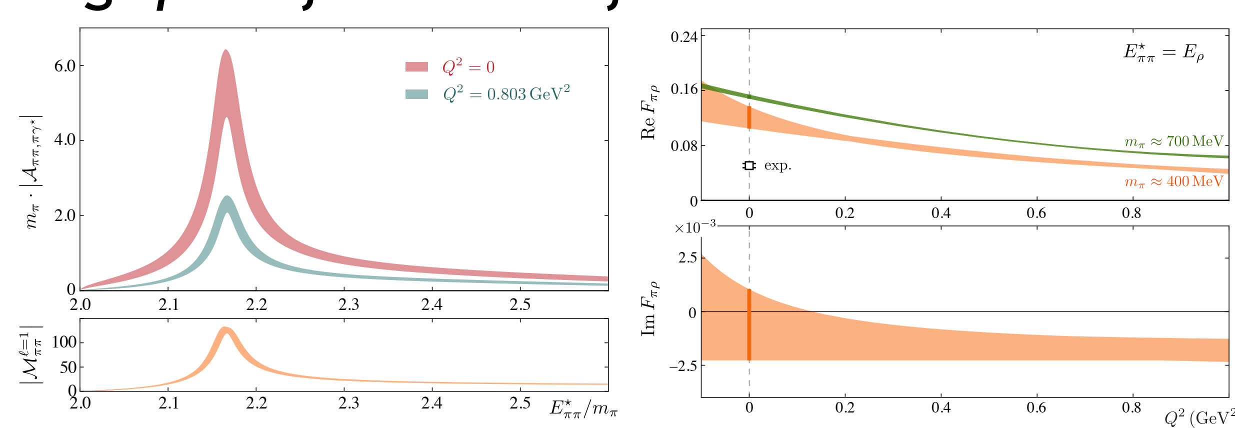
isospin separation is automatic

[are there independent linear combinations in the expt?]

can couple the resonances to external currents, study form-factors

e.g. $\rho\to\pi\gamma$ in $\pi\pi\to\pi\gamma$

PRL115 242001 (2015)
PRD93 114508 (2016)



state of play

what are the current limitations in the lattice approach

f.v. spectrum impacted by all open channels, can't 'turn things off'

issue when **three-body channels** open — no complete formalism as yet

for πK taken literally, mainly restricted to below $\pi\pi K$ threshold

at physical pion mass, 633 — 772 MeV !

amplitude parameterizations may not build in all relevant constraints

for cases with broad resonances, certainly room for improvement here

but much experience (at least for one-channel case) in 'dispersive community'

hadspec
hadron spectrum collaboration
hadspec.org

JEFFERSON LAB

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MESON SPECTRUM

***PRL**103 262001 (2009)*	*I=1*
PRD82 034508 (2010)	*I=1, K**
PRD83 111502 (2011)	*I=0*
JHEP07 126 (2011)	$c\bar{c}$
PRD88 094505 (2013)	*I=0*
JHEP05 021 (2013)	*D, Ds*
JHEP12 089 (2016)	$c\bar{c}$*, D, Ds*

BARYON SPECTRUM

PRD84 074508 (2011)	*(N,Δ)**
PRD85 054016 (2012)	$(N,\Delta)_{hyb}$
PRD87 054506 (2013)	*(N…Ξ)**
PRD90 074504 (2014)	$\Omega_{ccc}{}^*$
PRD91 094502 (2015)	$\Xi_{cc}{}^*$

HADRON SCATTERING

PRD83 071504 (2011)	*ππ I=2*
PRD86 034031 (2012)	*ππ I=2*
PRD87 034505 (2013)	*ππ I=1 | ρ*
***PRL**113 182001 (2014)*	*πK,ηK | K**
PRD91 054008 (2015)	*πK,ηK | K**
PRD92 094502 (2015)	$\pi\pi, K\bar{K}$ *| ρ*
PRD93 094506 (2016)	$\pi\eta, K\bar{K}$ | a_0
JHEP10 011 (2016)	$D\pi, D\eta, D_s\bar{K}$
***PRL**118 022002 (2017)*	*ππ I=0 | σ*

MATRIX ELEMENTS

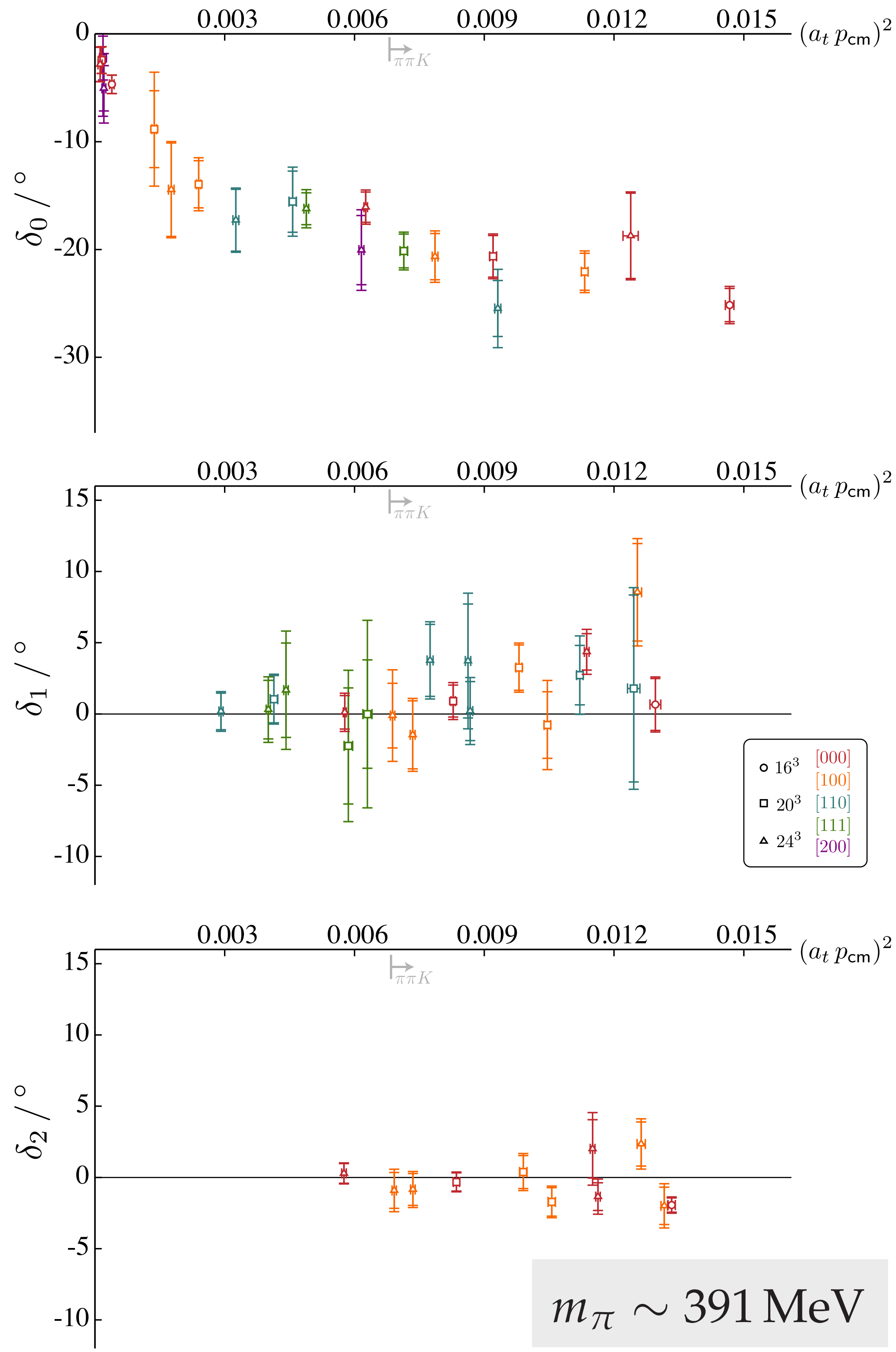
PRD90 014511 (2014)	$f_{\pi^\star}$
PRD91 114501 (2015)	$M' \to \gamma M$
***PRL**115 242001 (2015)*	$\gamma^\star\pi \to \pi\pi$
PRD93 114508 (2016)	$\gamma^\star\pi \to \pi\pi$

LATTICE TECH.

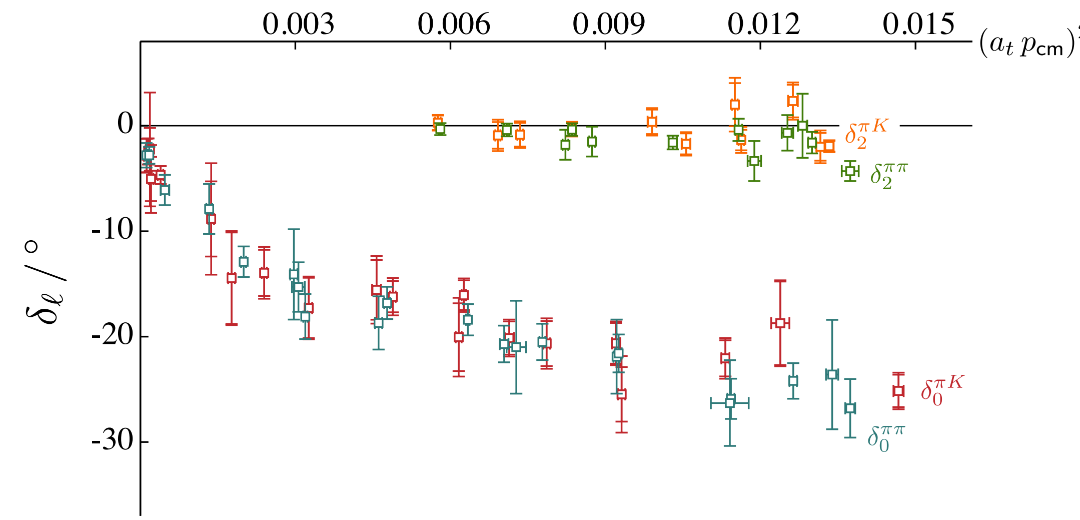
PRD79 034502 (2009)	lattices
PRD80 054506 (2009)	distillation
PRD85 014507 (2012)	$\vec{p} > 0$
JHEP (IN PRESS)	tetraquarks

πK $I=3/2$

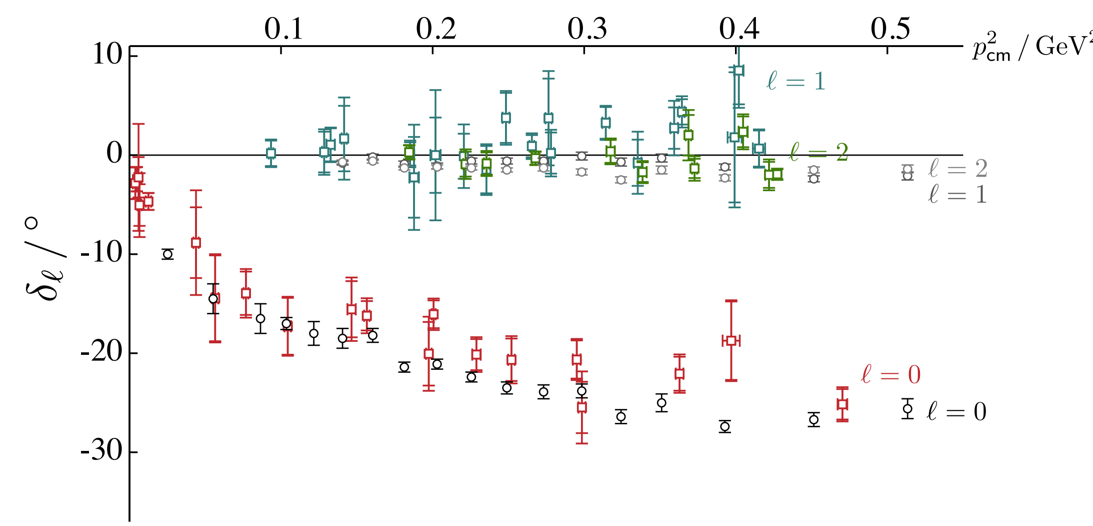
PHASE-SHIFTS



VERSUS $\pi\pi$ $I=2$



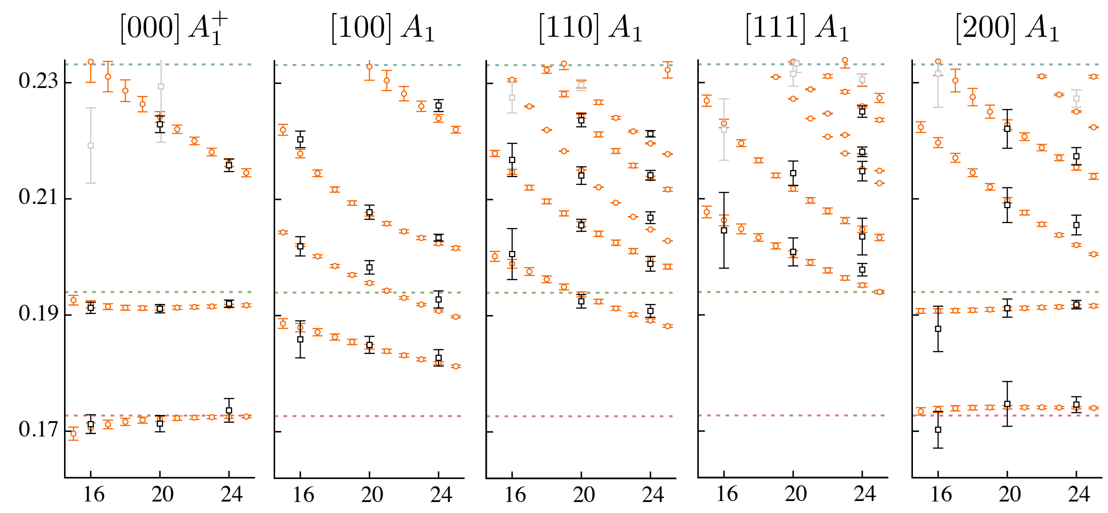
VERSUS EXPT



an example amplitude

$$\mathbf{K}(s) = \frac{1}{m^2 - s}\begin{bmatrix} g_{\pi\eta}^2 & g_{\pi\eta}\, g_{K\bar{K}} \\ g_{\pi\eta}\, g_{K\bar{K}} & g_{K\bar{K}}^2 \end{bmatrix} + \begin{bmatrix} \gamma_{\pi\eta,\pi\eta} & \gamma_{\pi\eta,K\bar{K}} \\ \gamma_{\pi\eta,K\bar{K}} & \gamma_{K\bar{K},K\bar{K}} \end{bmatrix}$$

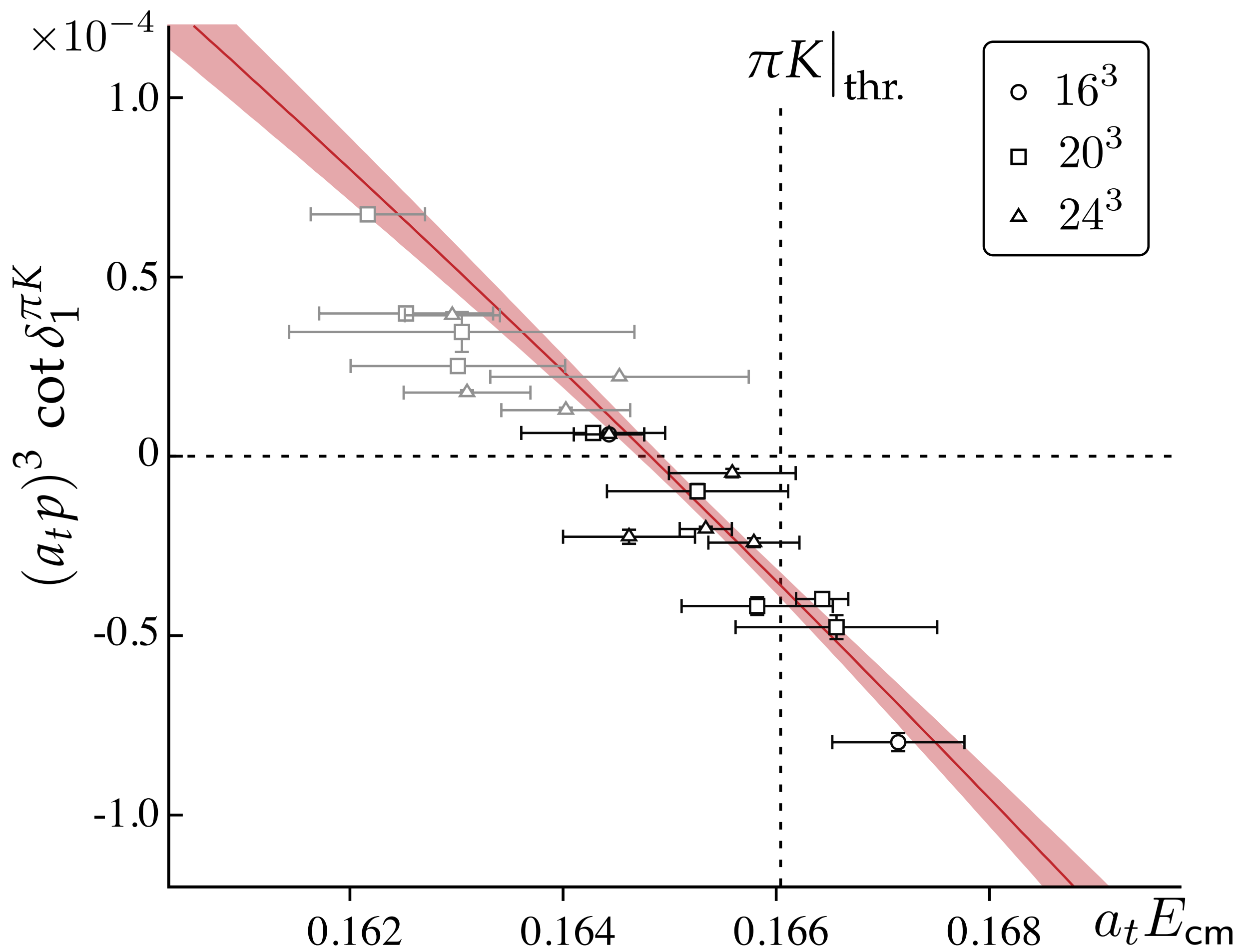
vary {*m*, *g*'s, *γ*'s} ...



