S-MATRIX APPROACH TO THE THERMODYNAMICS OF HADRONS

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PKI2018 WORKSHOP
14 FEB, 2018
JLAB, NEWPORT NEWS, VA

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CONCLUSION

- S-matrix approach to thermodynamics

$$\Delta \ln Z = \int dE \, e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \mathrm{tr} \, (\delta_E) \, .$$

- change in density of state / time delay

Broad resonances

Repulsive channels

IN COLLABORATION WITH

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Michal Szymanski (Wroclaw)

Bengt Friman (GSI)

Pasi Huovinen (Wroclaw)

Chihiro Sasaki (Wroclaw)

Krzysztof Redlich (Wroclaw)

HRG &
QCD EQUATION OF STATE

HADRON RESONANCE GAS MODEL

- Confinement

physical quantities → hadronic states representation

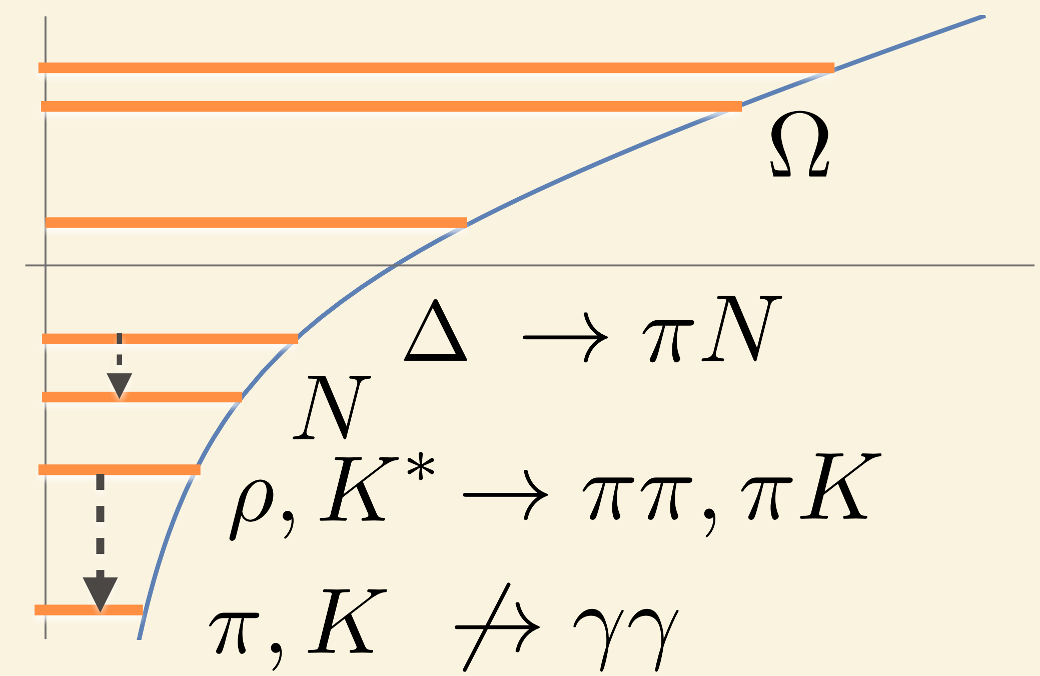
$$Z = \sum_{\alpha = B, M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

HADRONIC STAT
REPRESENTATIO

- Confinement

physical quantities

QCD spectrum



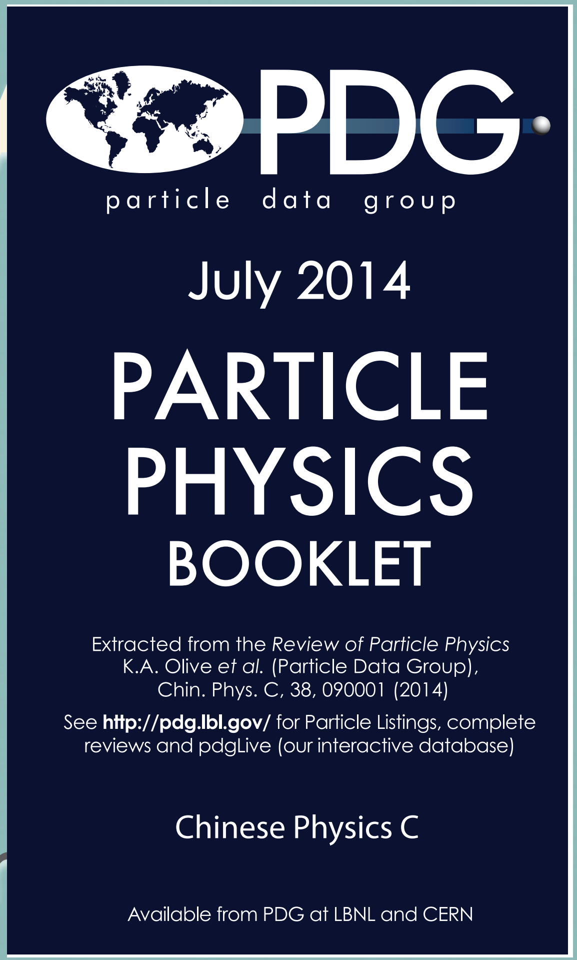
$$Z = \sum_{\alpha=B,M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

HADRON RESONANCE GA MODEL

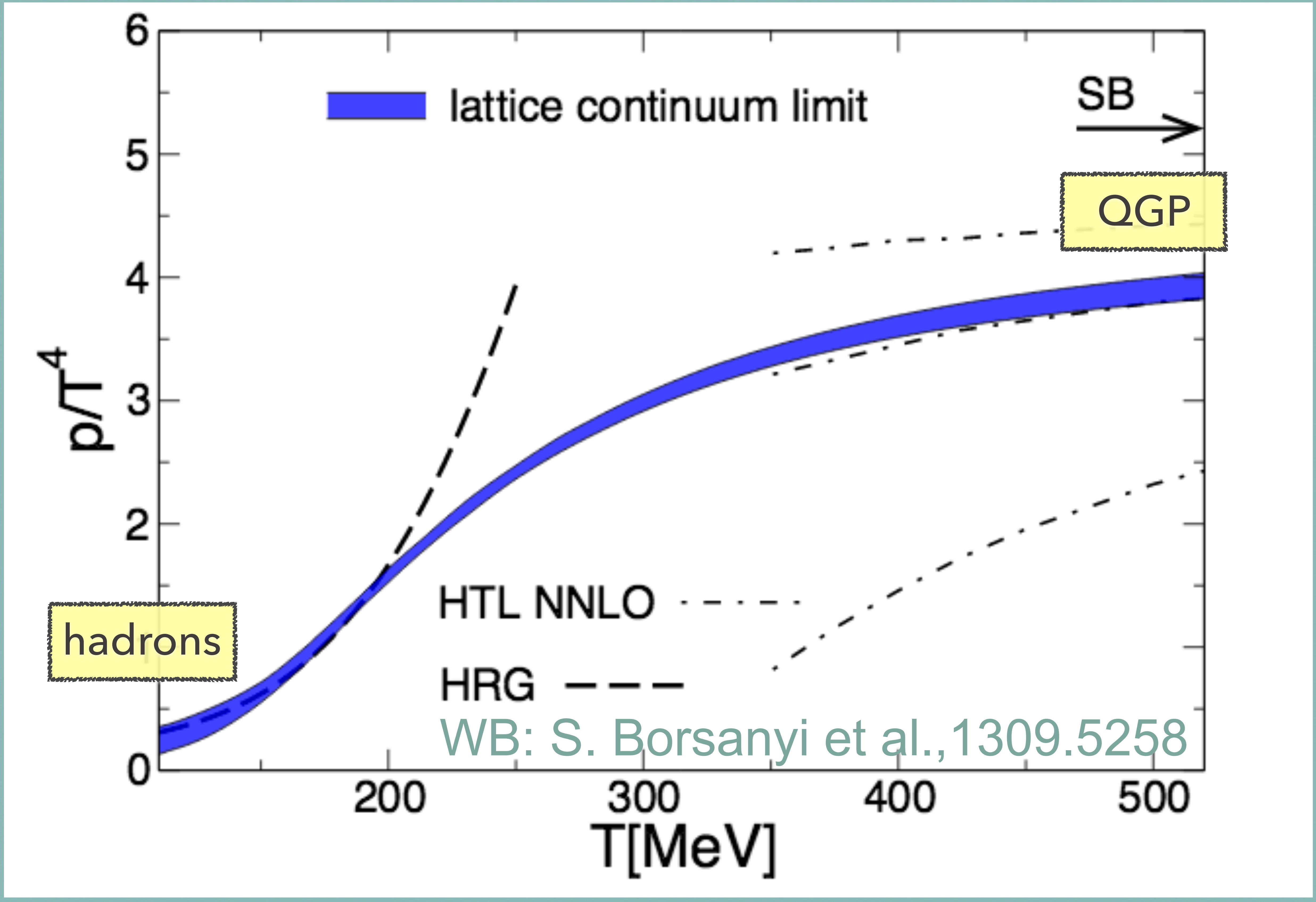
- Ground states $\pi, K, P, N...$

- Resonance formation dominates therm

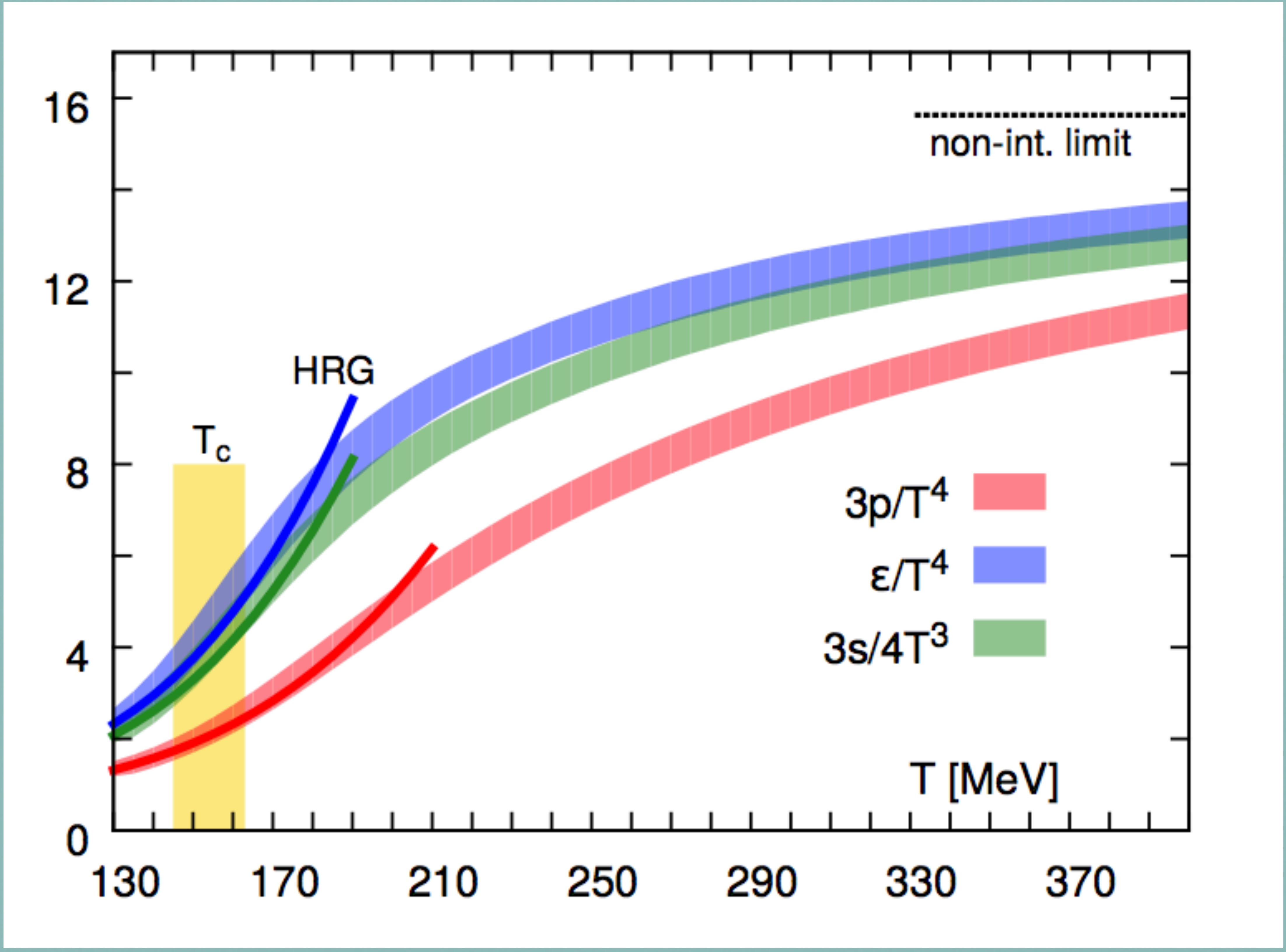
- Resonances treated as point-like particles



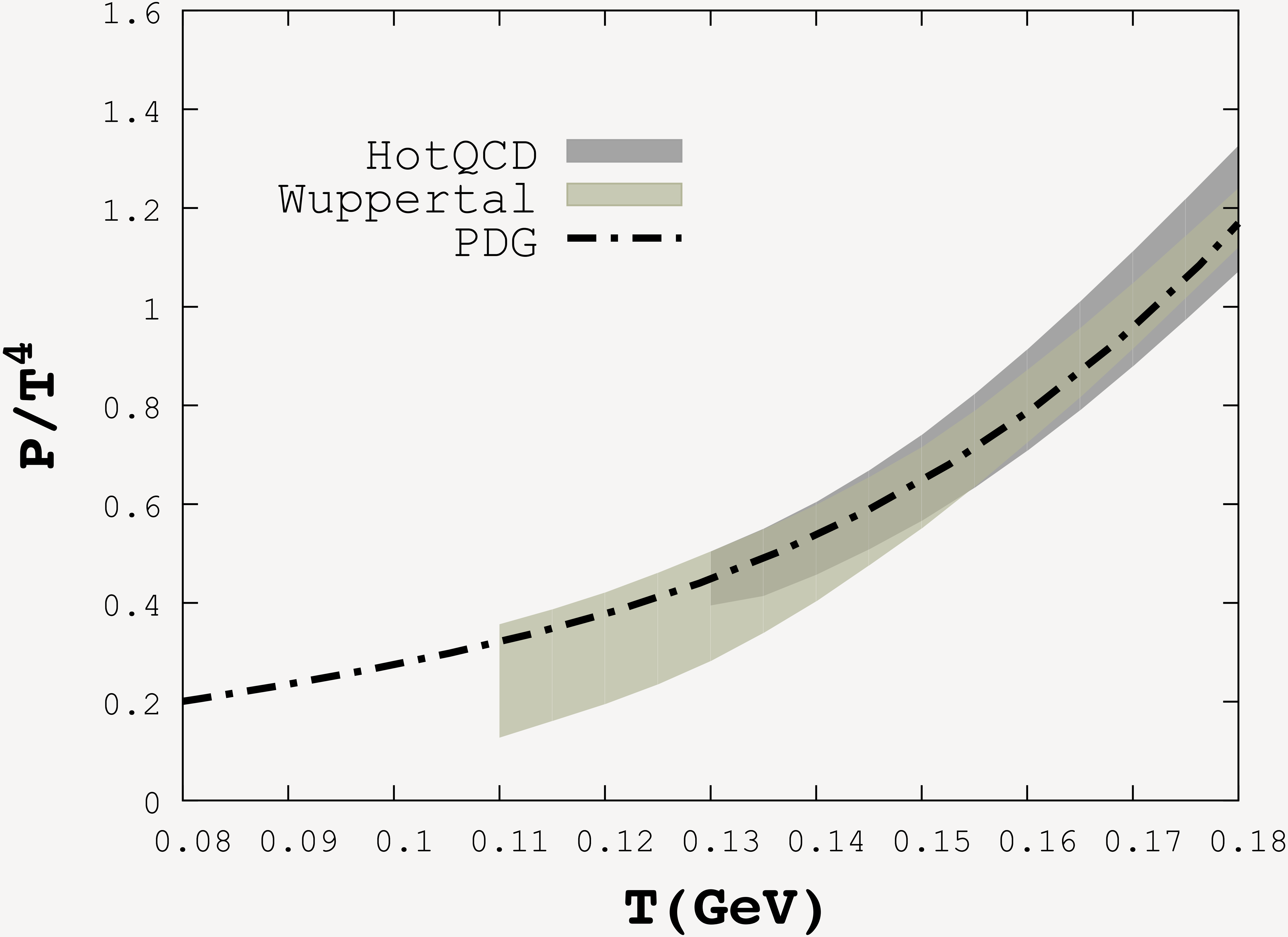
$$P = T \sum_{\alpha=M,B} g_\alpha \int \frac{d^3k}{(2\pi)^3} \mp \ln(1 \mp e^{-\beta\sqrt{k^2+M_\alpha^2}})$$



lattice continuum limit
SB
QGP
HTL NNLO
HRG
hadrons
WB: S. Borsanyi et al.,1309.5258
p/T^4
T[MeV]
0
2
3
4
5
6
200
300
400
500



A. Bazavov et al. (hotQCD), PRD90 (2014) 094503



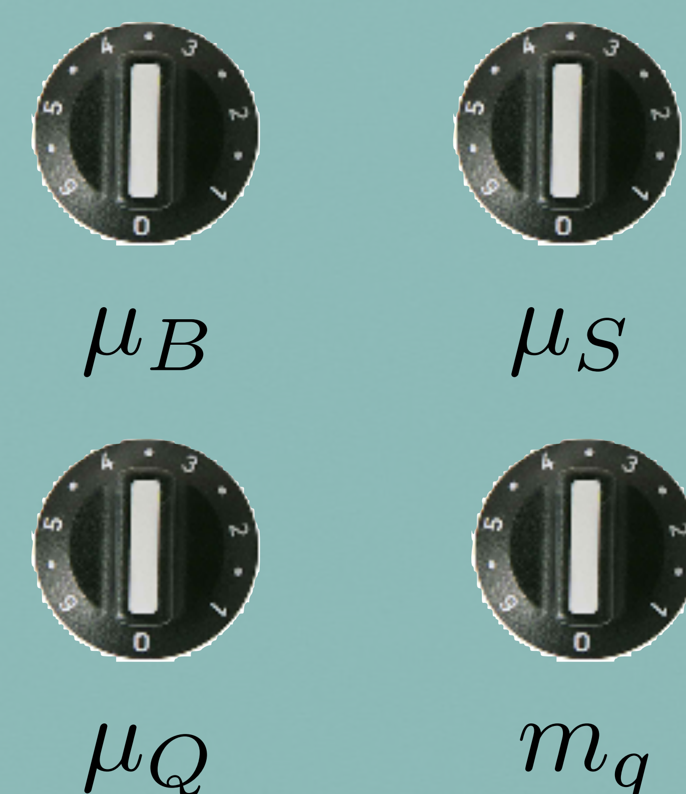
HotQCD
Wuppertal
PDG
P/T^4
T(GeV)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0.08
0.09
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18

FLUCTUATIONS

- studying the system by linear response

$$\mu = \mu_B B + \mu_Q Q + \mu_S S$$

$$\chi_{B,S,\ldots} = \frac{1}{\beta V} \frac{\partial^2}{\partial \bar{\mu}_B \partial \bar{\mu}_S \ldots} \ln Z$$



FLUCTUATIONS

- Baryon sector

$$P = T \sum_{\alpha=\cancel{M},B} g_\alpha \int \frac{d^3k}{(2\pi)^3} \mp \ln(1 \mp e^{-\beta\sqrt{k^2+M_\alpha^2}})$$

$\boxed{\text{or introduce the chemical potential}}$

$$P = T \sum_{\alpha=B,\bar{B}} g_\alpha \int \frac{d^3k}{(2\pi)^3} \ln(1 + e^{-\beta\sqrt{k^2+M_\alpha^2} \pm \bar{\mu}_B})$$

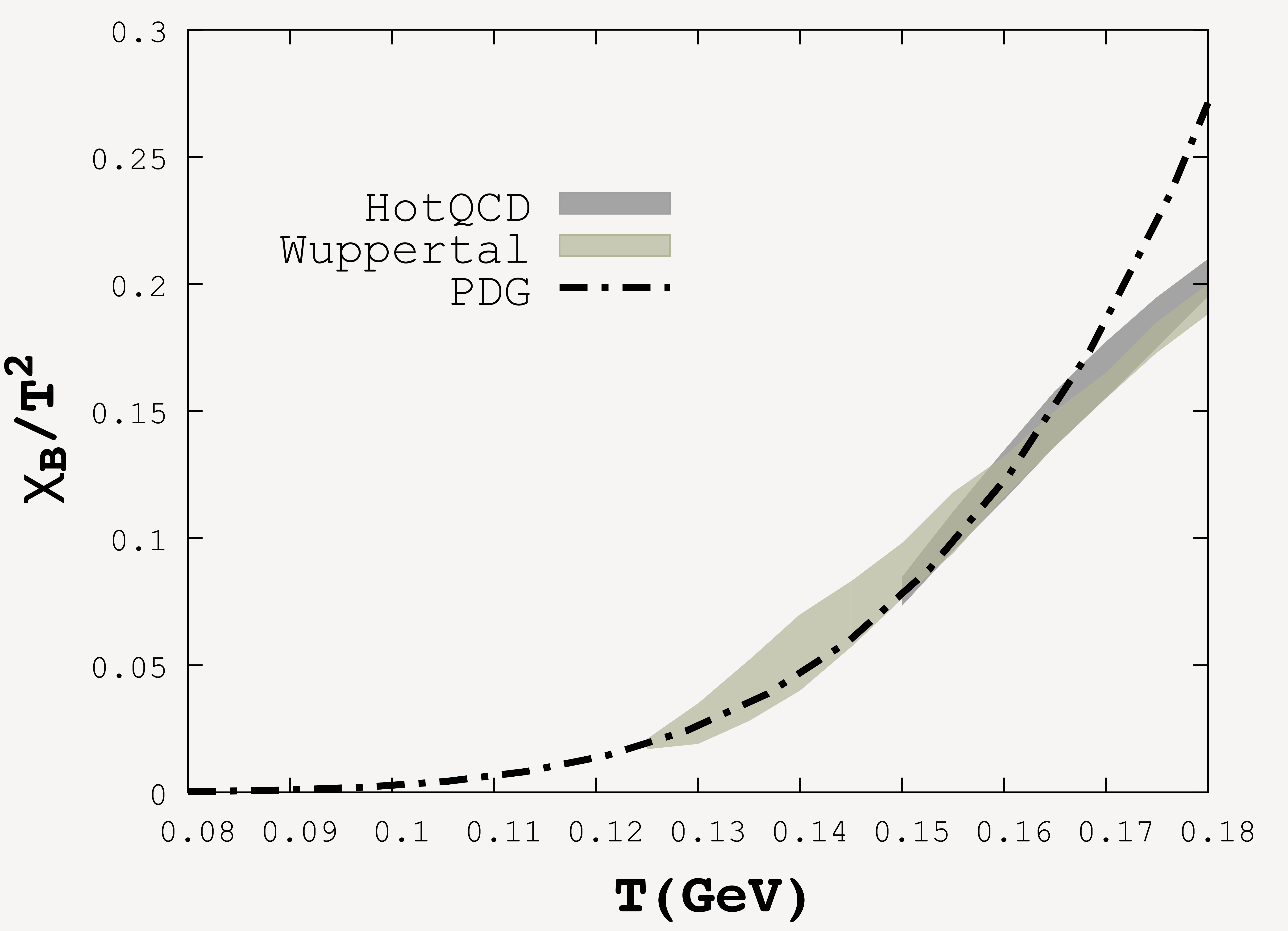
FLUCTUATIONS

- taking derivative

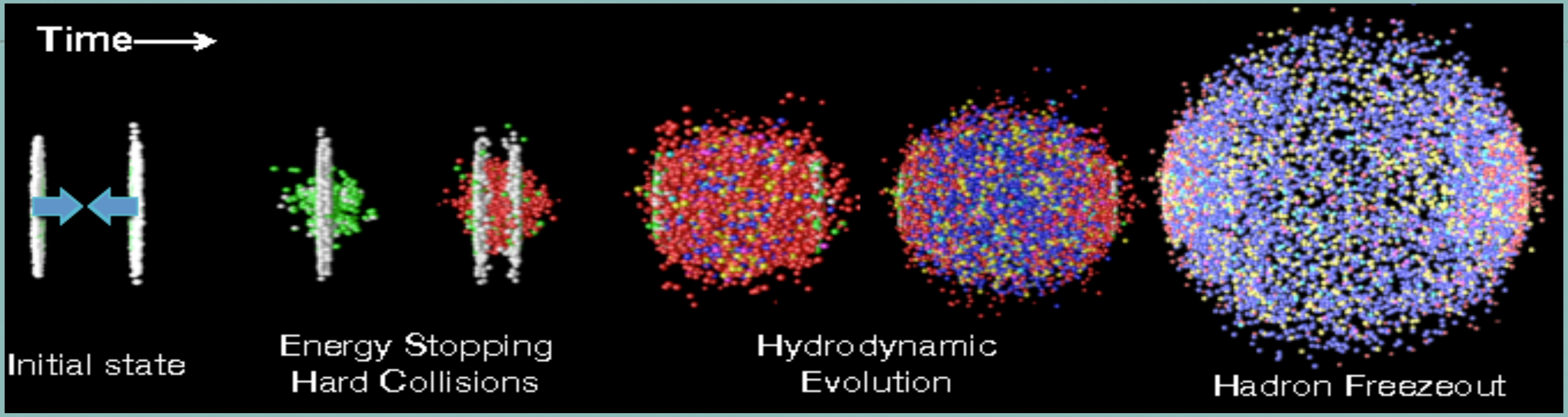
$$\chi_B = \frac{\partial^2}{\partial \bar{\mu}_B \partial \bar{\mu}_B} P$$ *at the limit* $\mu_B \to 0$

