Analyticity Constraints for Exotic Mesons

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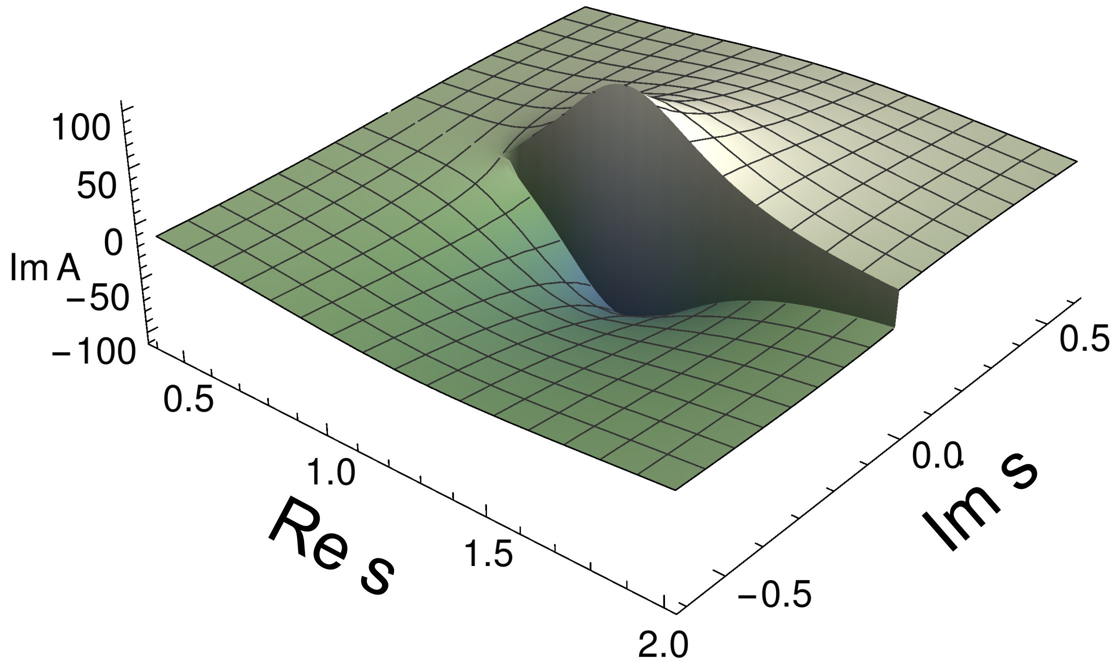
Jefferson Lab

Joint Physics Analysis Center

KL workshop
February 2018

Jefferson Lab
Thomas Jefferson National Accelerator Facility

Unitarity



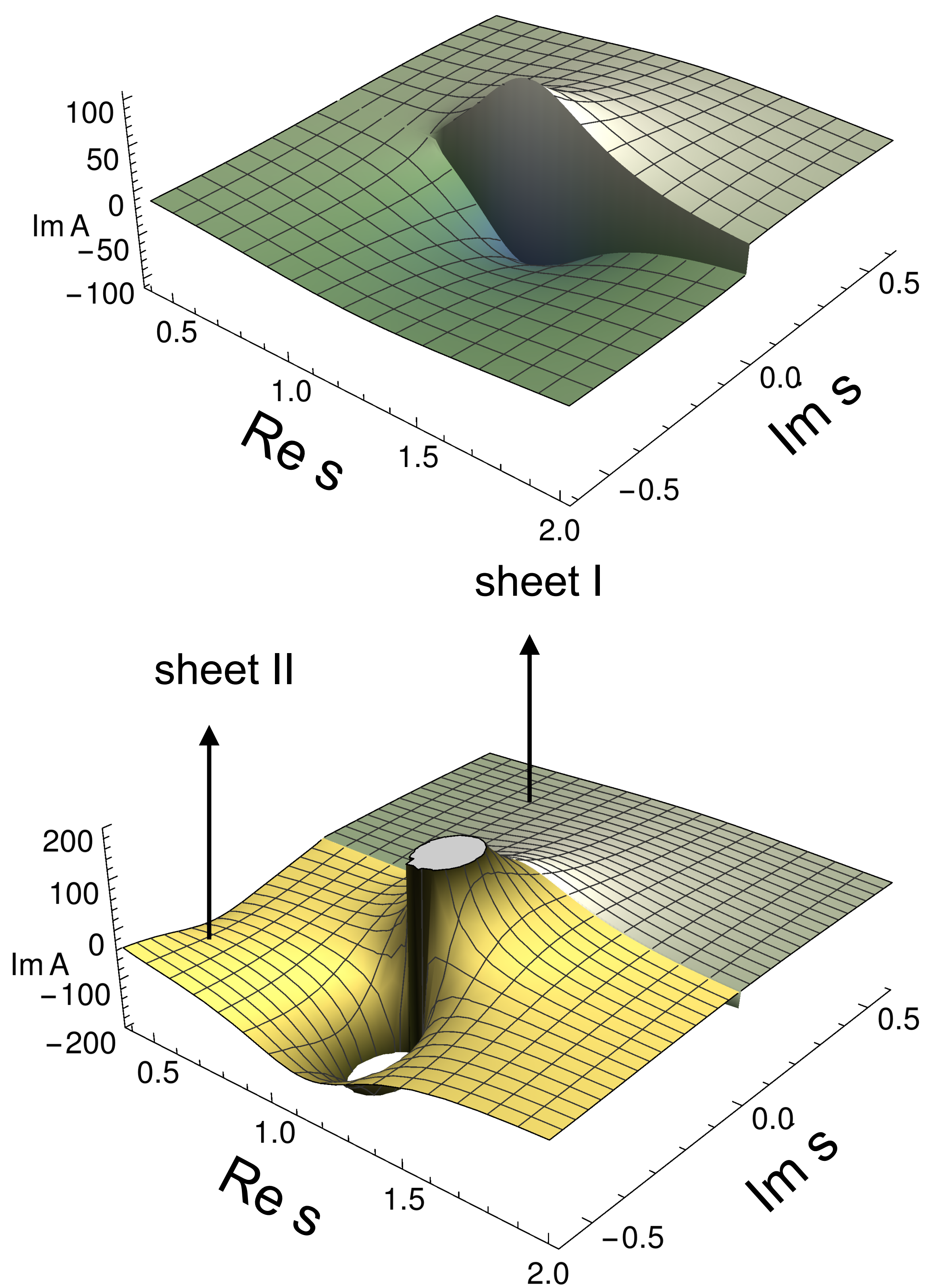
$$\mathrm{Im}\ t_\ell^{-1}(s) = -\rho(s)$$

$$t_\ell(s \pm i\epsilon) = \frac{1}{K(s) \mp i\rho(s)}$$

example: $$K(s) = \frac{m^2 - s}{m\Gamma}$$

satisfies causality
(regular outside the real axis)

Unitarity



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example: $$K(s) = \frac{m^2 - s}{m\Gamma}$$

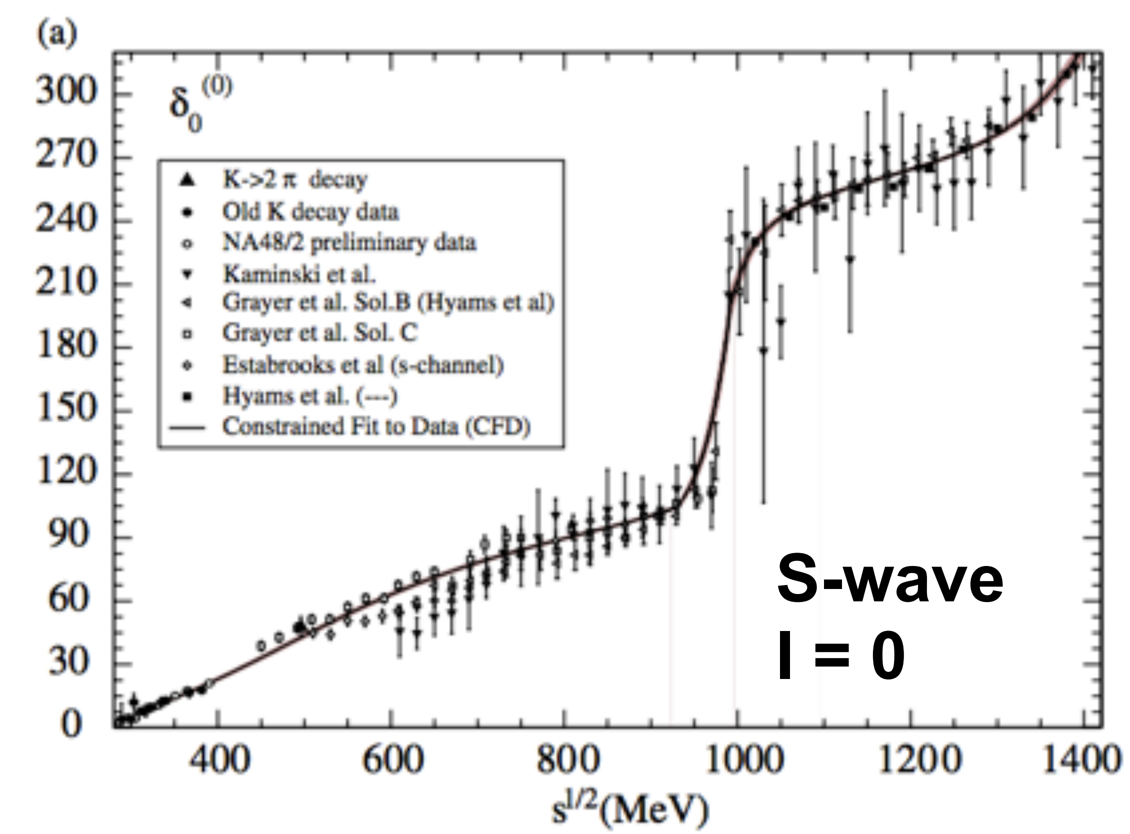
satisfies causality (regular outside the real axis)