$$\chi^2/N_{\mathrm{dof}} = \frac{58.0}{47-6} = 1.41$$

$\pi K/\eta K$ scattering amplitudes

P-WAVE πK SCATTERING



$$a_t m(K^\star) = 0.16482(15)$$

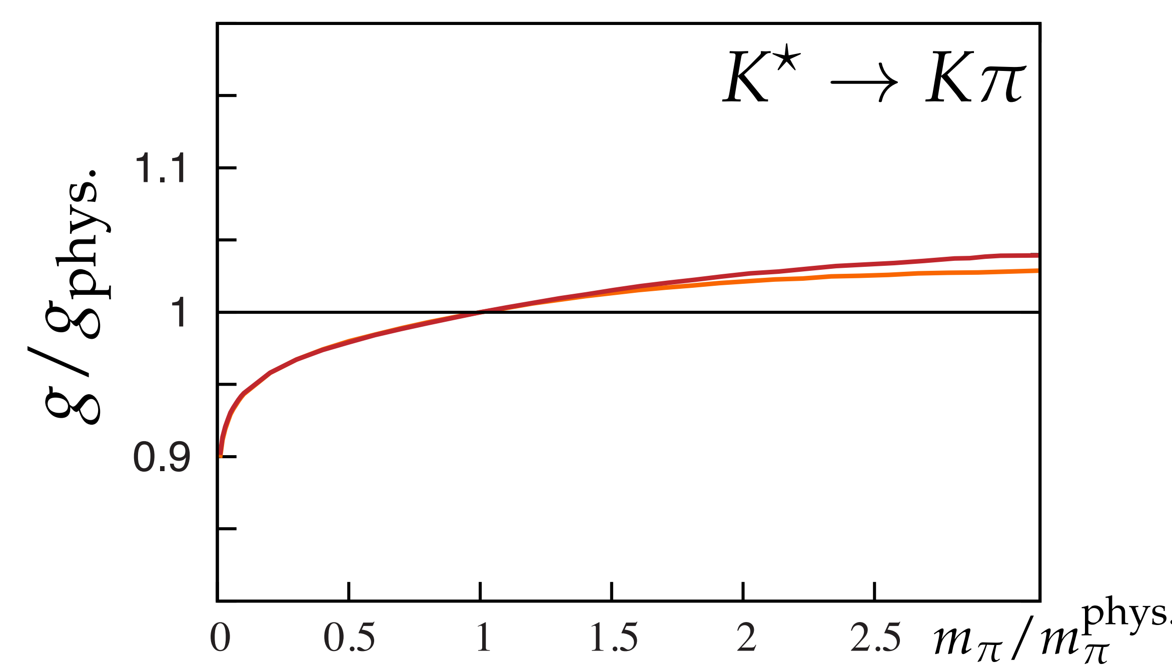
$$g = 5.93(30)$$

vector bound-state

$$a_t(m_\pi + m_K) = 0.16604$$

$$g_{\text{phys.}} = 5.5(2)$$ PDG

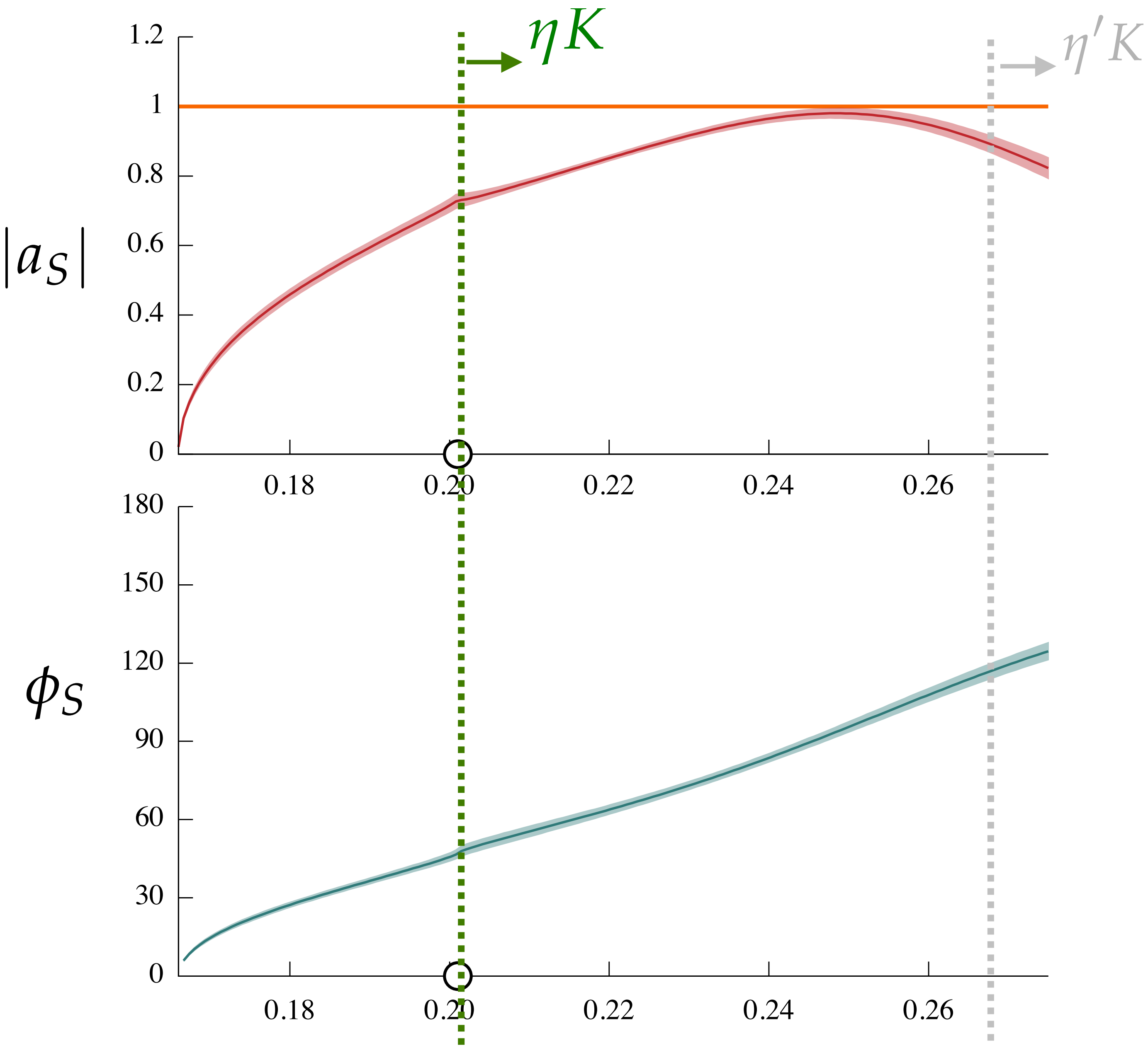
UNITARIZED χPT



NEBREDA & PELAEZ
PRD81 054035 (2010)

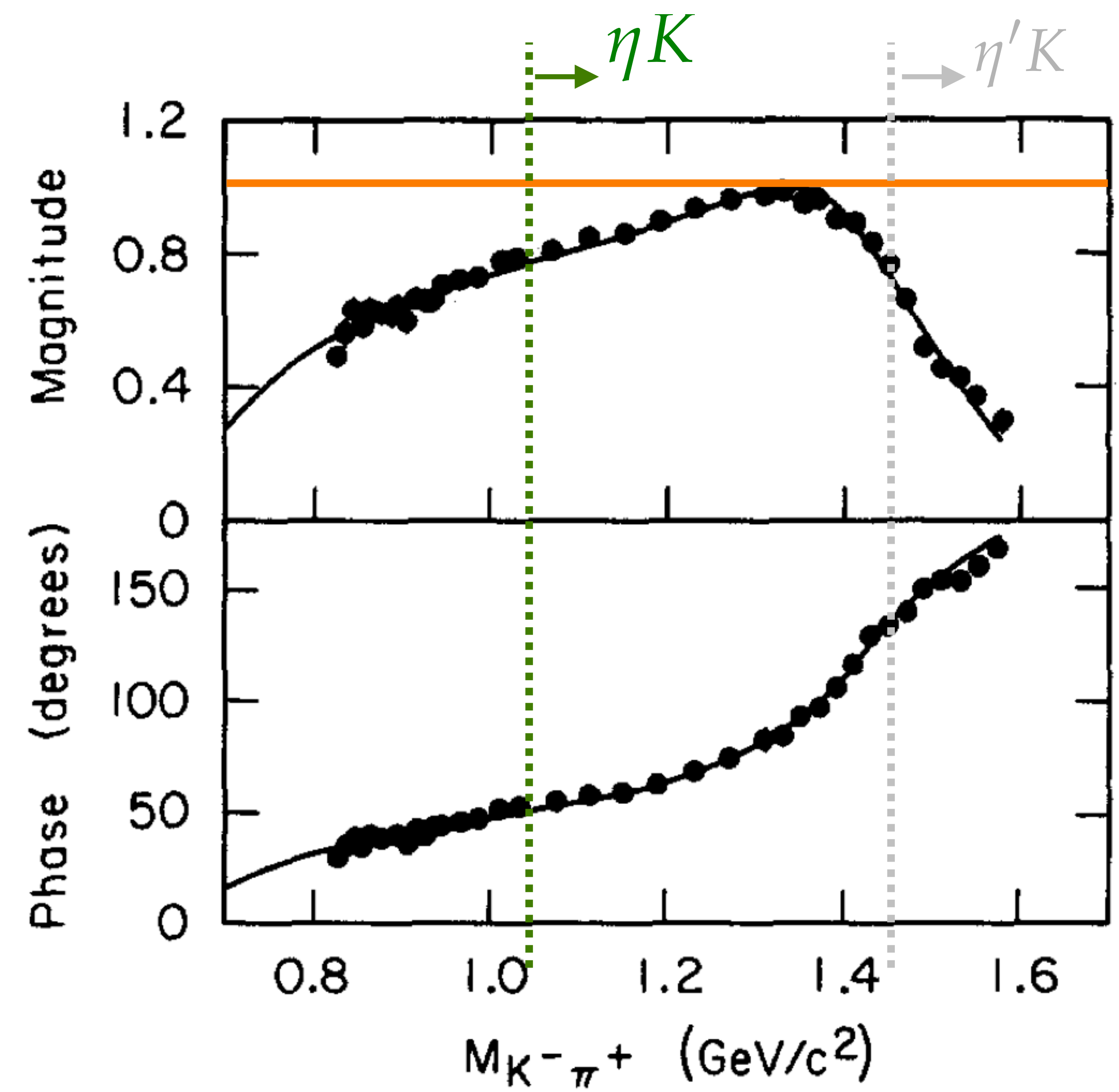
πK I=1/2

S-WAVE $\pi K \to \pi K$ AMPLITUDE



$m_\pi \sim 391$ MeV

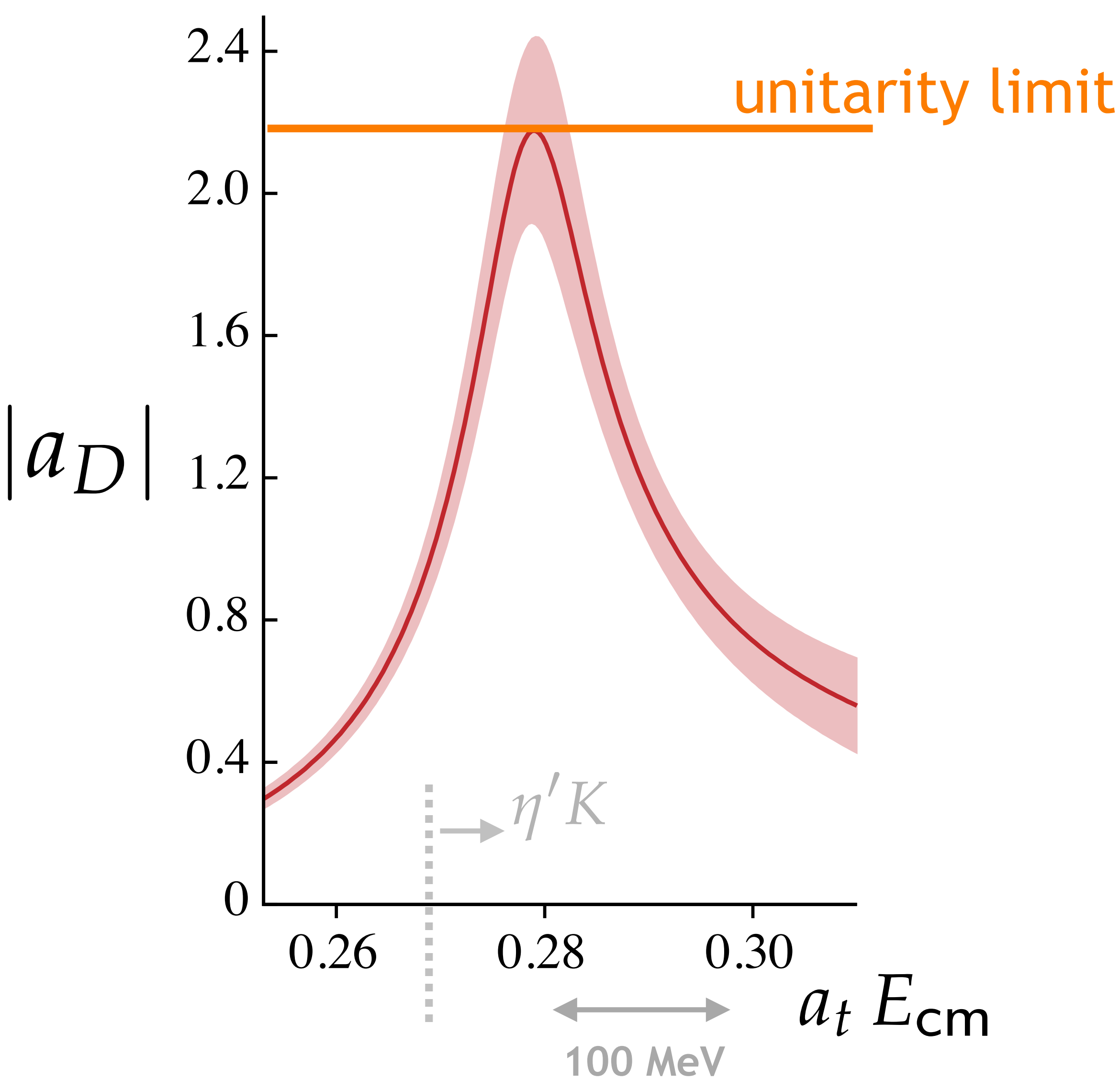
LASS S-WAVE



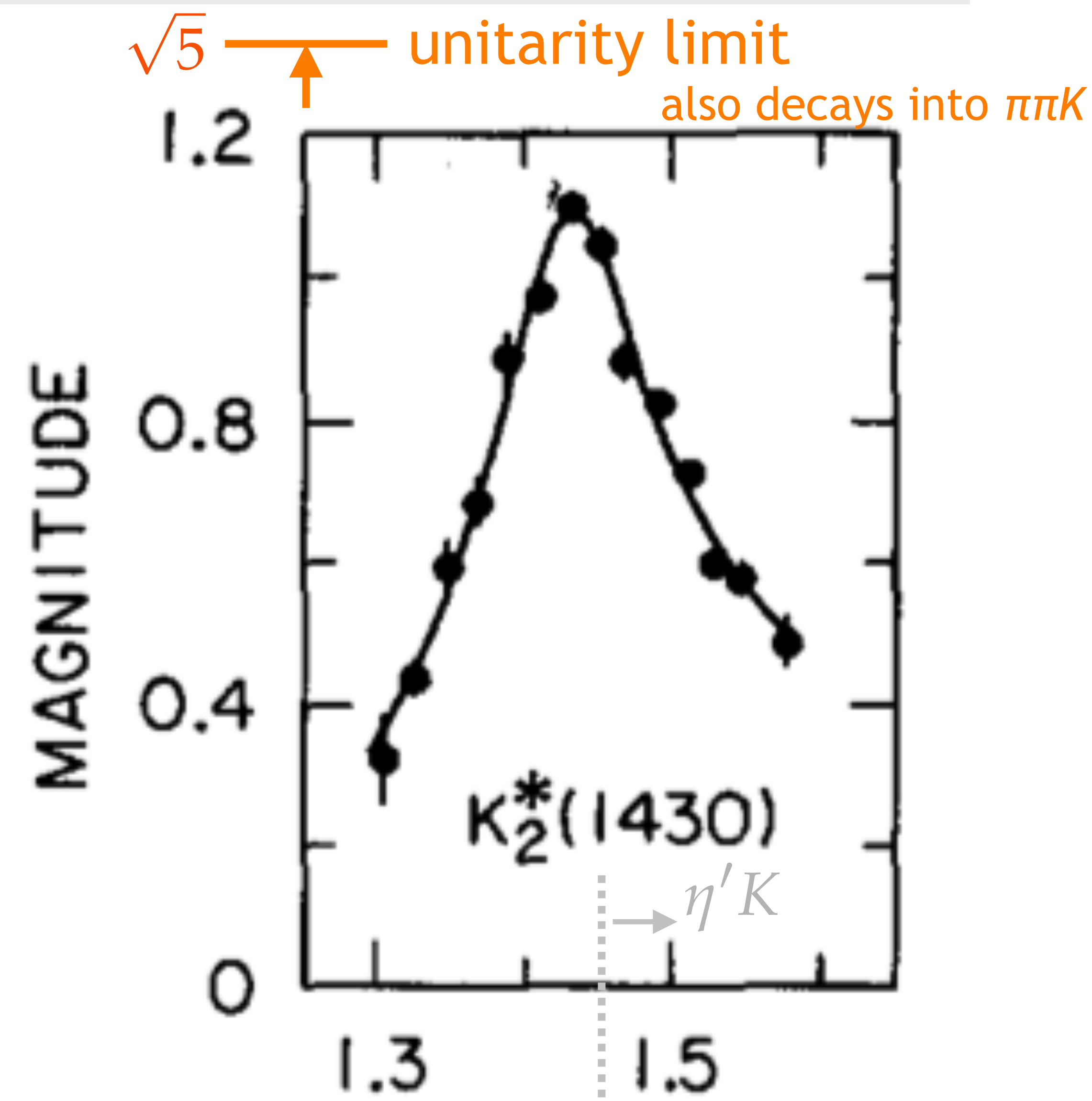
LASS, NPB296 493

πK $I=1/2$

D-WAVE $\pi K \to \pi K$ AMPLITUDE



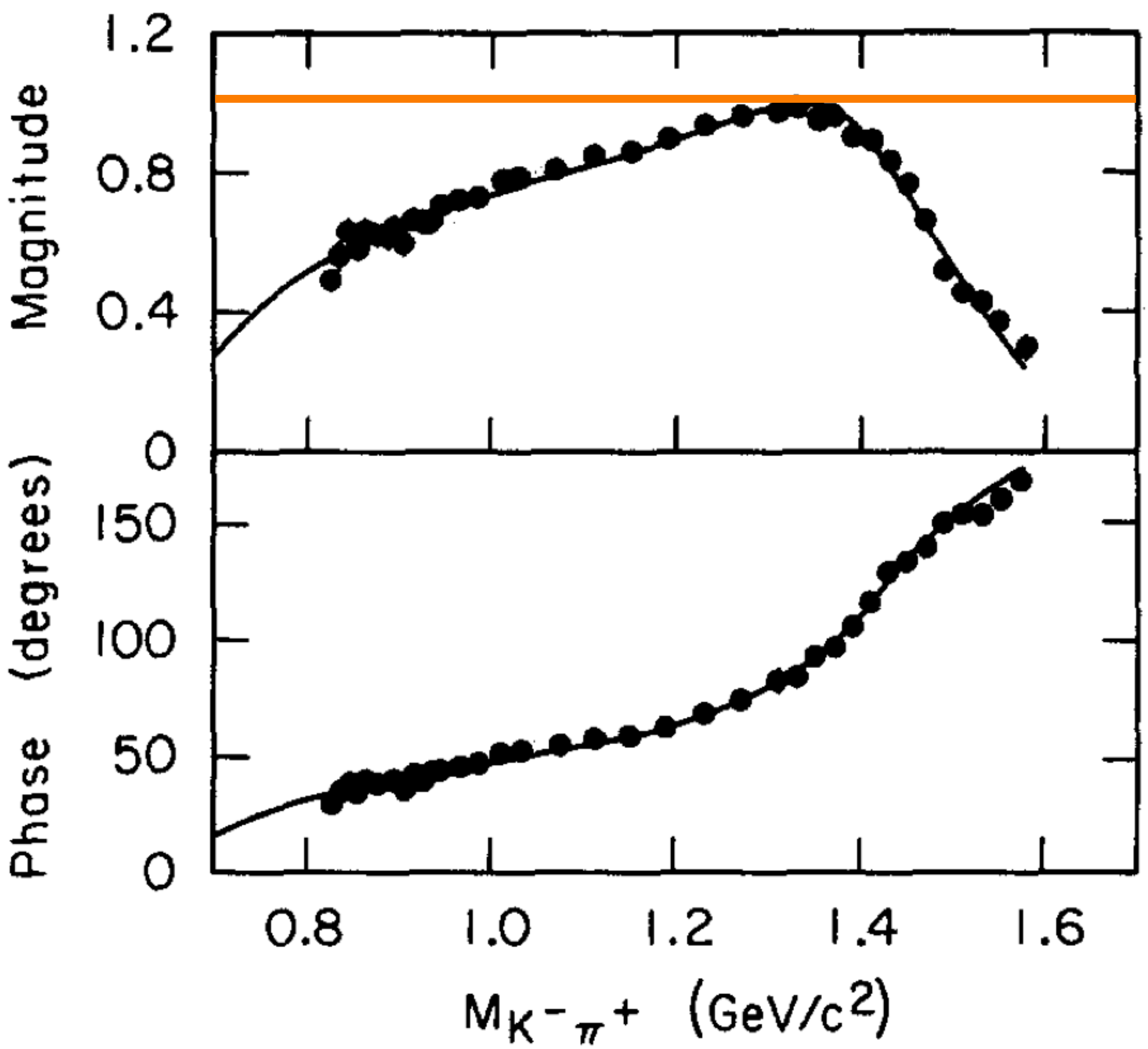
LASS D-WAVE



LASS, NPB296 493

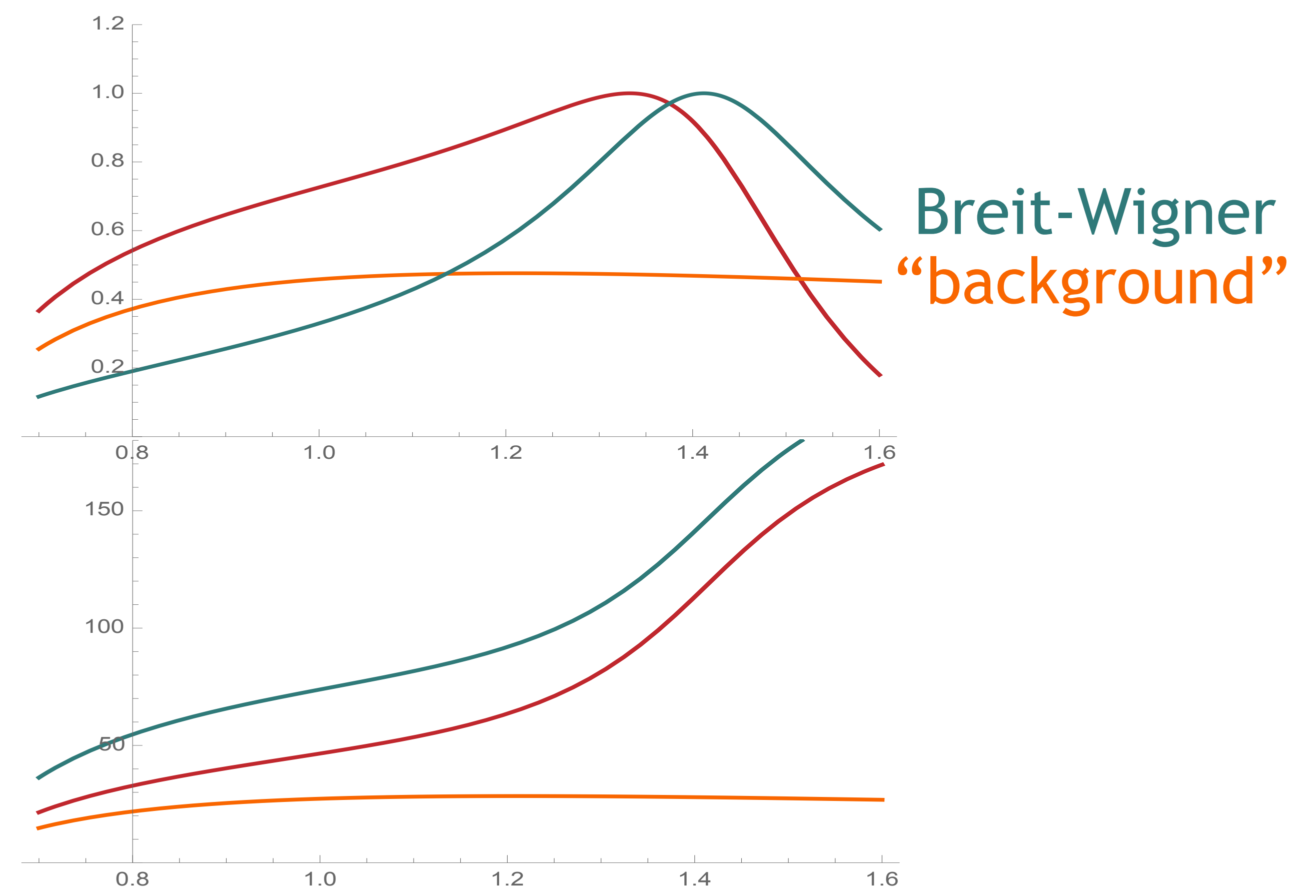
kaon beams

S-WAVE $\pi K \to \pi K$ AMPLITUDE



LASS, NPB296 493 (1988)