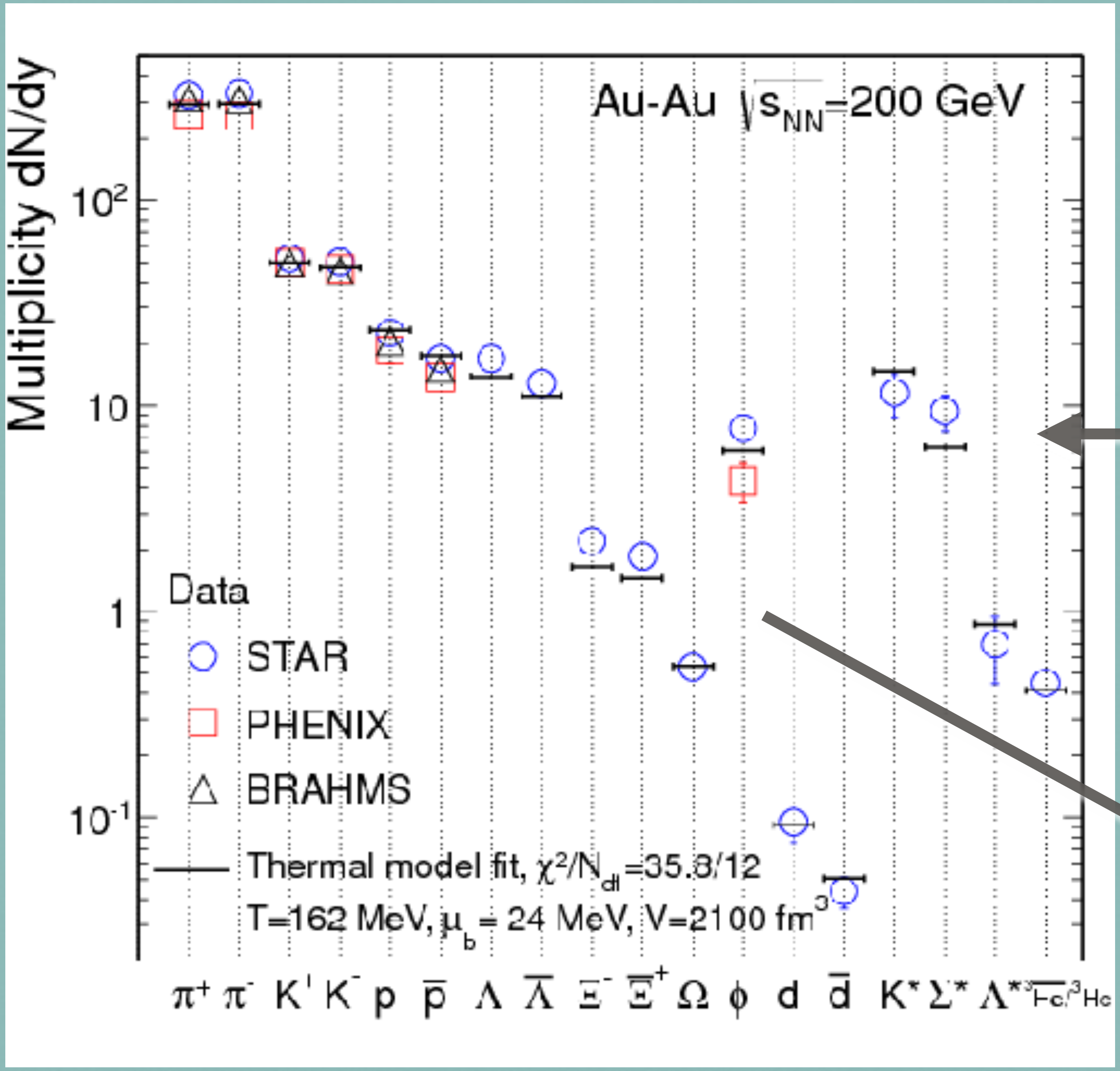
probes fluctuations

$$\chi_B = \frac{1}{\beta V} \frac{\partial^2}{\partial \bar{\mu}_B \partial \bar{\mu}_B} \ln Z$$
$$= T^2 \langle\langle \int d^4x\, \bar{\psi}(x)\gamma^0\psi(x)\bar{\psi}(0)\gamma^0\psi(0)\rangle\rangle_c$$



HotQCD
Wuppertal
PDG
χ_B/T^2
T(GeV)
0
0.05
0.1
0.15
0.2
0.25
0.3
0.08
0.09
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18





freezeout hadrons yields described by HRG

Freezeout parameters

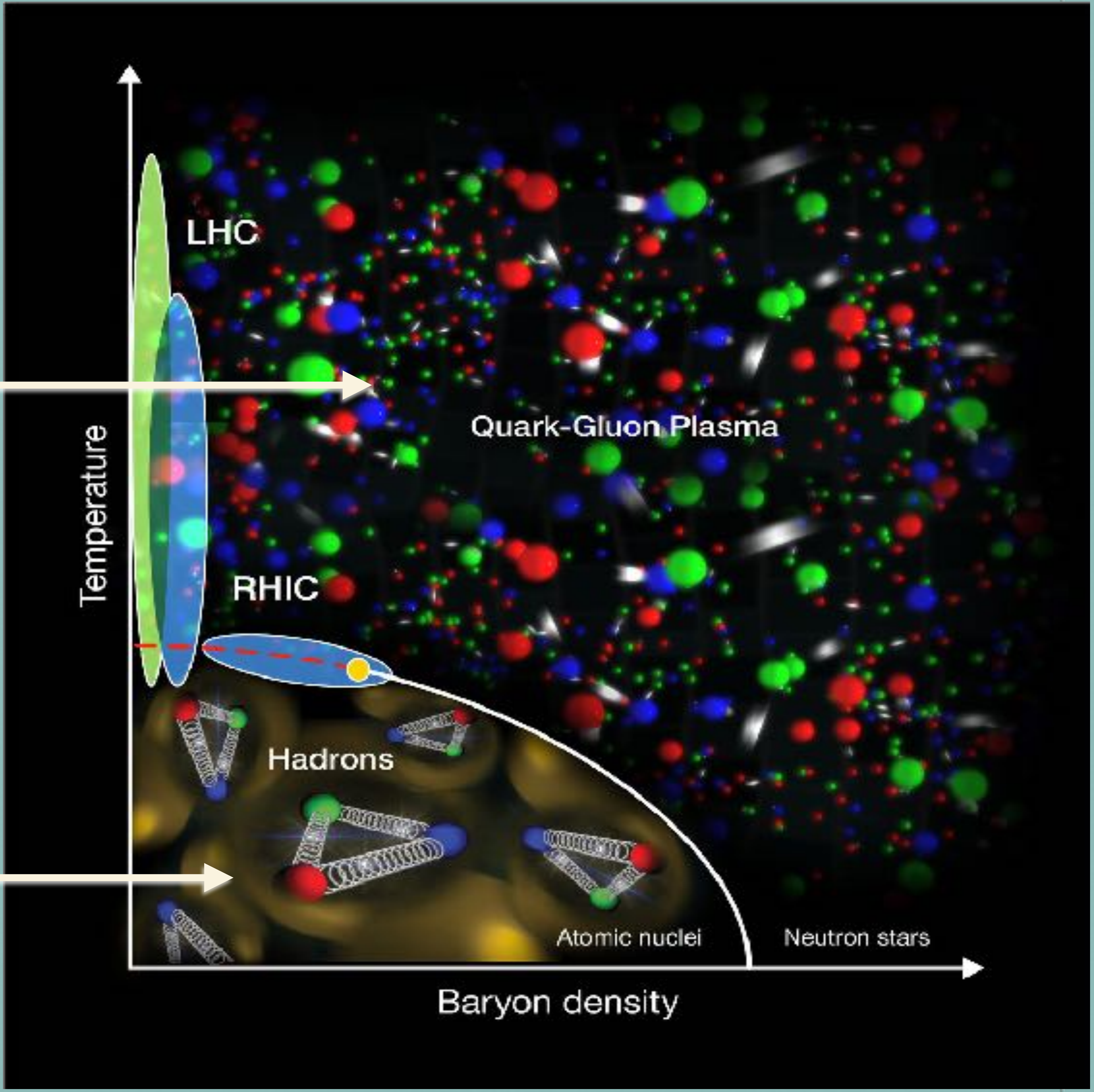
$$T^f, \mu_B^f, \mu_S^f, \mu_Q^f, \ldots$$

Andronic, A. *et al.* Nucl.Phys. A904-905 (2013)

QCD Phase Diagram

QGP:
quarks and gluons are deconfined.

Hadronic phase:
quarks are confined and massive.



Brookhaven National Laboratory

QCD Phase Diagram

QGP:
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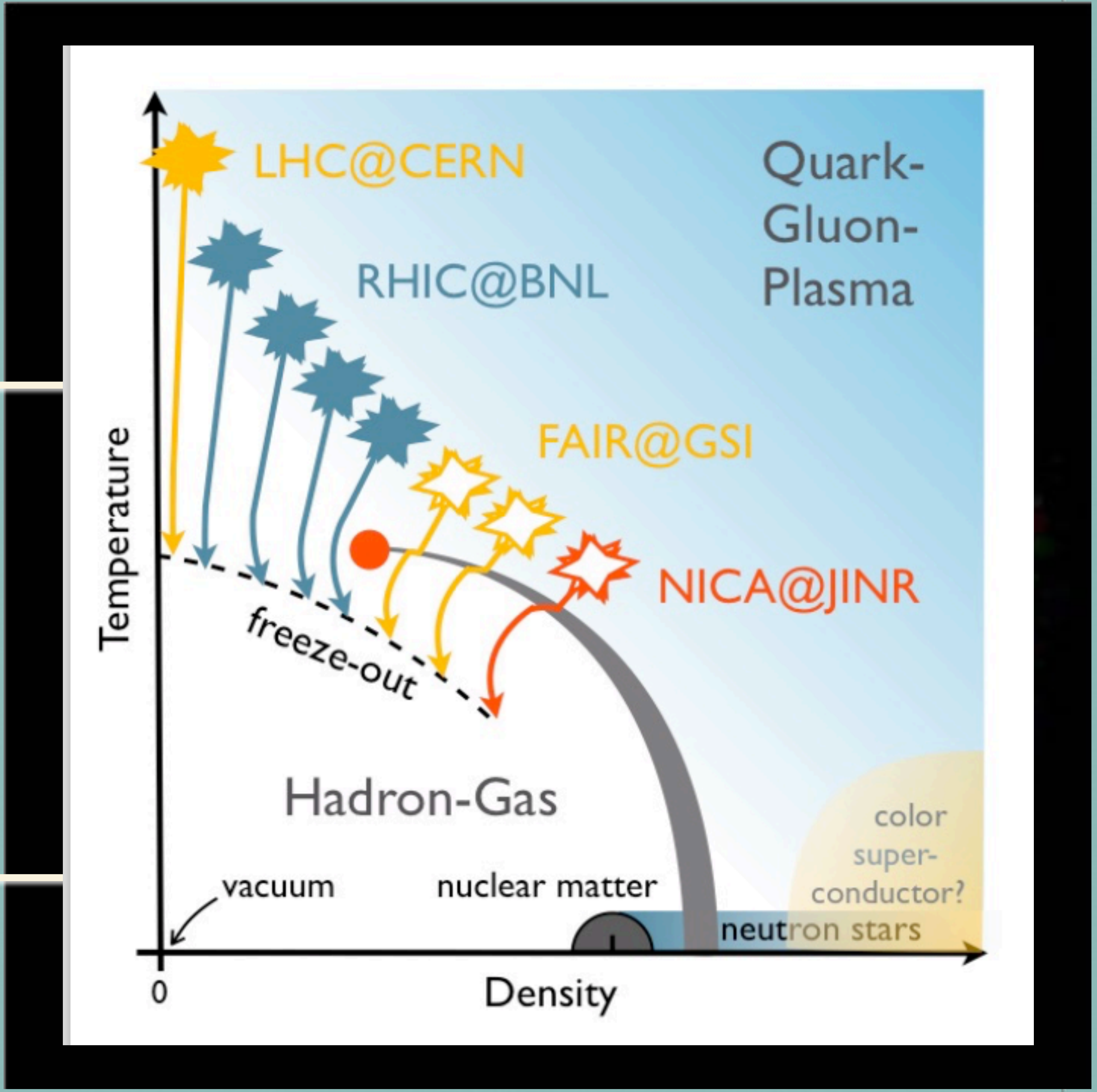

Courtesy of Brookhaven National Laboratory

QCD Phase Diagram

QGP:
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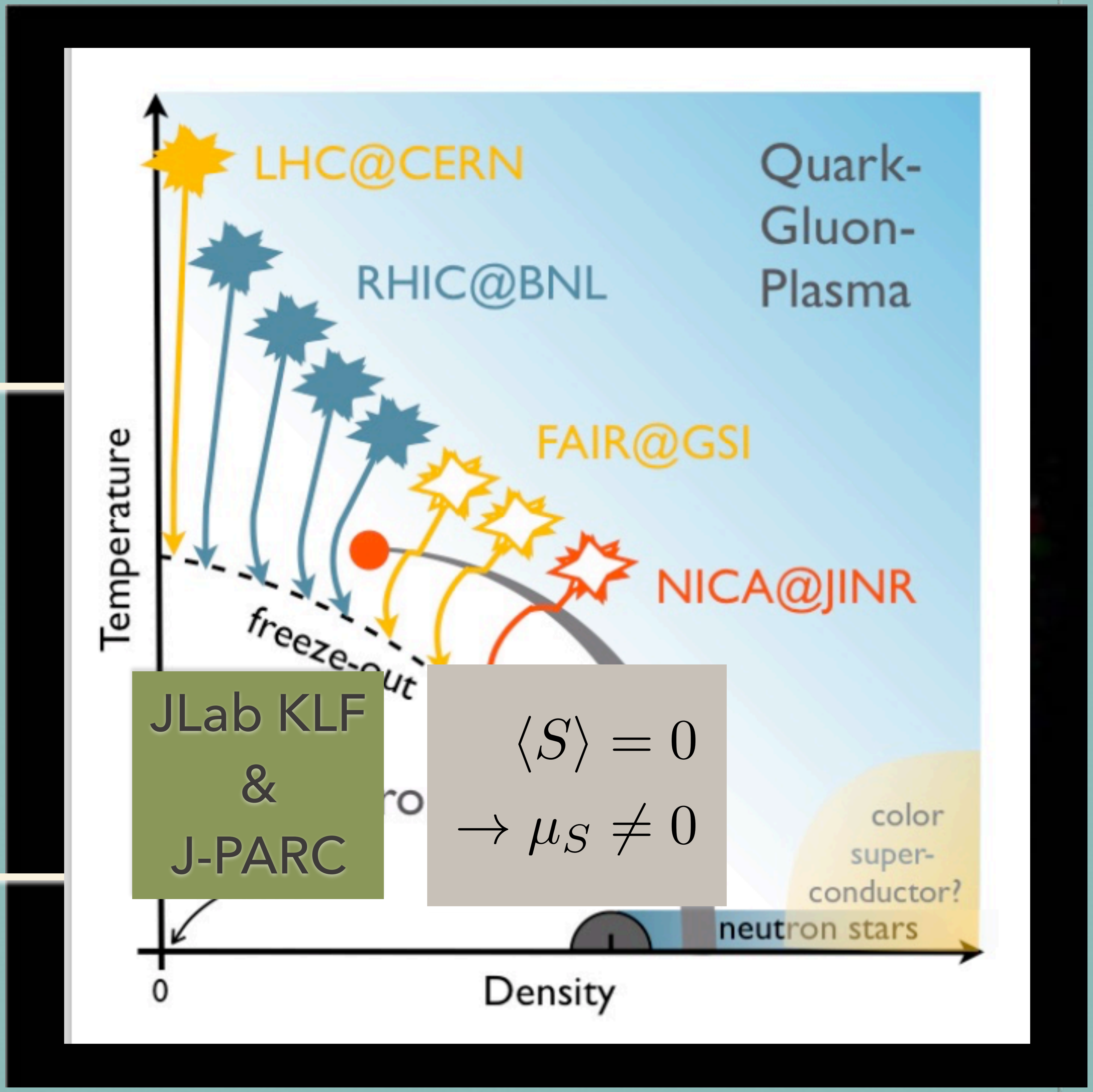
quarks are confined and massive.



Courtesy of Brookhaven National Laboratory

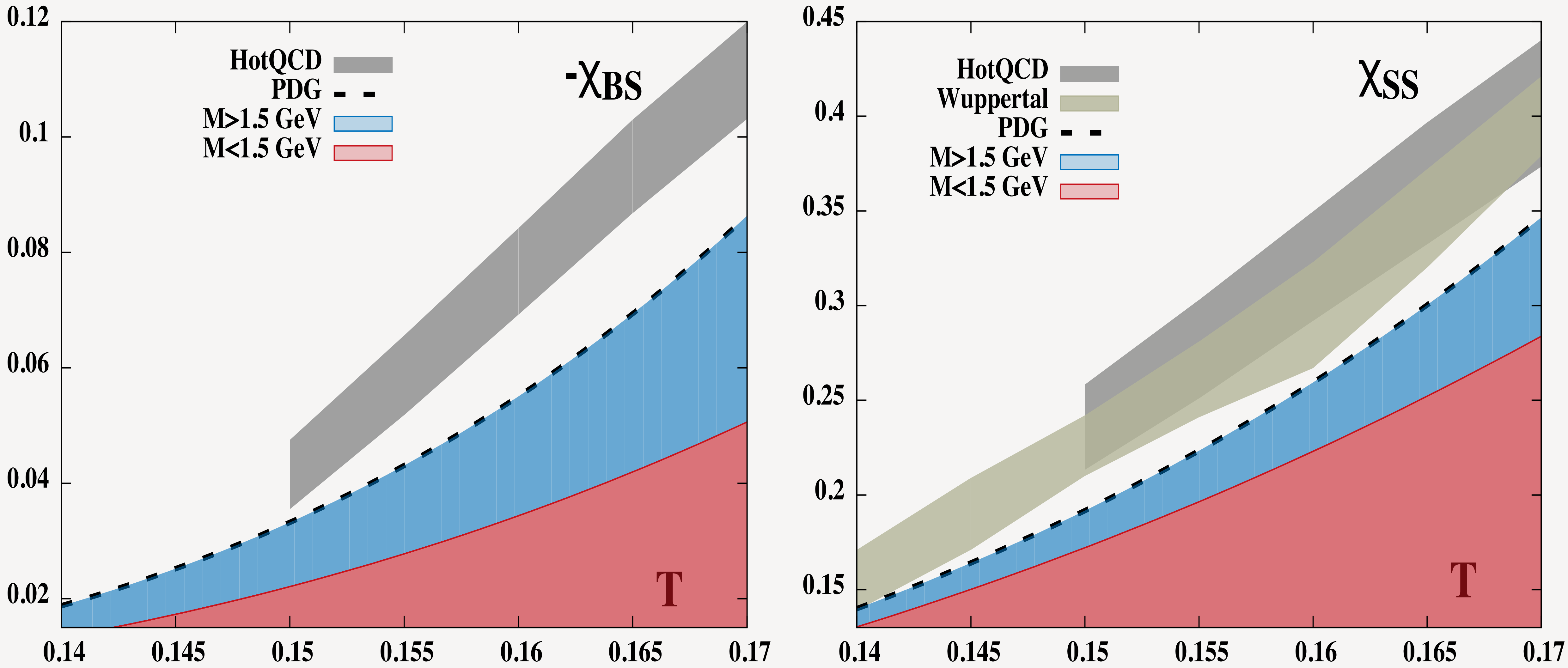
TOWARDS REAL HADRON GAS

- flavor content of hadrons in individual sectors

  -> the case of missing **strange baryons**

- Question the assumption of HRG treatment for resonances:
  -> non-interacting and point-like.