define function on sheet II on the lower half plane

$$t_\ell^{II}(s) = \frac{1}{K(s) - i\rho(s)}$$

$$= \frac{m\Gamma}{m^2 - s - i\rho(s) m\Gamma}$$

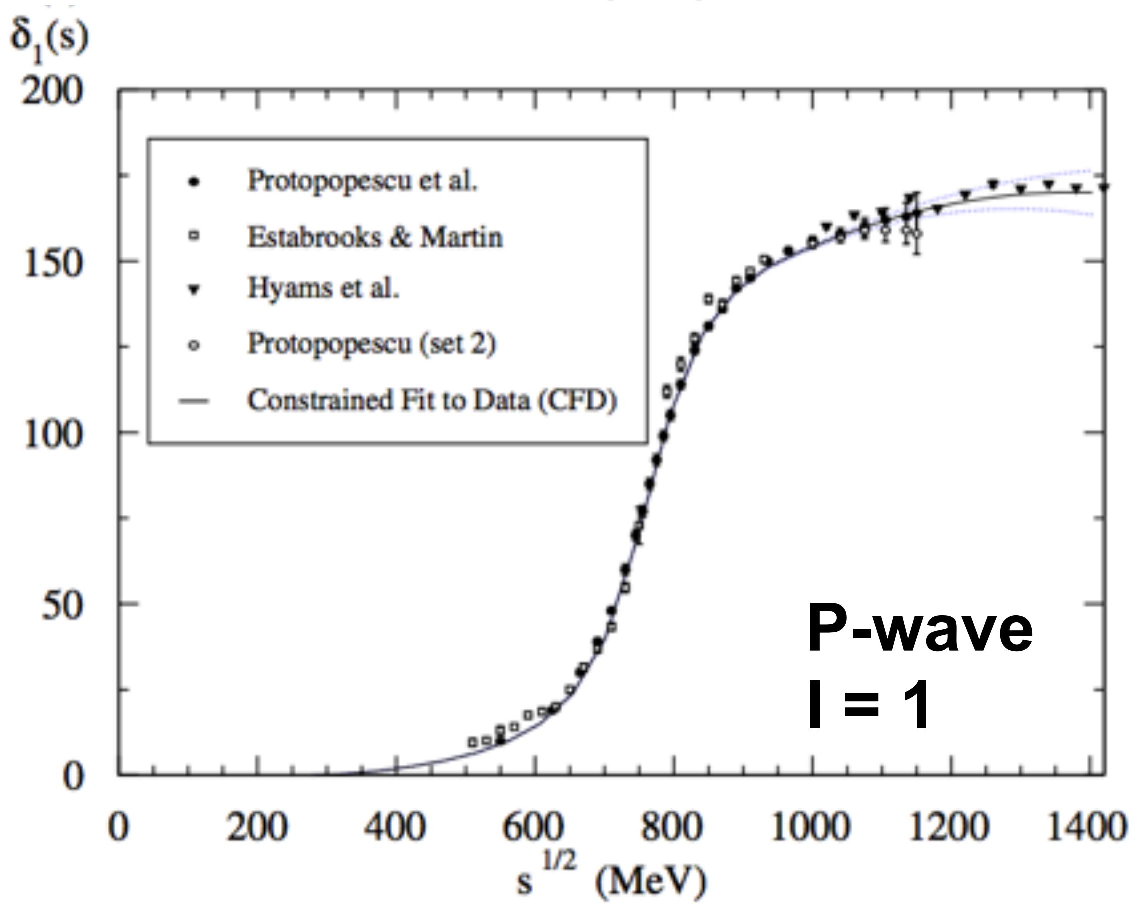
Unitarity

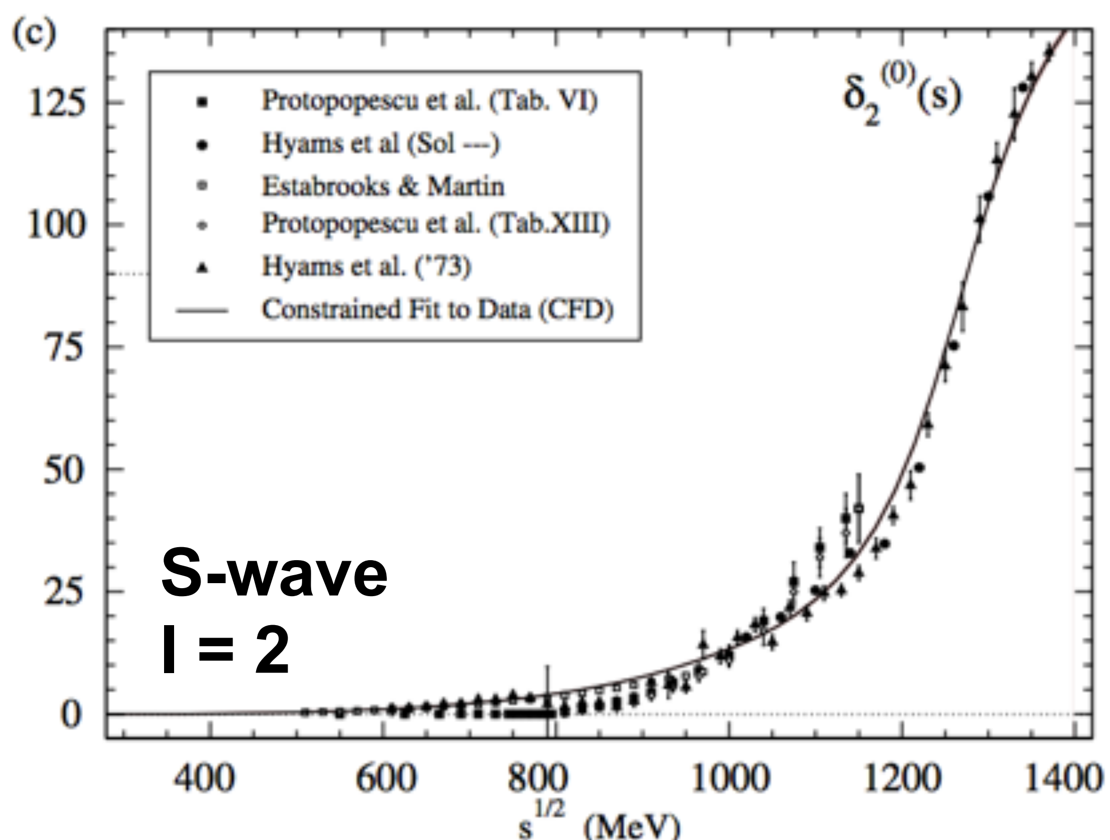


$$t_\ell\ (s) = \frac{1}{K(s) - i\rho(s)}$$

Unitarity used to construct parametrizations

Fit waves independently (first step)



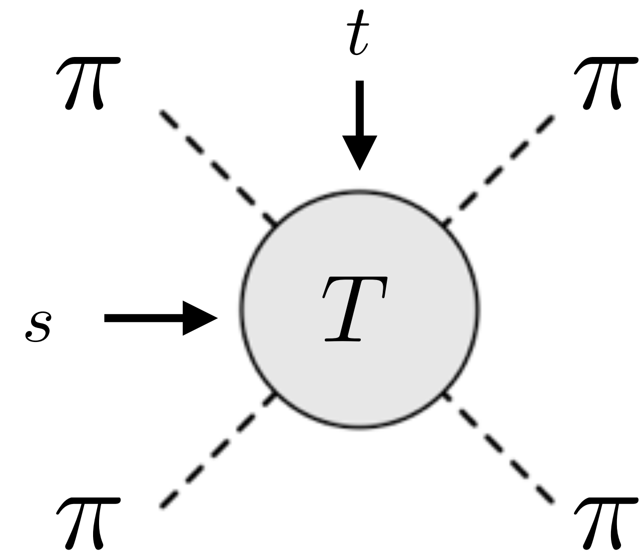


Phys. Rev. D77 054015 (2008)

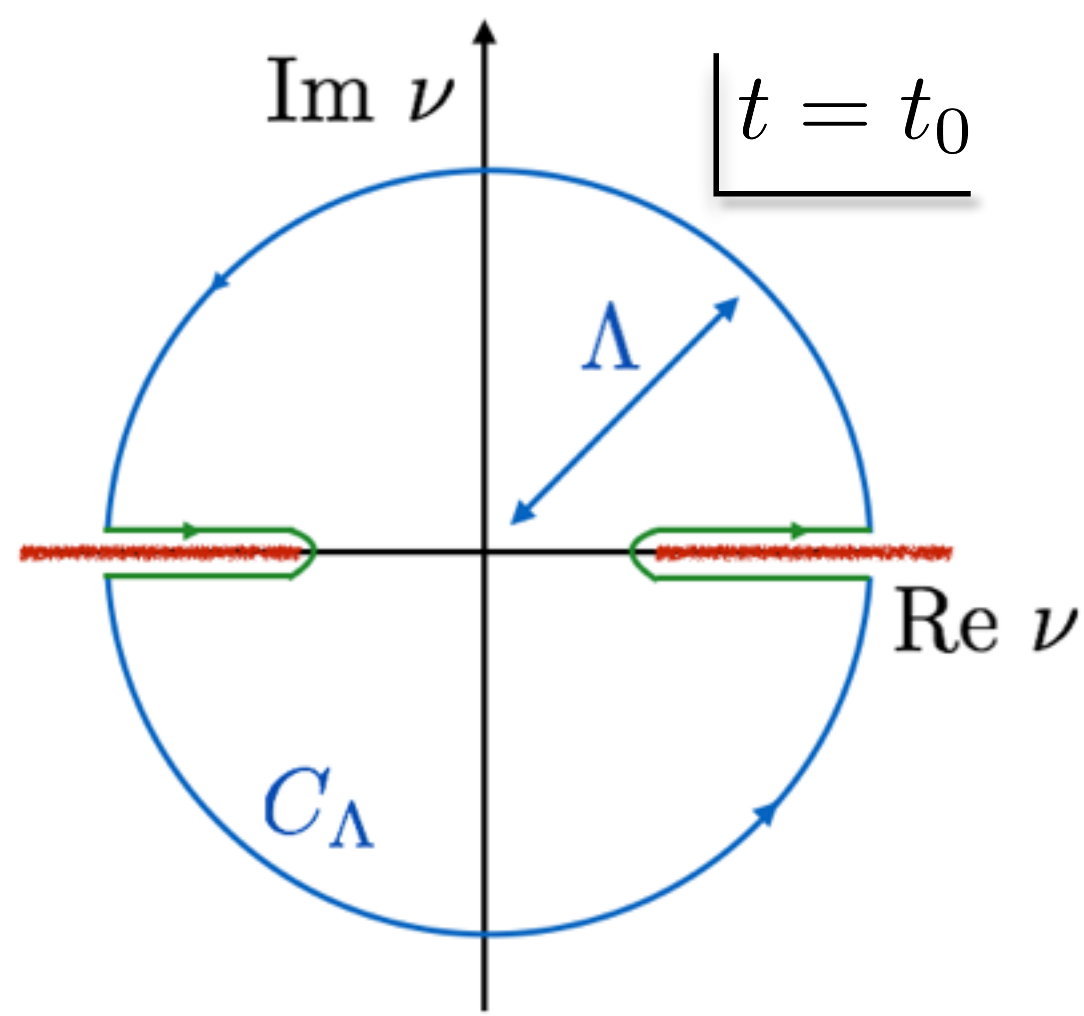
Dispersion Relations

$T(s,t)$ **has a right- and a left-hand cuts**

has no other singularity on sheet I



$$\nu = (s-u)/2$$



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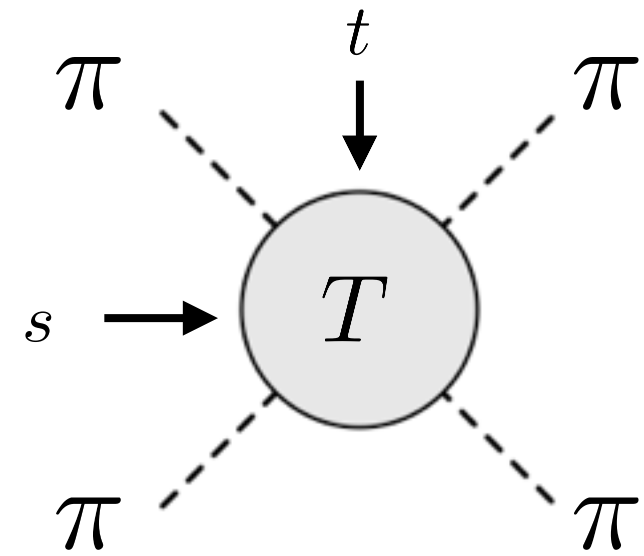
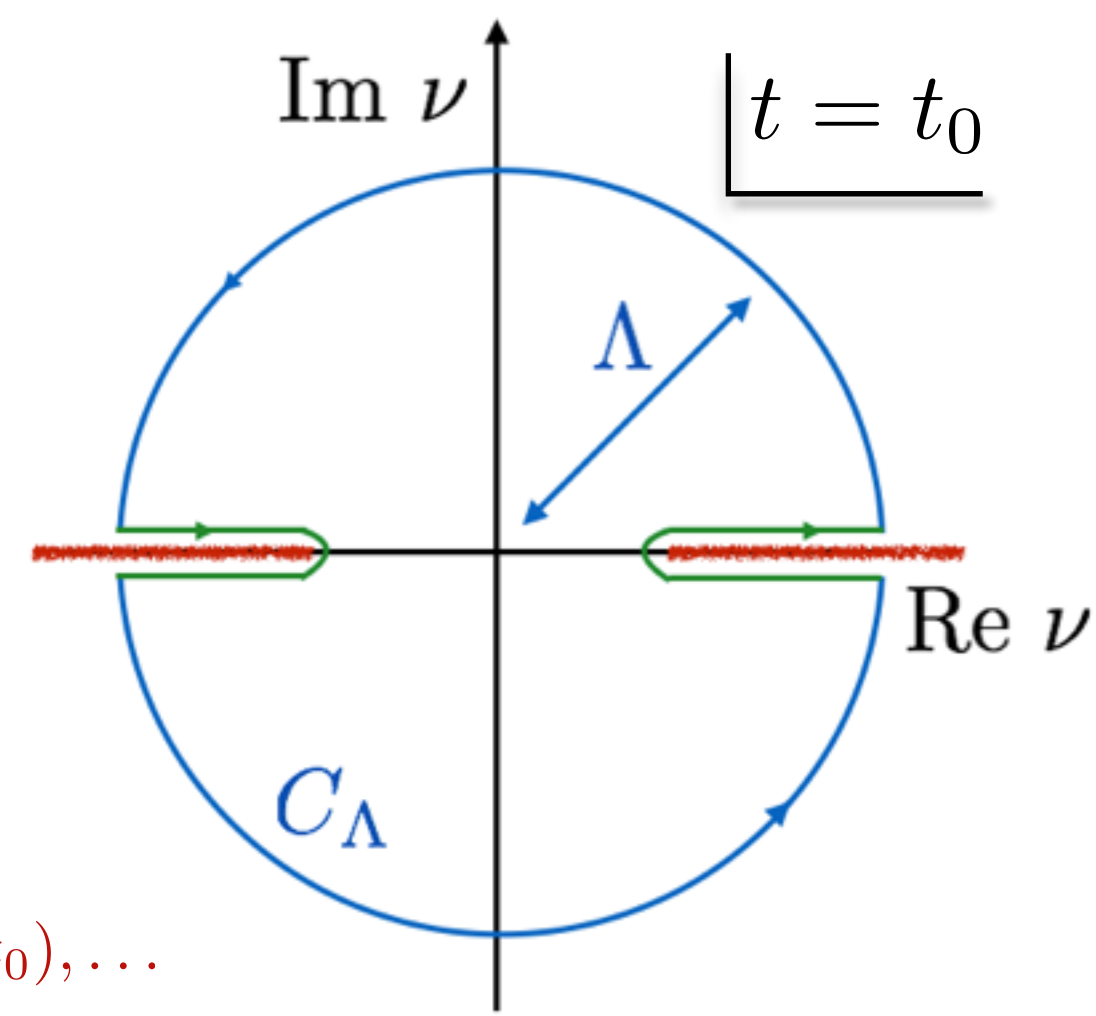