LASS PARAMETERIZATION



$SU(3)_F$ & $\pi K/\eta K$

- SU(3) flavor symmetry consequences
- assuming a pure octet η

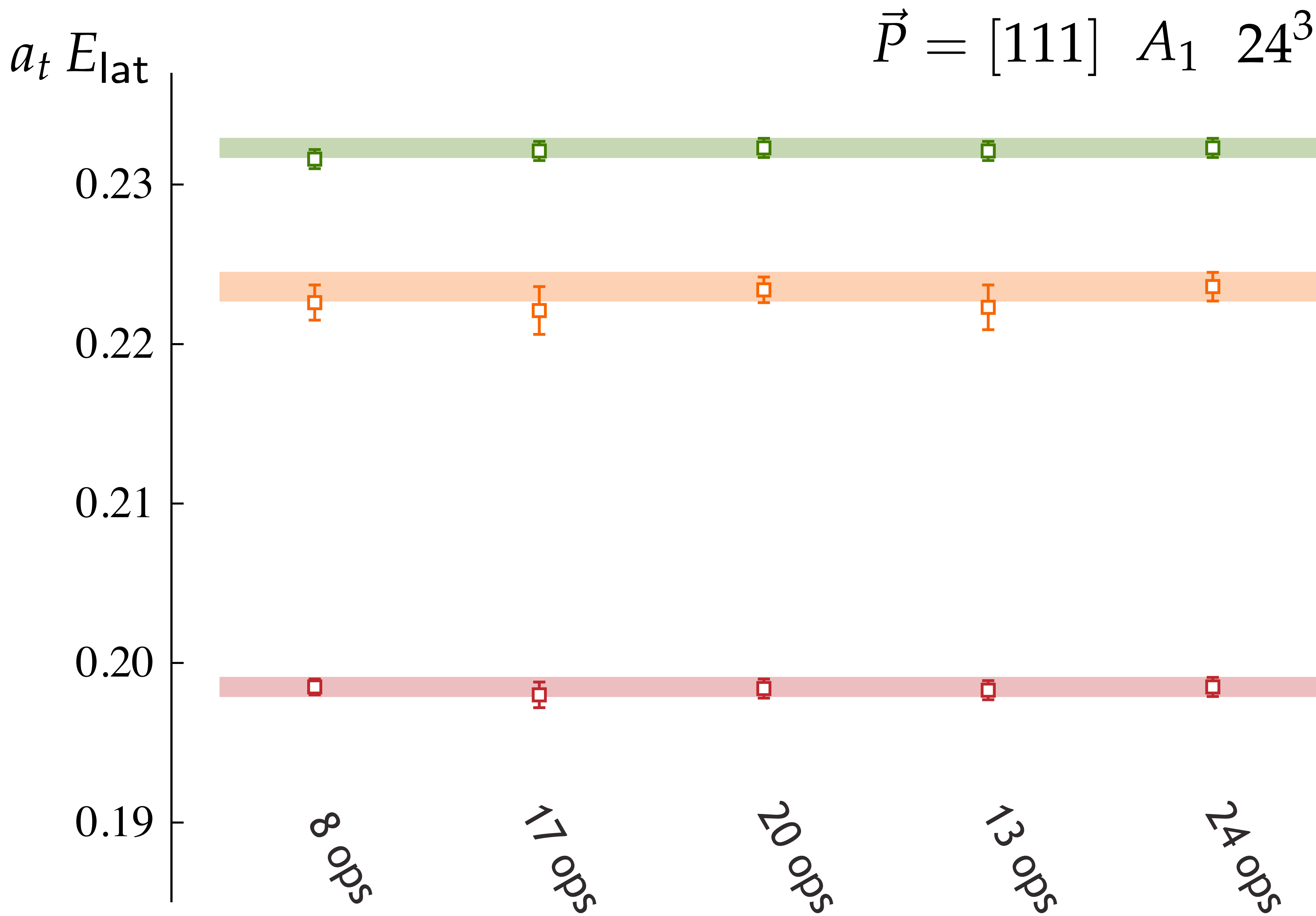
$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus \mathbf{8_1} \oplus \mathbf{8_2} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$$

$$\ell = \text{even} \quad \text{odd}$$

	$\pi K : \eta K$
$\|\mathbf{8}_1, \ell = \text{even}\rangle = -\frac{\sqrt{5}}{10}\left[3\left(\sqrt{\frac{2}{3}}\|K^0\pi^+\rangle + \sqrt{\frac{1}{3}}\|K^+\pi^0\rangle\right) + \|K^+\eta\rangle\right]$	3 : 1
$\|\mathbf{8}_2, \ell = \text{odd}\rangle = \frac{1}{2}\left[\left(-\sqrt{\frac{2}{3}}\|K^0\pi^+\rangle + \sqrt{\frac{1}{3}}\|K^+\pi^0\rangle\right) - \|K^+\eta\rangle\right]$	1 : 1

finite-volume spectrum

- varying the $\overline{\psi}\Gamma\psi$ content of the operator basis



3× ππ ops

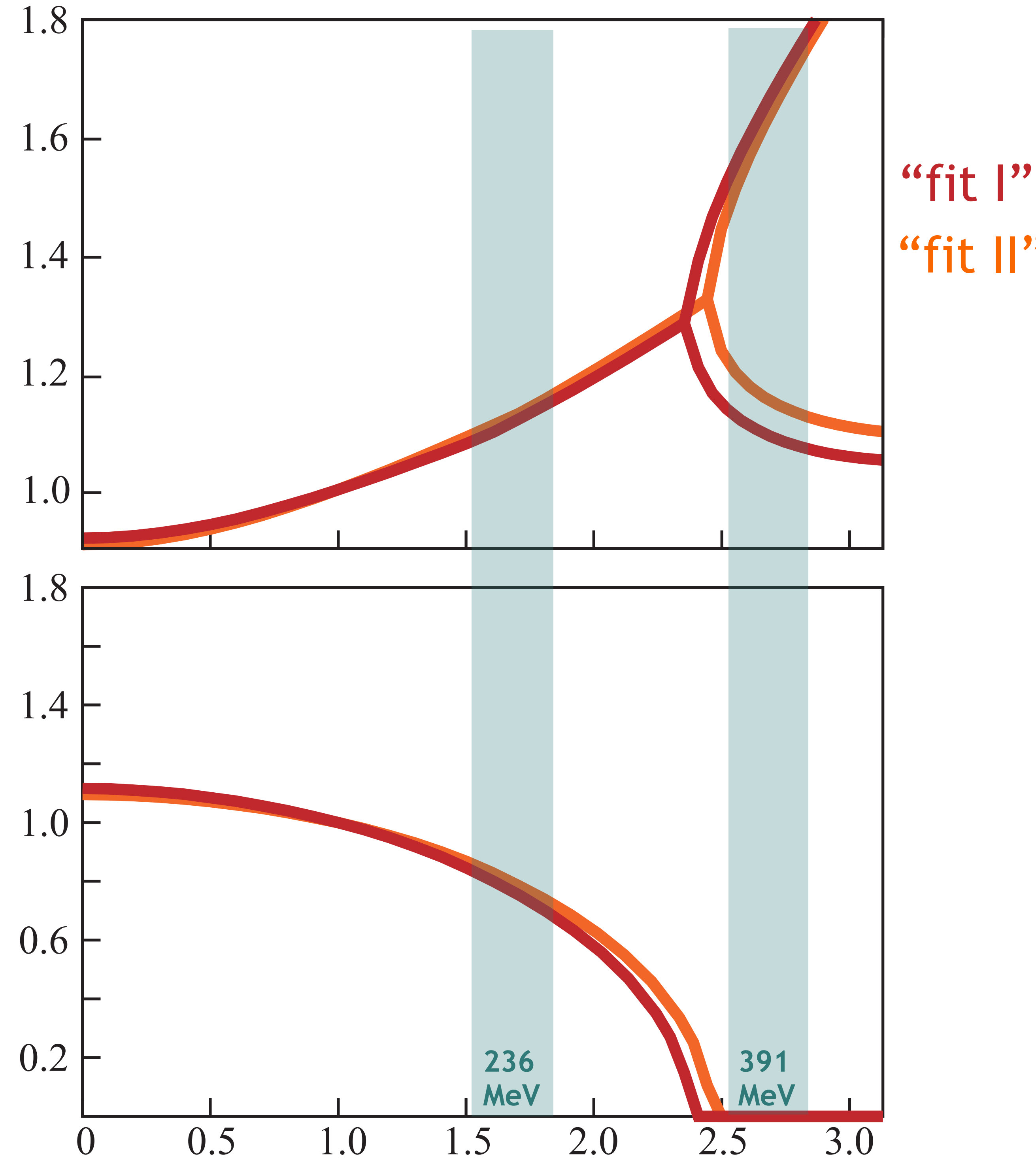
$m_\pi \sim 391\,\text{MeV}$

κ (kappa) pole with changing quark mass

- unitarized SU(3)F chiral perturbation theory
  - resonance poles become virtual bound states somewhere near $m_\pi \sim 2.5\ m_\pi^{phys}$

Nebreda & Pelaez PRD81 054035 (2010)

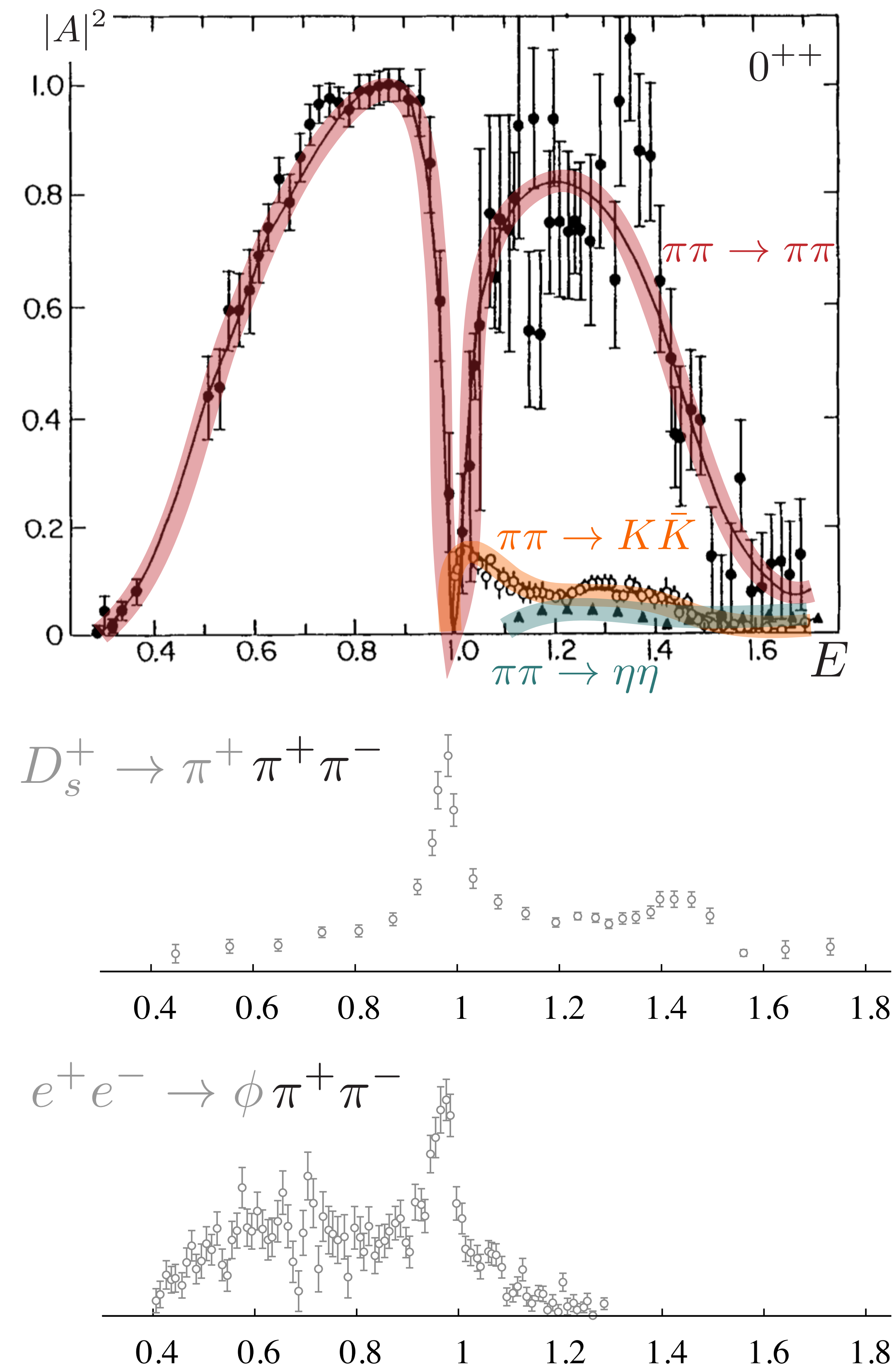
$$\sqrt{s_0} = m + \tfrac{i}{2}\Gamma$$



Descotes-Genon

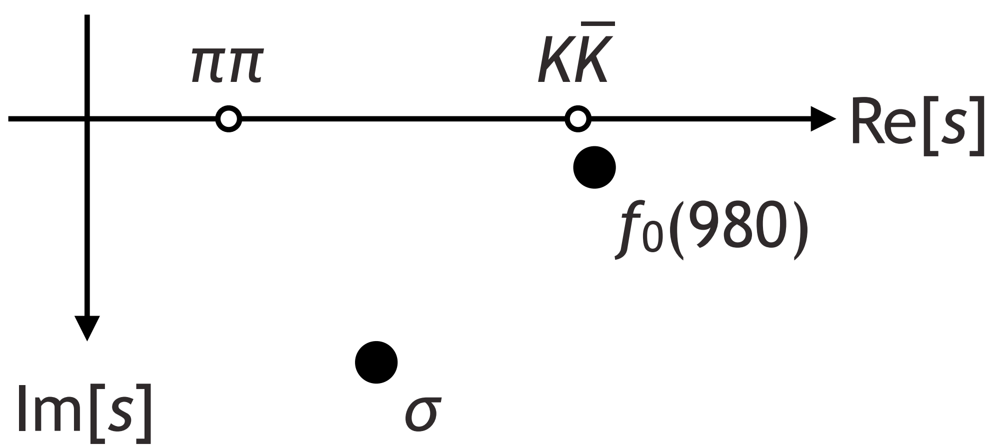
$$\sqrt{s_0} = 660(20) + \tfrac{i}{2}\,550(25)\,\text{MeV}$$

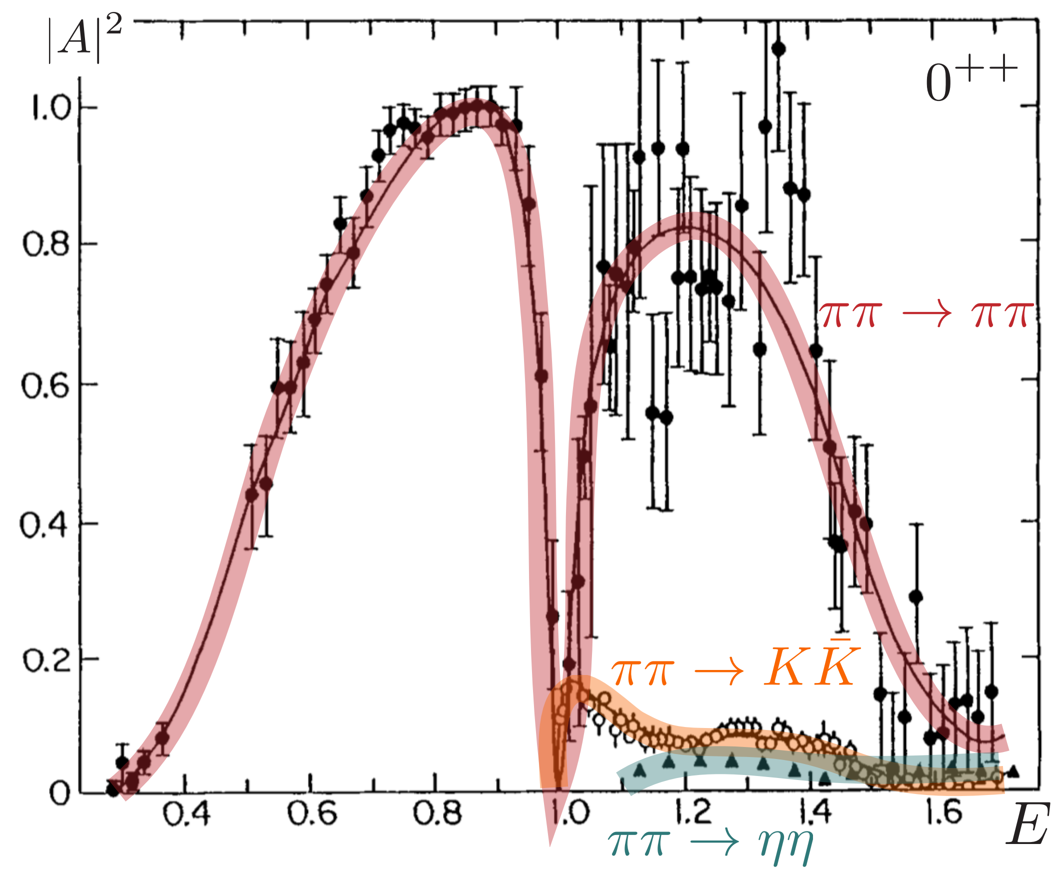
isoscalar meson resonances — scalars



in some processes
the **dip** is a **peak**

f_0 resonances ?