Missing resonances in the strange sector



see also Michael Doering

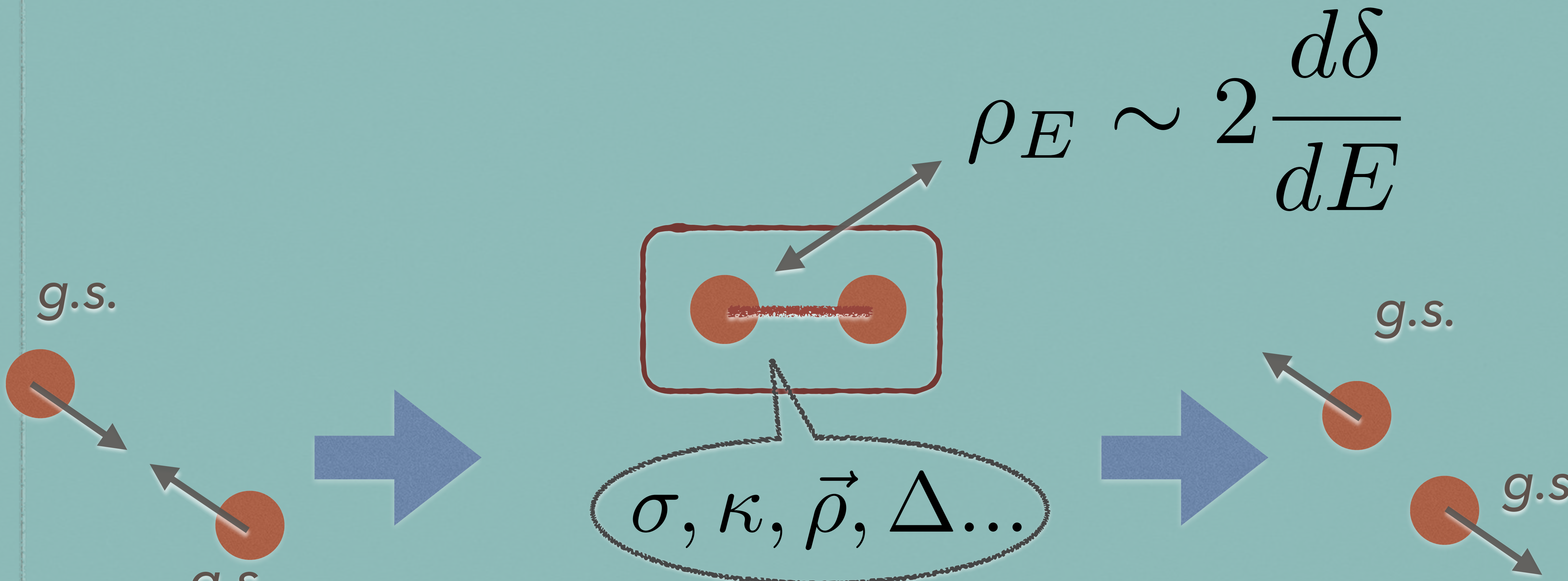
and Jose R. Pelaez

S-MATRIX APPROACH

R. Dashen, S. K. Ma and H. J. Bernstein, Phys. Rev. 187 (1969) 345.

Graham, Quandt, Weigel, Spectral Methods in QFT, Lect. Notes Phys. 777 (2009).

S-MATRIX APPROACH



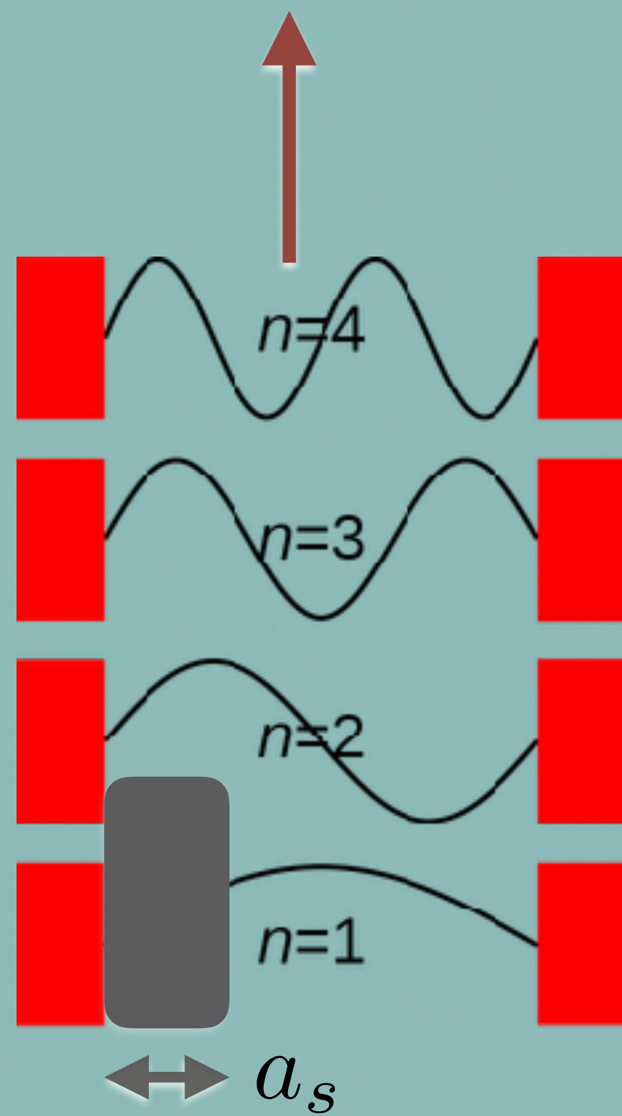
+ repulsive forces

consistent treatment of both attractive and repulsive forces

PHASE SHIFT AND DENSITY OF STATES

particle in a box

with an obstacle



density of states

$$kL + \delta(k) = n\pi \quad \Rightarrow \quad \frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$

S-MATRIX FORMULATION OF THERMODYNAMICS

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

R. Dashen, S. K. Ma and H. J. Bernstein, Phys. Rev. 187 (1969) 345.

A SIMPLE TRICK

$$S_E = e^{2i\delta_E}$$

$$\frac{1}{4\pi i}\,\mathrm{tr}\left\{S_E^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}S_E\right\}_c$$

$$= \frac{1}{2\pi}\times 2\,\frac{\partial}{\partial E}\,\boxed{\frac{1}{2}\,\mathrm{Im}\,\mathrm{tr}\left\{\ln S_E\right\}}$$

$$\Delta \ln Z = \int dE\, e^{-\beta E}\times\frac{1}{\pi}\frac{\partial}{\partial E}\mathrm{tr}\,(\delta_E)\,.$$

E. Beth and G. Uhlenbeck,
Physica (Amsterdam) 4, 915 (1937).

FORMULATION

given the exact phase shift $\delta_j(M)$

from theory
χ pt, LQCD
or
from experiment

thermodynamics

$$B_j = 2\frac{d}{dM}\delta_j$$

eff. spectral function

$$P = P^{(0)} + \Delta P^{\mathrm{B.U.}}$$

free gas + interaction

FORMULATION

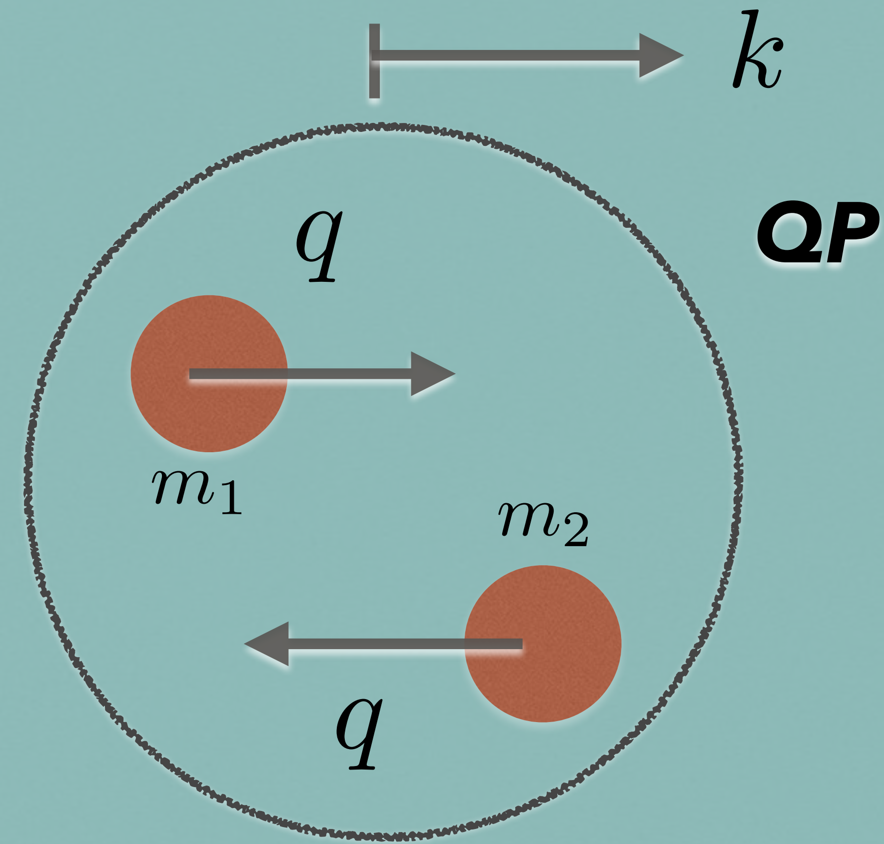
dynamical ***statistical (thermal weight)***

$$\Delta P^{\mathrm{B.U.}} = (2j+1) \int \frac{dM}{2\pi} B_j(M) \int \frac{d^3k}{(2\pi)^3}\, T \ln\left(1 + e^{-\beta E(k,q,m_i)}\right)$$

$$B_j = 2 \frac{d}{dM} \delta_j$$

BW $\rightarrow \dfrac{\gamma}{(M - m_{\mathrm{res}})^2 + \gamma^2/4}$

no width $\rightarrow 2\pi\delta(M - m_{\mathrm{res}})$



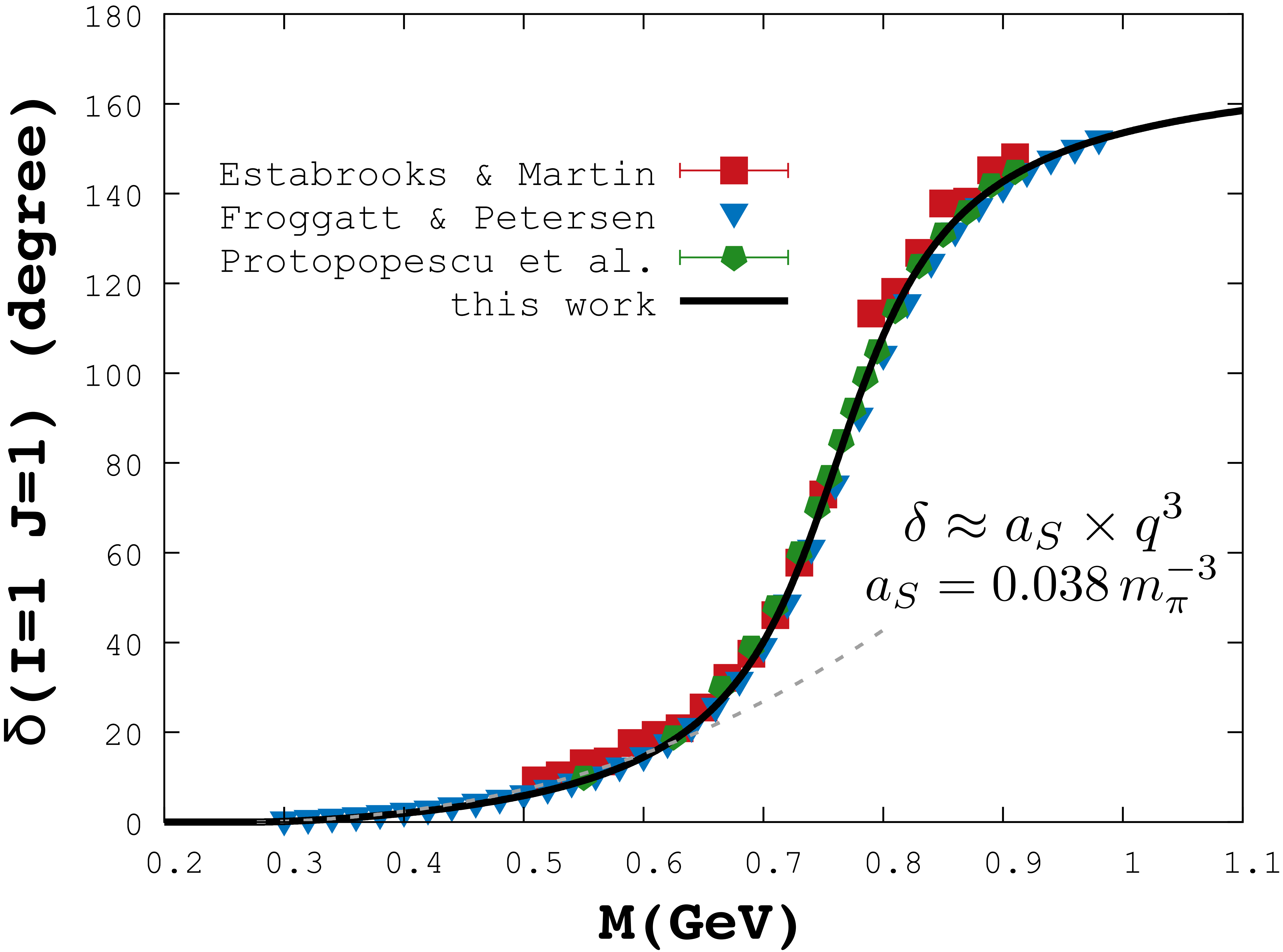
$$M(q) = \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2}$$

WHAT'S IN A NAME? THAT WHICH WE CALL A RESONANCES?

- A resonance is MORE than a MASS and a **WIDTH**

$\boxed{\boldsymbol{\rho(770)}\ ^{[h]}}$ $\qquad I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.26 \pm 0.25$ MeV
Full width $\Gamma = 149.1 \pm 0.8$ MeV
$\Gamma_{ee} = 7.04 \pm 0.06$ keV



δ(I=1 J=1) (degree)
M(GeV)
Estabrooks & Martin
Froggatt & Petersen
Protopopescu et al.
this work
$\delta \approx a_S \times q^3$
$a_S = 0.038\, m_\pi^{-3}$
0
20
40
60
80
100
120
140
160
180
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1

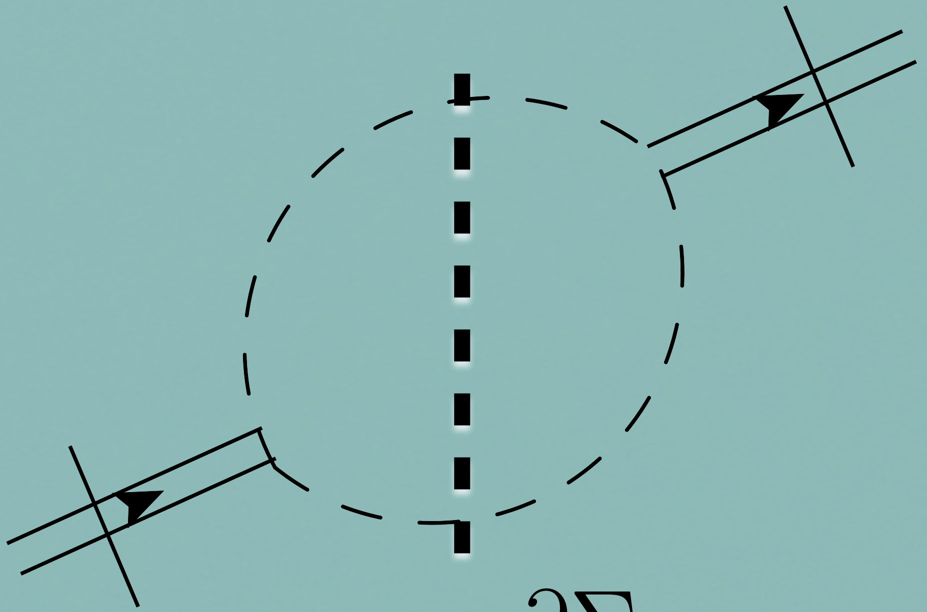
BETH-UHLENBECK APPROXIMATION

$$\delta = -\mathrm{Im}\,\mathrm{Tr}\ln G_\rho^{-1}$$

physical interpretation:

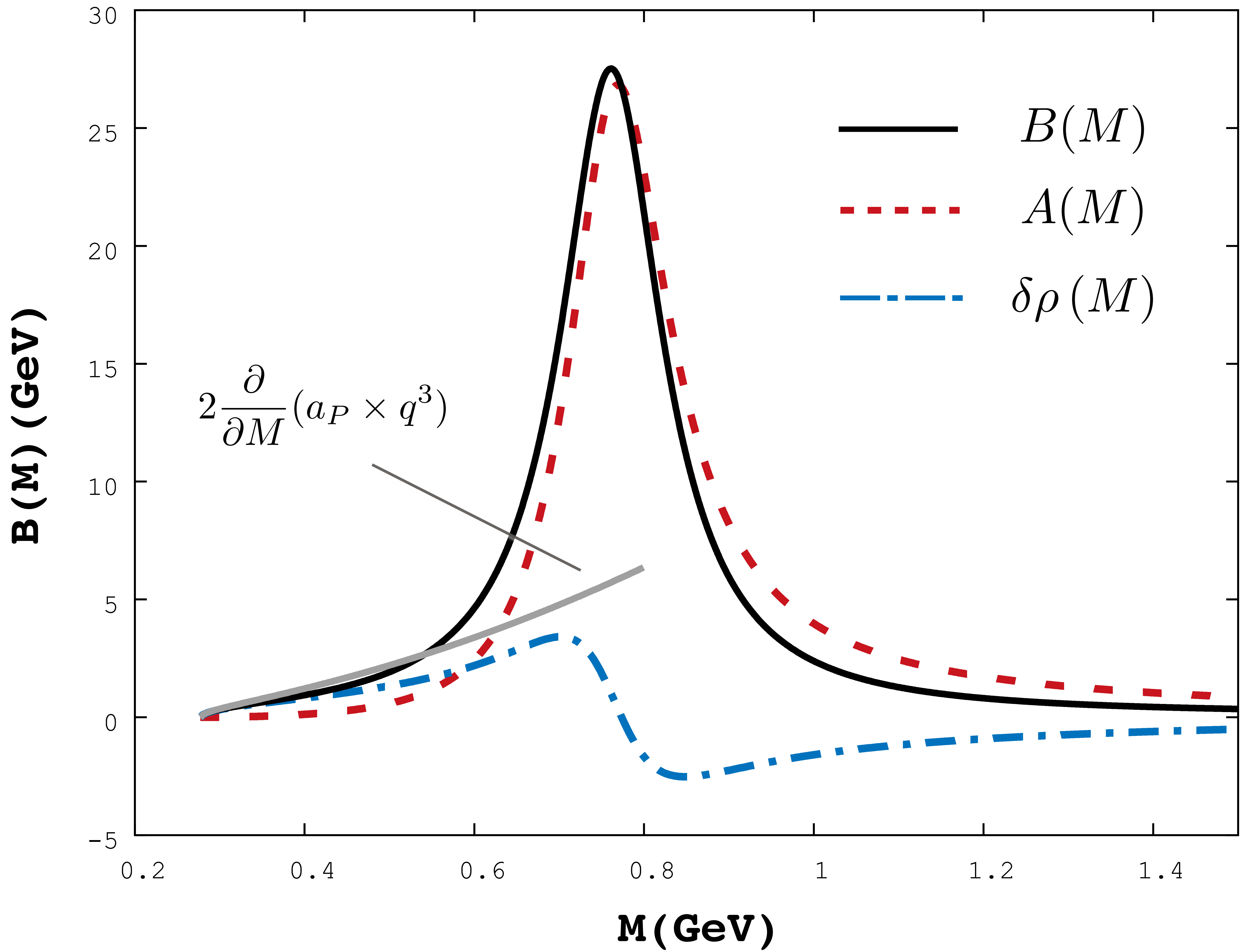
$$\begin{aligned} B &= 2\,\frac{\partial}{\partial E}\delta \\ &= -2\,\mathrm{Im}\frac{\partial}{\partial E}\ln G_\rho^{-1} \\ &= -2\,\mathrm{Im}[G_\rho](2E) + 2\,\mathrm{Im}[\frac{\partial \Sigma_\rho}{\partial E} G_\rho] \end{aligned}$$

contribution from correlated pi pi pair



$$\frac{\partial \Sigma_\rho}{\partial E}$$

$$=> \boxed{\rho_\rho(E) + \delta\rho_\rho(E)}$$

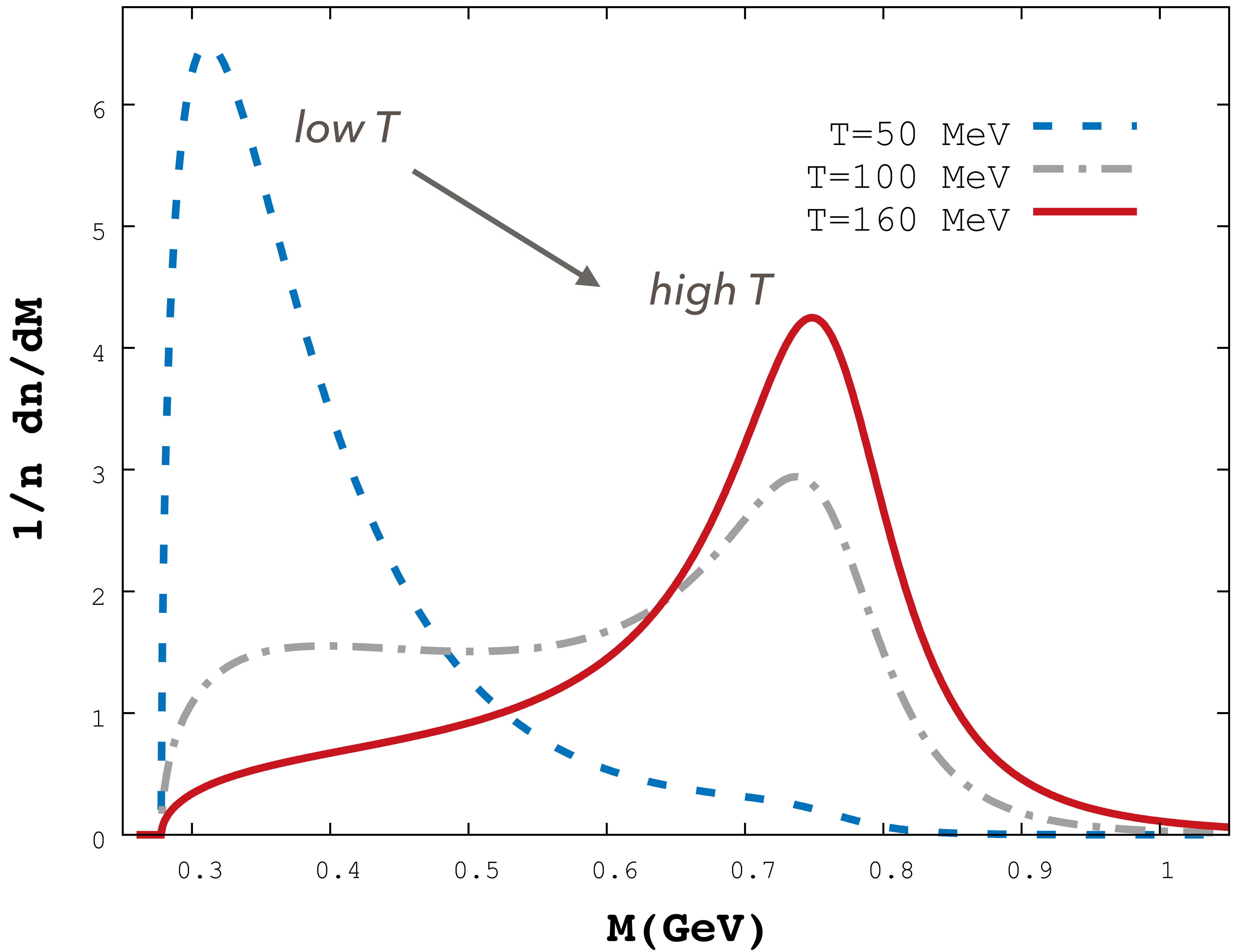


$B(M)$
$A(M)$
$\delta\rho\,(M)$
$2\frac{\partial}{\partial M}(a_P \times q^3)$
B(M) (GeV)
M(GeV)
30
25
20
15
10
5
0
-5
0.2
0.4
0.6
0.8
1
1.2
1.4