$$\nu = (s-u)/2$$

$$f(\nu) = \int_{\nu_0}^{\Lambda} \left(\frac{\mathrm{Im}\ f(\nu')}{\nu' - \nu} + \frac{\mathrm{Im}\ f(-\nu')}{\nu' + \nu} \right) \frac{d\nu'}{\pi}$$
$$+ \oint_{C_\Lambda} \frac{f(\nu')}{\nu' - \nu} \frac{d\nu'}{2i\pi} \qquad (+\mathrm{sub.}) \qquad (I)$$

$$\int_{\nu_0}^{\Lambda} \mathrm{Im}\ f(\nu') + \mathrm{Im}\ f(-\nu') d\nu' = \oint_{C_\Lambda} f(\nu') d\nu' \quad (II)$$



$$f(\nu) = \ldots, \frac{T(\nu,t_0)}{\nu^2}, \frac{T(\nu,t_0)}{\nu}, T(\nu,t_0), \nu T(\nu,t_0), \nu^2 T(\nu,t_0), \ldots$$

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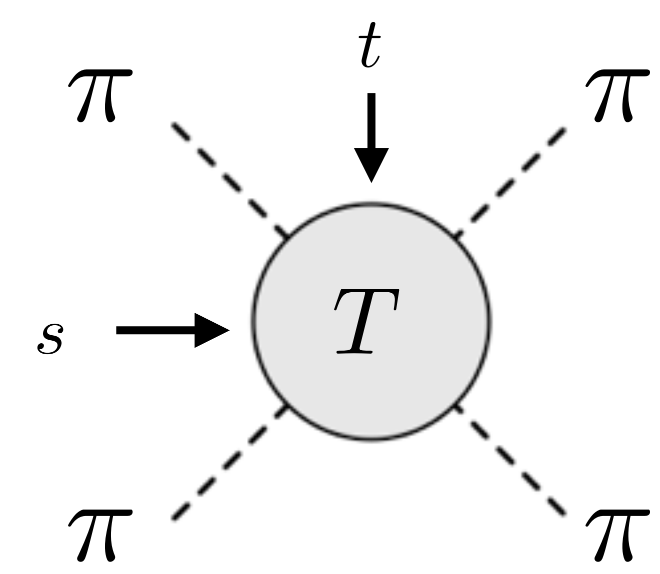

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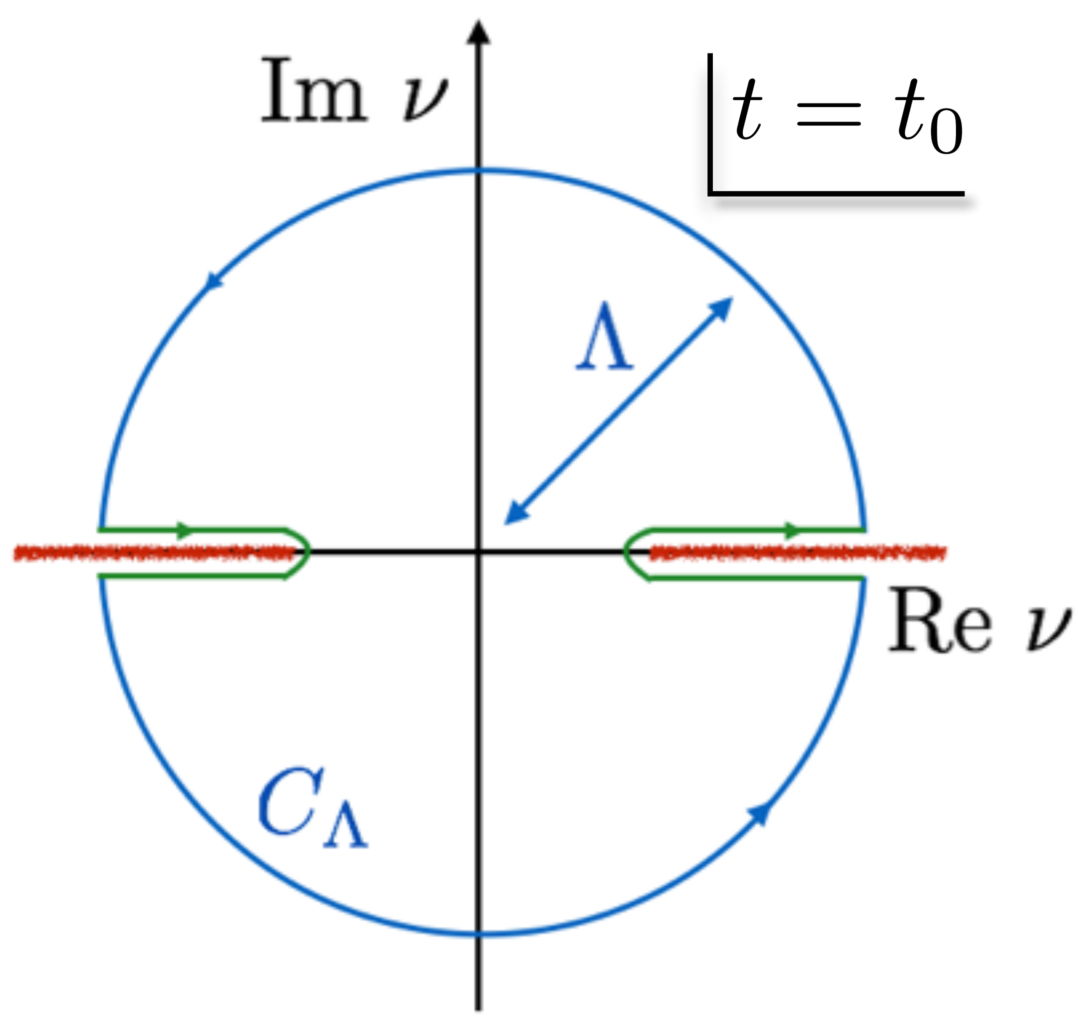