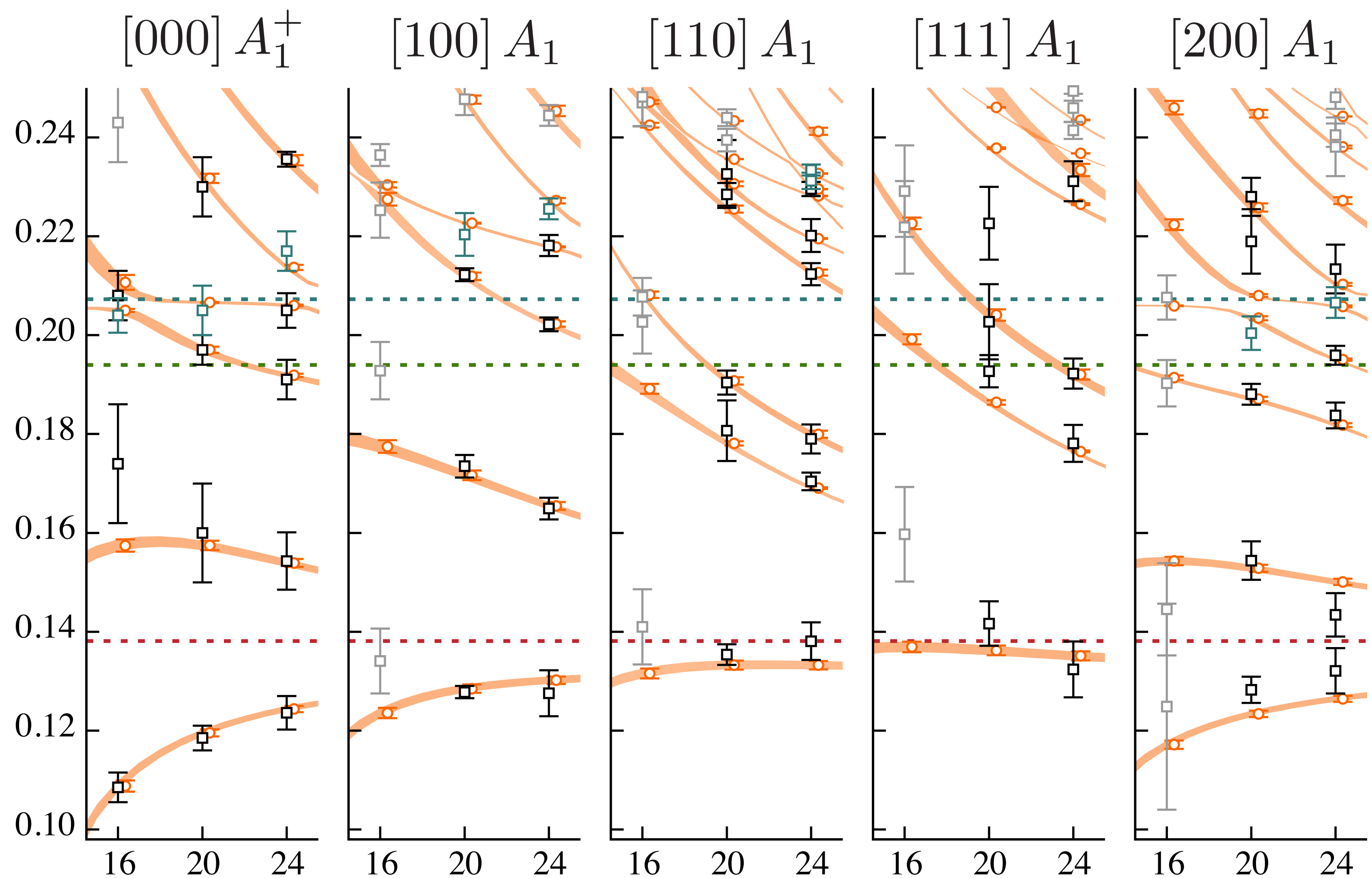
$f_0(980)$ large coupling to $K\bar{K}$

a *K*-matrix amplitude description

$\pi\pi$ $K\overline{K}$ $\eta\eta$

$$\mathbf{K}^{-1} = \begin{pmatrix} a + b\,s & c + d\,s & e \\ c + d\,s & f & g \\ e & g & h \end{pmatrix}$$

+ Chew-Mandelstam phase-space



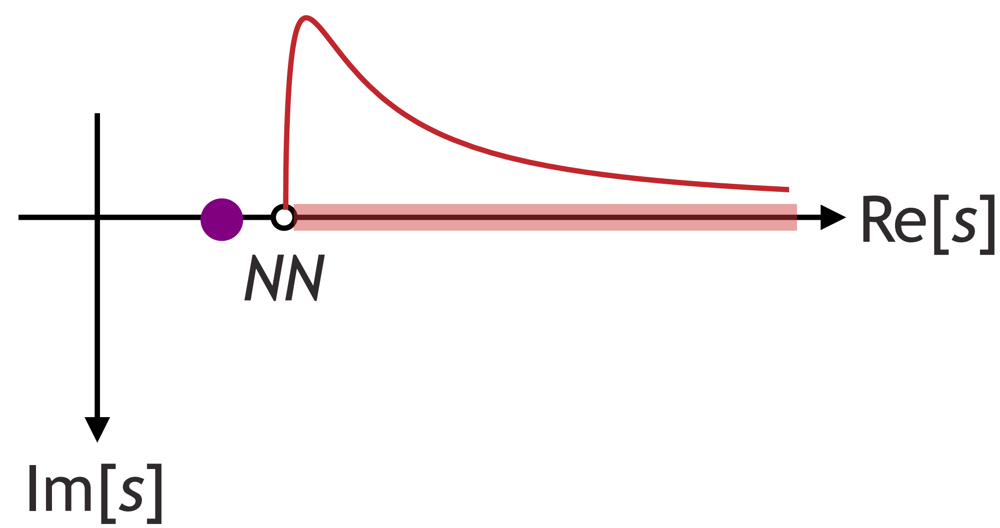
$\chi^2/N_{\mathrm{dof}} = 44.0/(57-8) = 0.90$

resonance content ?

a rigorous definition – pole singularity in a partial-wave amplitude $t^{(\ell)}_{ij}(s) \sim \frac{c_i\, c_j}{s_0 - s}$

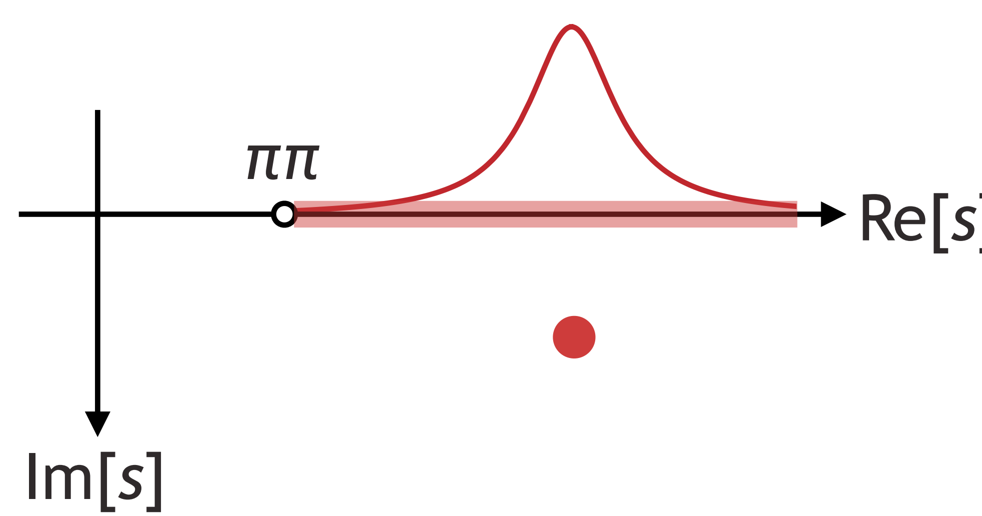
– bound state: $s_0 = M^2$

e.g. deuteron



– resonance: $\sqrt{s_0} = M - i\frac{1}{2}\Gamma$

e.g. ρ meson



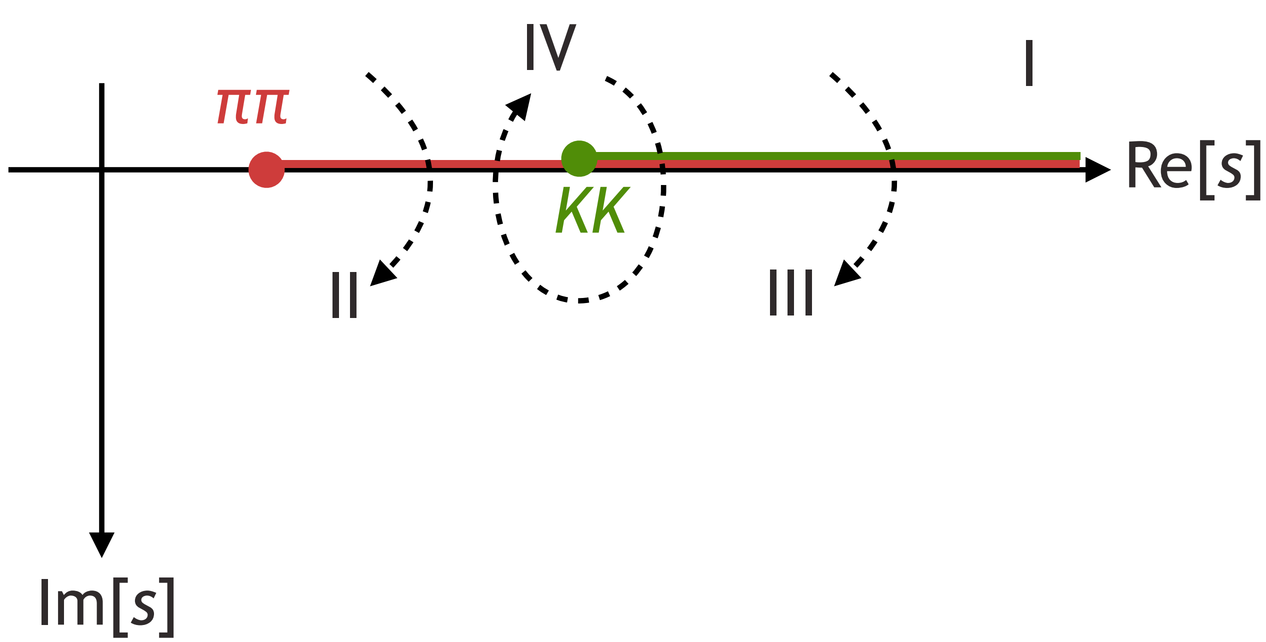
sheets ?

complex *s*-plane actually multi-sheeted

unitarity $\mathrm{Im}\big[t_{ij}(s)\big] = -\delta_{ij}\,\rho_i(s)$

$$\rho_i(s) = \sqrt{1 - \frac{4m_i^2}{s}}$$

square-root branch-point at each threshold



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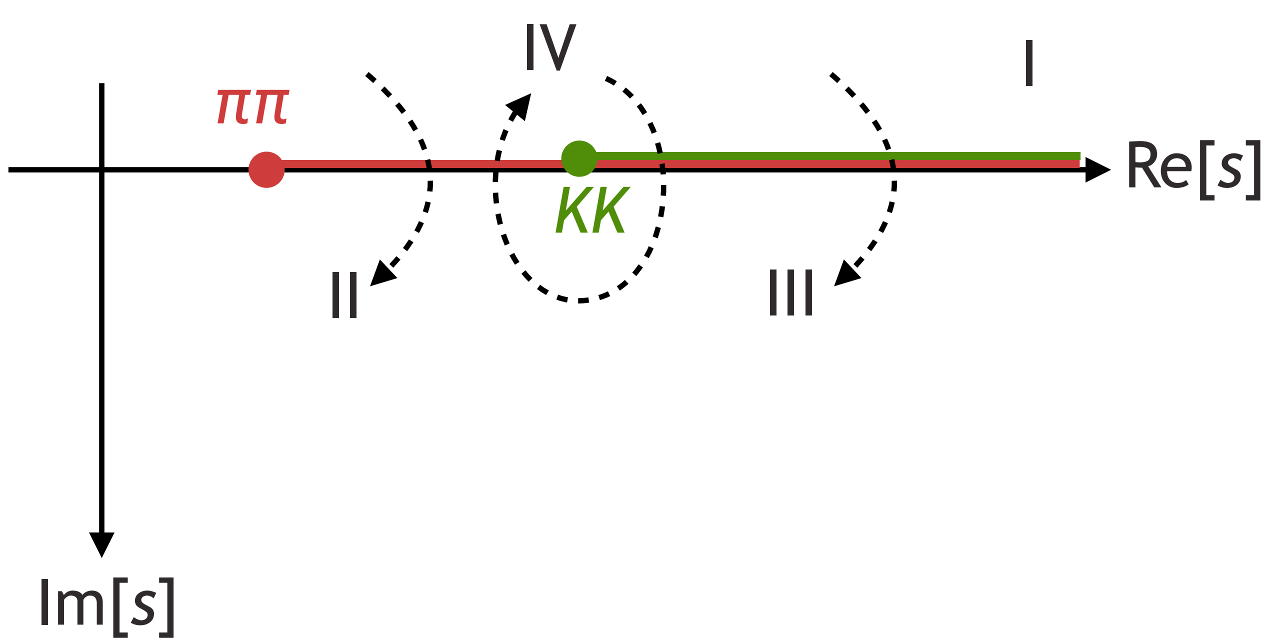
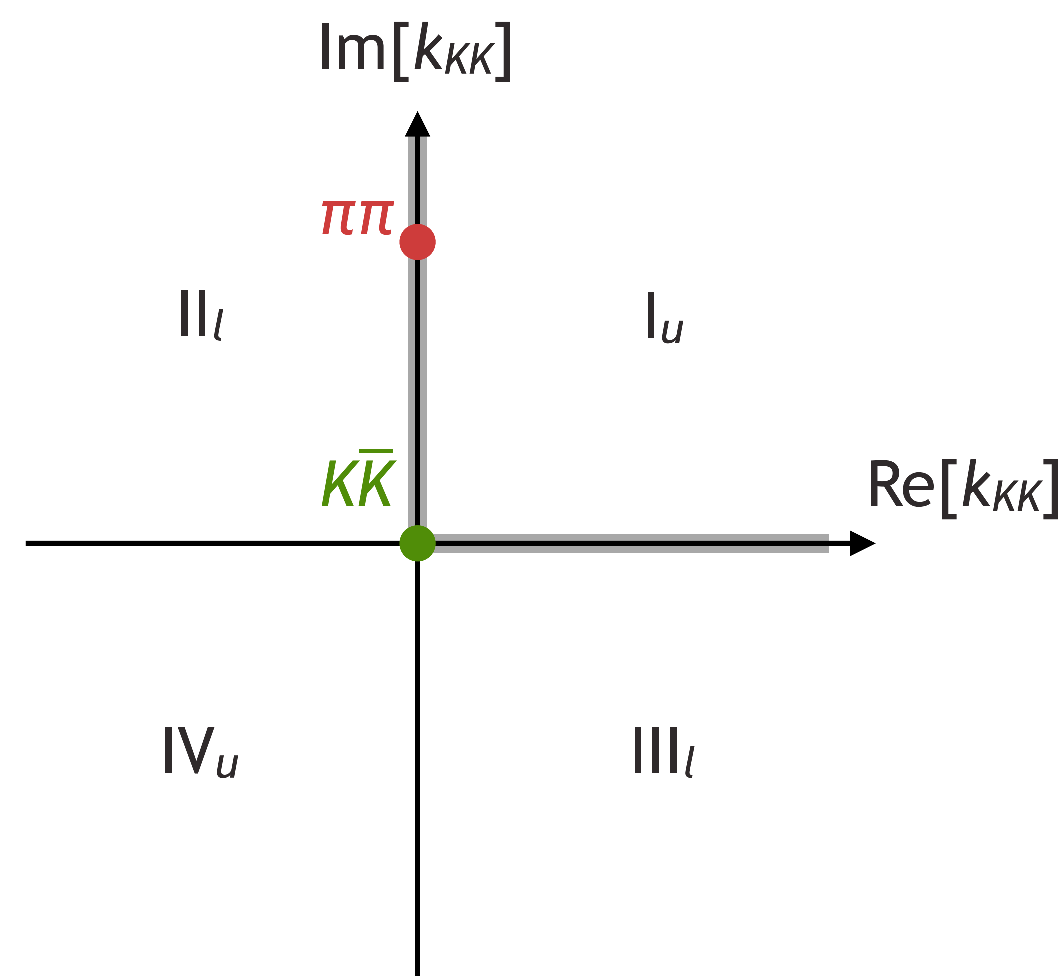




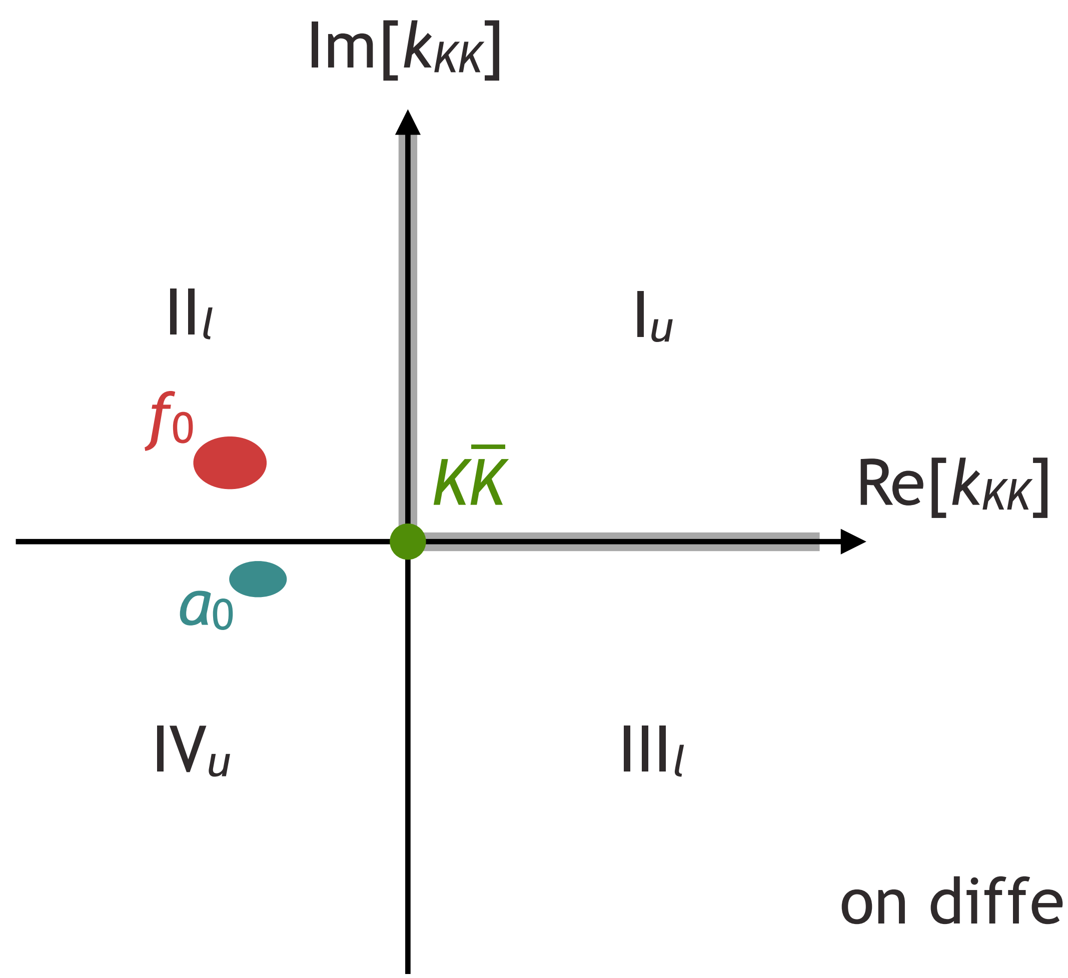
sheets ?

$m_\pi \sim 391\,\mathrm{MeV}$

$$m_R(f_0) = 1166(45)\,\mathrm{MeV}, \quad \Gamma_R(f_0) = 181(68)\,\mathrm{MeV},$$
$$m_R(a_0) = 1177(27)\,\mathrm{MeV}, \quad \Gamma_R(a_0) = 49(33)\,\mathrm{MeV}.$$

$$\left|c(a_0 \to K\overline{K})\right| \approx \left|c(f_0 \to K\overline{K})\right| \quad \sim 850\,\mathrm{MeV}$$
$$\left|c(a_0 \to \pi\eta)\right| \approx \left|c(f_0 \to \pi\pi)\right| \quad \sim 700\,\mathrm{MeV}.$$

look very similar (in mass and couplings), but …



on different sheets ?