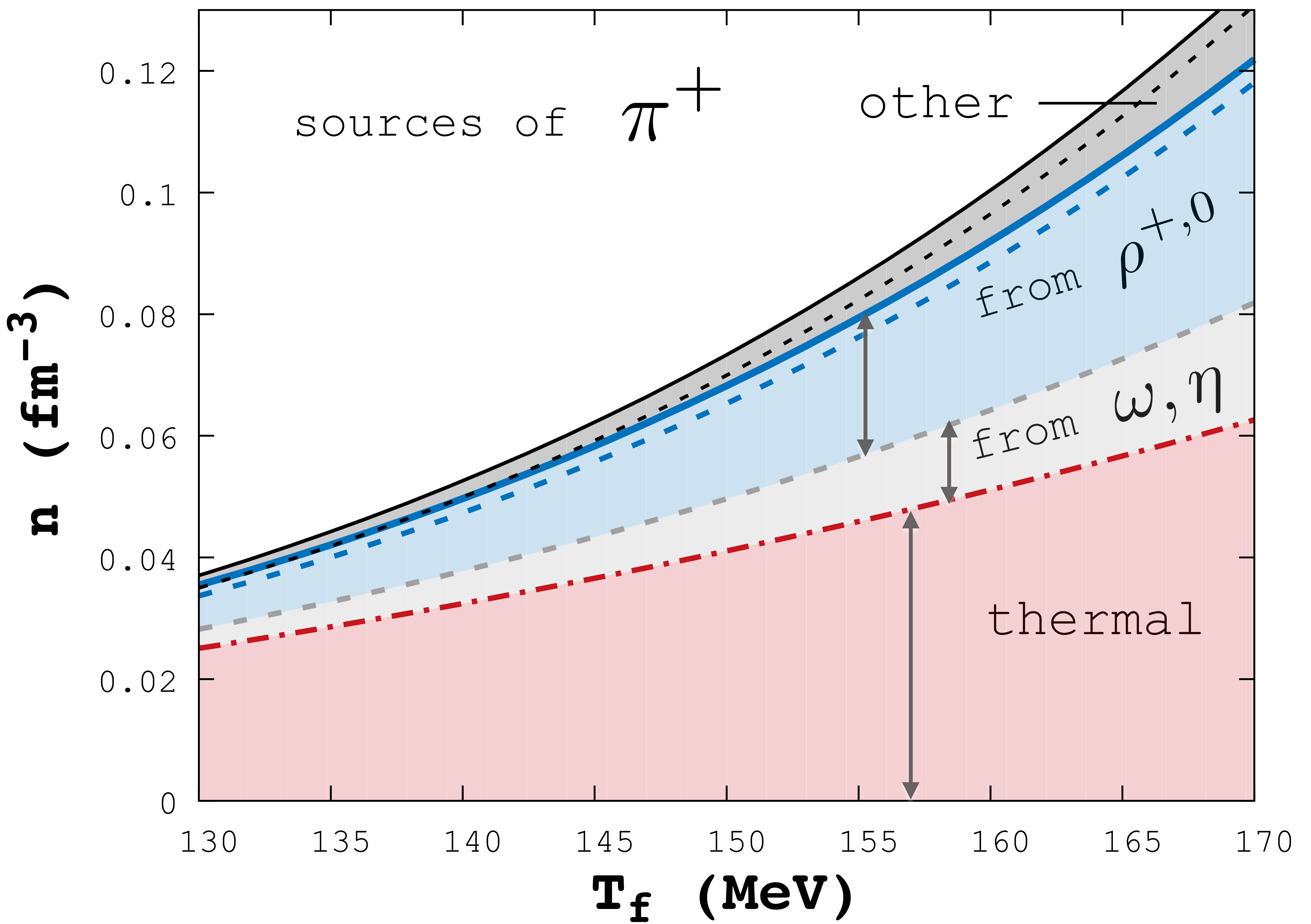
BOLTZMANN SUPPRESSION

$$\Delta P \approx \frac{T^2}{2\pi^2} \int \frac{dM}{2\pi} B(M) \times \boxed{\big(M^2 K_2(M/T)\big)}$$

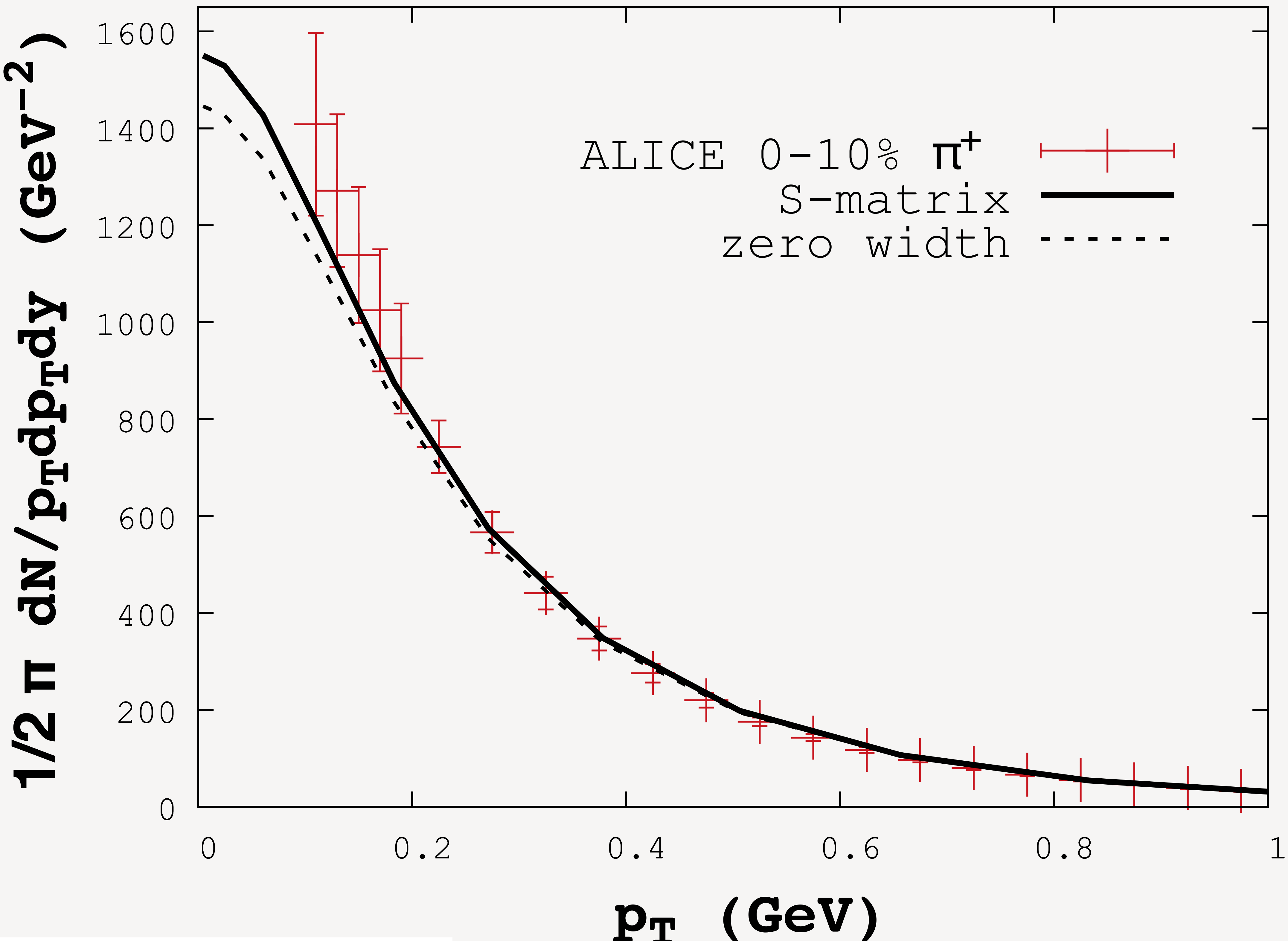
Boltzmann suppression



low T
high T
T=50 MeV
T=100 MeV
T=160 MeV
1/n dn/dM
M(GeV)
0
1
2
3
4
5
6
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



sources of π^+
other
from $\rho^{+,0}$
from ω,η
thermal
n (fm^{-3})
T_f (MeV)
0.12
0.1
0.08
0.06
0.04
0.02
0
130
135
140
145
150
155
160
165
170



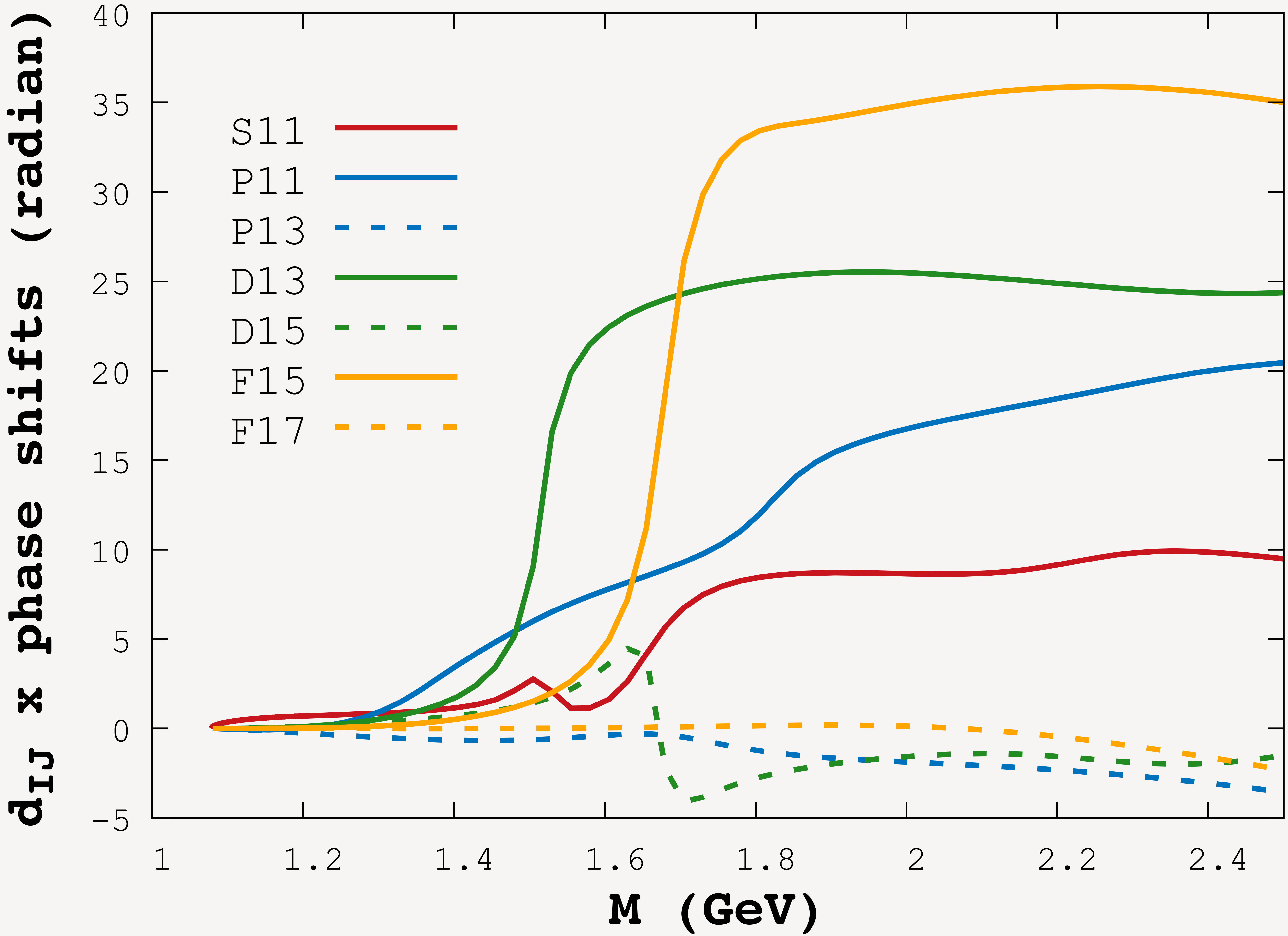
H. Pasi, PML, M. Marczenko,
K. Morita, K. Redlich and C. Sasaki
PLB 769 (2017) 509-512

PI-N SYSTEM

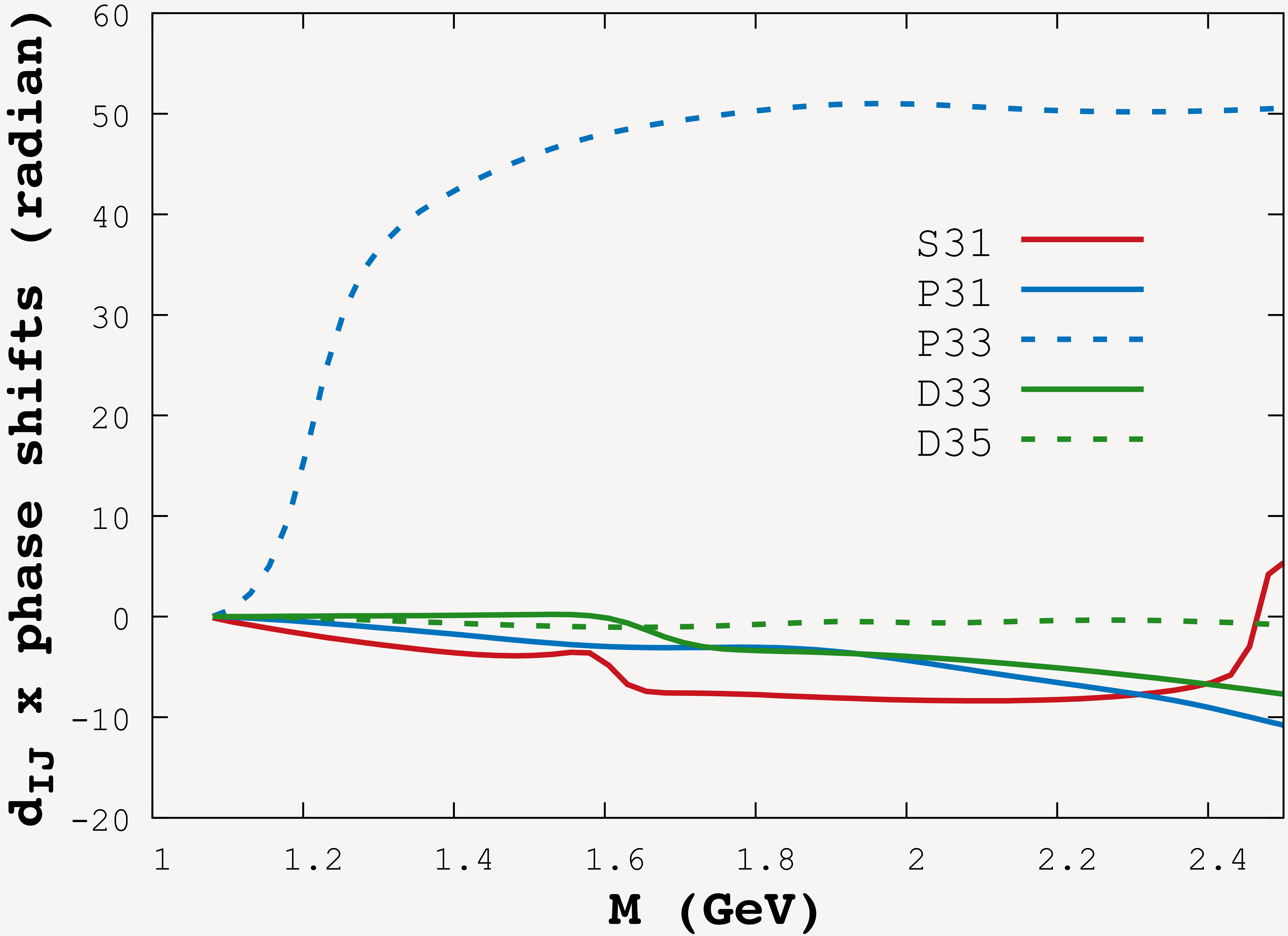
PML, B. Friman, K. Redlich, C. Sasaki,
PLB 778 (2018) 454-458

N* AND DELTAS

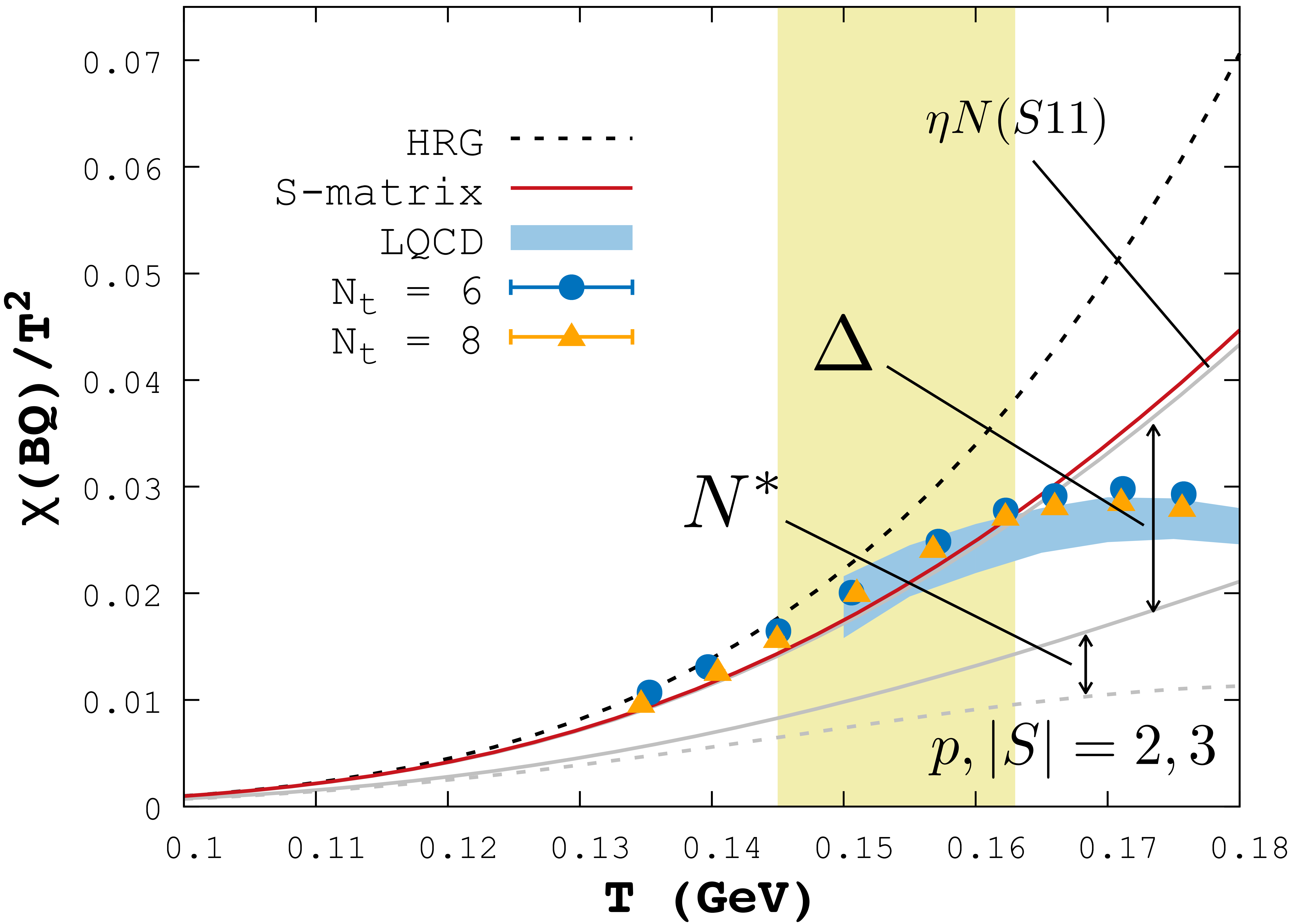
- N*: 1535 (S11), 1440 (P11), 1520 (D13) …
  Δ : 1232 (P33), 1620 (S31) …
- Repulsive forces between pions and nucleons
- BQ-correlation: S = -1 hyperons are excluded!



S11
P11
P13
D13
D15
F15
F17
d_IJ x phase shifts (radian)
M (GeV)
40
35
30
25
20
15
10
5
0
-5
1
1.2
1.4
1.6
1.8
2
2.2
2.4



d_{IJ} x phase shifts (radian)
60
50
40
30
20
10
0
−10
−20
1
1.2
1.4
1.6
1.8
2
2.2
2.4
M (GeV)
S31
P31
P33
D33
D35



HRG
S-matrix
LQCD
$N_t = 6$
$N_t = 8$
$\eta N(S11)$
Δ
N^*
$p, |S| = 2, 3$
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
$\chi(BQ)/T^2$
T (GeV)

KNOWN UNKNOWNS ???