$$f(\nu) = \frac{T(\nu, t_0)}{\nu^2}, \frac{T(\nu, t_0)}{\nu}$$

$$\Lambda \to \infty$$

Roy Equations

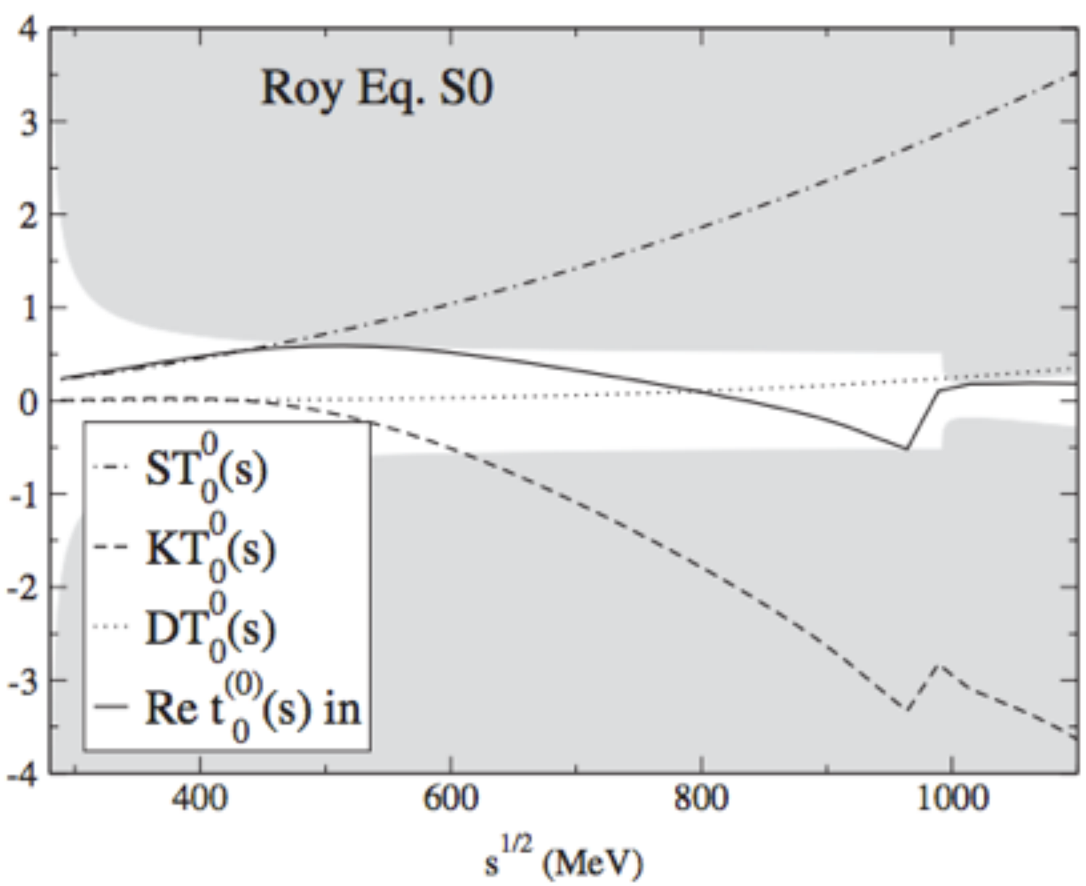
S-,P-wave above smax and other waves

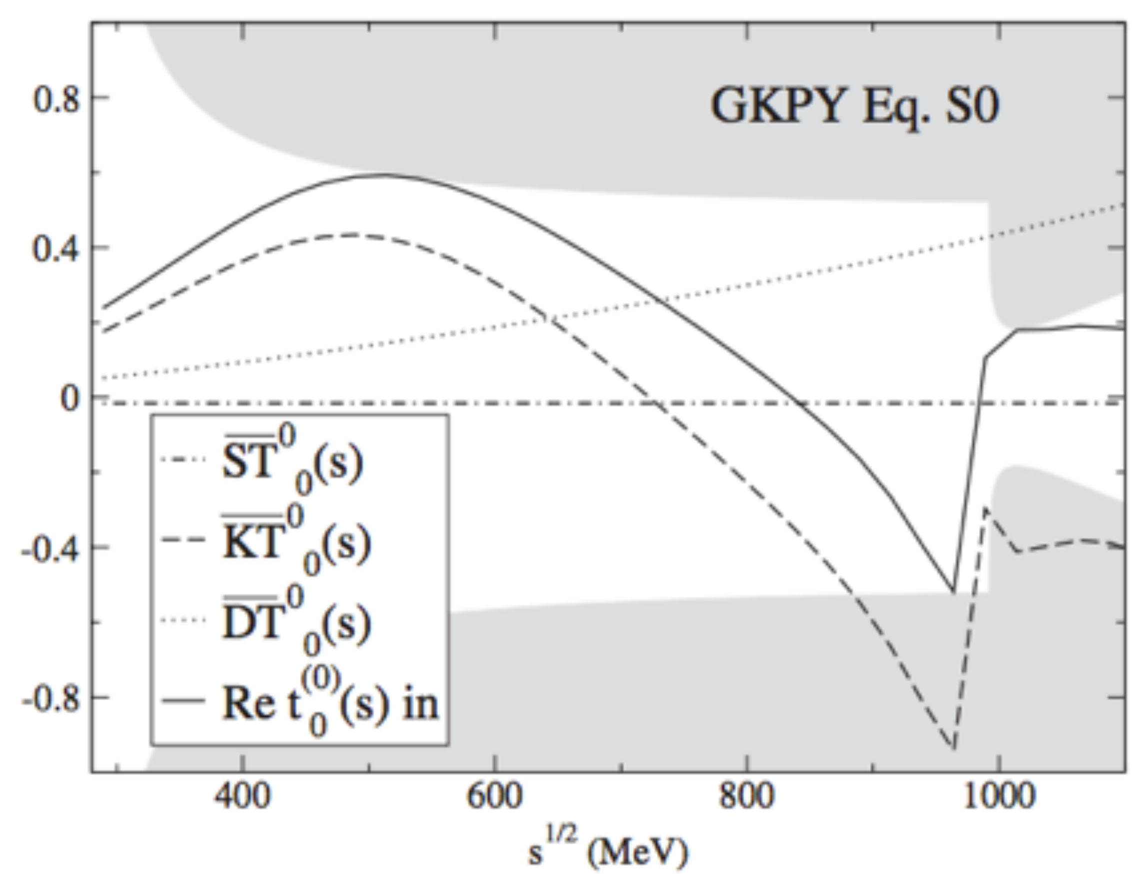
$$\mathrm{Re}\, t_\ell^{(I)}(s) = \overline{ST}_\ell^I + \overline{DT}_\ell^I(s) + \sum_{I'=0}^{2} \sum_{\ell'=0}^{1} \mathrm{P.P.} \int_{4M_\pi^2}^{s_{\max}} ds' \overline{K}_{\ell\ell'}^{II'}(s,s')\, \mathrm{Im} t_{\ell'}^{I'}(s'),$$

$$\sqrt{s_{\max}} = 0.8 - 1.4\ \mathrm{GeV}$$

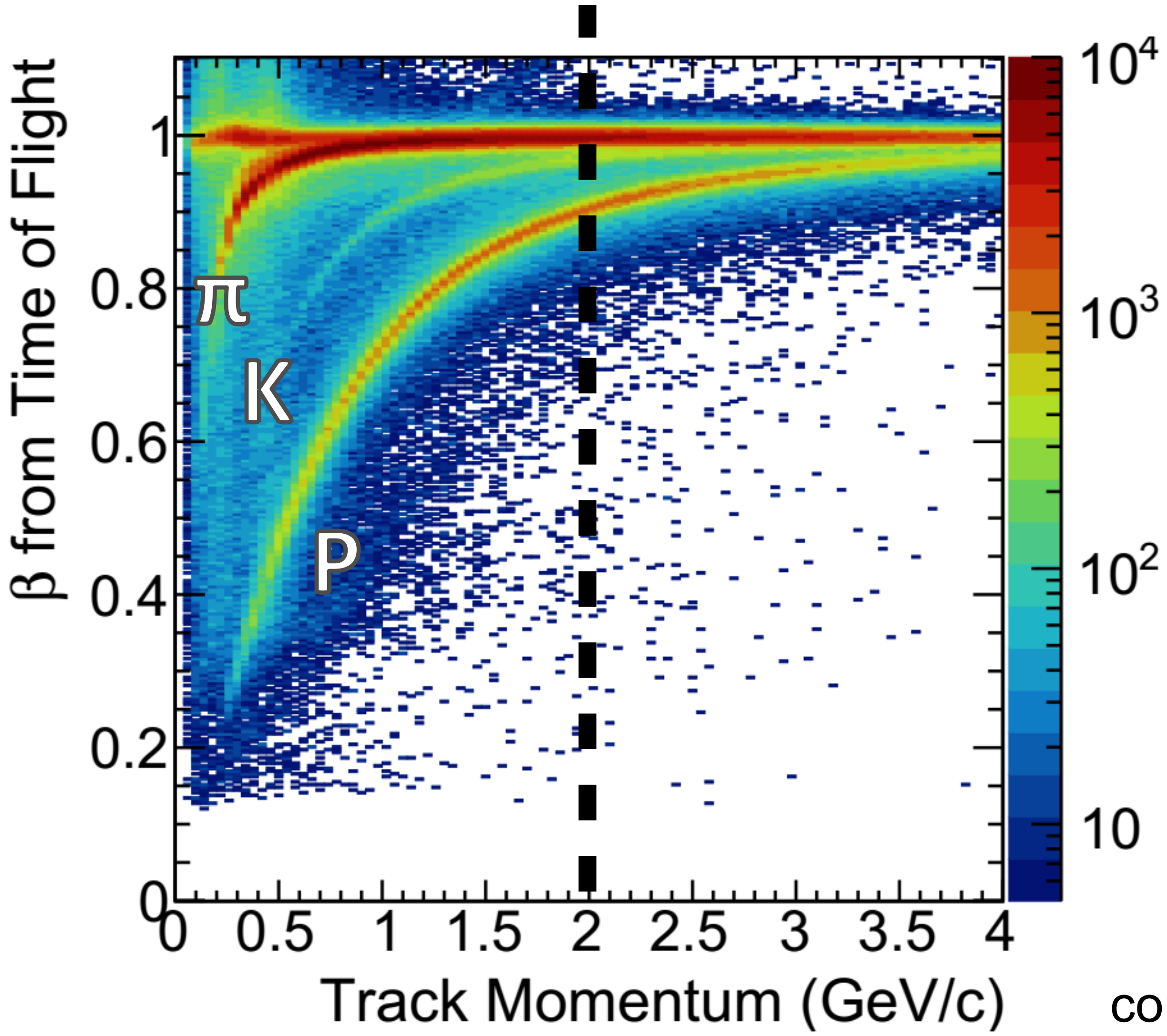
Relevant for sigma and kappa resonances

Constrained fit to data: Phys. Rev. D83 074004 (2011)
Solving Roy equations: Phys. Rev. Lett. 96 132001 (2006)