‘explaining’ the sheet distribution

e.g. Flatté form $D(s) = m_0^2 - s - i g_1^2\, \rho_1(s) - i g_2^2\, \rho_2(s)$

has poles

$$\sqrt{s_0} \approx m_0 \pm \frac{i}{2} \frac{g_2^2\, \rho_2}{m_0} \left[\left(\frac{g_1}{g_2} \right)^2 \frac{\rho_1}{\rho_2} - 1 \right] \quad \text{on sheet II, if} \quad \left(\frac{g_1}{g_2} \right)^2 \frac{\rho_1}{\rho_2} > 1, \text{ or,}$$

$$\sqrt{s_0} \approx m_0 \pm \frac{i}{2} \frac{g_2^2\, \rho_2}{m_0} \left[1 - \left(\frac{g_1}{g_2} \right)^2 \frac{\rho_1}{\rho_2} \right] \quad \text{on sheet IV, if} \quad \left(\frac{g_1}{g_2} \right)^2 \frac{\rho_1}{\rho_2} < 1, \text{ and,}$$

$$\sqrt{s_0} \approx m_0 \pm \frac{i}{2} \frac{g_2^2\, \rho_2}{m_0} \left[1 + \left(\frac{g_1}{g_2} \right)^2 \frac{\rho_1}{\rho_2} \right] \quad \text{on sheet III, in all cases,}$$

$$m_R(f_0) = 1166(45)\,\text{MeV}, \quad \Gamma_R(f_0) = 181(68)\,\text{MeV},$$

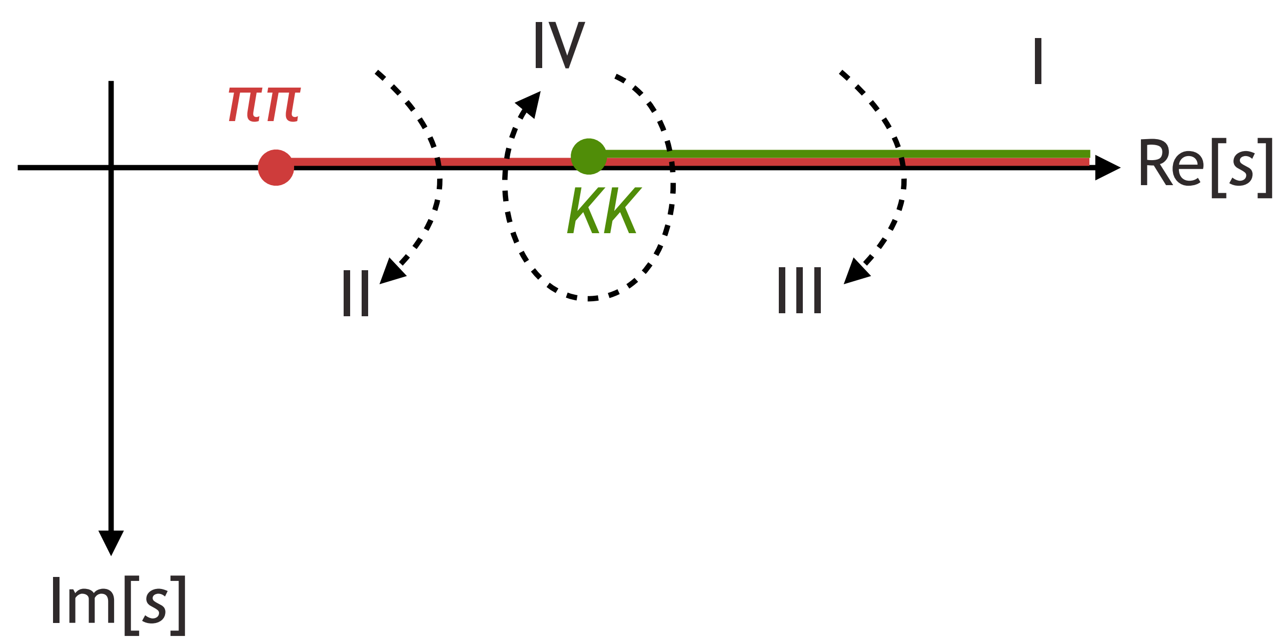
$$m_R(a_0) = 1177(27)\,\text{MeV}, \quad \Gamma_R(a_0) = 49(33)\,\text{MeV}.$$

$$\begin{aligned} \left|c(a_0 \to K\overline{K})\right| &\approx \left|c(f_0 \to K\overline{K})\right| \quad &\sim 850\,\text{MeV} \\ \left|c(a_0 \to \pi\eta)\right| &\approx \left|c(f_0 \to \pi\pi)\right| \quad &\sim 700\,\text{MeV}. \end{aligned}$$

but larger phase-space for $\pi\pi$ than $\pi\eta$

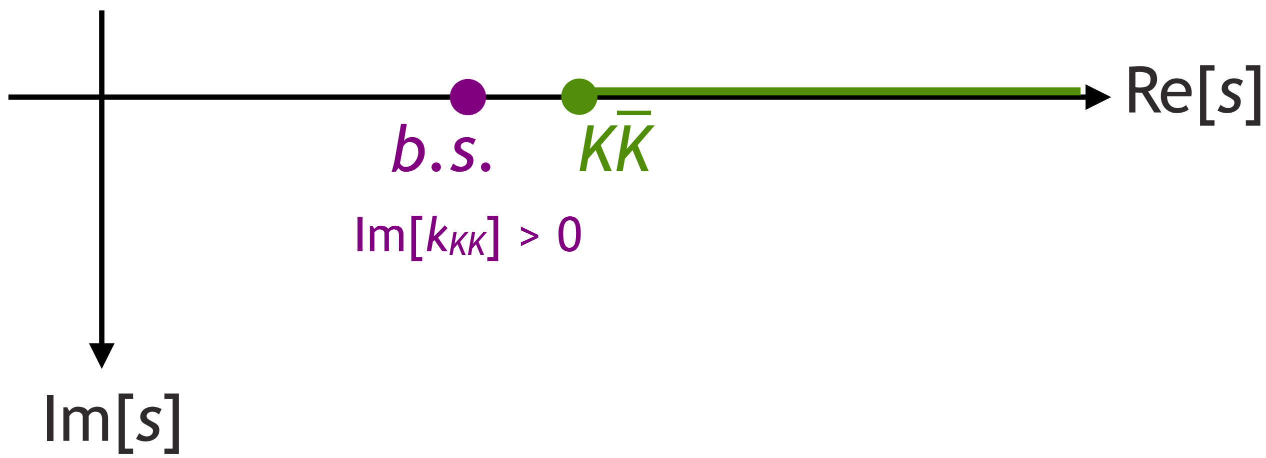
interpreting the sheet distribution ?

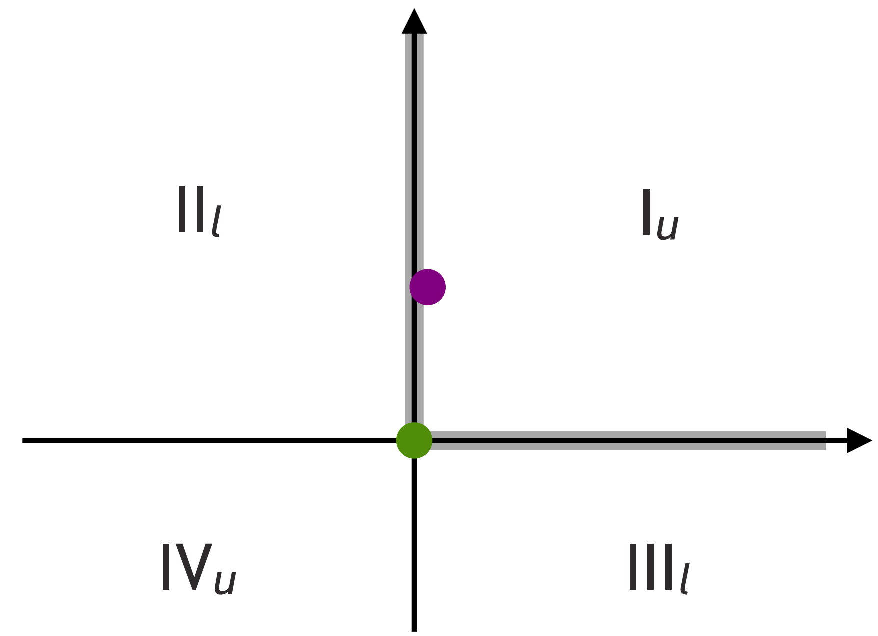
a pole on **only** sheet II or sheet IV ⇒ ‘molecular resonance’ ?



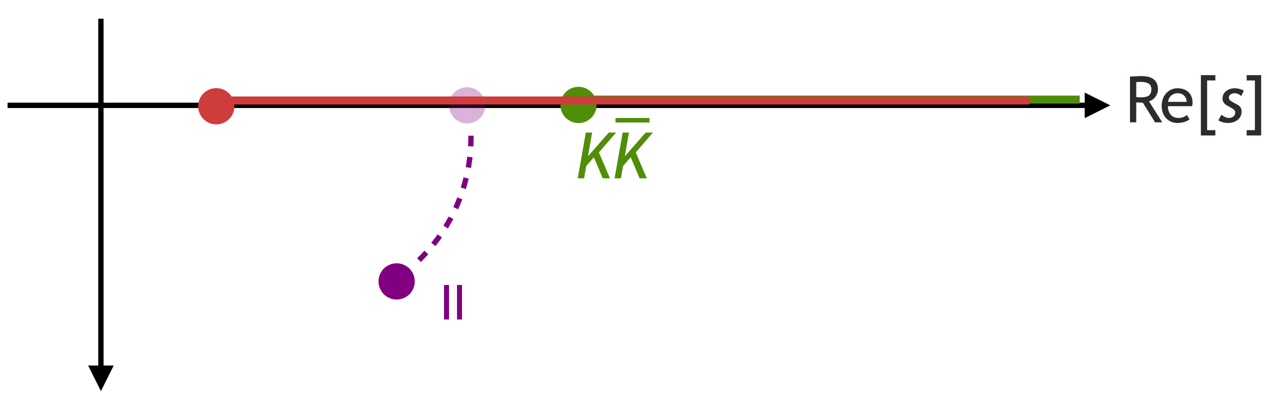
interpreting the sheet distribution ?

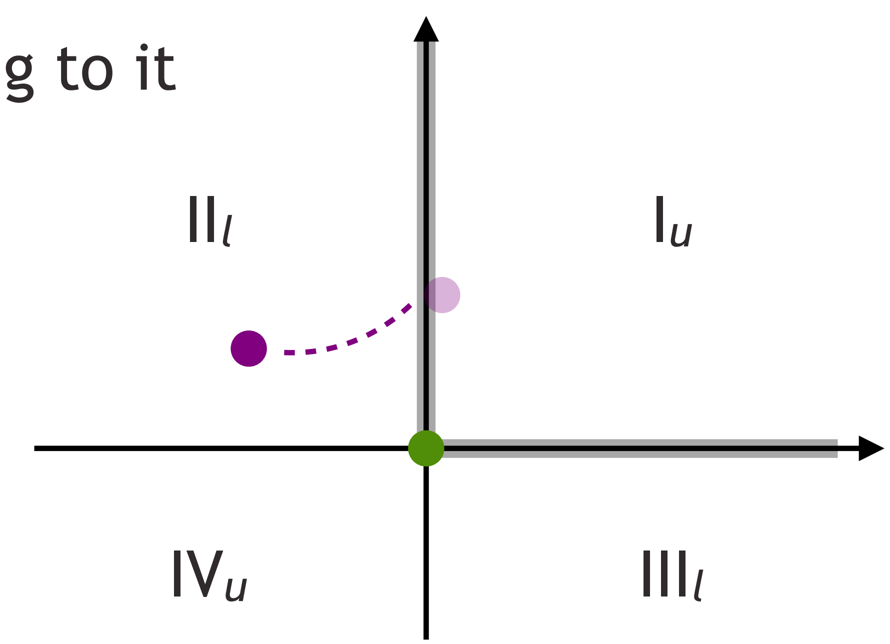
imagine no lower channel
and binding dynamics in $K\bar{K}$





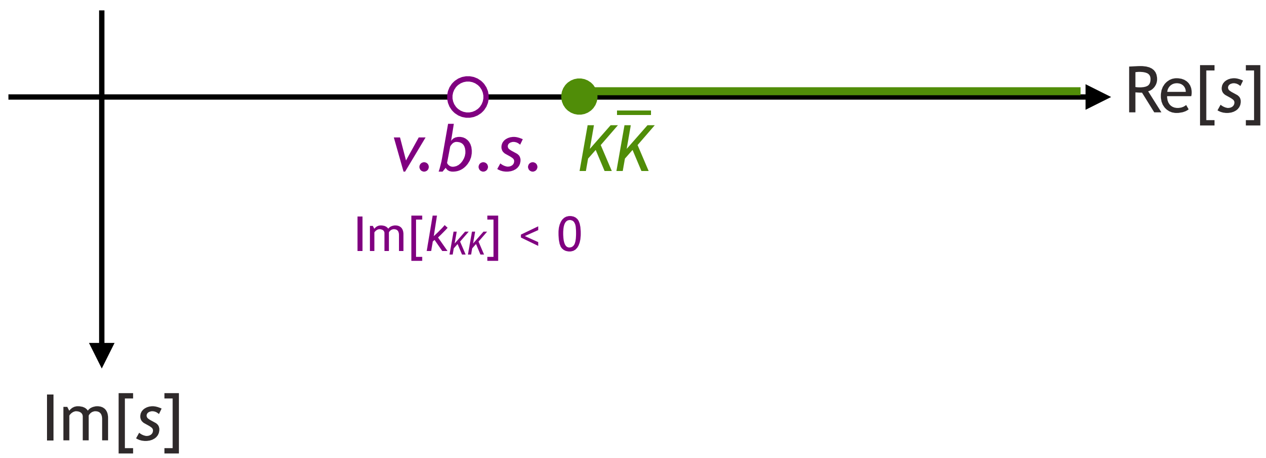
now ‘turn on’ the lower channel and allow a coupling to it

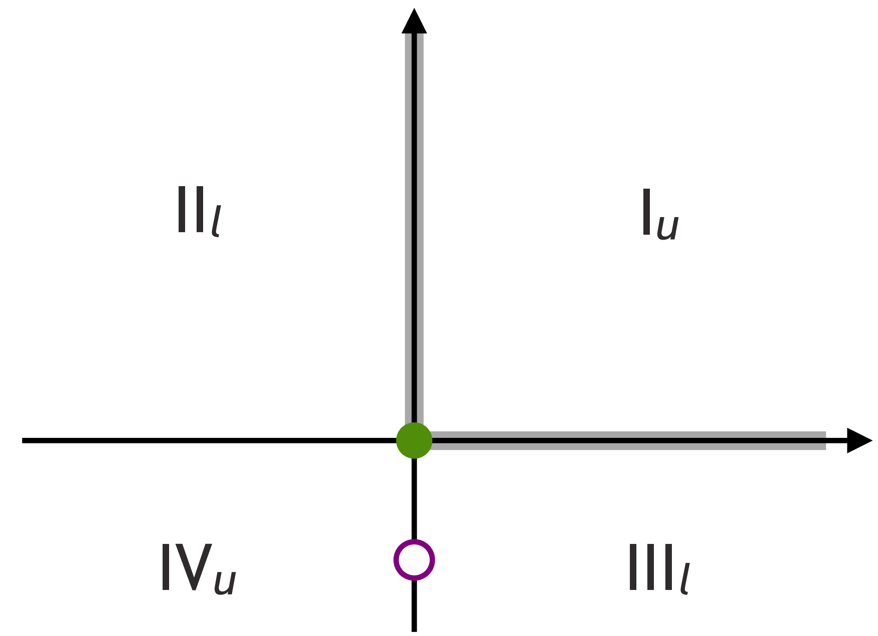




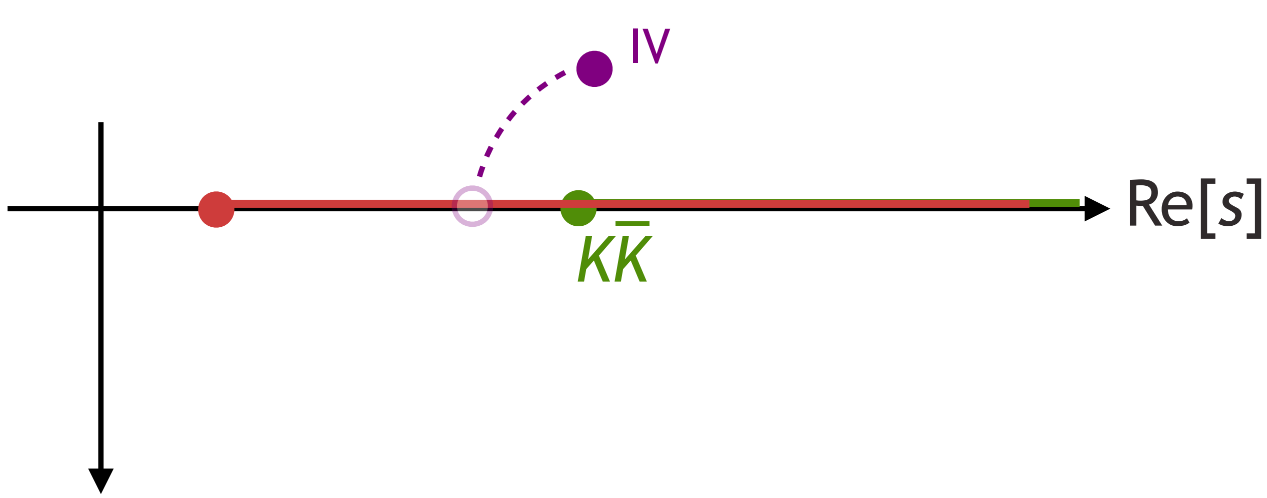
interpreting the sheet distribution ?

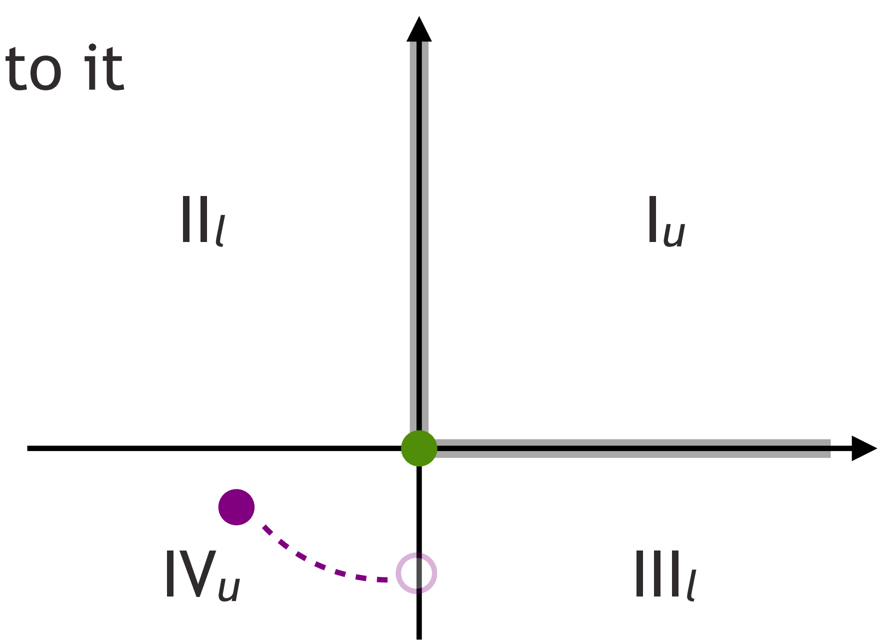
imagine no lower channel
and weaker attraction in $K\bar{K}$





now turn on the lower channel and allow a coupling to it





interpreting the sheet distribution ?

on the other hand …

an 'ordinary' resonance is expected to have 'mirror' poles:

e.g. Flatté form

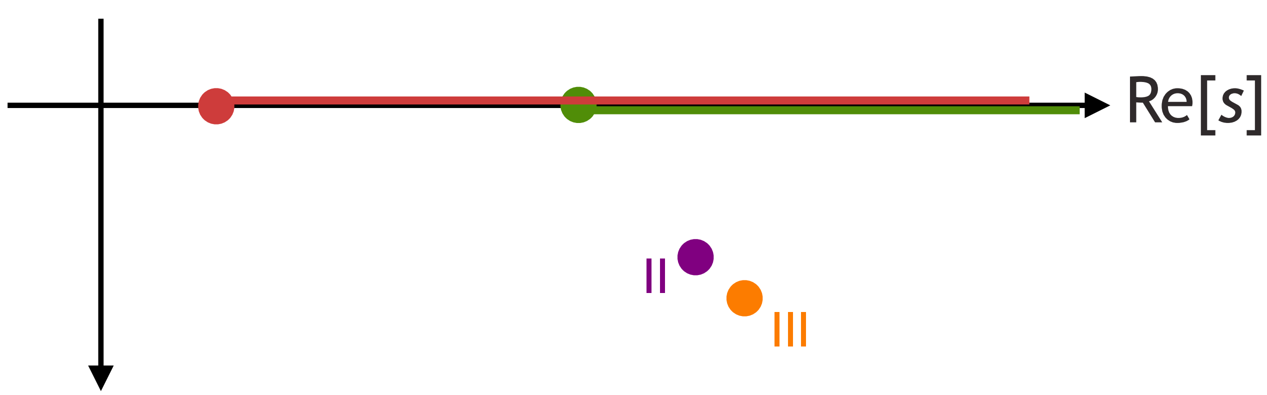
$$D(s) = m_0^2 - s - i g_1^2 \rho_1(s) - i g_2^2 \rho_2(s)$$