- Inelasticity:

  η production (*ok*)

  multi-pions states (*in progress*)

COUPLED-CHANNEL PROBLEM

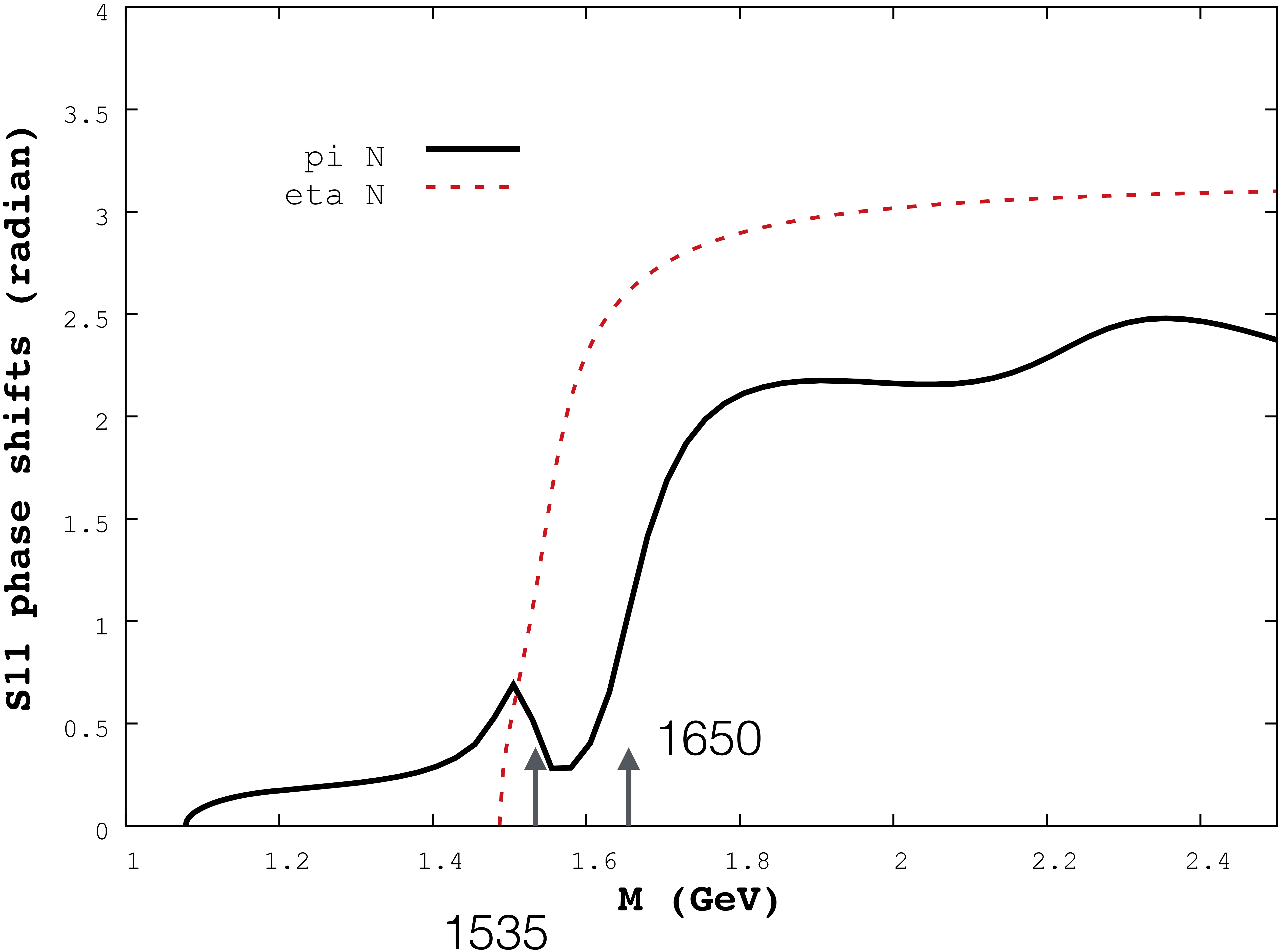
$$S = \begin{pmatrix} \eta\, e^{2\,i\,\delta_I} & i\sqrt{1-\eta^2}\, e^{i\,(\delta_I+\delta_{II})} \\ i\sqrt{1-\eta^2}\, e^{i\,(\delta_I+\delta_{II})} & \eta\, e^{2\,i\,\delta_{II}} \end{pmatrix}$$

$$\begin{aligned} \mathcal{Q}(M) &\equiv \frac{1}{2}\,\mathrm{Im}\,(\mathrm{tr}\ln\,\mathrm{S}) \\ &= \frac{1}{2}\,\mathrm{Im}\,(\ln\,\det\,[\mathrm{S}]) \\ &= \delta_I + \delta_{II}. \end{aligned}$$

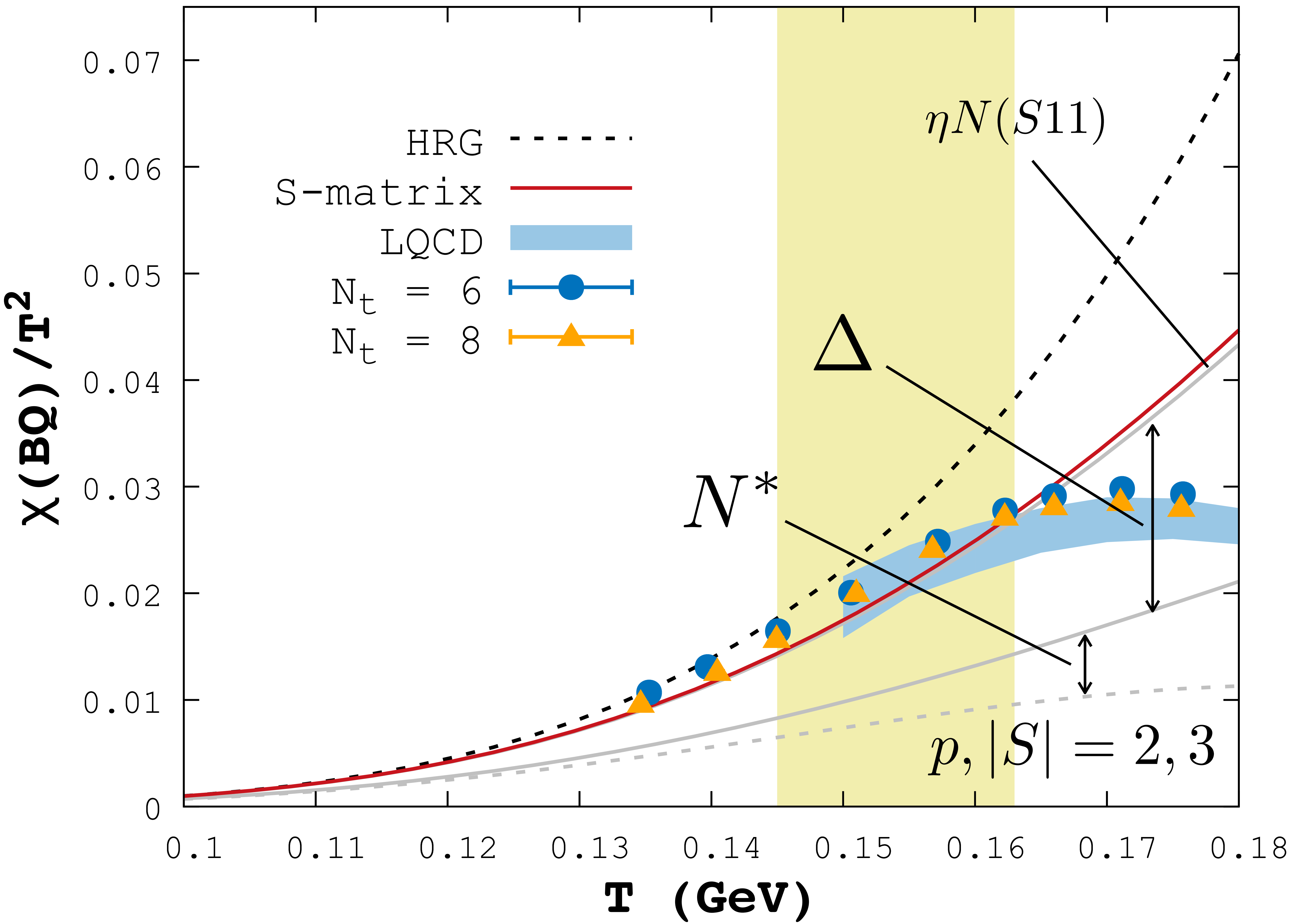
πN *system*

$$\pi N \to \begin{pmatrix} \pi N \\ \eta N \end{pmatrix} \to \pi N$$

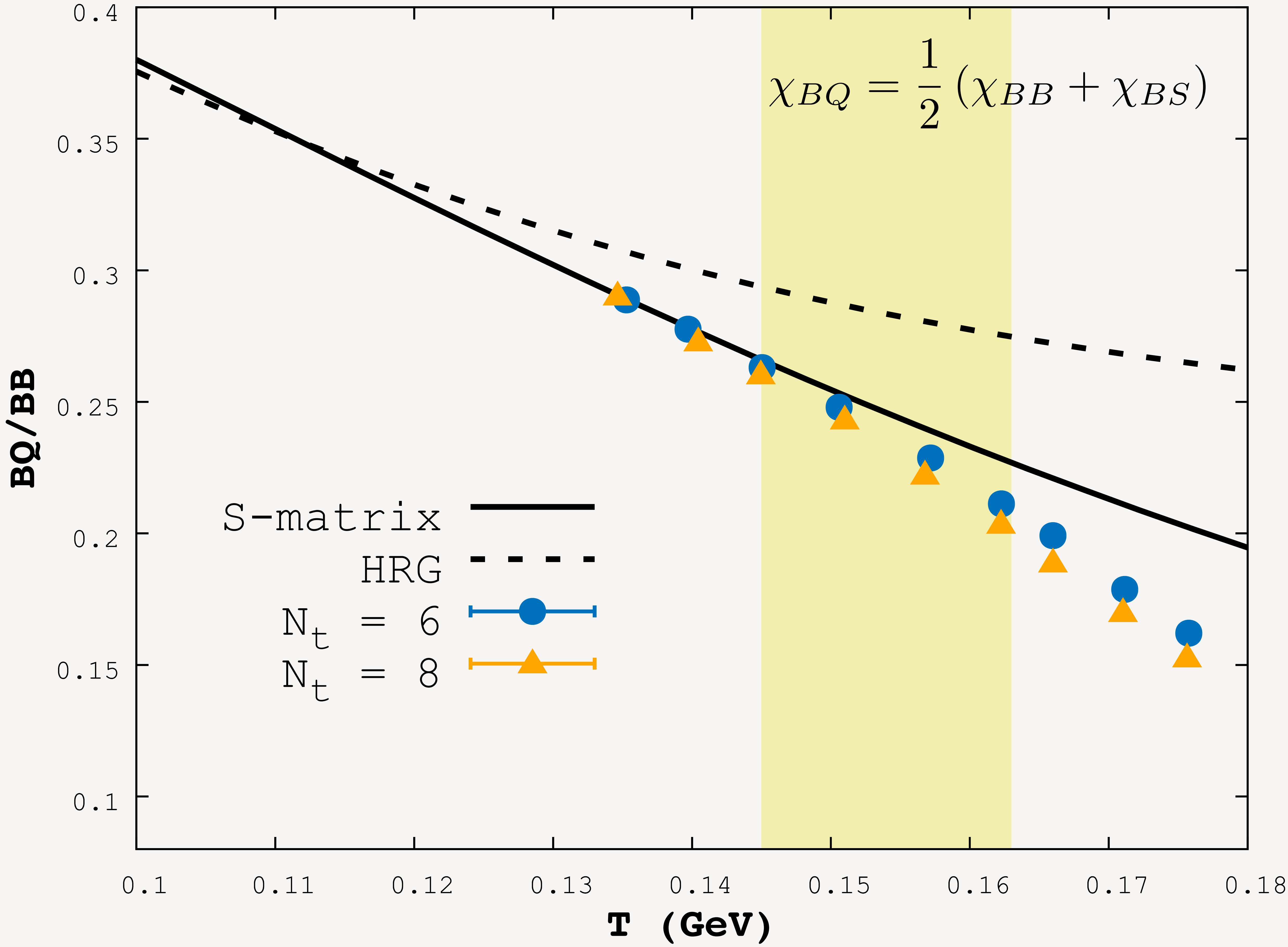
$$\eta N \to \begin{pmatrix} \pi N \\ \eta N \end{pmatrix} \to \eta N$$



S11 phase shifts (radian)
4
3.5
3
2.5
2
1.5
1
0.5
0
pi N
eta N
1650
1
1.2
1.4
1.6
1.8
2
2.2
2.4
M (GeV)
1535



HRG
S-matrix
LQCD
$N_t = 6$
$N_t = 8$
$\eta N(S11)$
Δ
N^*
$p, |S| = 2, 3$
$\chi(BQ)/T^2$
T (GeV)
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18



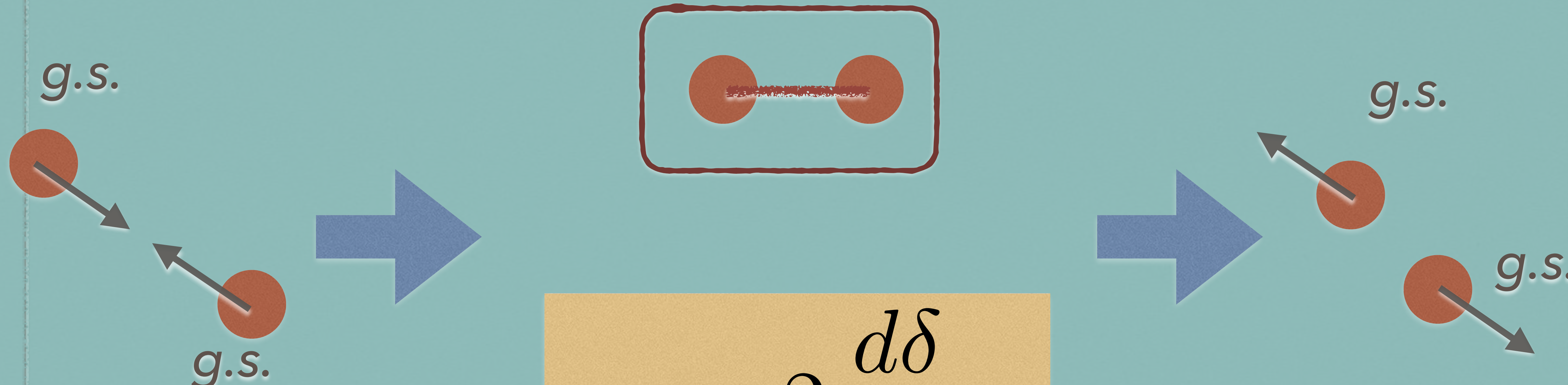
$\chi_{BQ} = \frac{1}{2}(\chi_{BB} + \chi_{BS})$
S-matrix
HRG
$N_t = 6$
$N_t = 8$
BQ/BB
T (GeV)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18

TIME DELAY

P. Danielewicz and S. Pratt
Phys.Rev. C53 (1996) 249-266

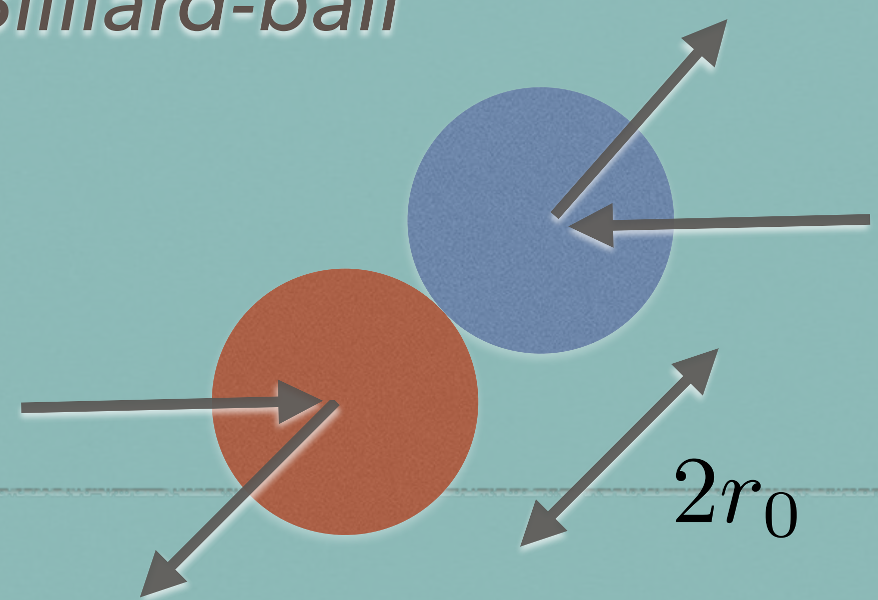
S. Leupold
Nucl.Phys. A695 (2001) 377-394