Current resolution is ~ 2 GeV

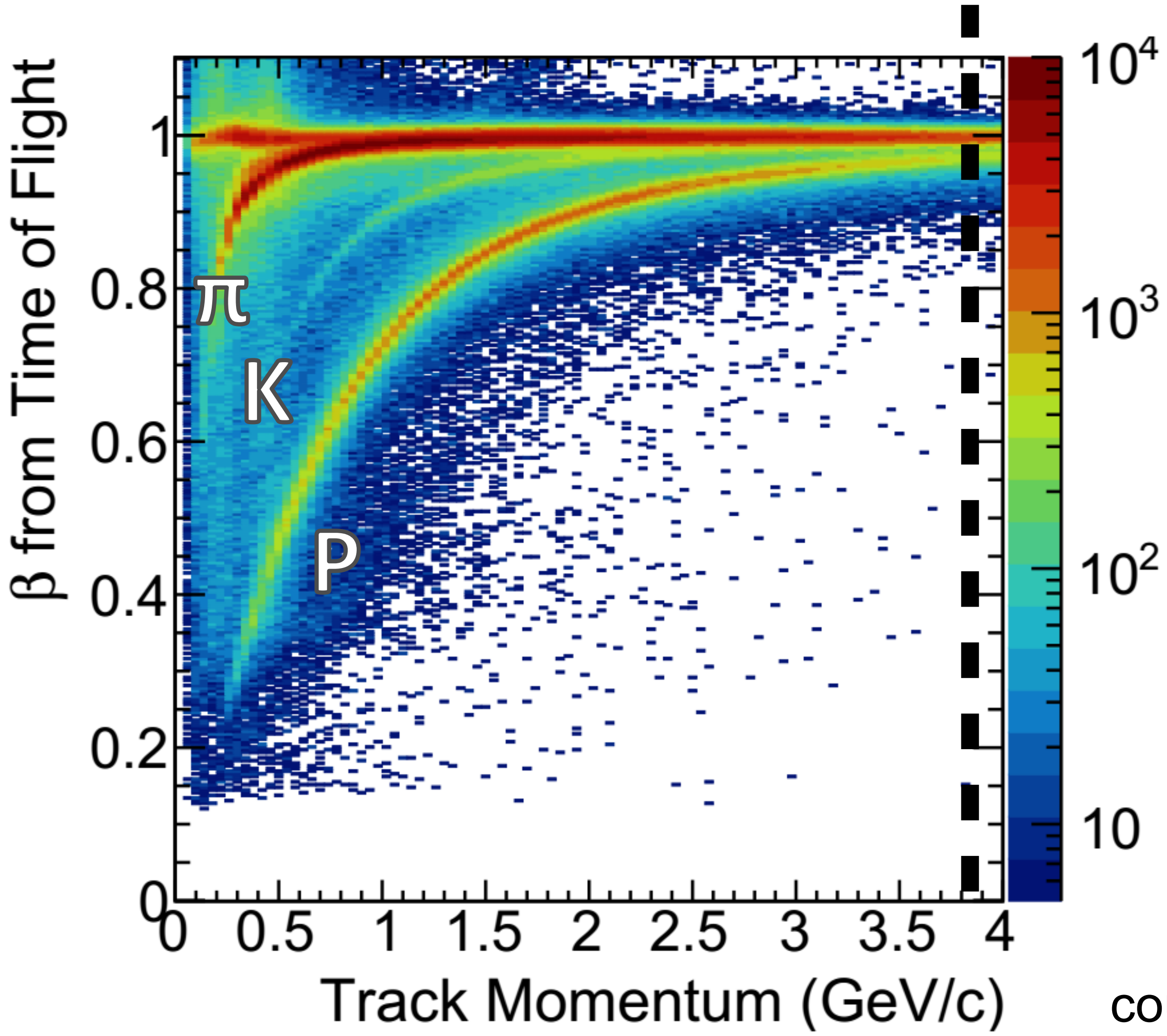


courtesy of M. Williams

Current resolution is ~ 2 GeV

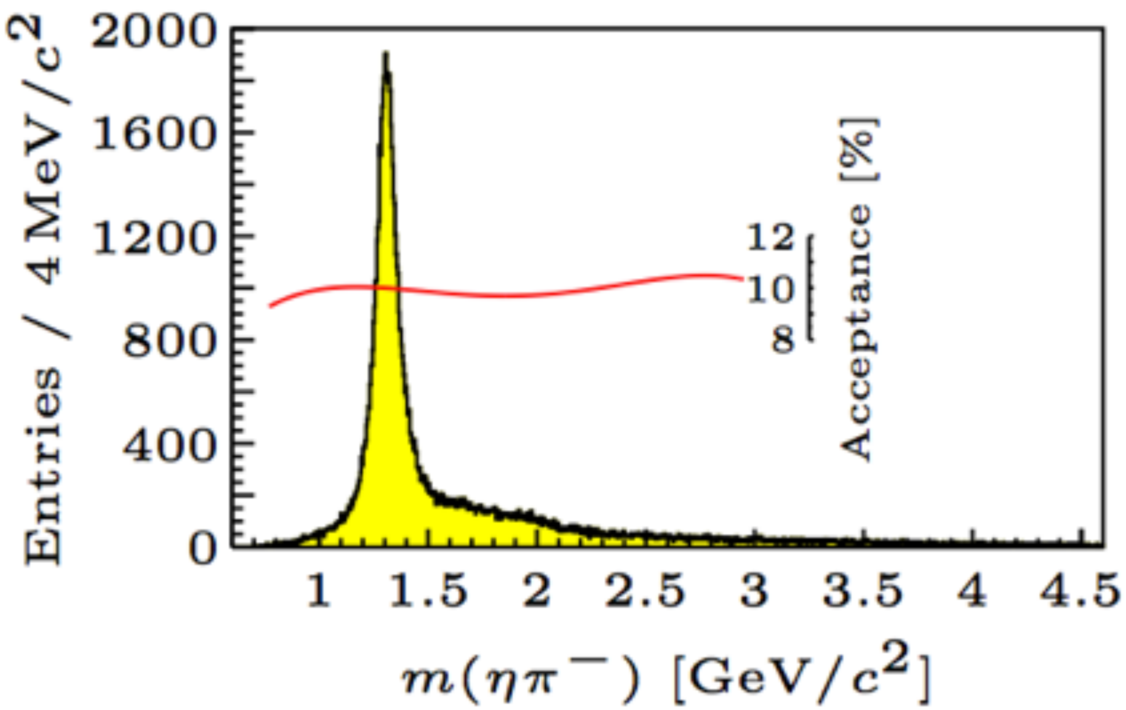
DIRC will extend resolution to ~ 4 GeV

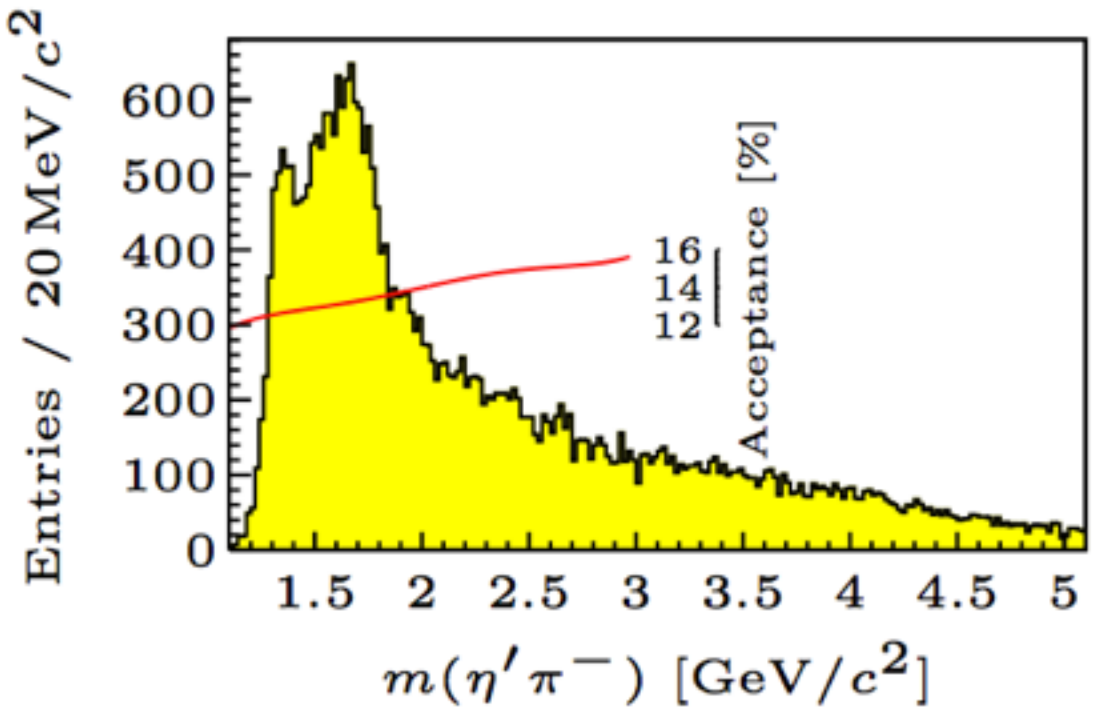
Data in high mass region will be available!

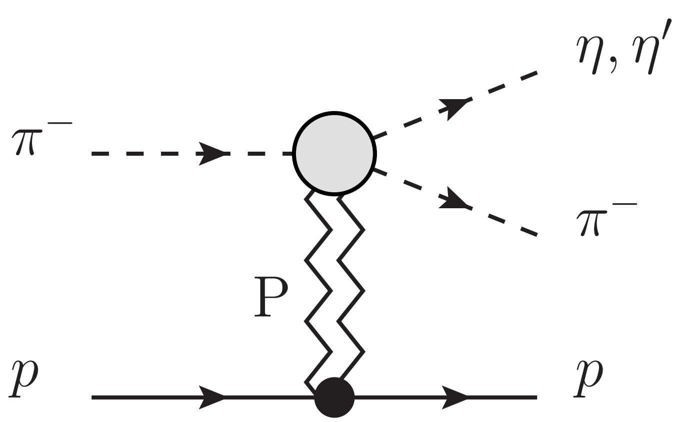


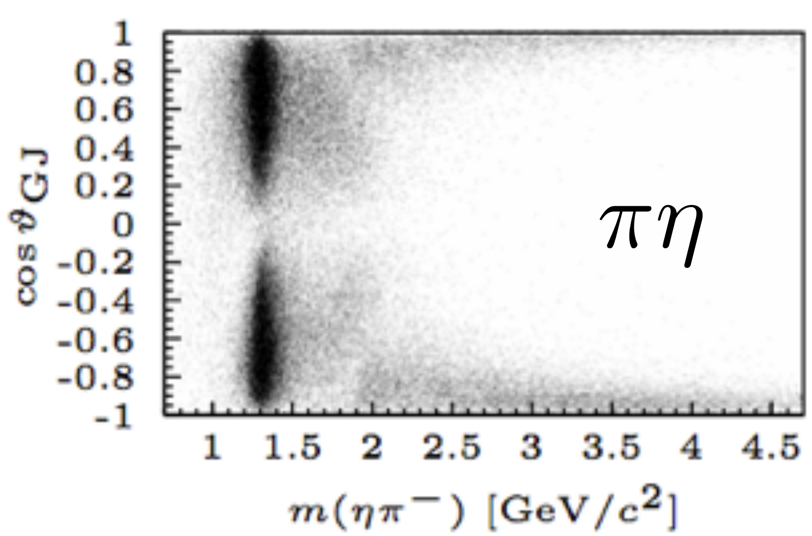
courtesy of M. Williams

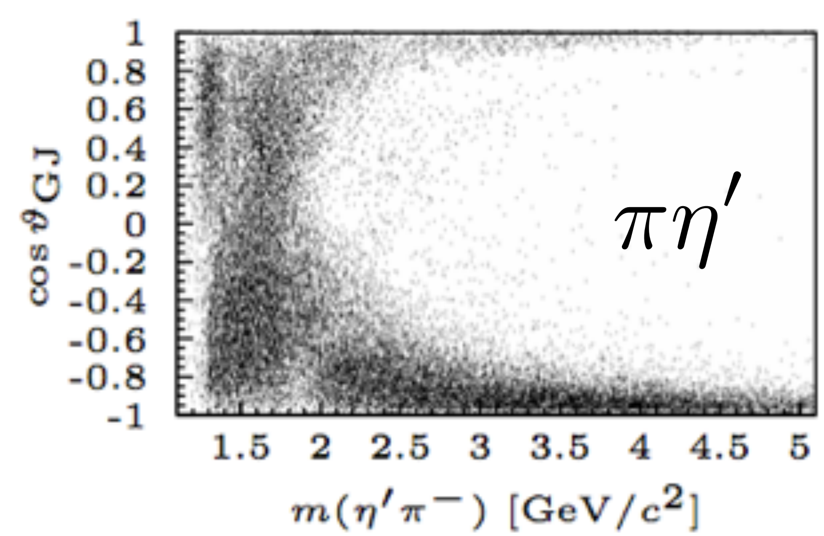
Eta-Pi @COMPASS







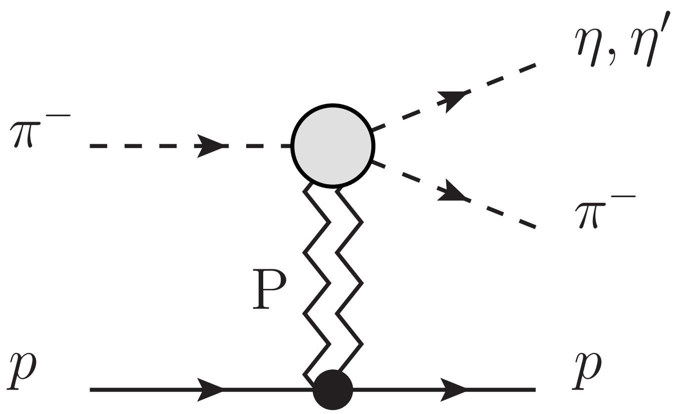


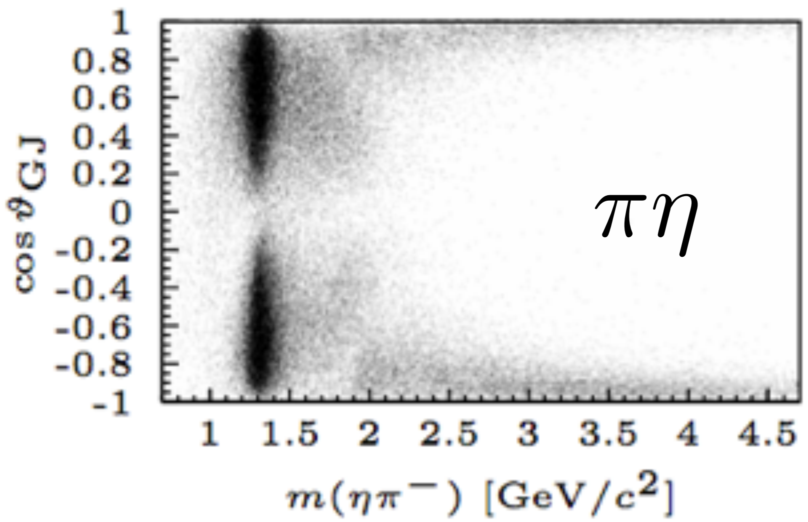


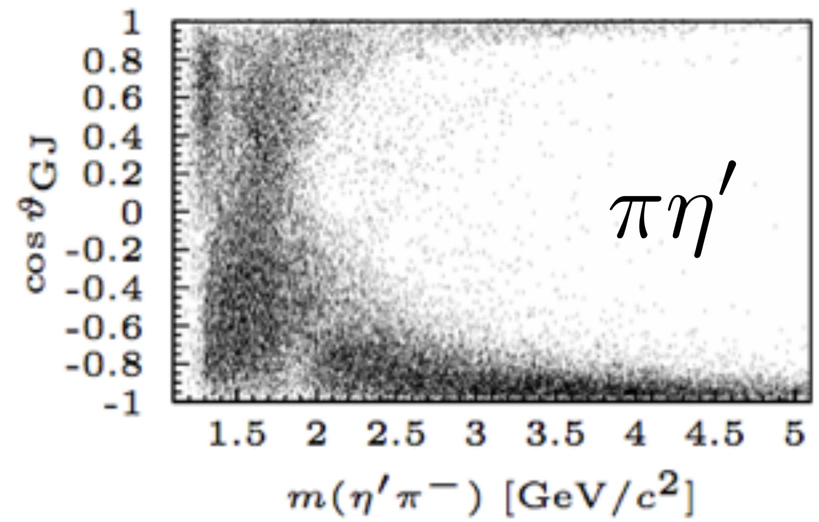
Odd waves have exotic quantum numbers (in the quark model sense)

COMPASS Phys. Lett. B740 (2015)