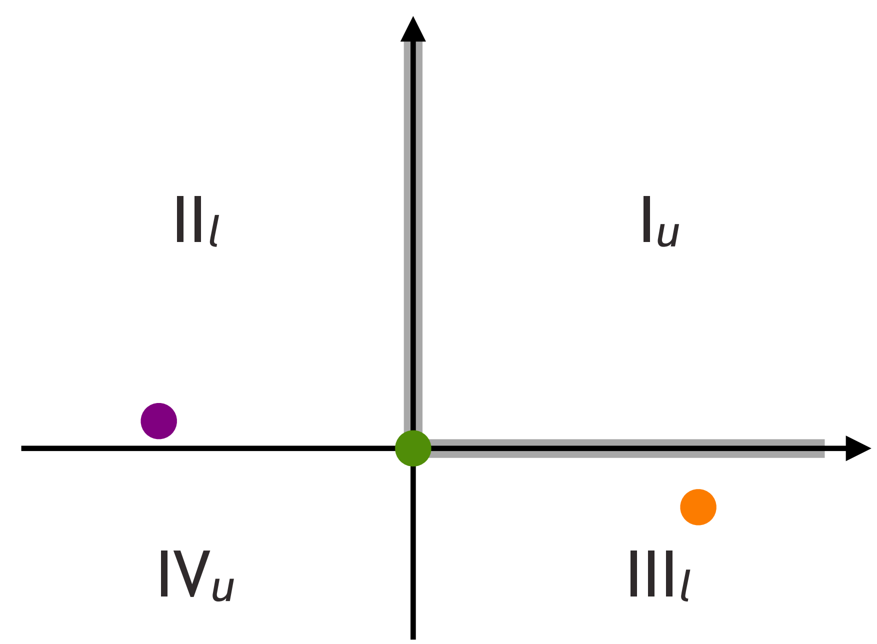
has poles

$$\sqrt{s_0} \approx m_0 \pm \frac{i}{2}\frac{g_2^2\rho_2}{m_0}\left[\left(\frac{g_1}{g_2}\right)^2\frac{\rho_1}{\rho_2} - 1\right] \quad \text{on sheet II, if } \left(\frac{g_1}{g_2}\right)^2\frac{\rho_1}{\rho_2} > 1, \text{ or,}$$

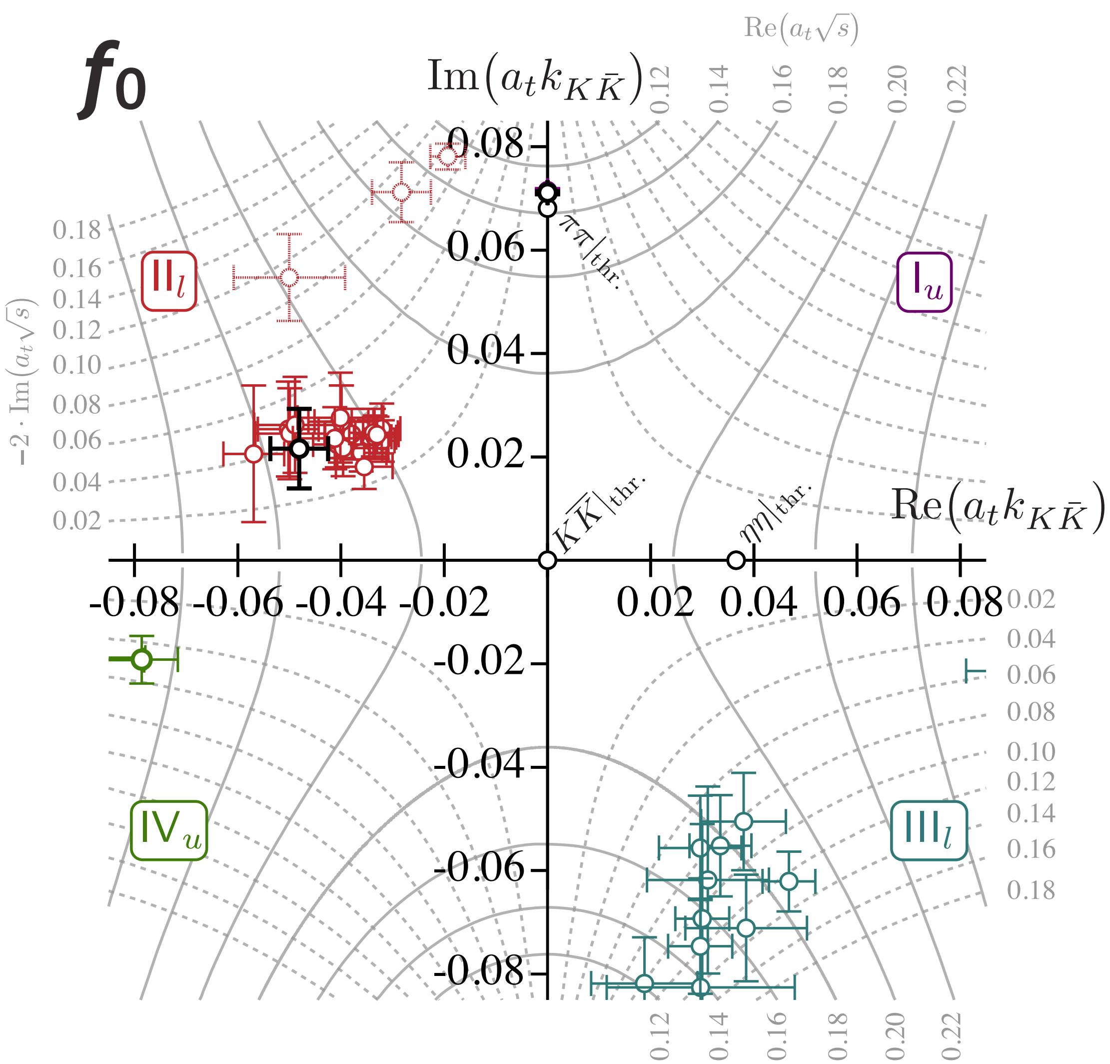
$$\sqrt{s_0} \approx m_0 \pm \frac{i}{2}\frac{g_2^2\rho_2}{m_0}\left[1 - \left(\frac{g_1}{g_2}\right)^2\frac{\rho_1}{\rho_2}\right] \quad \text{on sheet IV, if } \left(\frac{g_1}{g_2}\right)^2\frac{\rho_1}{\rho_2} < 1, \text{ and,}$$

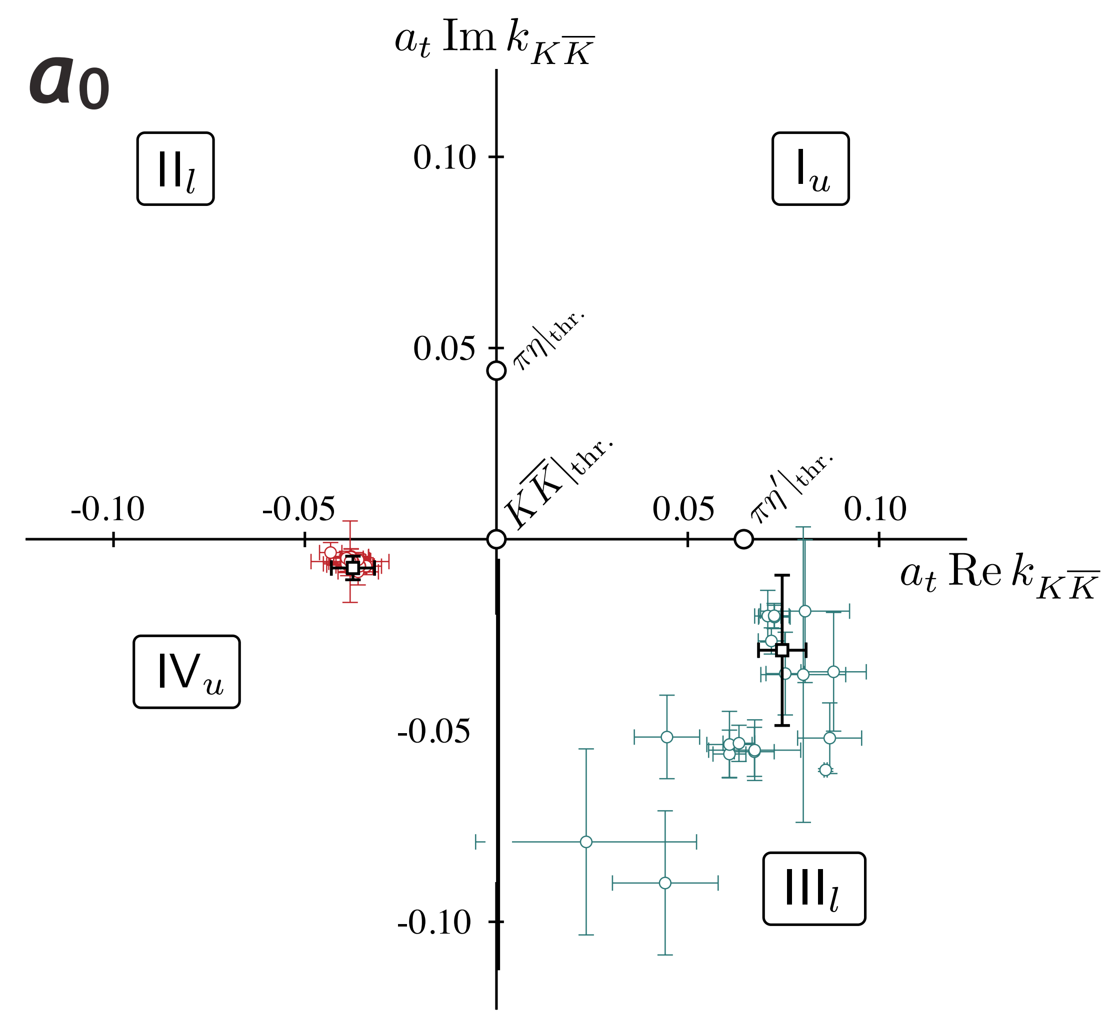
$$\sqrt{s_0} \approx m_0 \pm \frac{i}{2}\frac{g_2^2\rho_2}{m_0}\left[1 + \left(\frac{g_1}{g_2}\right)^2\frac{\rho_1}{\rho_2}\right] \quad \text{on sheet III, in all cases,}$$





poles on other sheets in the lattice calc ?

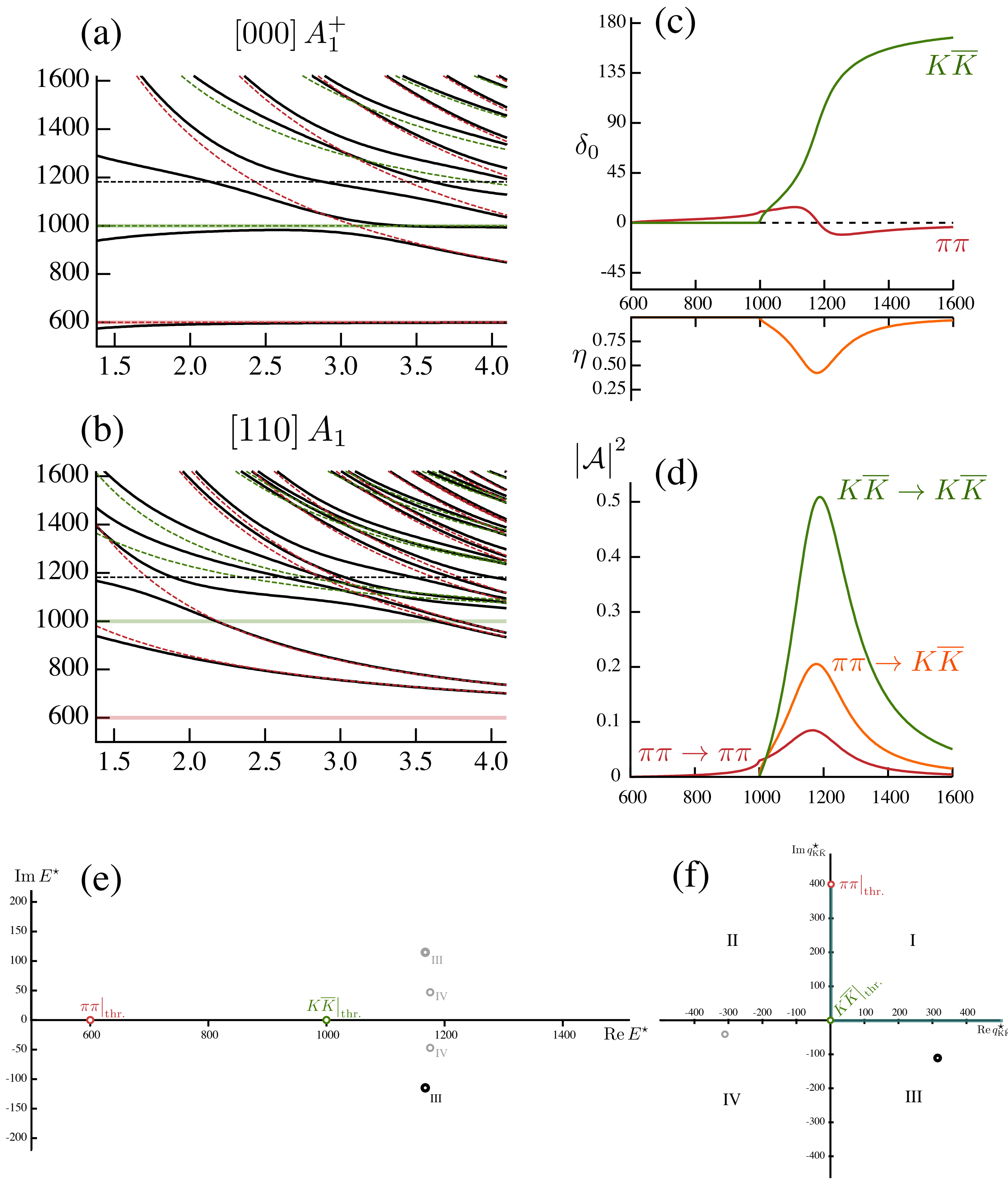




parameterization dependent
distant poles on sheet III

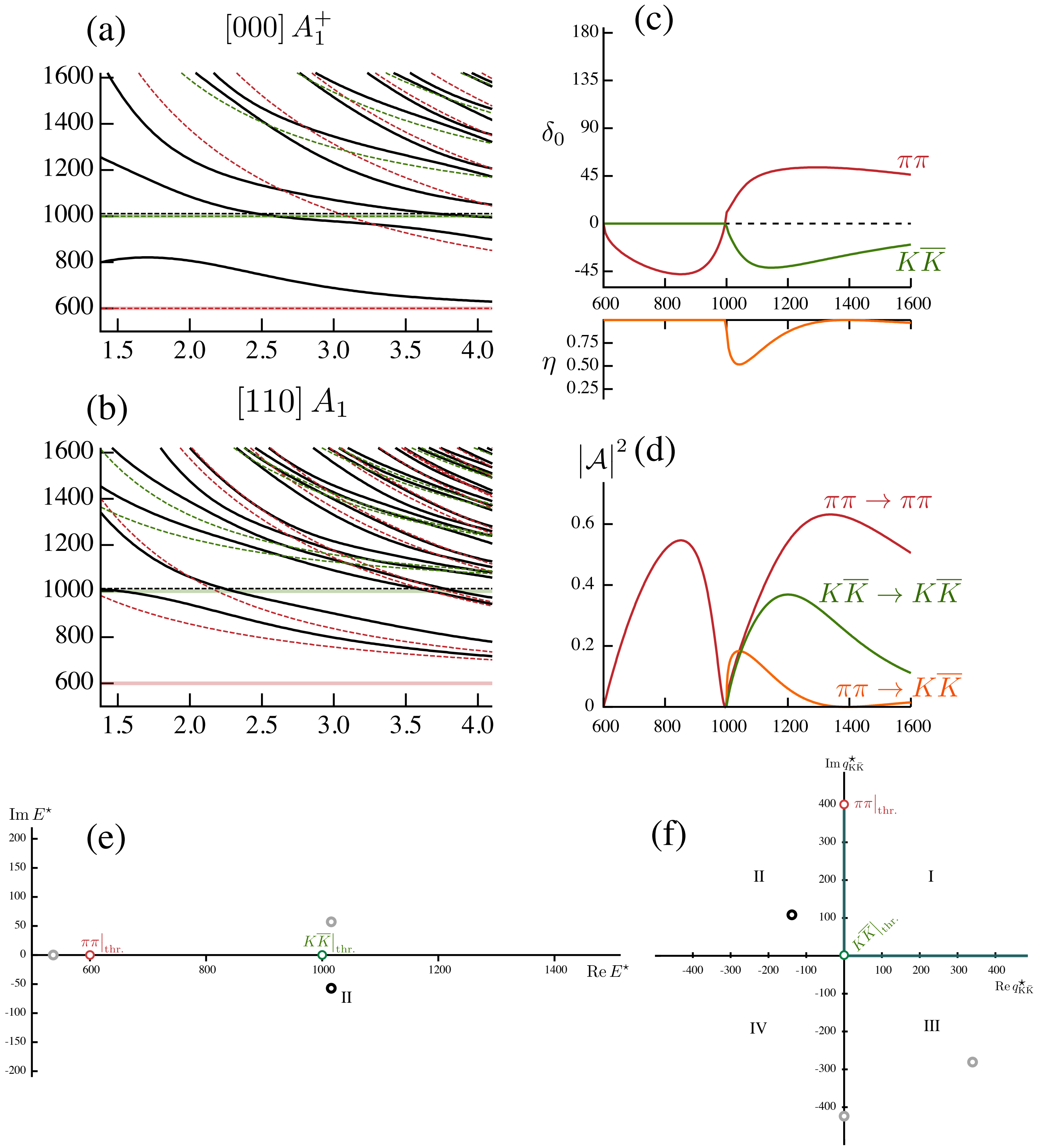
looks more like
one pole ⇒ ‘molecular resonance’ ?

finite-volume Flatté



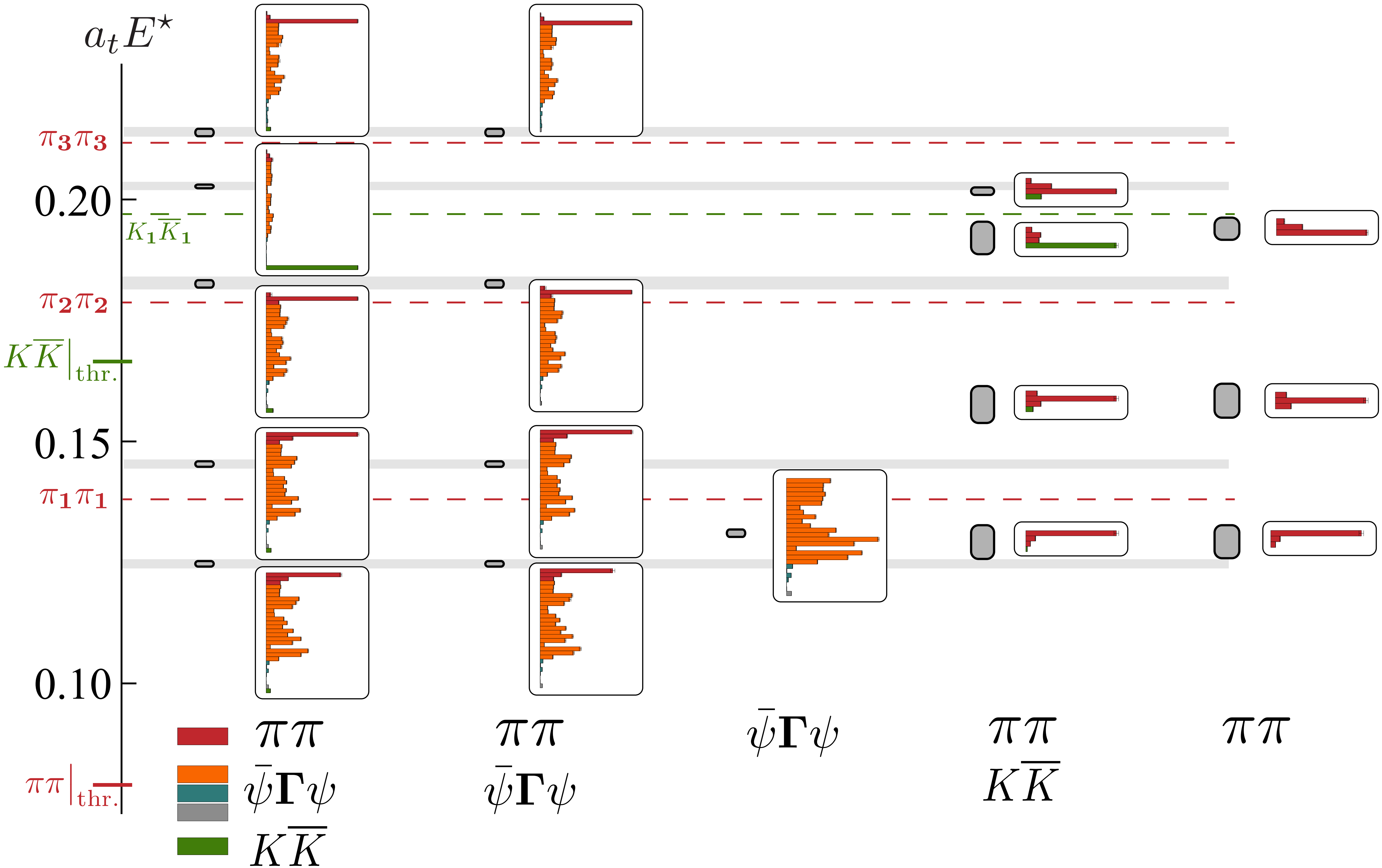
modelling an
'ordinary resonance'

finite-volume ‘dip-like’



‘cartoon’ of $f_0(980)$?