Yu. B. Ivanov et al
Phys.Atom.Nucl.64:652-669,2001

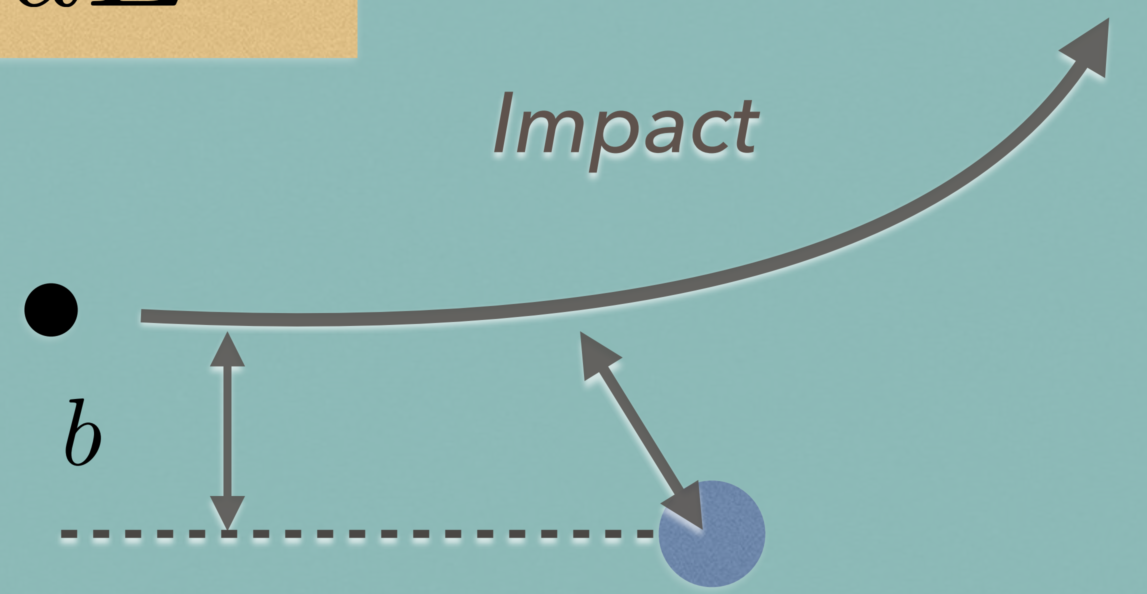


$$\tau \sim 2\frac{d\delta}{dE}$$

Billiard-ball



Impact



CURRENT STATUS

S-MATRIX TREATMENT OF RESONANCES

- mesonic

$\pi\pi \to \sigma, \rho$

$\pi K \to \kappa, K^\star$

$\pi\pi\pi \to \omega$

$\pi\pi \to KK$

$\eta, \phi, \ldots$

$3\pi \to 3\pi$ `triangle`

$4\pi \to 4\pi$ `box`

- baryonic

$\pi N \to N^\star, \Delta$

$NN \to NN$

$\pi N \to \eta N$ `s-wave`

$KN, \pi\Sigma, \pi\Lambda \to \Lambda^\star, \Sigma^\star$

$\pi\pi N \to \rho N, \pi N^\star, \pi\Delta$

STRANGE BARYONS
K N, PI LAMBDA, PI SIGMA …

PHASE SHIFT FROM PWA

```
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Coupled Channels partial wave calculator for KN scattering

by the Joint Physics Analysis Center (JPAC)

Version: September 1, 2015

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Authors:

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Igor V. Danilkin (Jefferson Lab)
Vincent Mathieu (Indiana University)
Adam P. Szczepaniak (Indiana University and Jefferson Lab)

Citation: Fernandez-Ramirez et al., arxiv:1510.07065 [hep-ph]

First version: Cesar Fernandez-Ramirez (Jefferson Lab)
This version: Cesar Fernandez-Ramirez (Jefferson Lab)

Contact: cefera@gmail.com (Cesar Fernandez-Ramirez)

Disclaimers:

1 - This code follows the 'garbage in, garbage out' philosphy. If your
parameters do not make sense, the output will not make sense either.
2 - You can use, share and modify this code under your own responsability.
3 - This code is distributed in the hope that it will be useful,
but WITHOUT ANY WARRANTY; without even the implied warranty of
MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.
4 - No PhD students or postdocs were severely damaged during the development
of this project.

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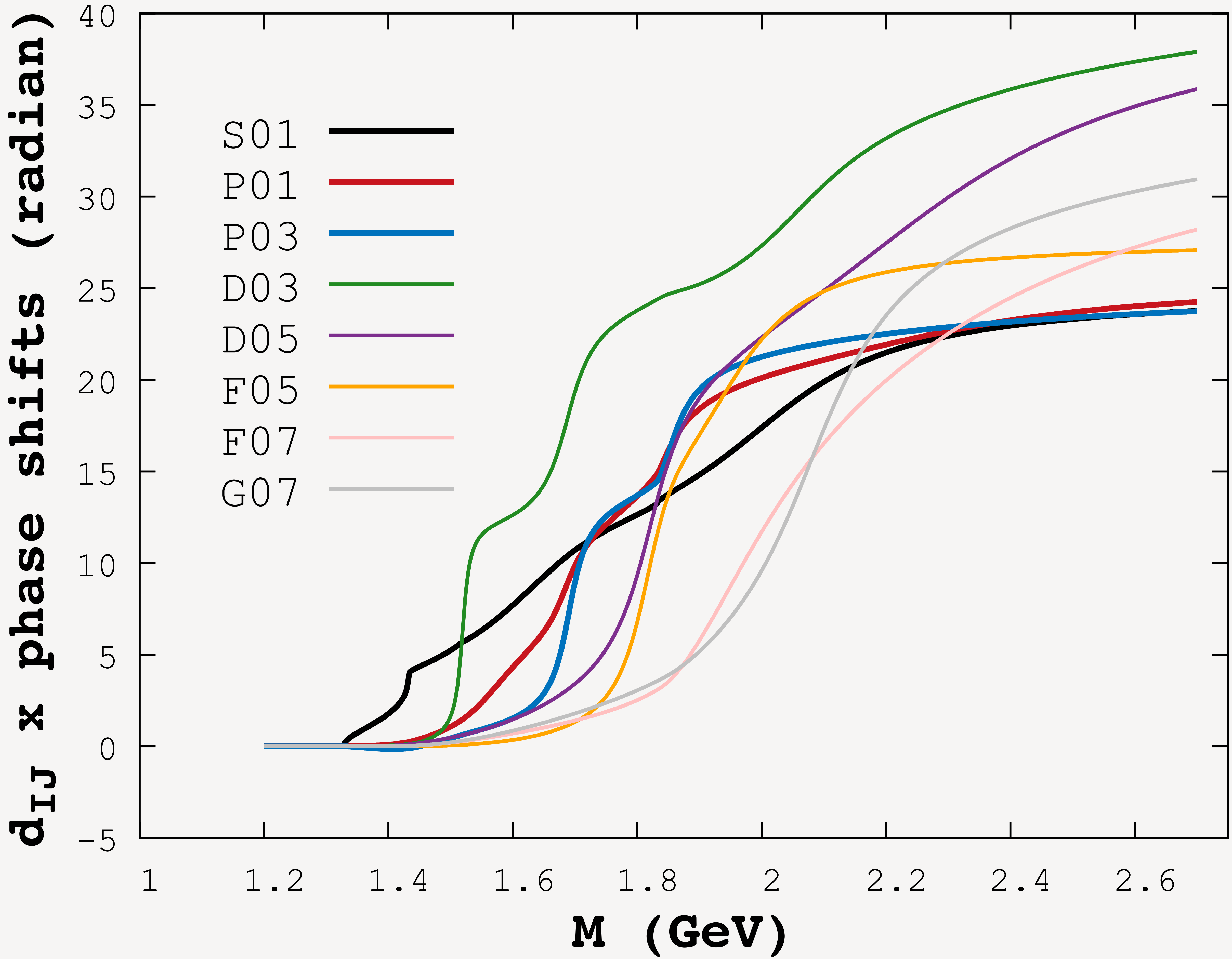