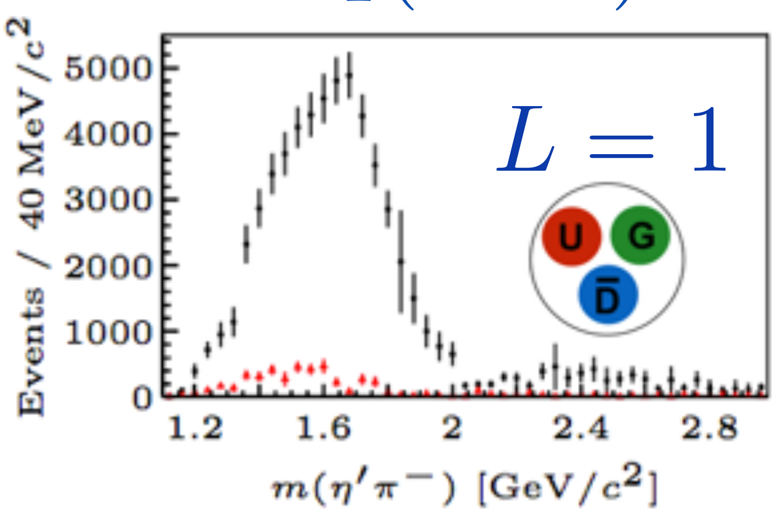
Eta-Pi @COMPASS



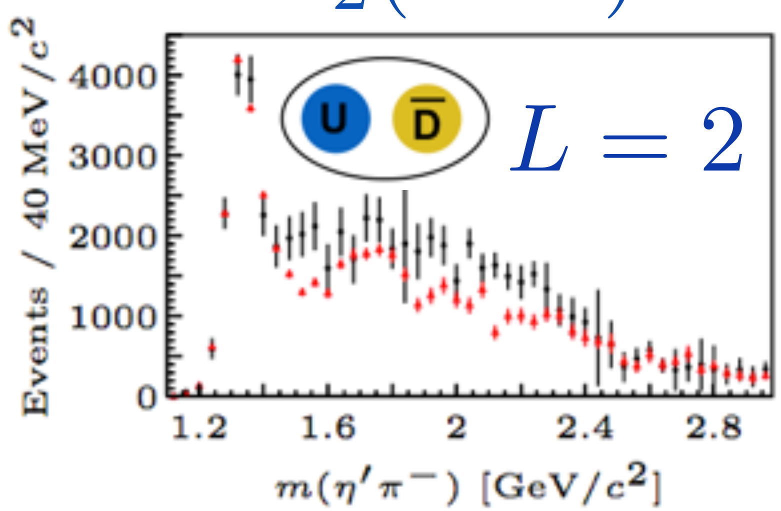




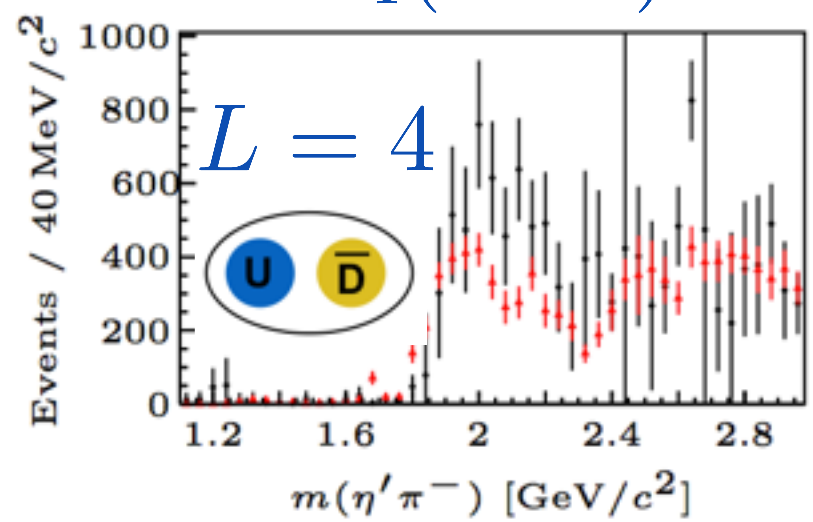
$\pi_1(1600)$?



$a_2(1320)$



$a_4(2040)$



black: $\pi\eta'$

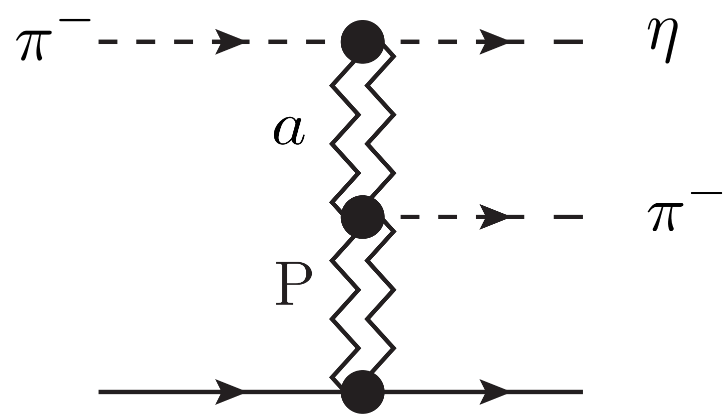
red: $\pi\eta$ **(scaled)**

Resonance in angular mom. L = 1 ?

P-wave originates from asymmetry in high mass region

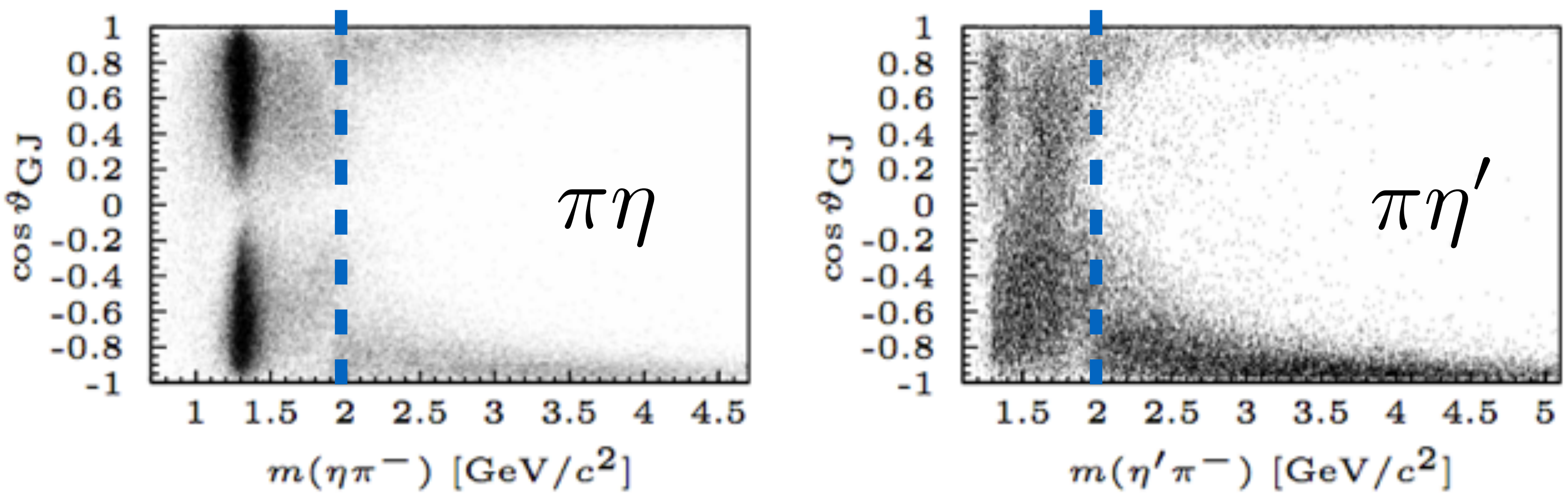
COMPASS Phys. Lett. B740 (2015)

High Mass Region

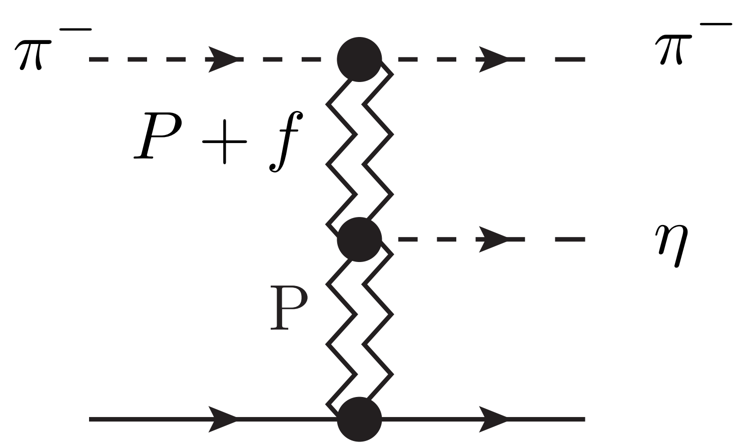


$a : I^G J^{PC} = 1^-(0, 2, 4, 6, \ldots)^{++}$

$\cos\theta_{GF} \sim 1 \rightarrow \eta$ forward



$\cos\theta_{GF} \sim -1 \rightarrow \eta$ backward

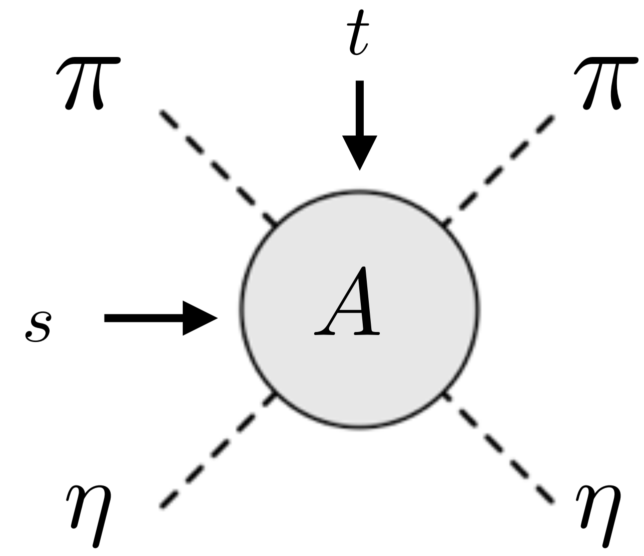


$f : I^G J^{PC} = 0^+(0, 2, 4, 6, \ldots)^{++}$

Dispersion Relations

$A(s,t)$ **has a right- and a left-hand cuts**

has no other singularity on sheet I



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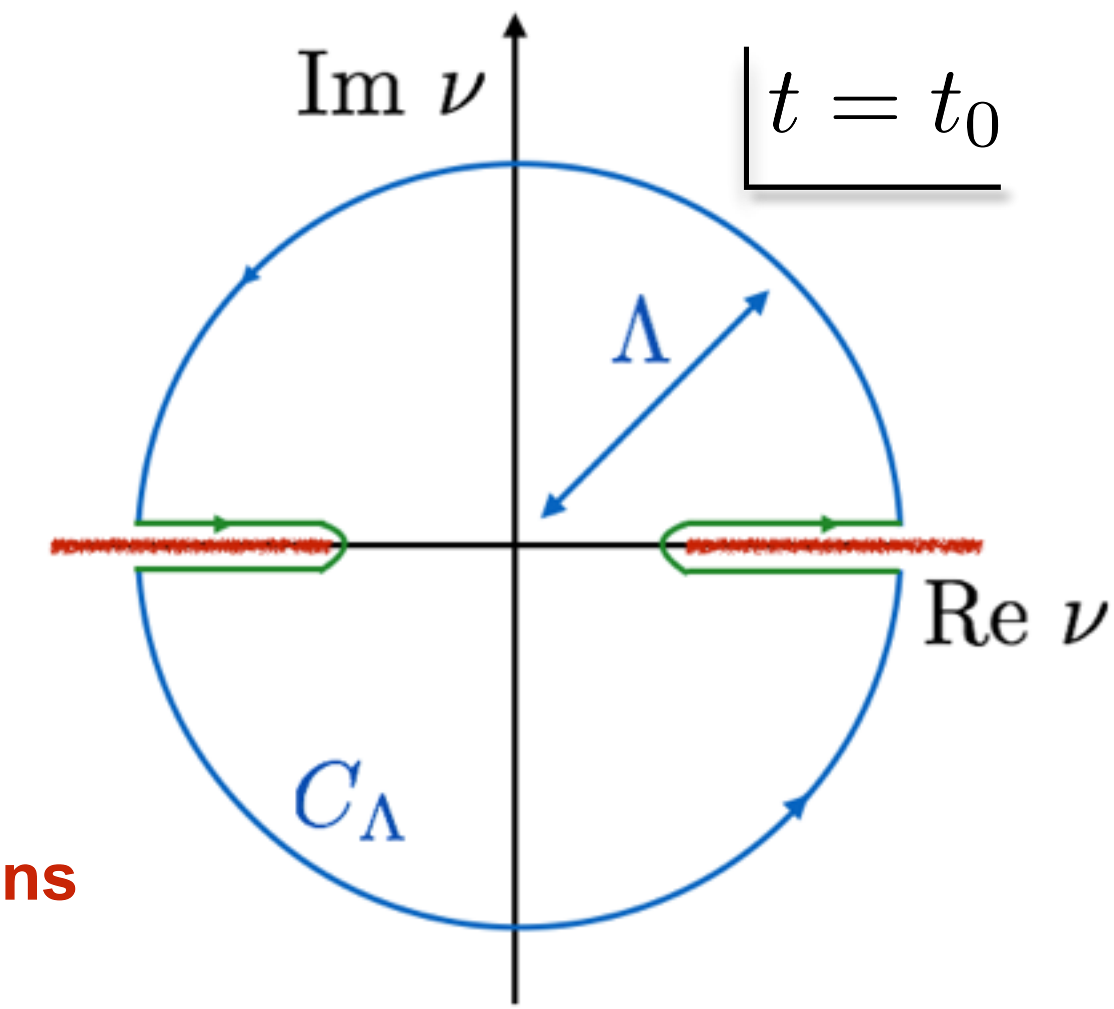