meson-meson ops are vital



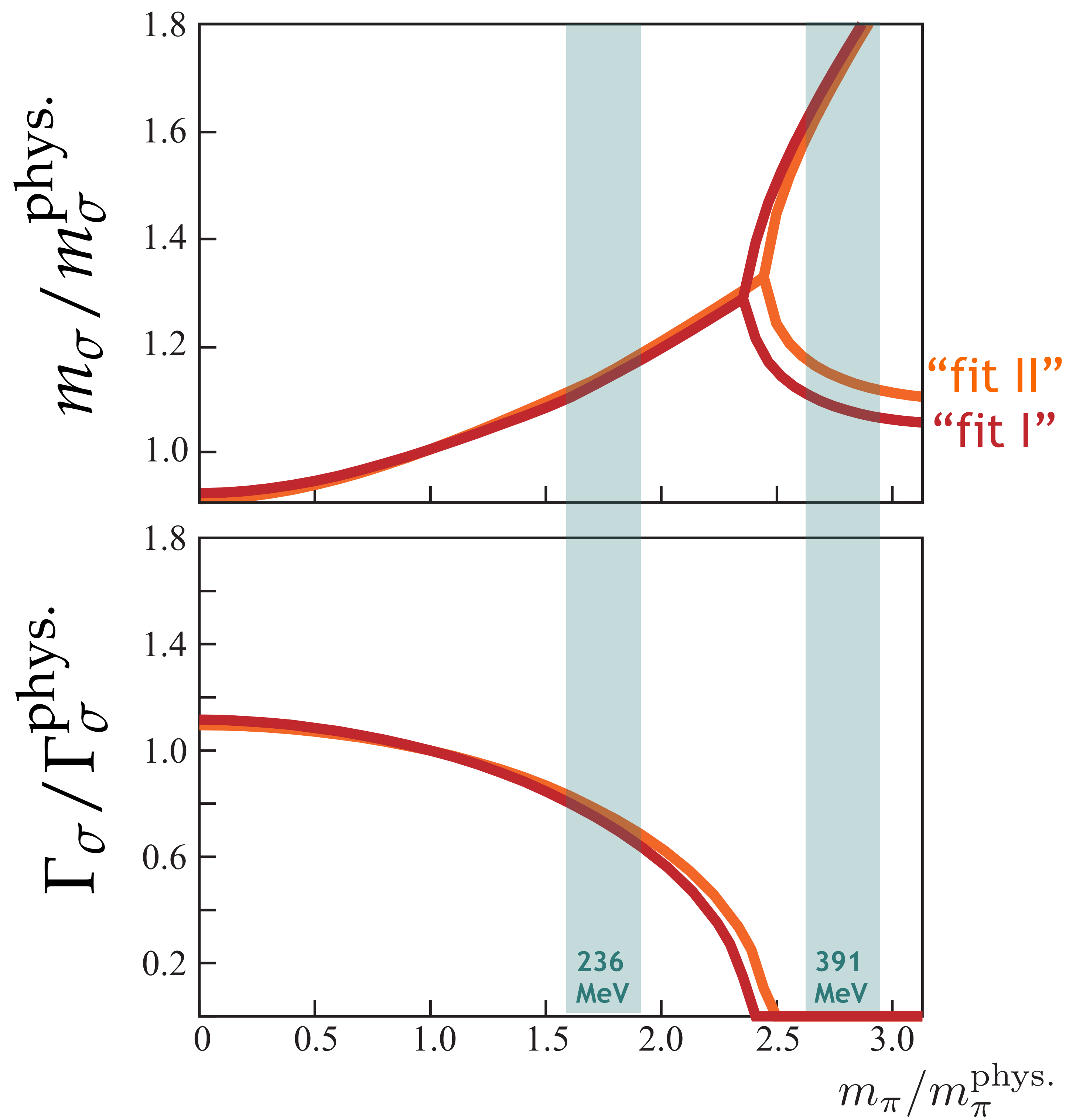
σ pole with changing quark mass

- unitarized SU(3)F chiral perturbation theory
  - resonance poles become virtual bound states somewhere near $m_\pi \sim 2.5\, m_\pi^{phys}$
  - lowest mass v.b.s becomes a bound state somewhere slightly above $m_\pi \sim 3.0\, m_\pi^{phys}$

    "the exact m_π value when this happens is not very reliable"

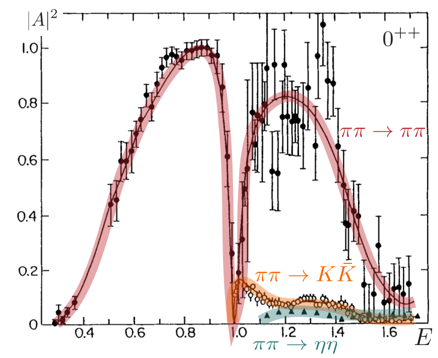
NEBREDA & PELAEZ
PRD81 054035 (2010)

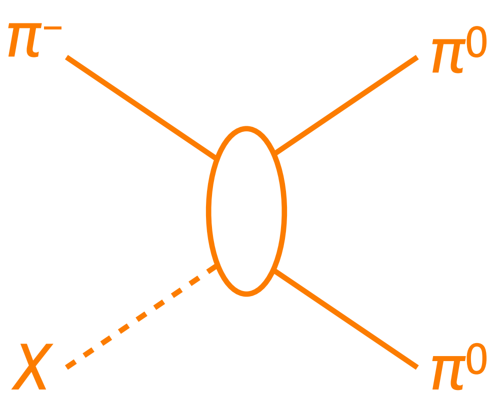
$$\sqrt{s_0} = m + \frac{i}{2}\Gamma$$



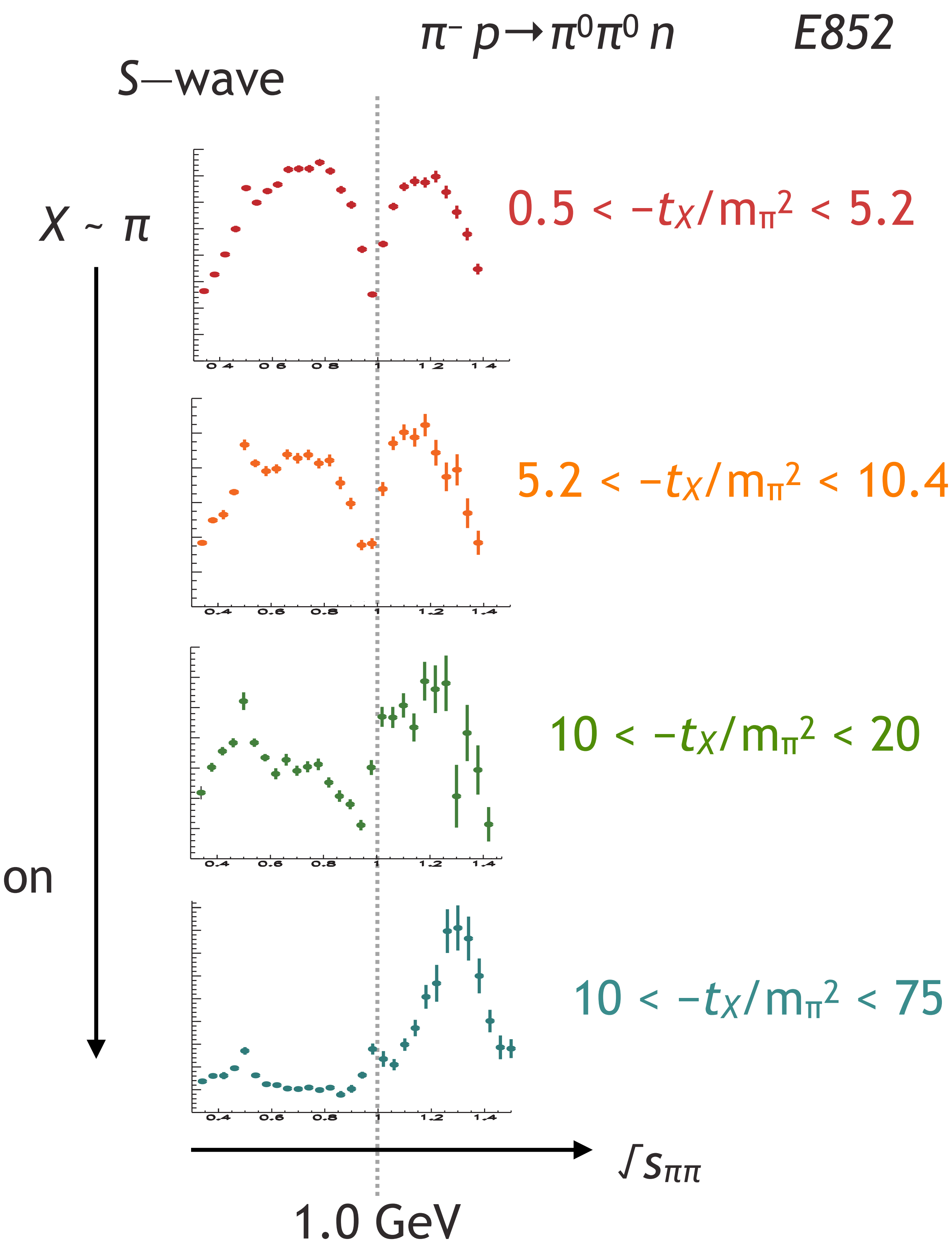
$f_0(980)$ dip — peak

can even look different in 'same' process





other exchanges modify the production



tetraquark operators

JHEP 1711 033 (2017)

generic local diquark operator

$$\delta^{J[\Gamma]}_{RF} = \langle \mathbf{3} r_a ; \mathbf{3} r_b | R r \rangle \langle F_a f_a ; F_b f_b | F f \rangle \, q^T_{r_a f_a} (C\Gamma) \, q_{r_b f_b}$$

color reps. $R = \bar{\mathbf{3}}, \mathbf{6}$

spins $J^P = 0^{\pm}, 1^{\pm}$

no assumptions made at this point about good/bad diquarks

generic local tetraquark operator

$$\mathcal{T}^{J[\Gamma_1 \Gamma_2]}_{\mathbf{1}[R_1 R_2] \, F[F_1 F_2]} = \langle J_1 m_1 ; J_2 m_2 | J m \rangle \langle R_1 r_1 ; R_2 r_2 | \mathbf{1} \rangle \langle F_1 f_1 ; F_2 f_2 | F f \rangle \, \delta^{J_1[\Gamma_1]}_{R_1 F_1} \, \bar{\delta}^{J_2[\Gamma_2]}_{R_2 F_2}$$

(+ *C*/*G*-parity symmetrisation ...)

spins $J \leq 2$

smeared quark fields, but otherwise **local**,
certainly not sampling the whole lattice volume

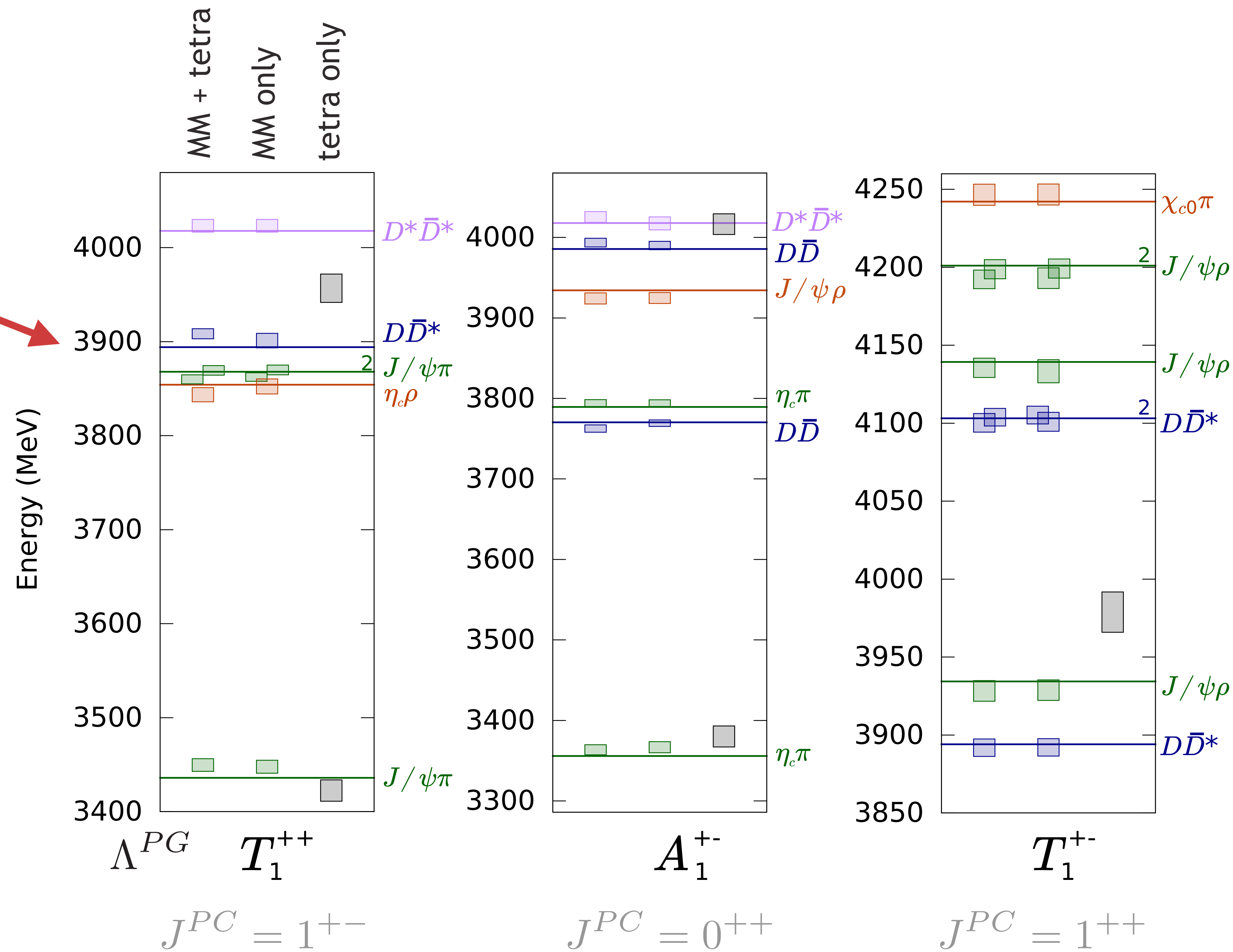
(diquark construction just makes fermion antisymmetry manifest)

tetraquark operators — hidden charm *I*=1

JHEP 1711 033 (2017)

all ‘expected’ meson-meson operators + several tetraquark operators

Zc(‘3900’) ?



$m_\pi \sim 391\,\mathrm{MeV}$

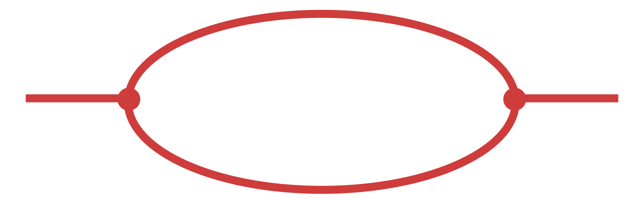
Chew-Mandlestam

$$\left[t^{-1}(s)\right]_{ij} = \left[K^{-1}(s)\right]_{ij} + \delta_{ij}\, I_i(s)$$

- equal mass case

$$I(s) = -C(s)$$

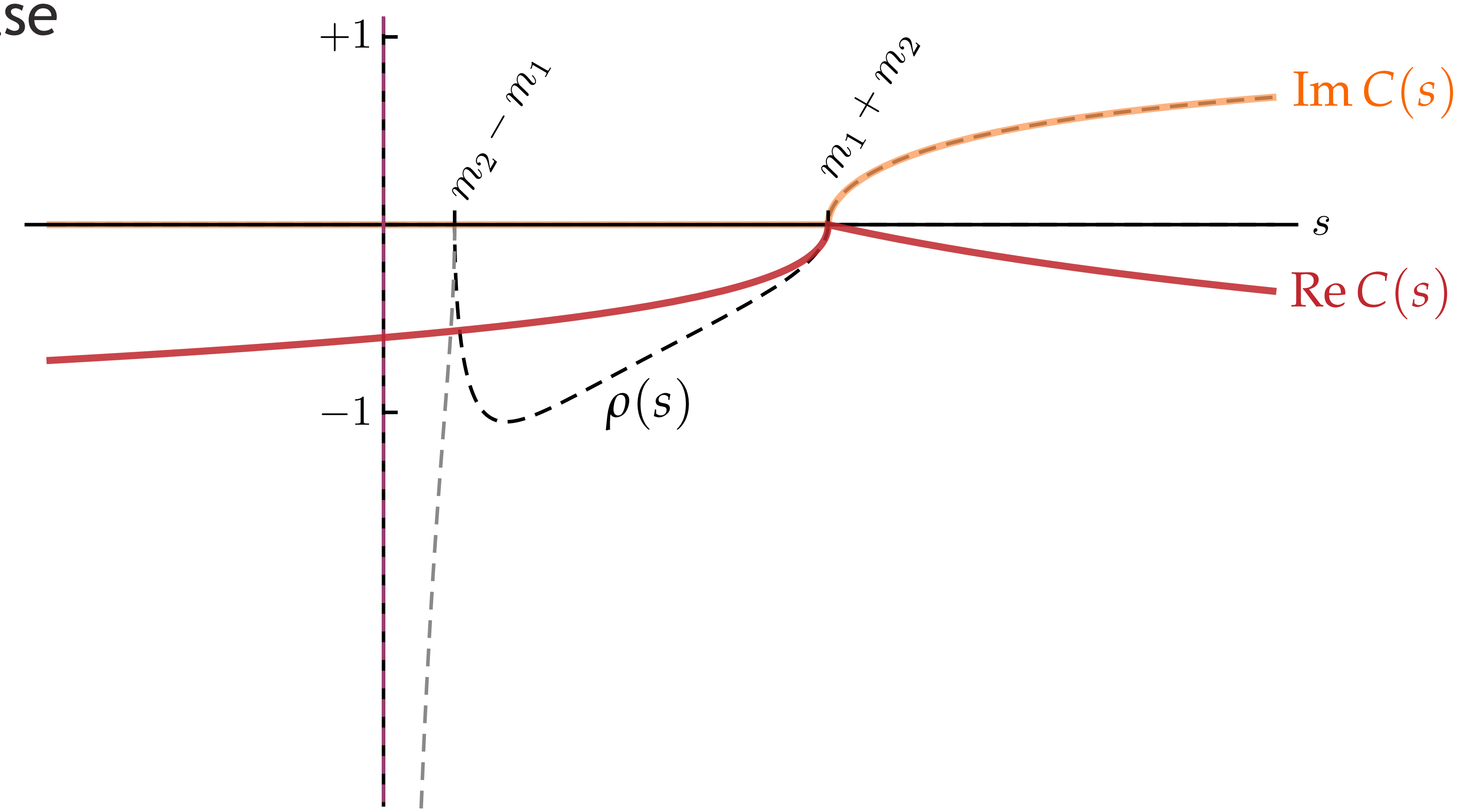
$$C(s) = C(0) + \frac{s}{\pi}\int_{s_{\text{th}}}^{\infty} ds' \sqrt{1 - \frac{s_{\text{th}}}{s'}}\, \frac{1}{s'(s'-s)}$$



$$C(s) = \frac{\rho(s)}{\pi} \log\left[\frac{\rho(s)-1}{\rho(s)+1}\right]$$

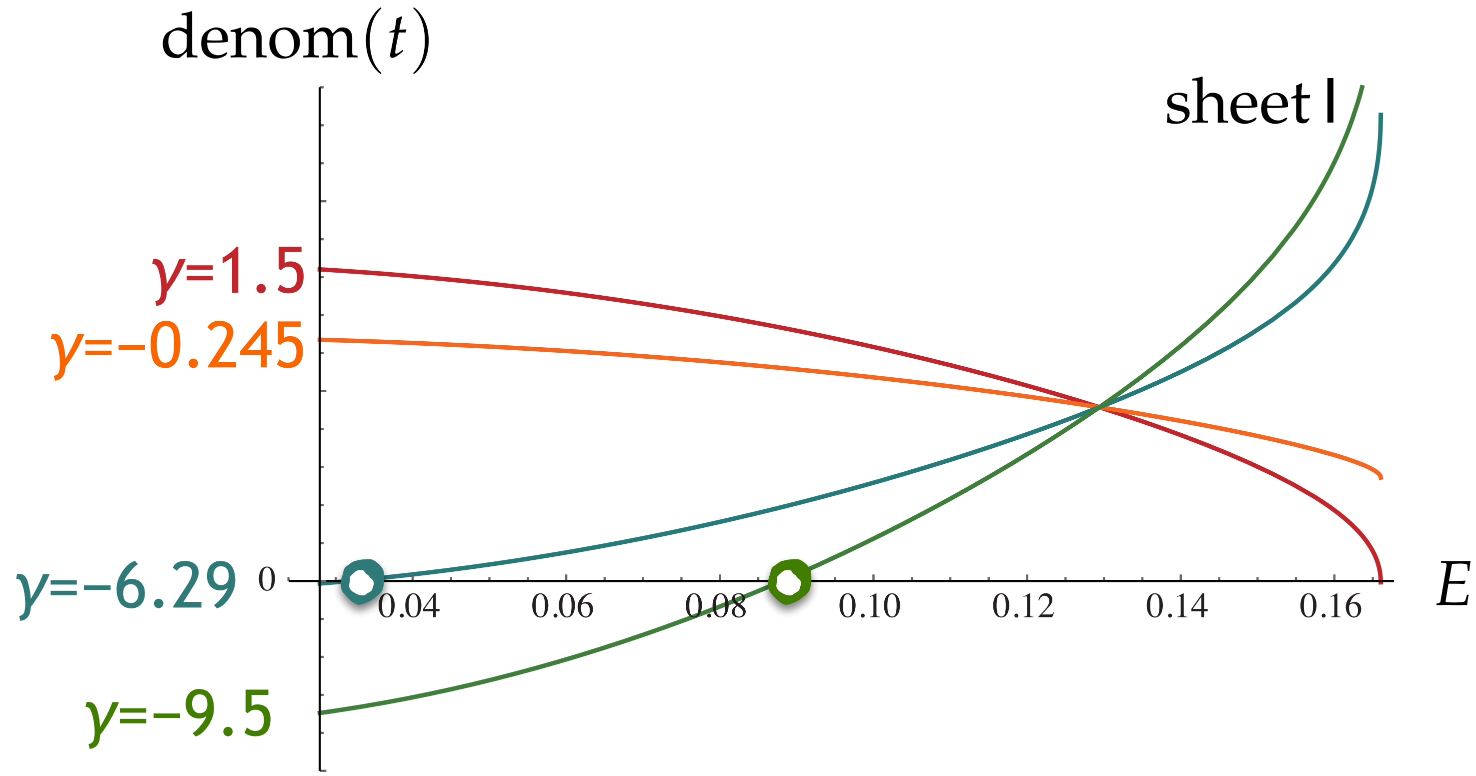
subtracting at threshold $C(s_{\text{th}}) = 0$