STRANGENESS CONTENT IN A HADRON GAS

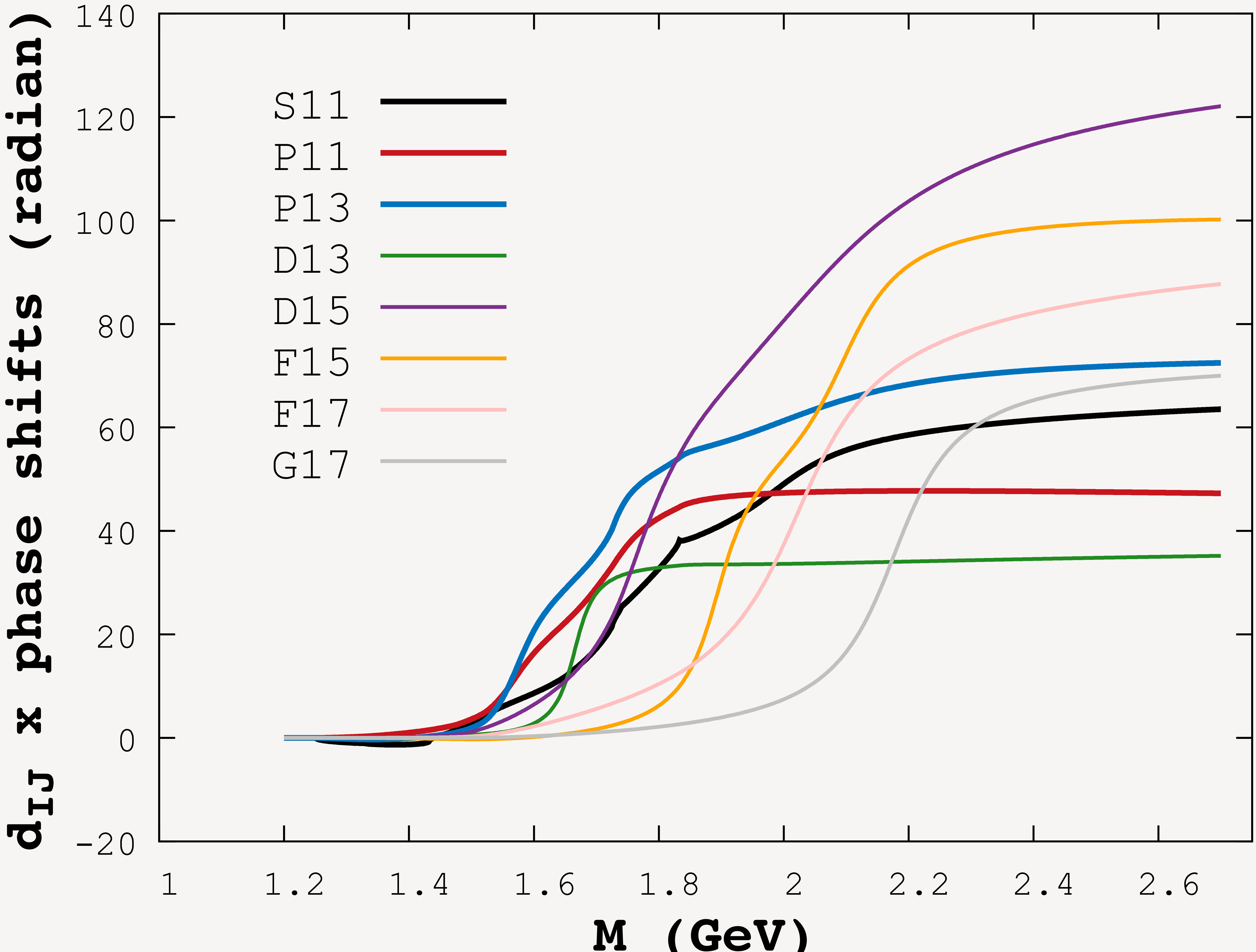
- K-N system requires a coupled channel analysis

$|\bar{K}N\rangle, |\pi\Sigma\rangle, |\pi\Lambda\rangle, |\eta\Lambda\rangle, \ldots$ *16 basis states*

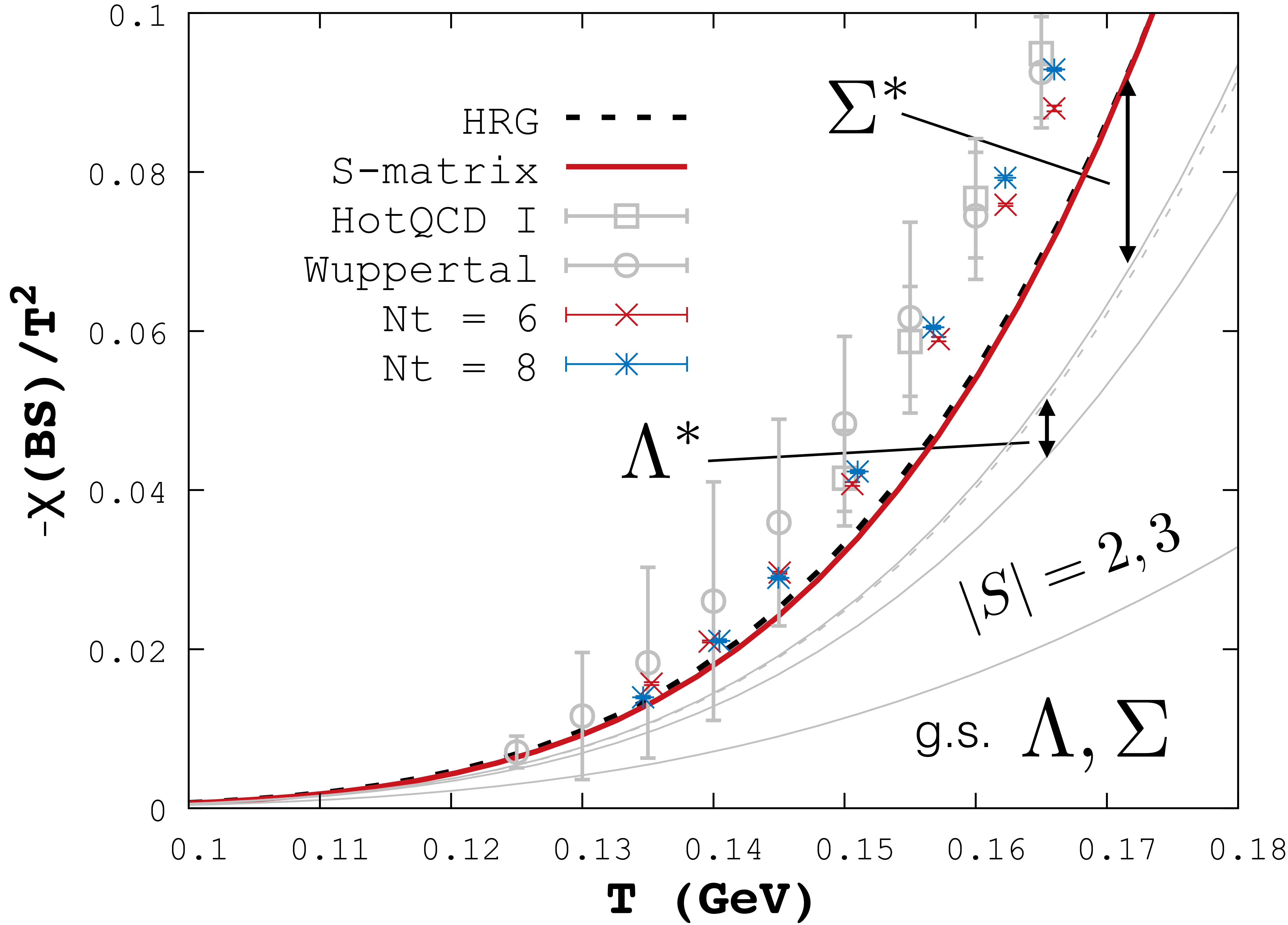
$$\begin{aligned}\mathcal{Q}(M) &\equiv \frac{1}{2}\,\mathrm{Im}\,(\mathrm{tr}\ln\,\mathrm{S}) \\ &= \frac{1}{2}\,\mathrm{Im}\,(\ln\,\det\,[\mathrm{S}]) \\ &= \delta_{KN} + \delta_{\pi\Sigma} + \delta_{\pi\Lambda} + \ldots\end{aligned}$$



d_{IJ} x phase shifts (radian)
M (GeV)
S01
P01
P03
D03
D05
F05
F07
G07
40
35
30
25
20
15
10
5
0
-5
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6

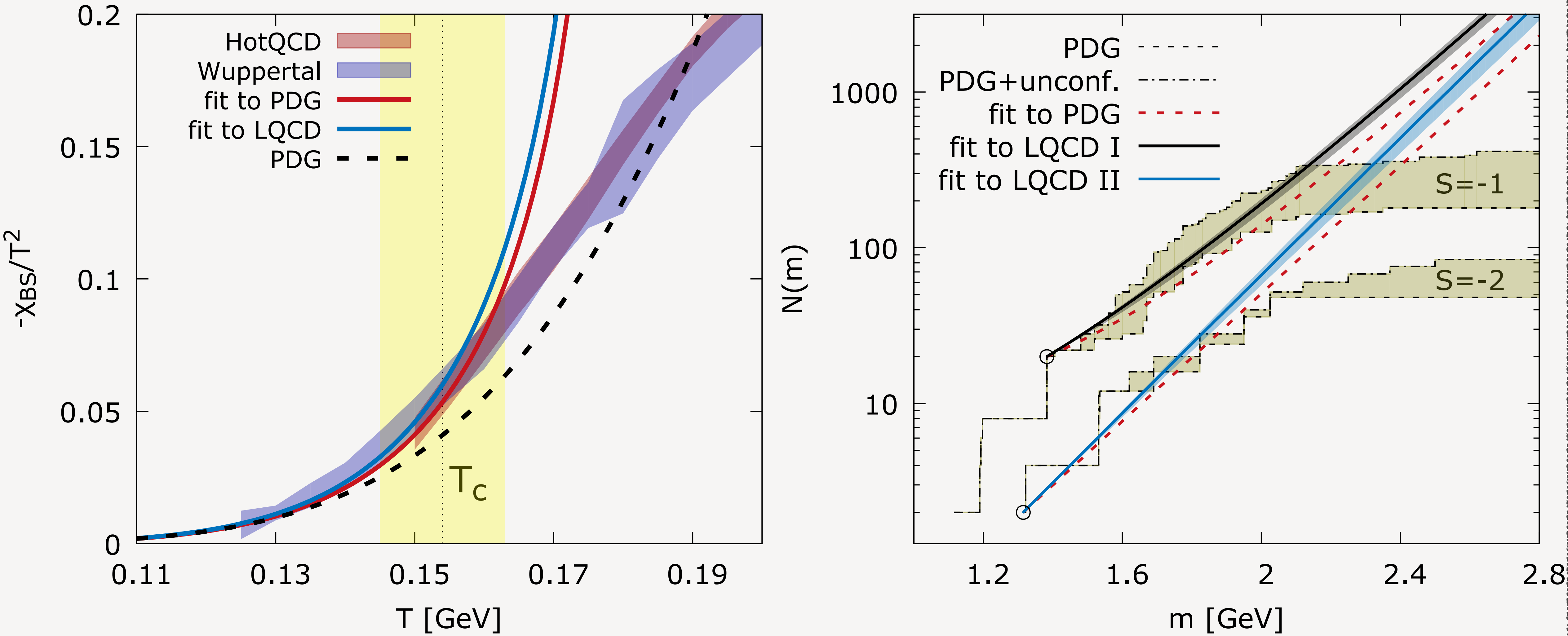


S11
P11
P13
D13
D15
F15
F17
G17
140
120
100
80
60
40
20
0
-20
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
M (GeV)
d_{IJ} x phase shifts (radian)

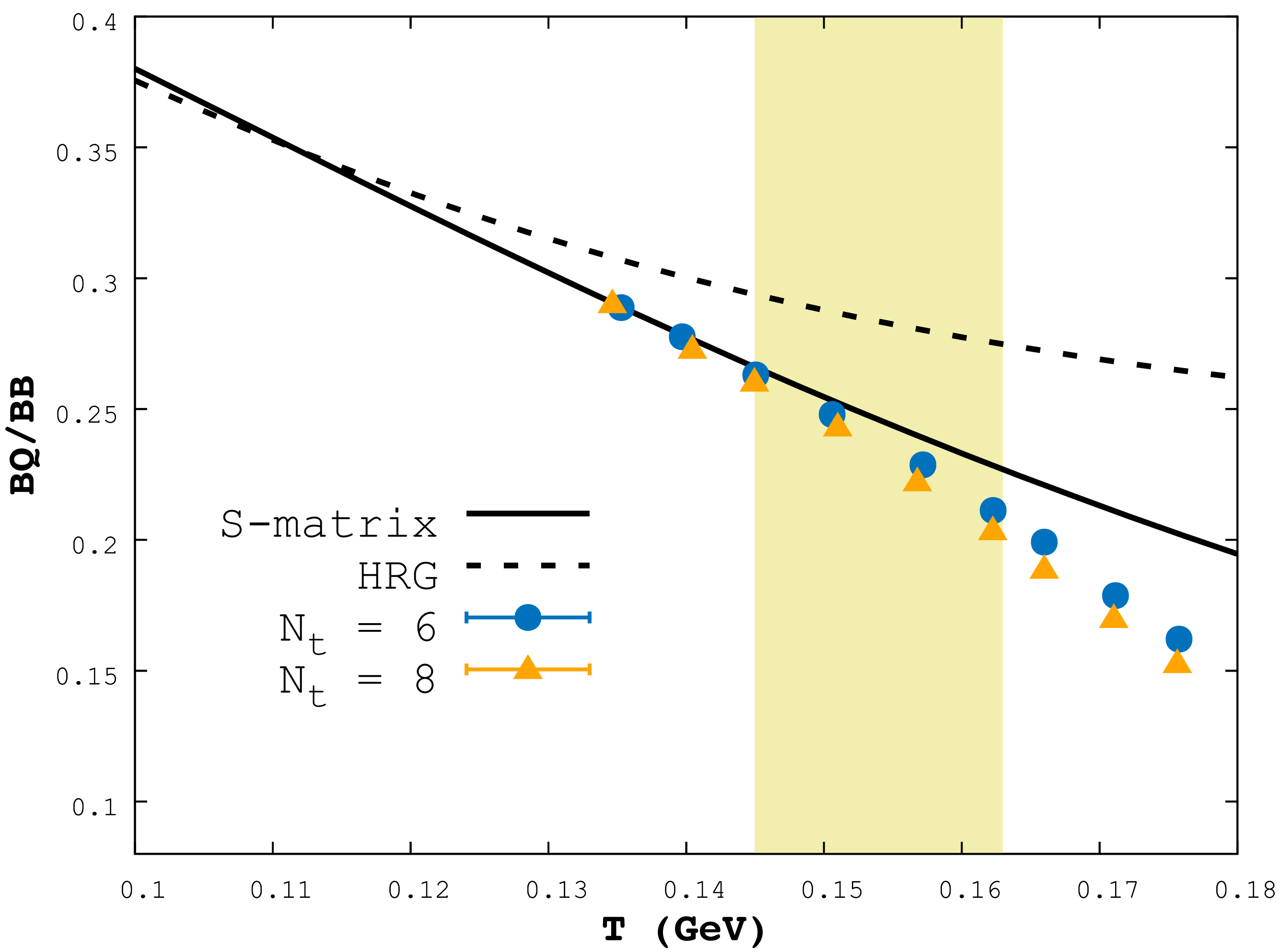


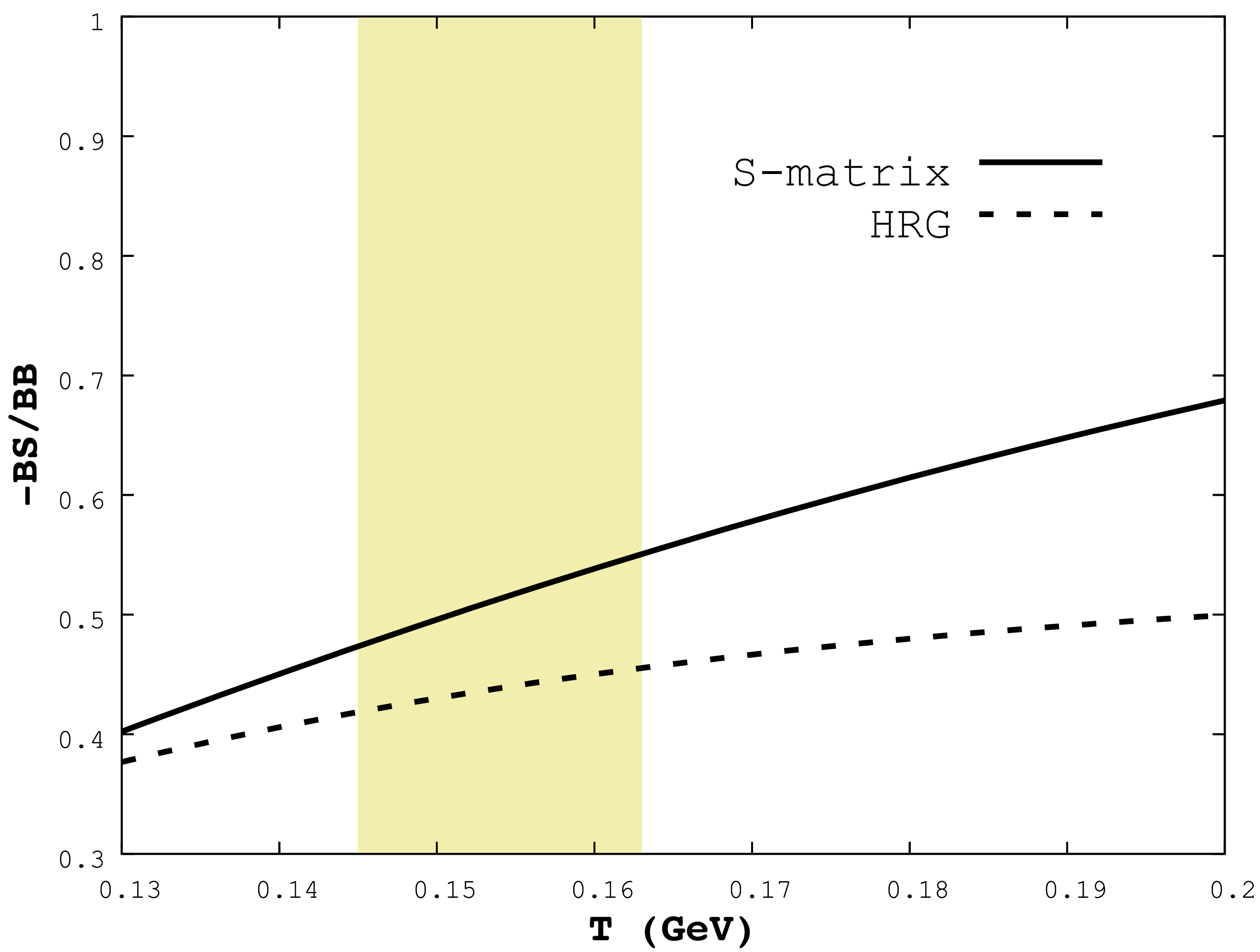
HRG
S-matrix
HotQCD I
Wuppertal
Nt = 6
Nt = 8
Σ^*
Λ^*
$|S| = 2, 3$
g.s. Λ, Σ
$-\chi(\mathrm{BS})/T^2$
T (GeV)
0.1
0.08
0.06
0.04
0.02
0
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18

strange mesons to be discovered...



PML, M. Marczenko, K. Redlich and C. Sasaki
PRC 92 (2015) no.5, 055206

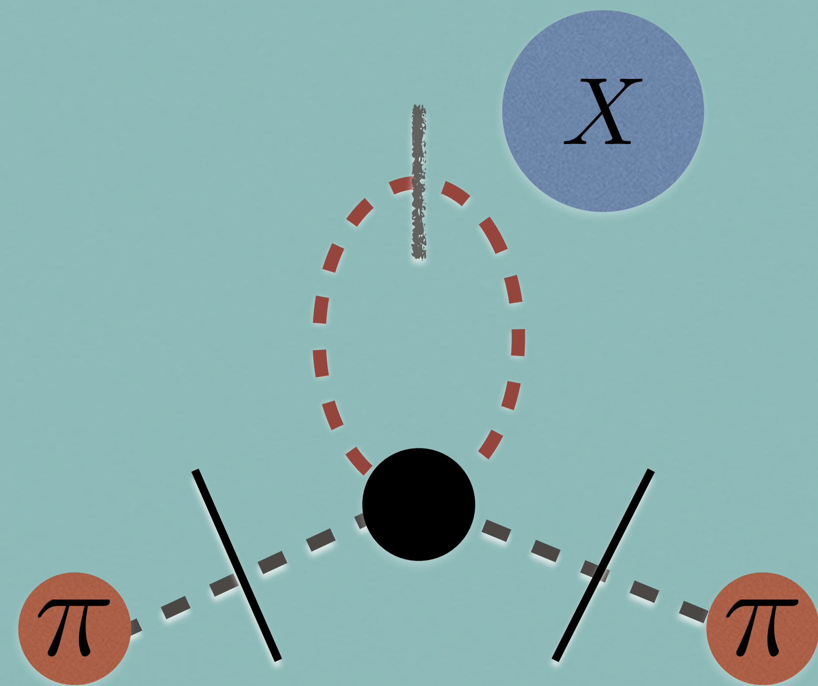




$$2 \times BQ/BB - BS/BB = 1$$

IN MEDIUM EFFECTS

$$\Sigma_\pi =$$

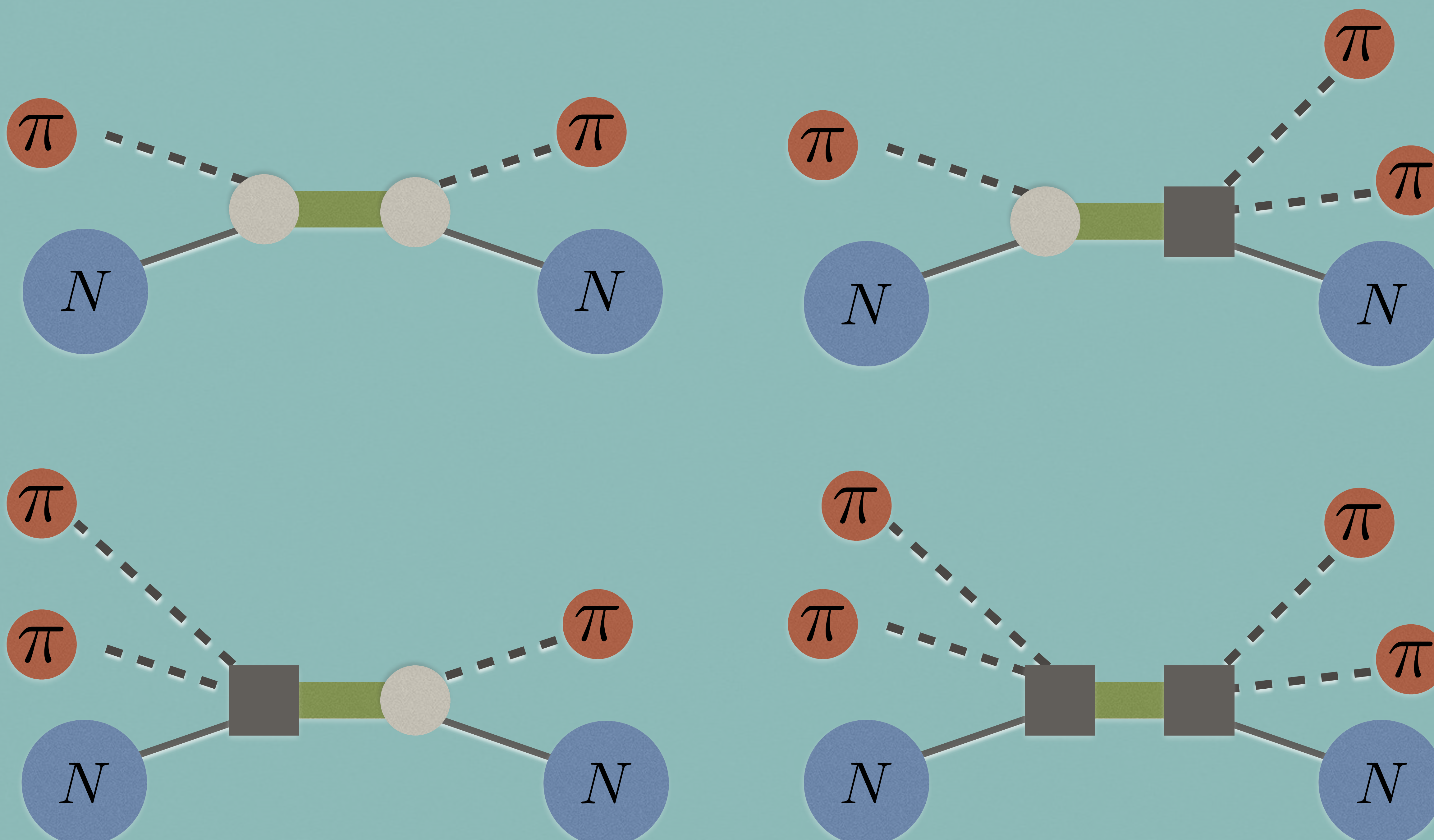


$$\propto \int \frac{d^3 q}{\omega_p \, \omega_q} \, n_X \times T_{\pi X}(s)$$

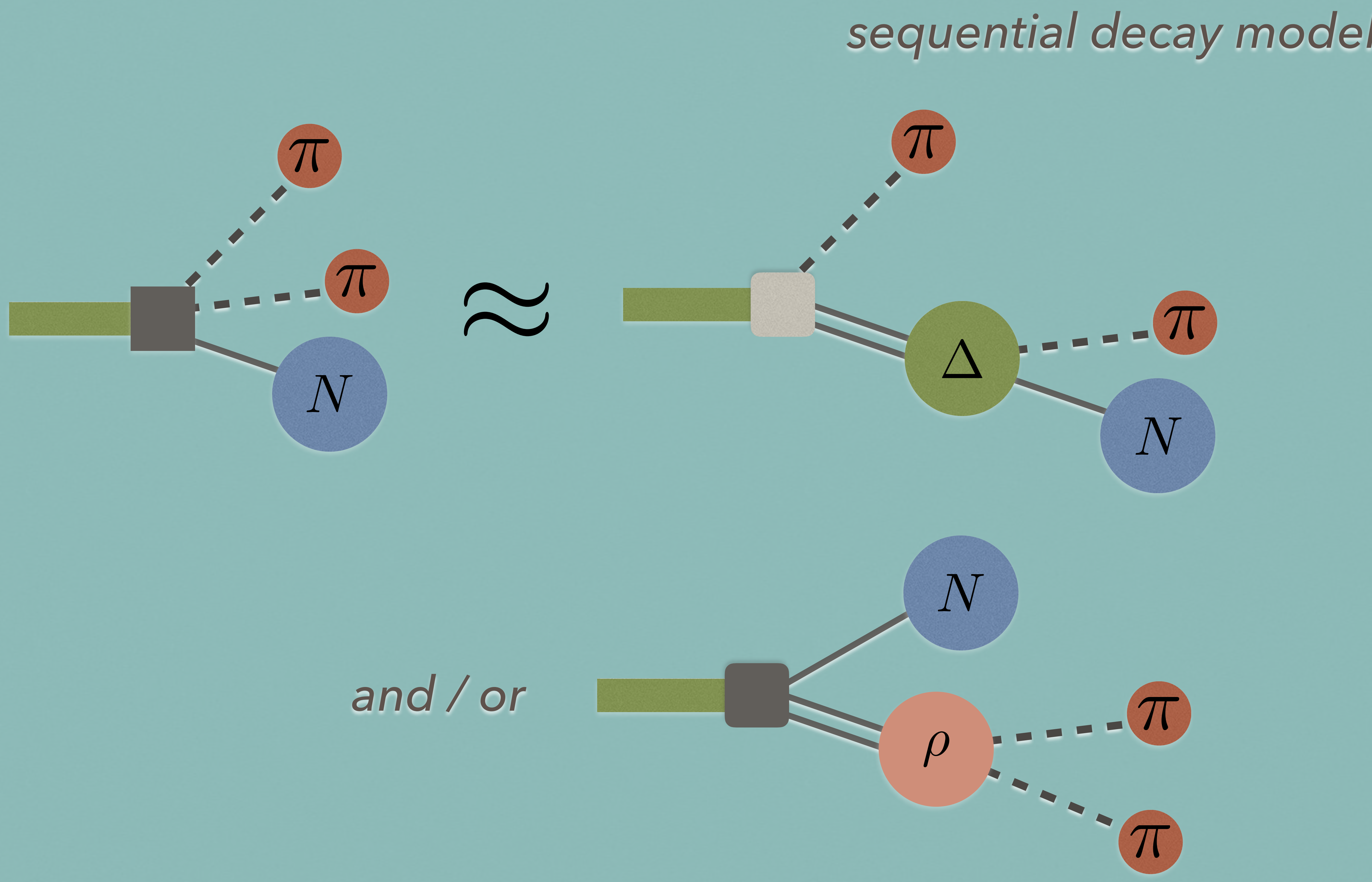
forward amplitude

A. Schenk NPB 363(1991)

ISOBAR MODEL



ISOBAR MODEL

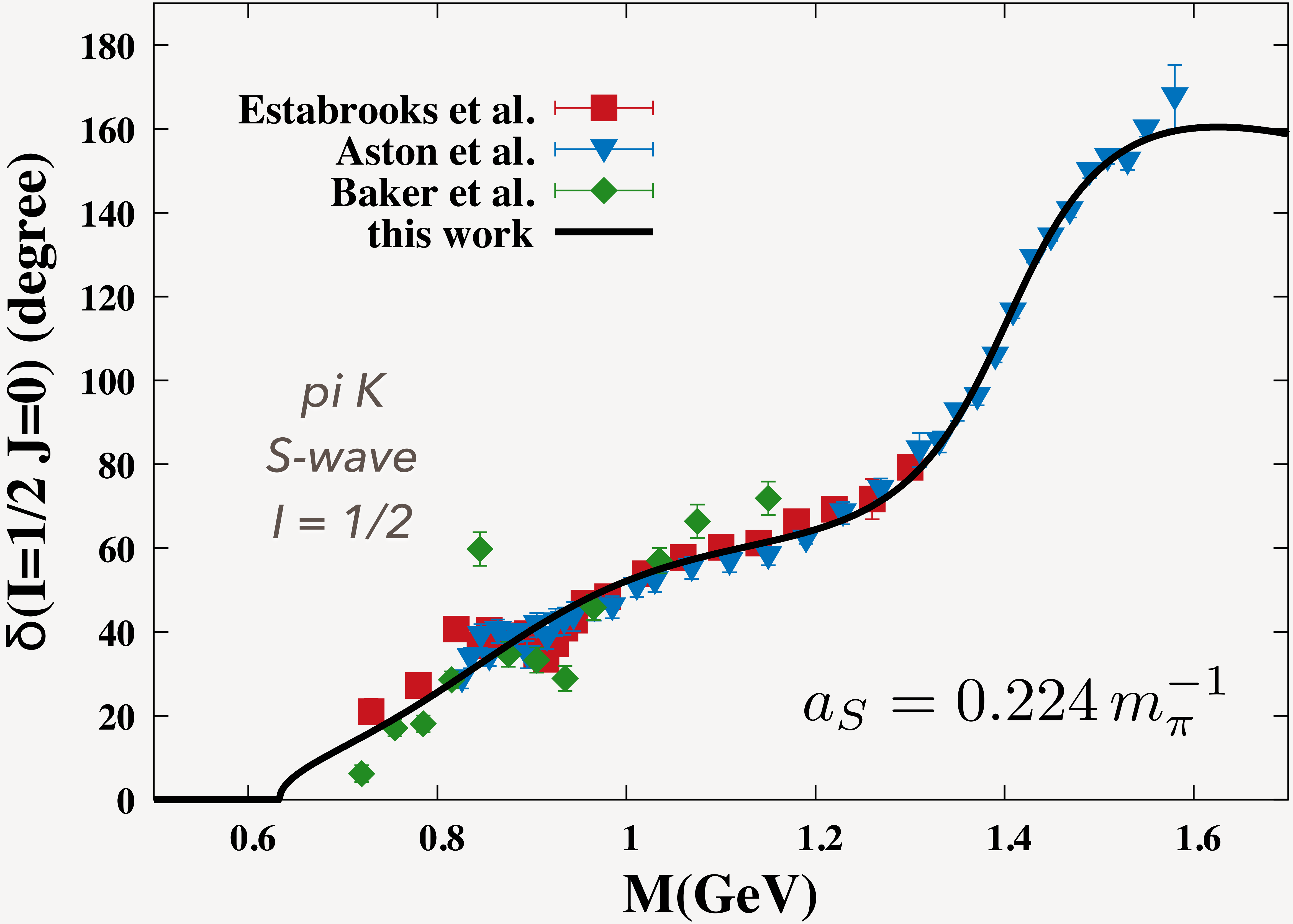


THANK YOU

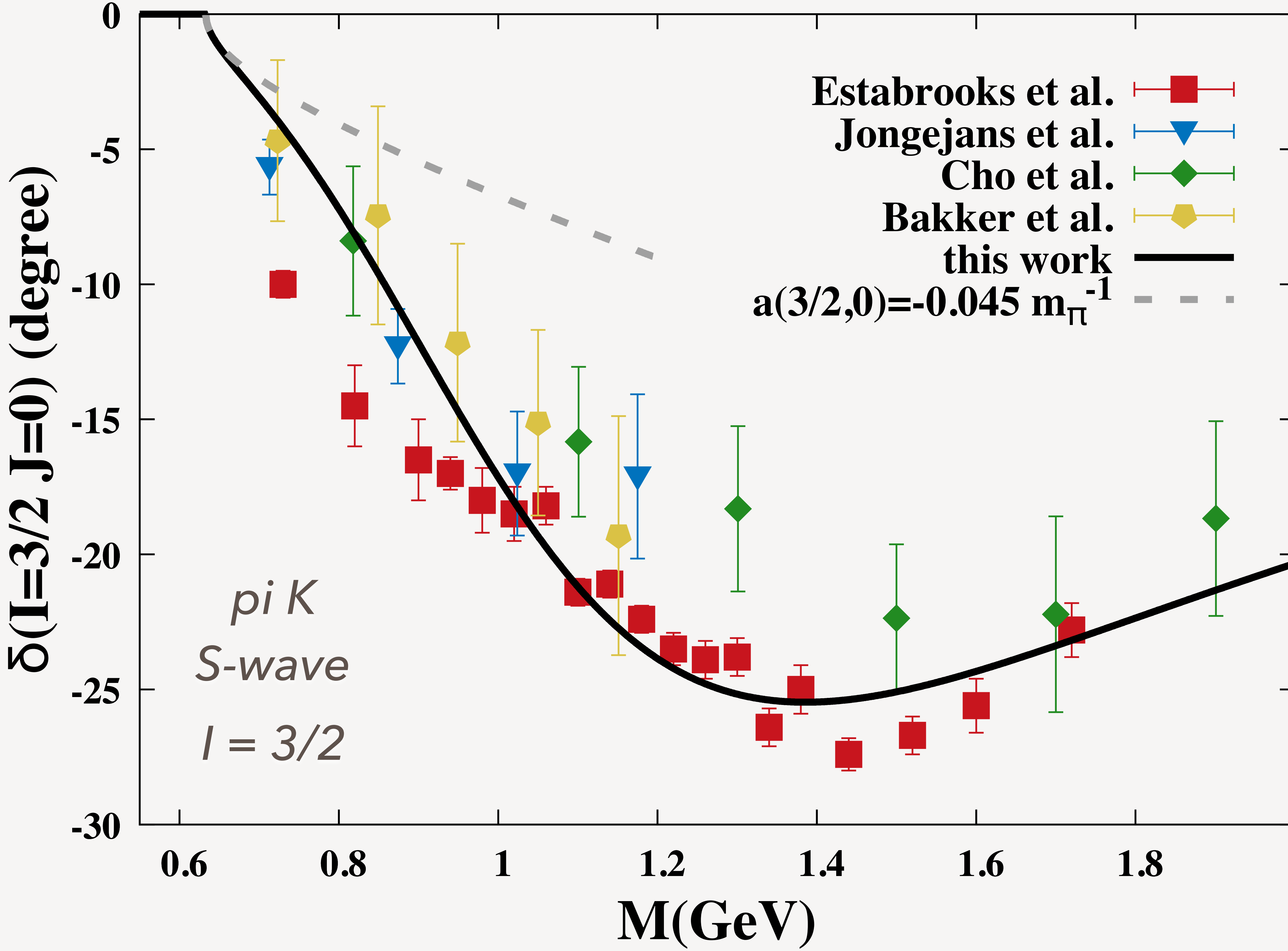
BACKUP

STRANGE MESONS

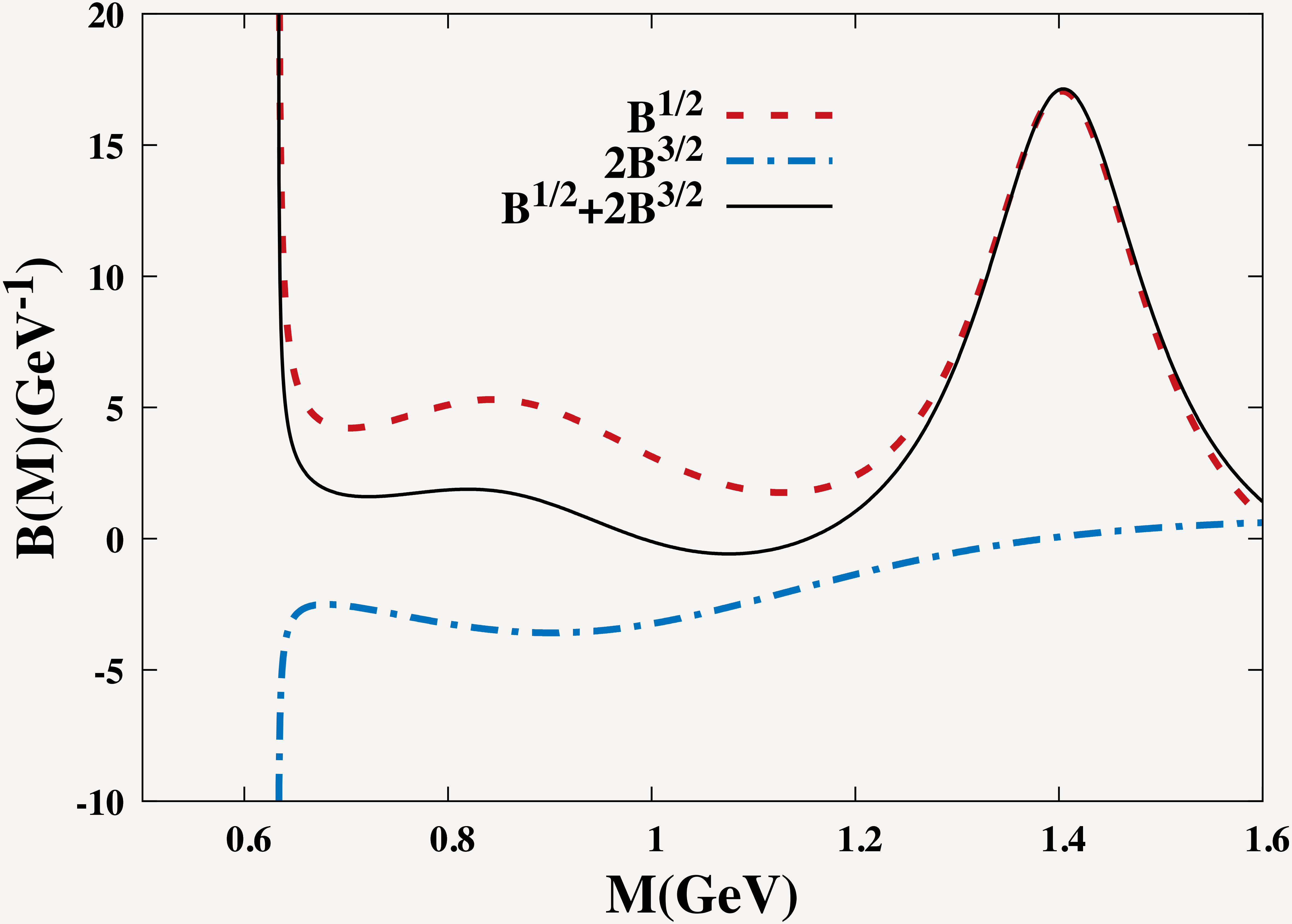
$$\pi K \to \kappa, K^\star$$



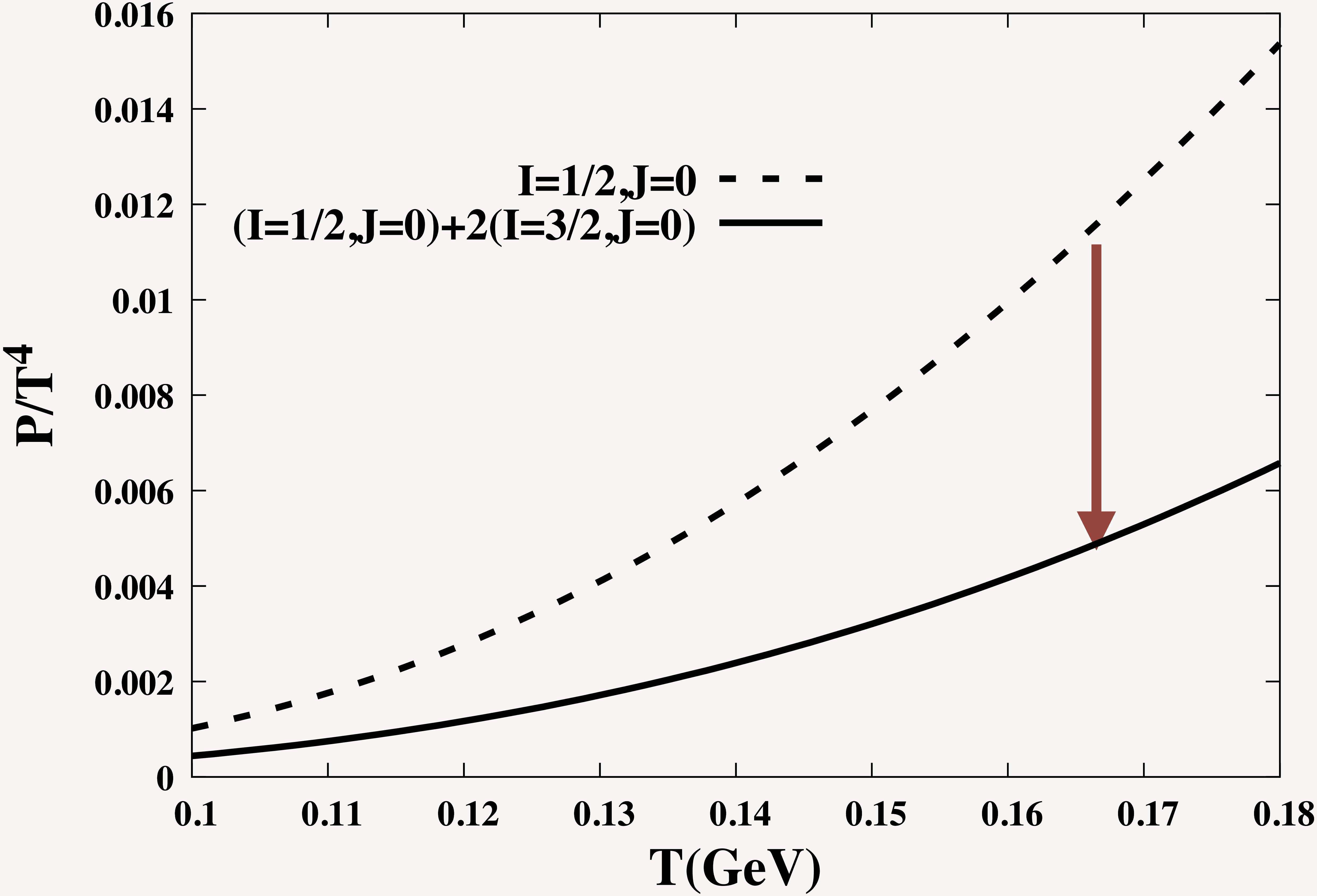
Estabrooks et al.
Aston et al.
Baker et al.
this work
pi K
S-wave