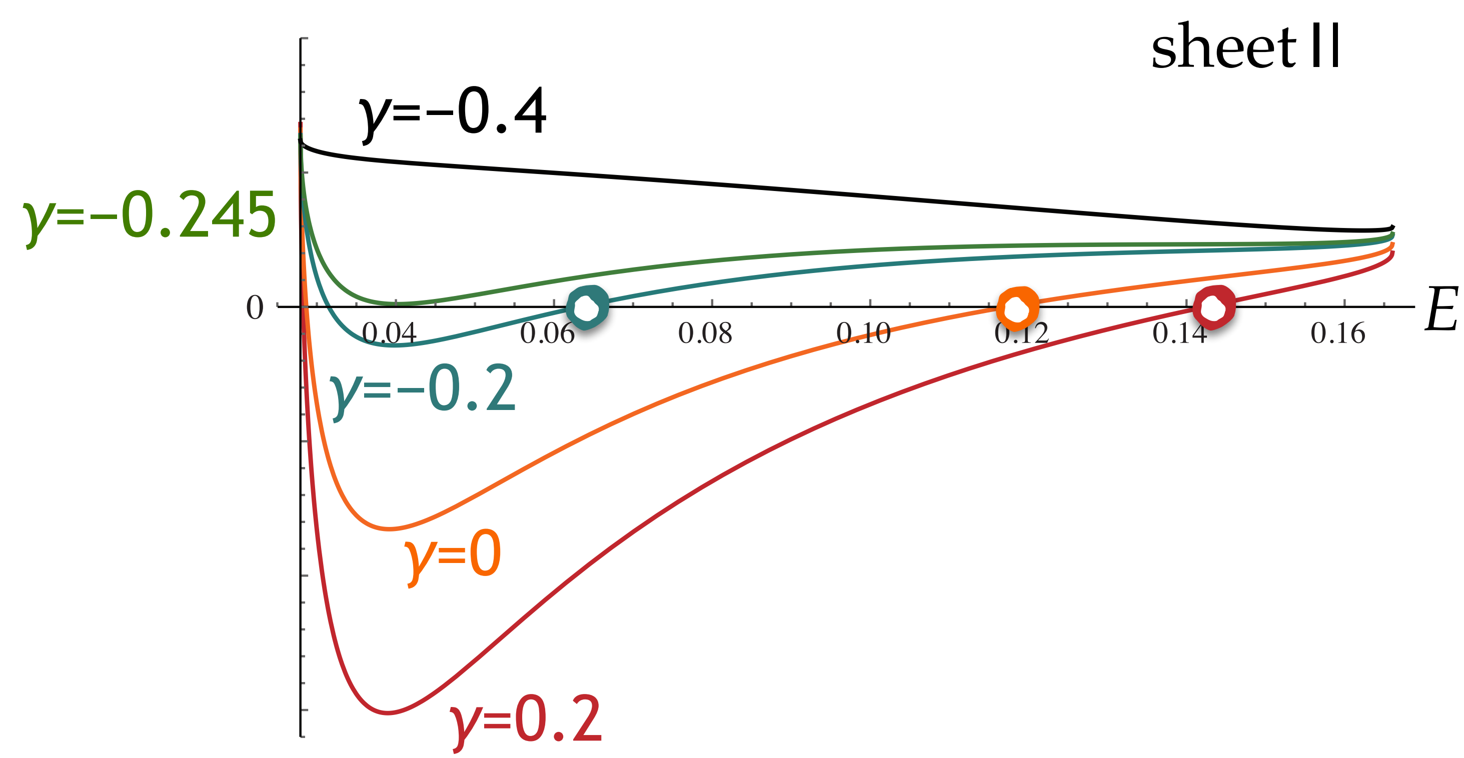
- unequal mass case



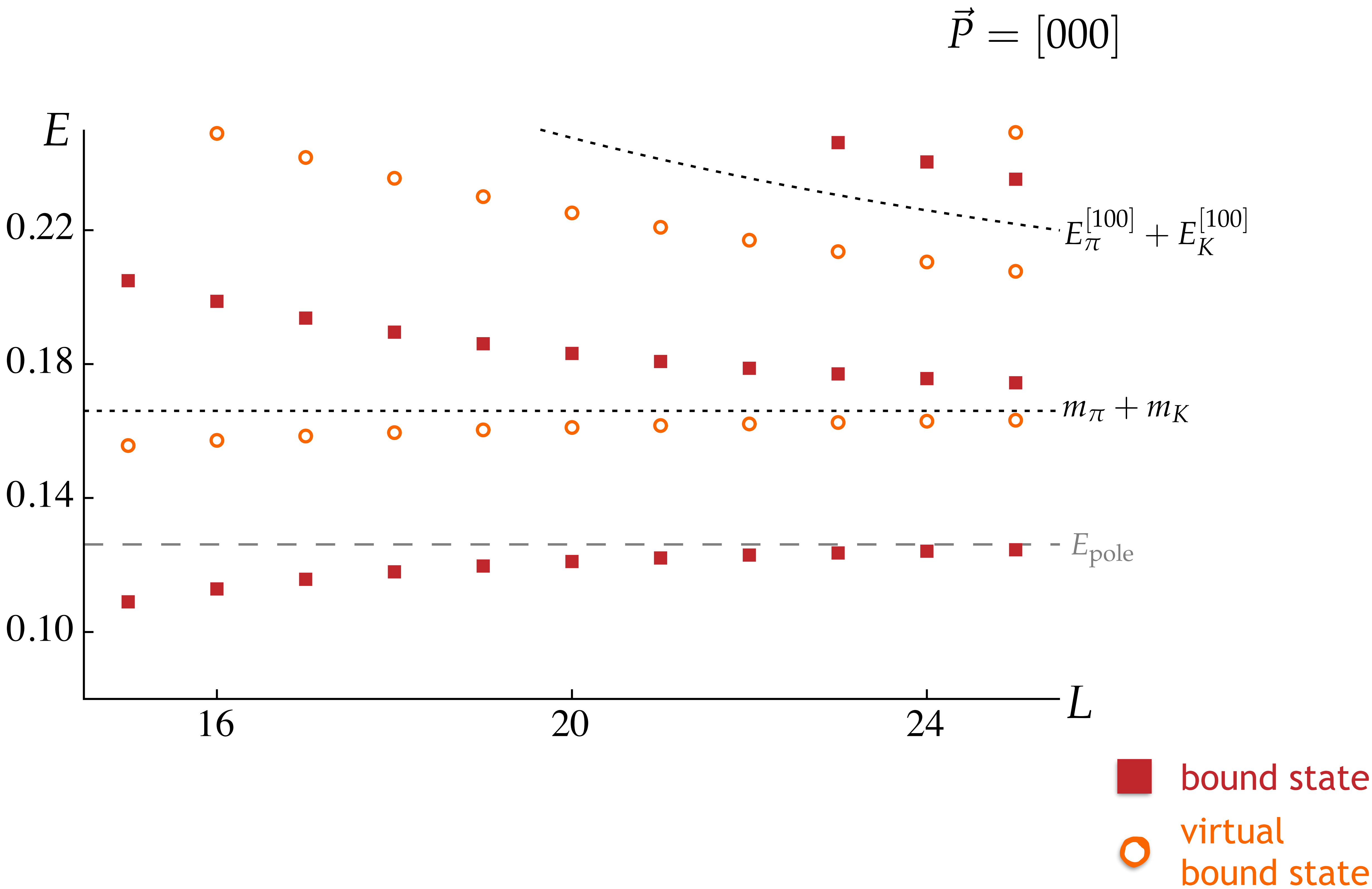
bound states & virtual bound-states

- $K(E) = \frac{g^2}{m^2 - E^2} + \gamma$ & Chew-Man
- fix $m = 0.25$, $g = 0.16$, and vary γ
- so this form can support
  a b.s., a v.b.s. or neither

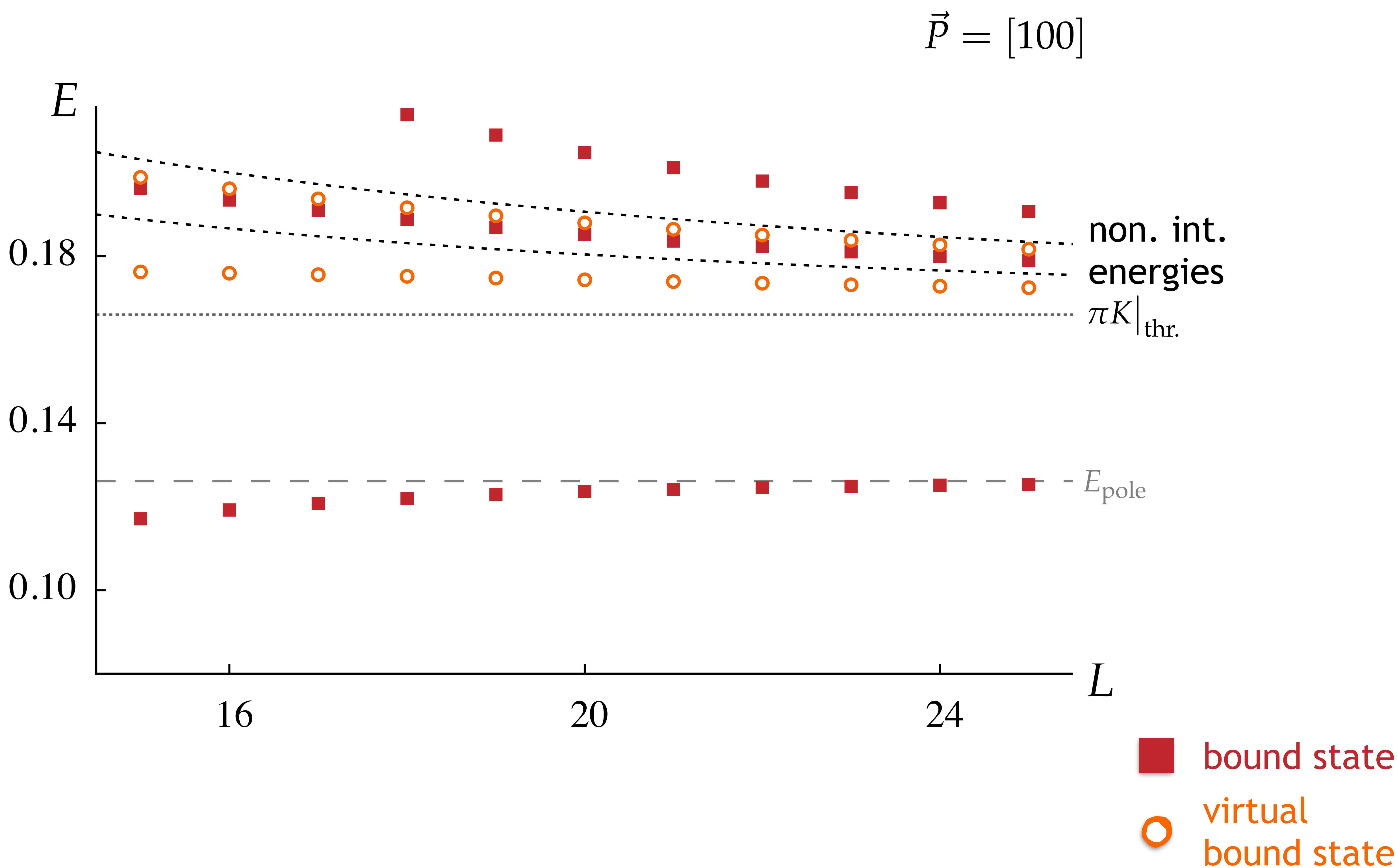




bound state versus virtual bound state

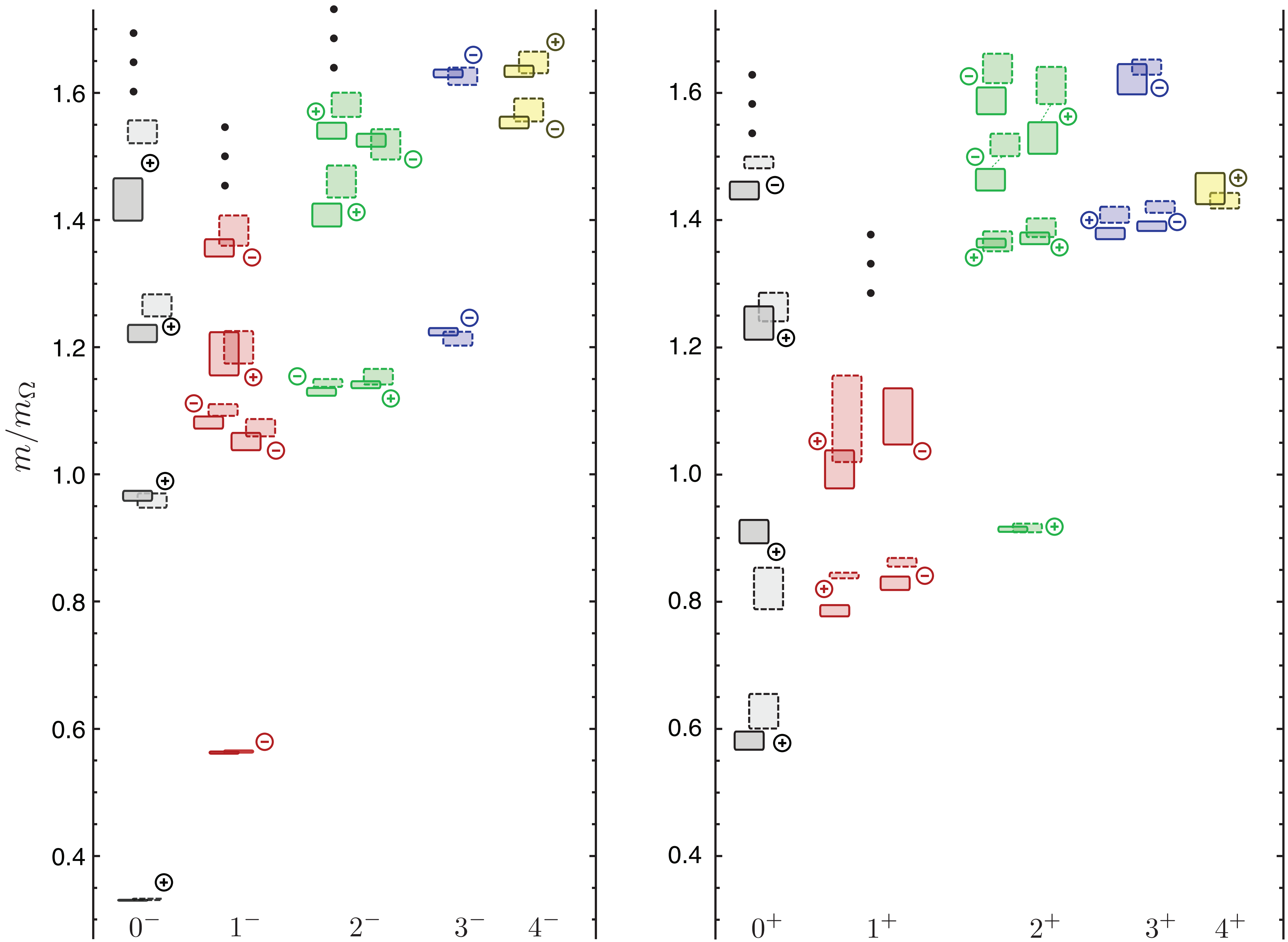


bound state versus virtual bound state



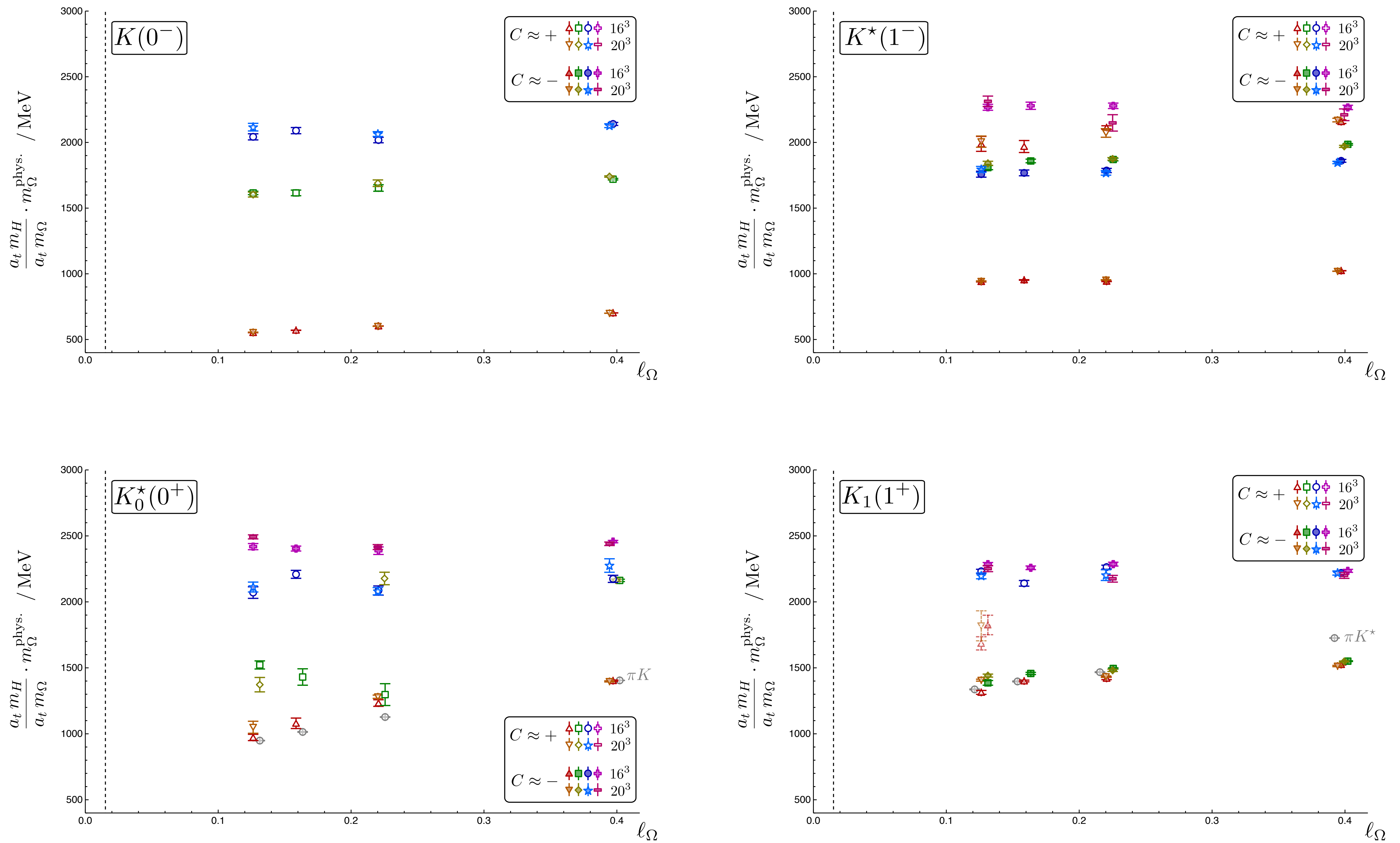
'single-hadron' kaon spectrum @ m_π~391 MeV

PRD82 034508 (2010)



‘single-hadron’ kaon spectrum @ m_π~391 MeV

PRD82 034508 (2010)



‘single-hadron’ kaon spectrum @ m_π~391 MeV

PRD82 034508 (2010)