I = 1/2
$a_S = 0.224\, m_\pi^{-1}$
δ(I=1/2 J=0) (degree)
M(GeV)
0
20
40
60
80
100
120
140
160
180
0.6
0.8
1
1.2
1.4
1.6



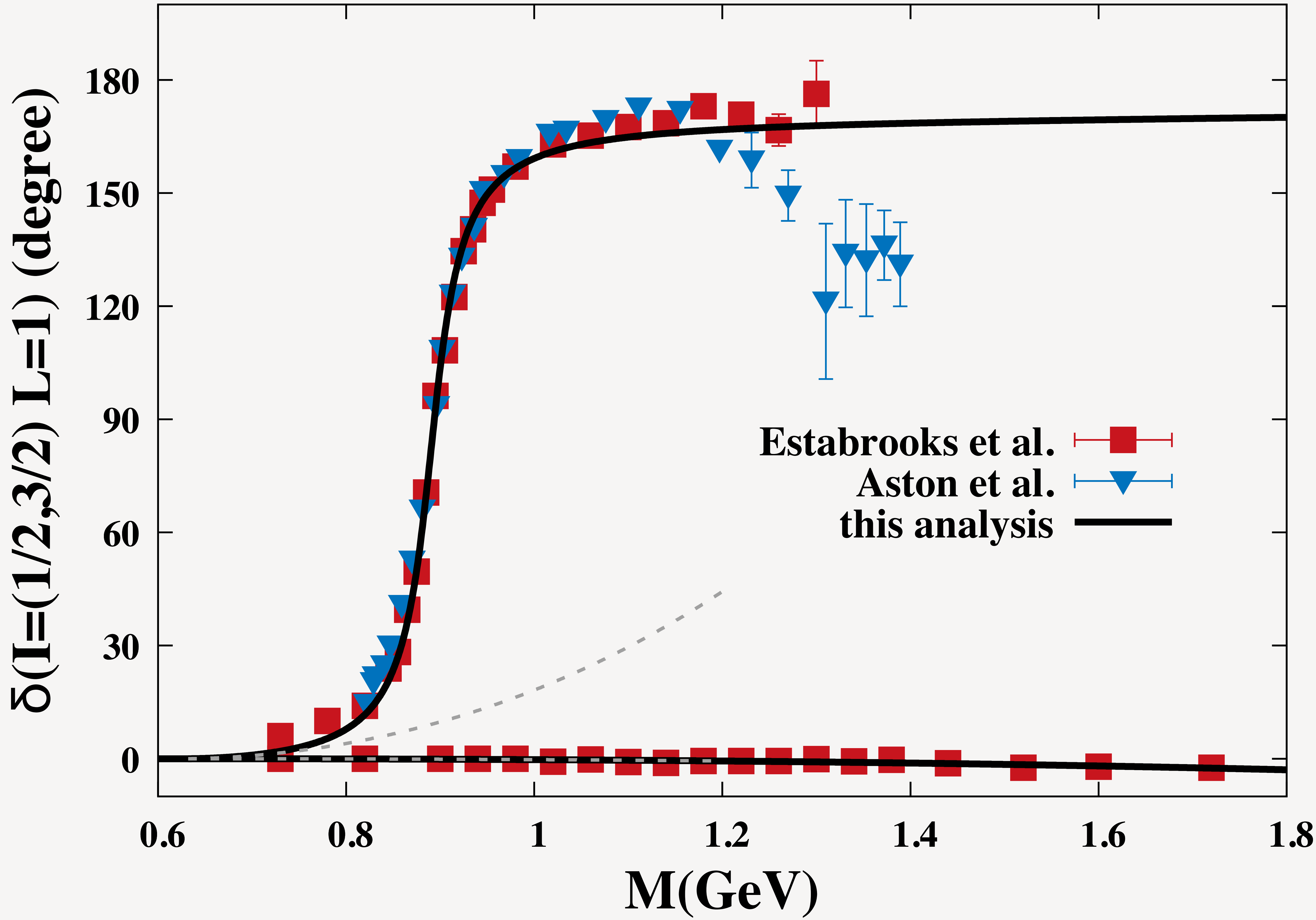
δ(I=3/2 J=0) (degree)
0
-5
-10
-15
-20
-25
-30
0.6
0.8
1
1.2
1.4
1.6
1.8
2
M(GeV)
Estabrooks et al.
Jongejans et al.
Cho et al.
Bakker et al.
this work
a(3/2,0)=-0.045 m$_\pi^{-1}$
pi K
S-wave
I = 3/2



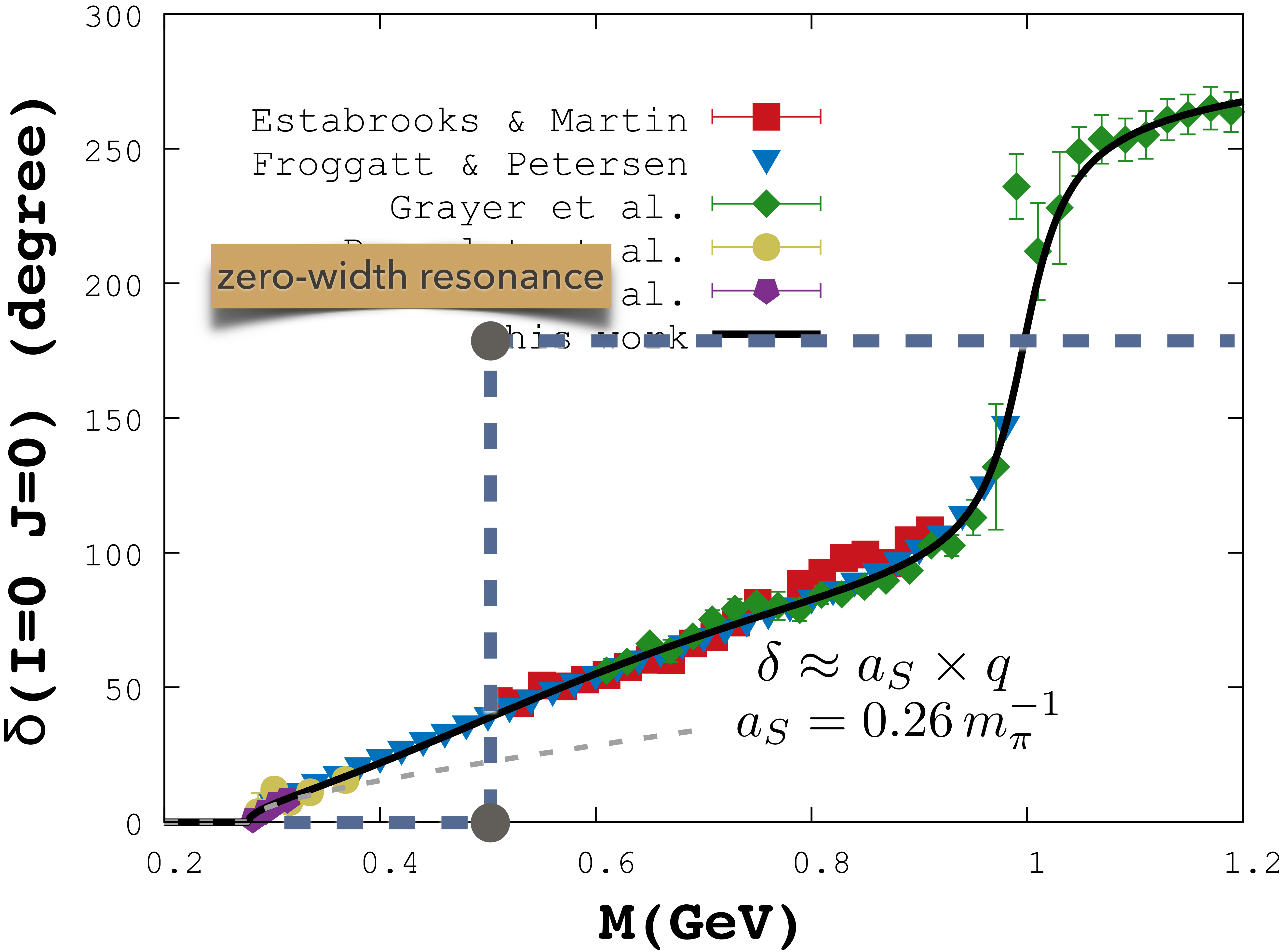
20
15
10
5
0
-5
-10
0.6
0.8
1
1.2
1.4
1.6
B(M)(GeV-1)
M(GeV)
B1/2
2B3/2
B1/2+2B3/2



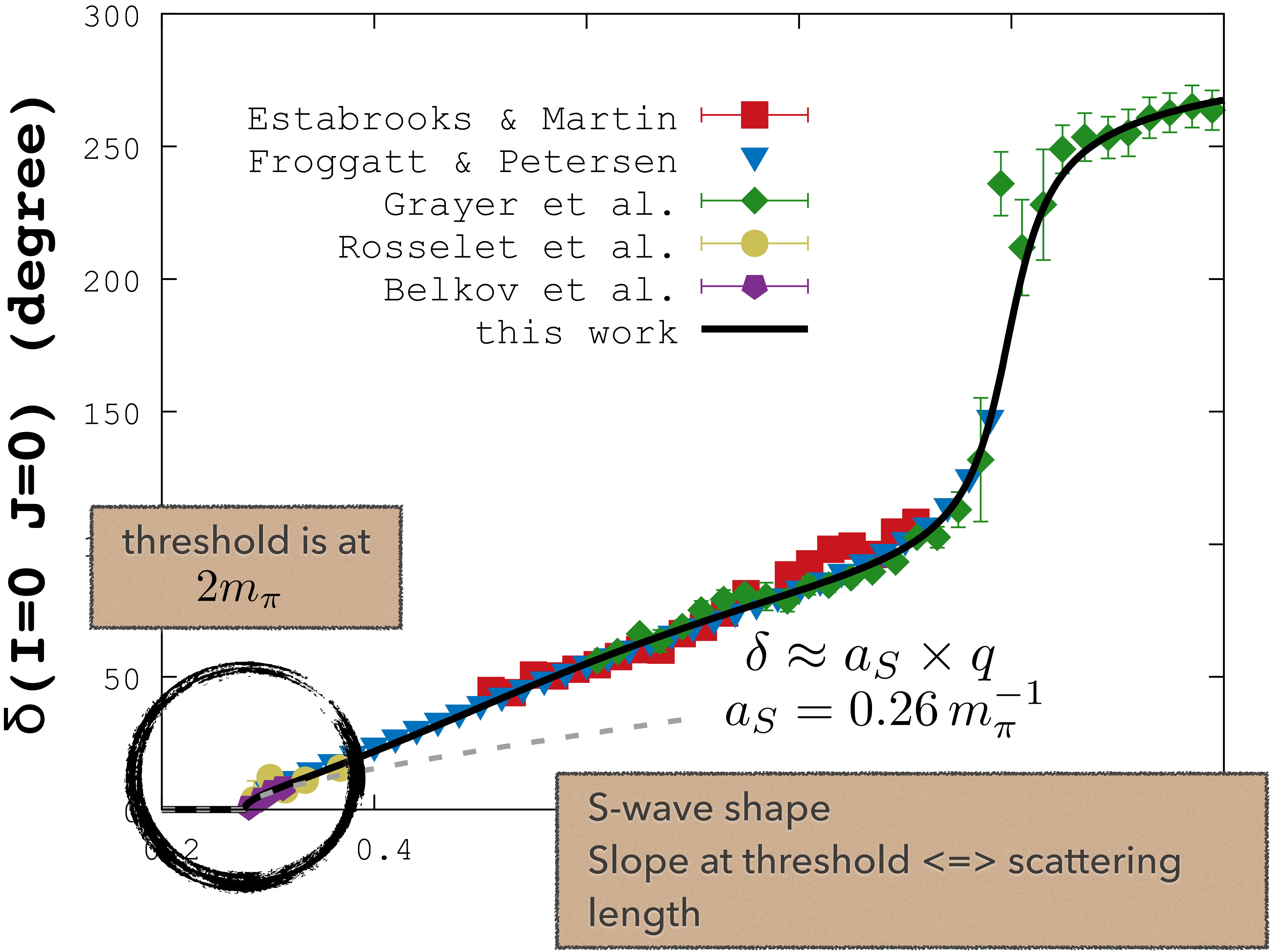
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
P/T4
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
T(GeV)
I=1/2,J=0
(I=1/2,J=0)+2(I=3/2,J=0)



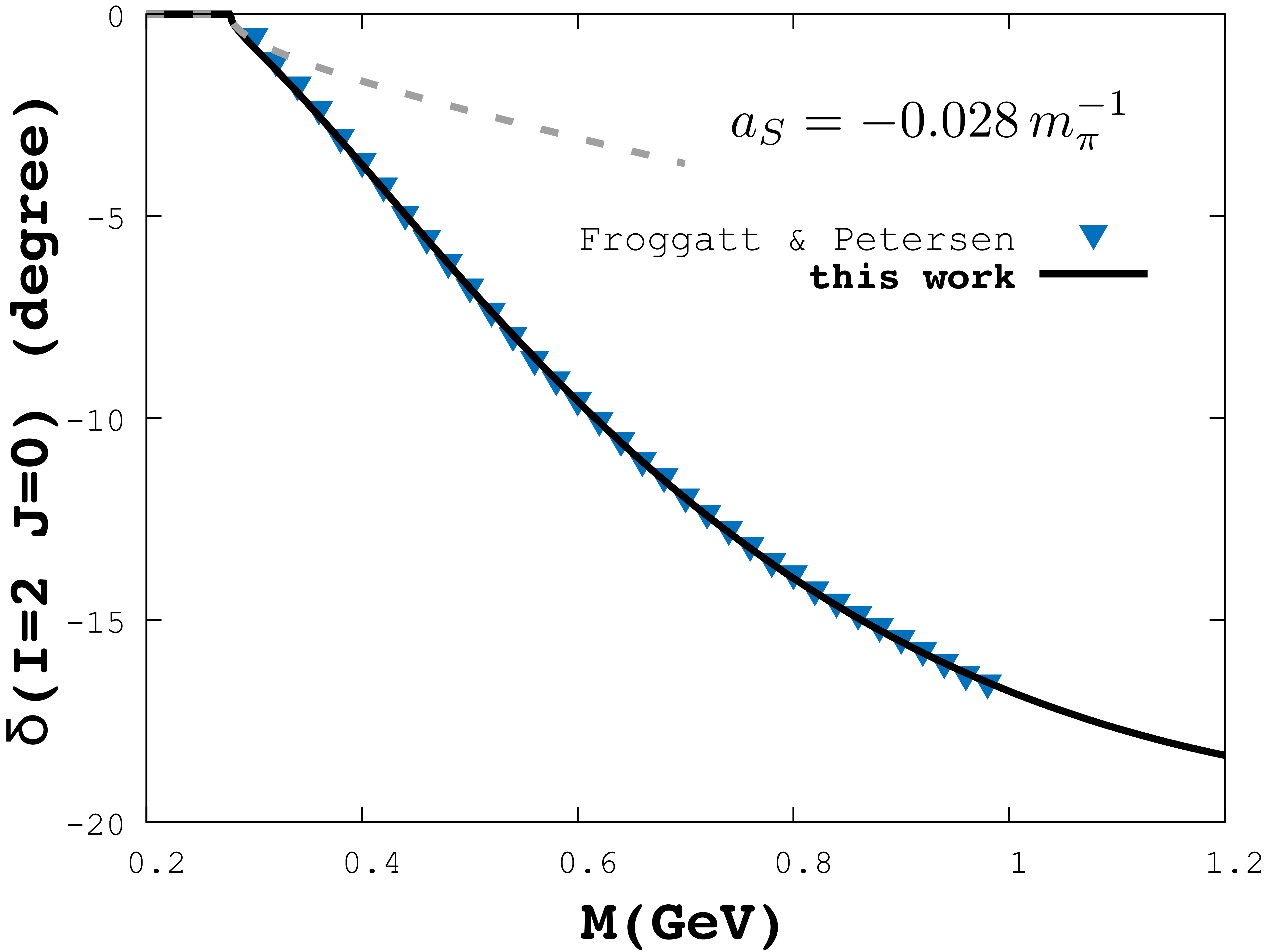
δ(I=(1/2,3/2) L=1) (degree)
180
150
120
90
60
30
0
0.6
0.8
1
1.2
1.4
1.6
1.8
M(GeV)
Estabrooks et al.
Aston et al.
this analysis



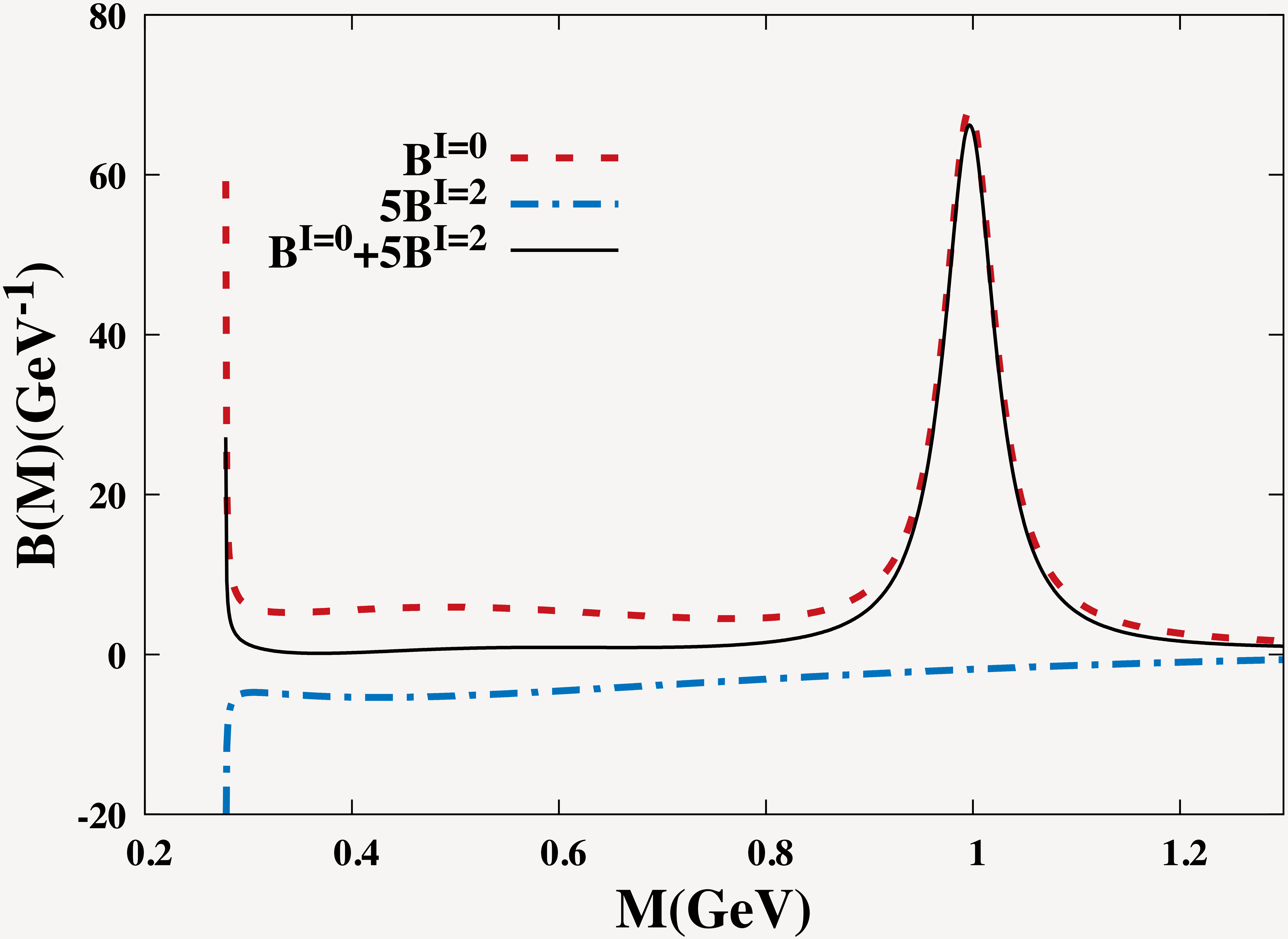
Estabrooks & Martin
Froggatt & Petersen
Grayer et al.
zero-width resonance
$\delta \approx a_S \times q$
$a_S = 0.26\, m_\pi^{-1}$
δ(I=0 J=0) (degree)
M(GeV)
300
250
200
150
100
50
0
0.2
0.4
0.6
0.8
1
1.2



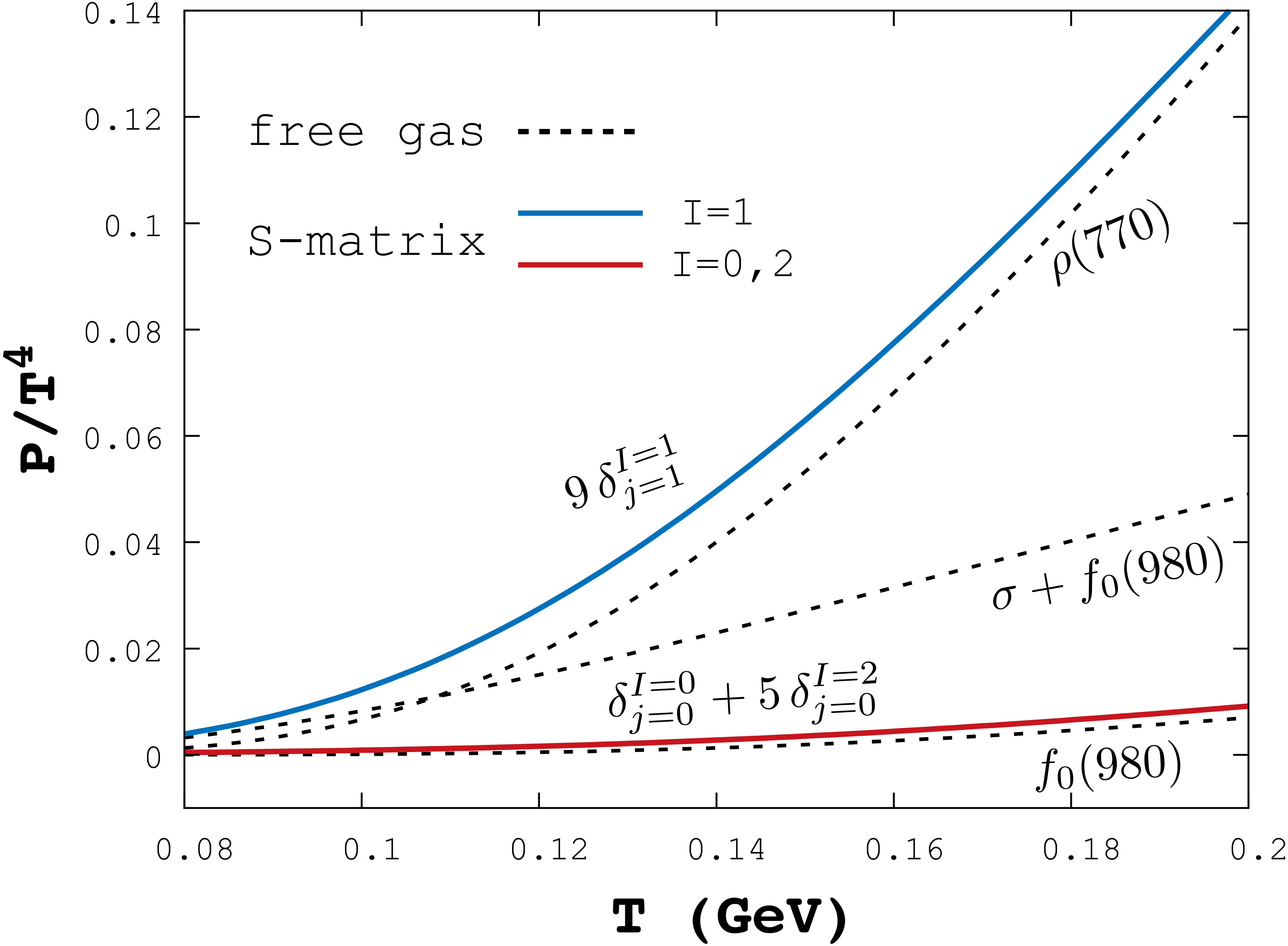
δ(I=0 J=0) (degree)
300
250
200
150
50
0.2
0.4
Estabrooks & Martin
Froggatt & Petersen
Grayer et al.
Rosselet et al.
Belkov et al.
this work
threshold is at
$2m_\pi$
$\delta \approx a_S \times q$
$a_S = 0.26\, m_\pi^{-1}$
S-wave shape
Slope at threshold <=> scattering
length



$a_S = -0.028\, m_\pi^{-1}$
Froggatt & Petersen
this work
0
-5
-10
-15
-20
δ(I=2 J=0) (degree)
0.2
0.4
0.6
0.8
1
1.2
M(GeV)



80
60
40
20
0
-20
0.2
0.4
0.6
0.8
1
1.2
M(GeV)
B(M)(GeV^{-1})
$B^{I=0}$
$5B^{I=2}$
$B^{I=0}+5B^{I=2}$



P/T^4
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
free gas
S-matrix
I=1
I=0,2
$\rho(770)$
$9\,\delta^{I=1}_{j=1}$
$\sigma + f_0(980)$
$\delta^{I=0}_{j=0} + 5\,\delta^{I=2}_{j=0}$
$f_0(980)$
0.08
0.1
0.12
0.14
0.16
0.18
0.2
T (GeV)