$$\int_{\nu_0} \mathrm{Im}\ A(\nu, t_0) d\nu = \beta(t_0) \frac{\Lambda^{\alpha(t_0)+1}}{\alpha(t_0)+1}$$

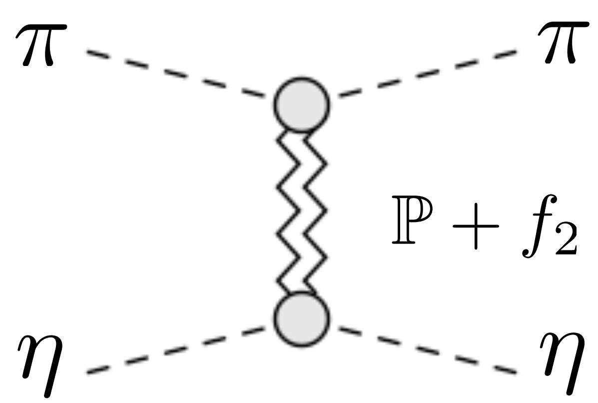
Use this relation in forward and backward regions

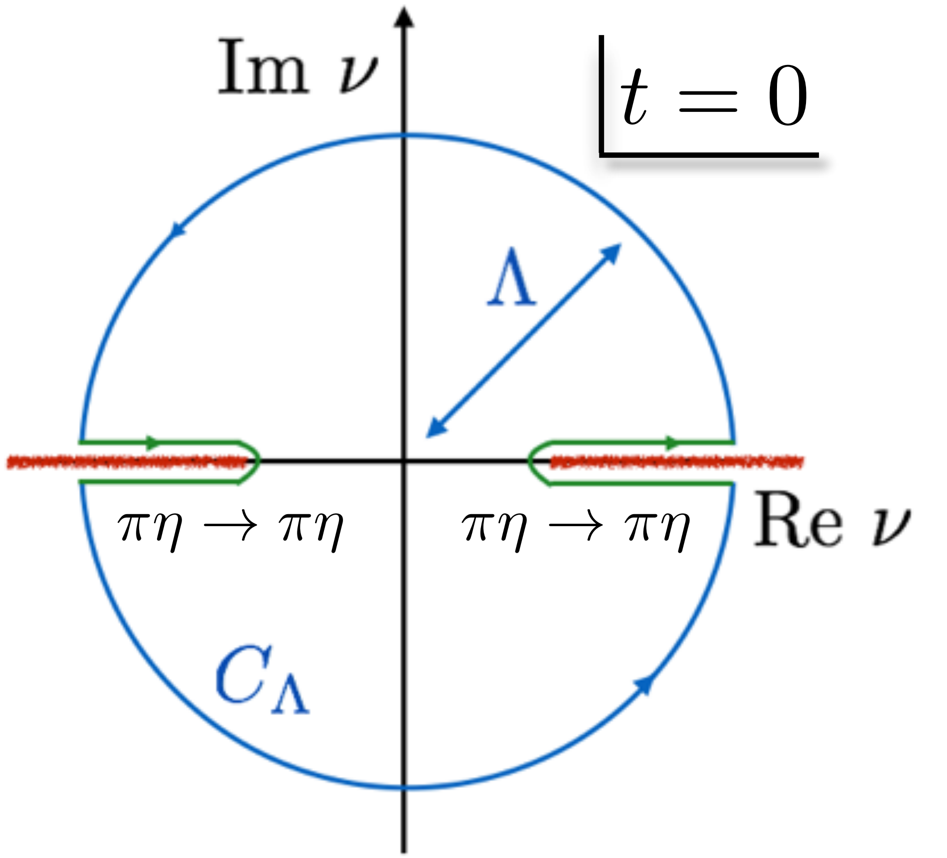
Forward and Backward Sum Rules

Forward:

$$\int_{\nu_0}^{\Lambda} \text{Im } A^{\pi\eta\to\pi\eta}(\nu, t=0)d\nu = \beta_{\mathbb{P}}\frac{\Lambda^{\alpha_{\mathbb{P}}+1}}{\alpha_{\mathbb{P}}+1} + \beta_a\frac{\Lambda^{\alpha_a+1}}{\alpha_a+1}$$

only even
signature exchange





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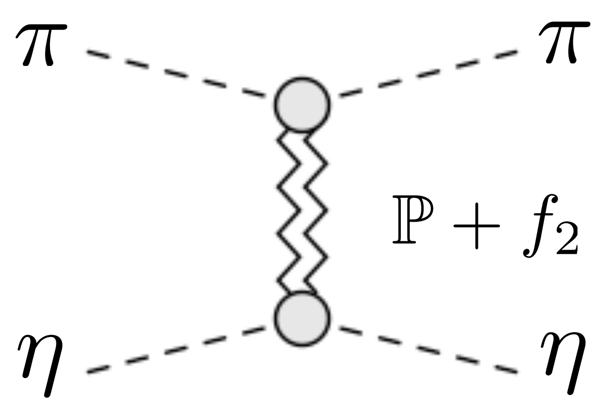
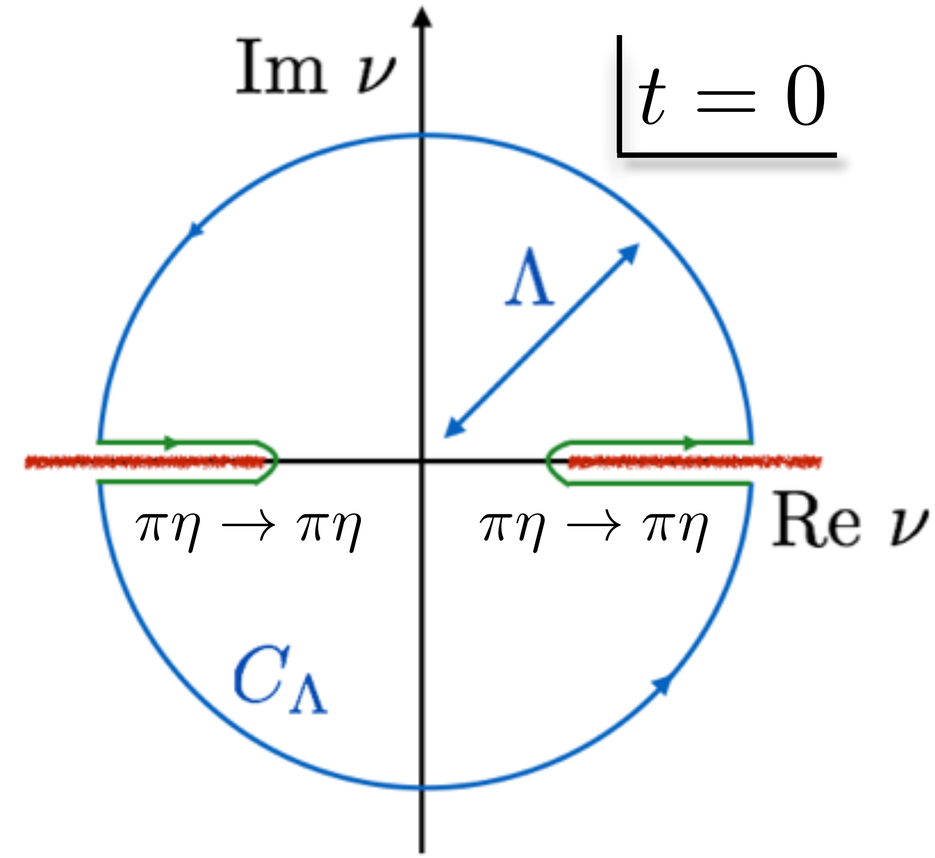


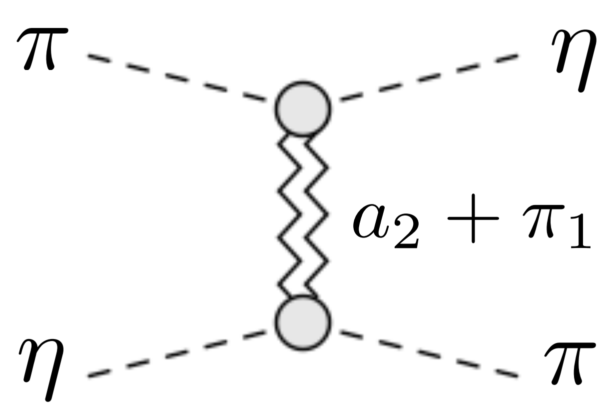


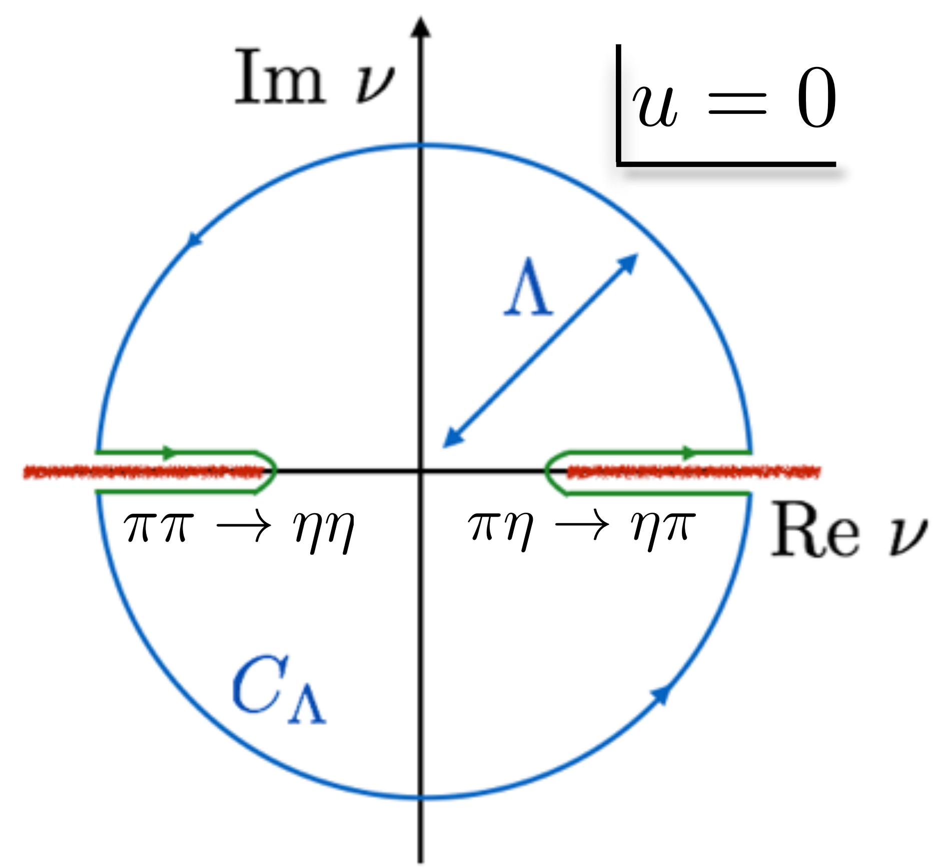
Backward:

$$\frac{1}{2}\int_{\nu_0}^{\Lambda} \text{Im } \left[A^{\pi\eta\to\pi\eta}(\nu, u=0) + A^{\pi\pi\to\eta\eta}(\nu, u=0)\right] d\nu = \beta_a \frac{\Lambda^{\alpha_a+1}}{\alpha_a+1}$$

$$\frac{1}{2}\int_{\nu_0}^{\Lambda} \text{Im } \left[A^{\pi\eta\to\pi\eta}(\nu, u=0) - A^{\pi\pi\to\eta\eta}(\nu, u=0)\right] \frac{\nu}{\Lambda} d\nu = \beta_\pi \frac{\Lambda^{\alpha_\pi+1}}{\alpha_\pi+1}$$

even and odd
signature exchange





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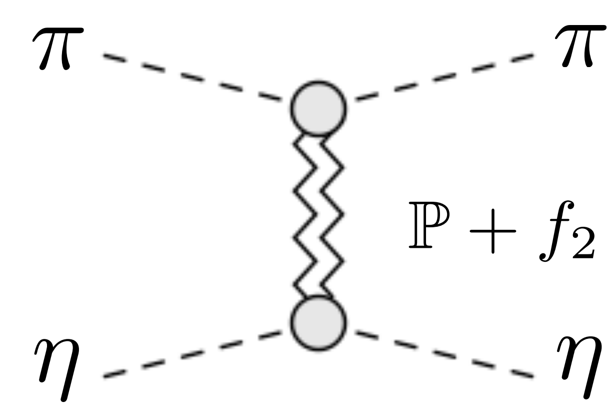

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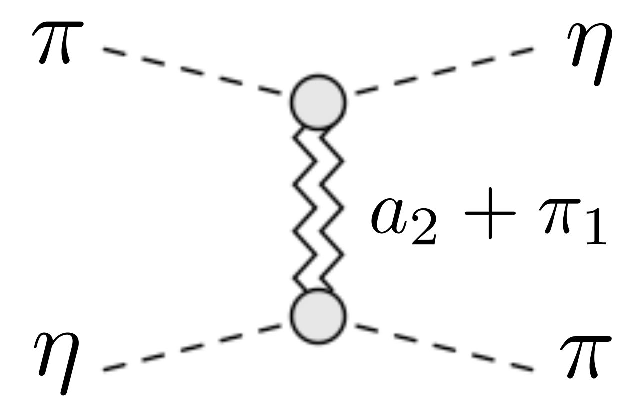
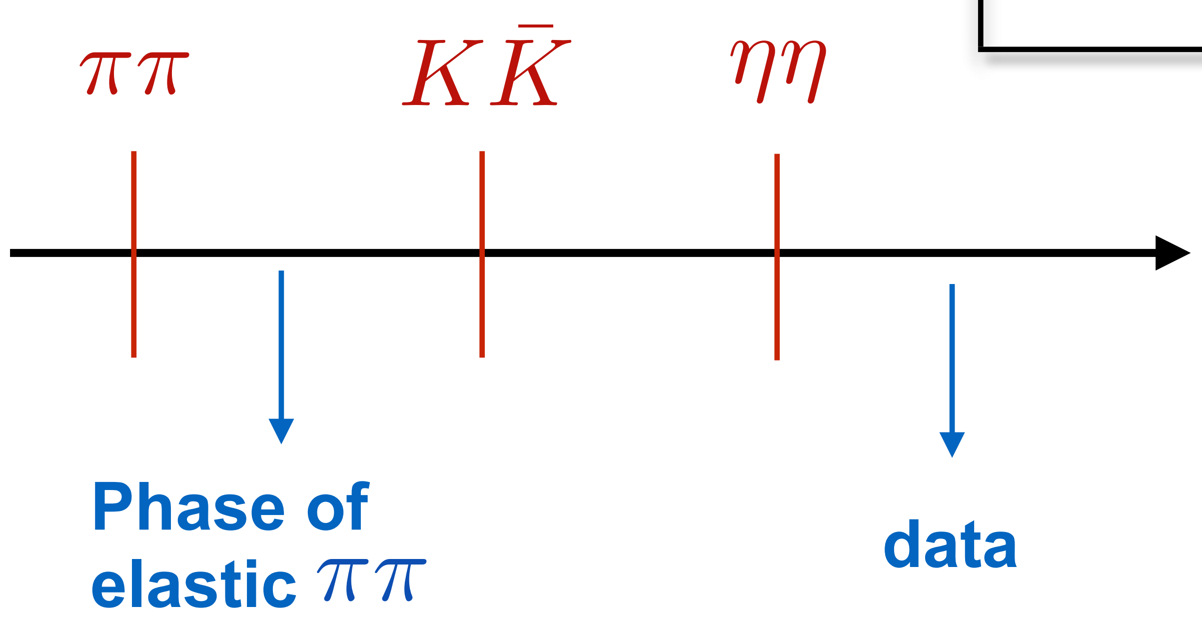


$\pi\pi \to \eta\eta$



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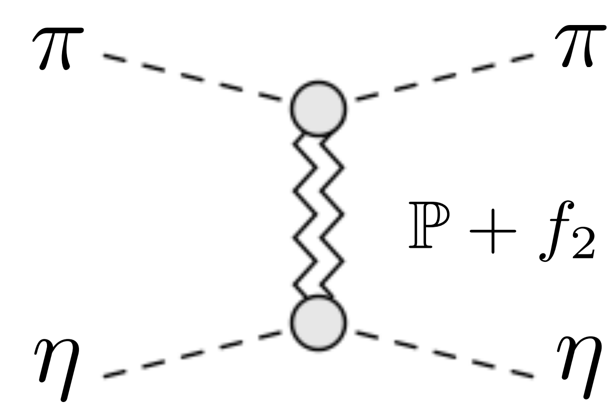

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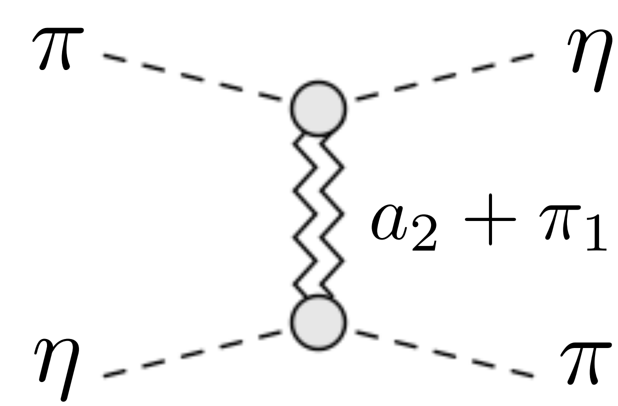
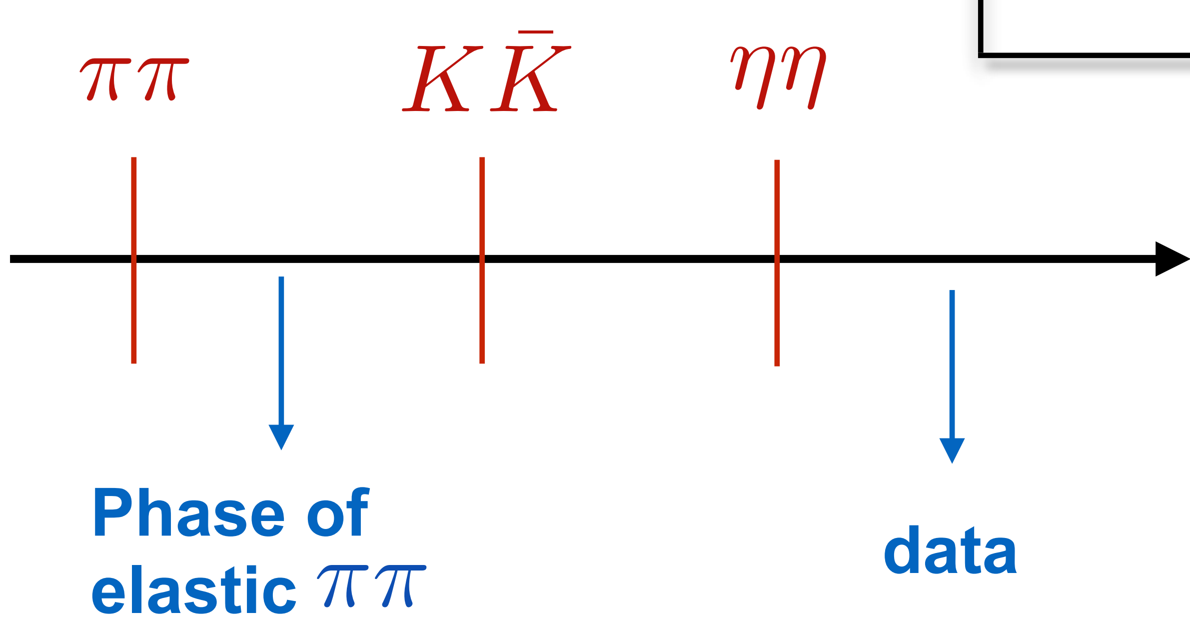
$$\frac{1}{2}\int_{\nu_0}^{\Lambda} \mathrm{Im}\ \left[A^{\pi\eta\to\pi\eta}(\nu, u=0) - A^{\pi\pi\to\eta\eta}(\nu, u=0)\right] \frac{\nu}{\Lambda}d\nu = \beta_\pi\frac{\Lambda^{\alpha_\pi+1}}{\alpha_\pi+1} \approx 0$$



$$\mathrm{Im}\ A^{\pi\eta\to\pi\eta}(\nu, u=0) \approx \mathrm{Im}\ A^{\pi\pi\to\eta\eta}(\nu, u=0)$$

$\pi\pi \to \eta\eta$



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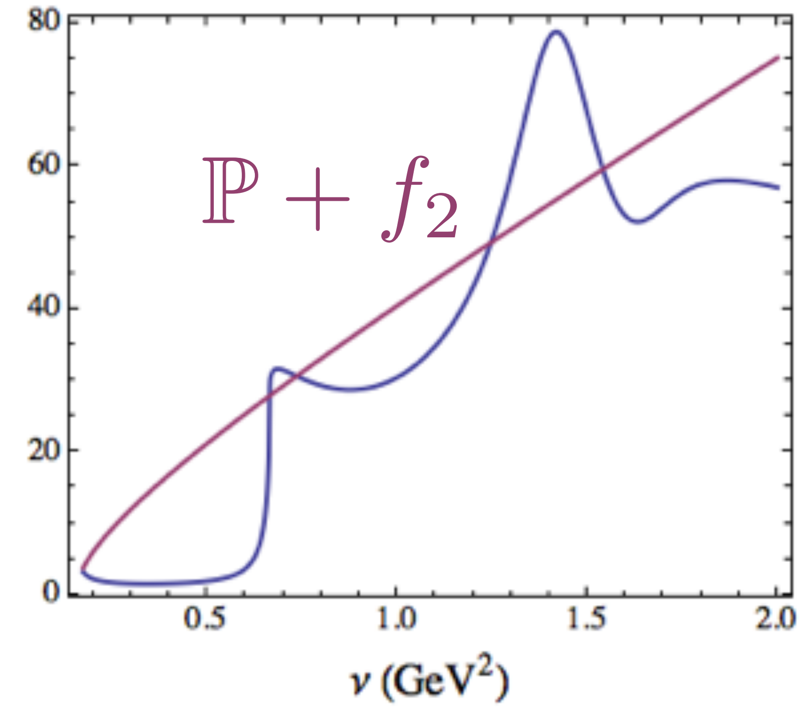

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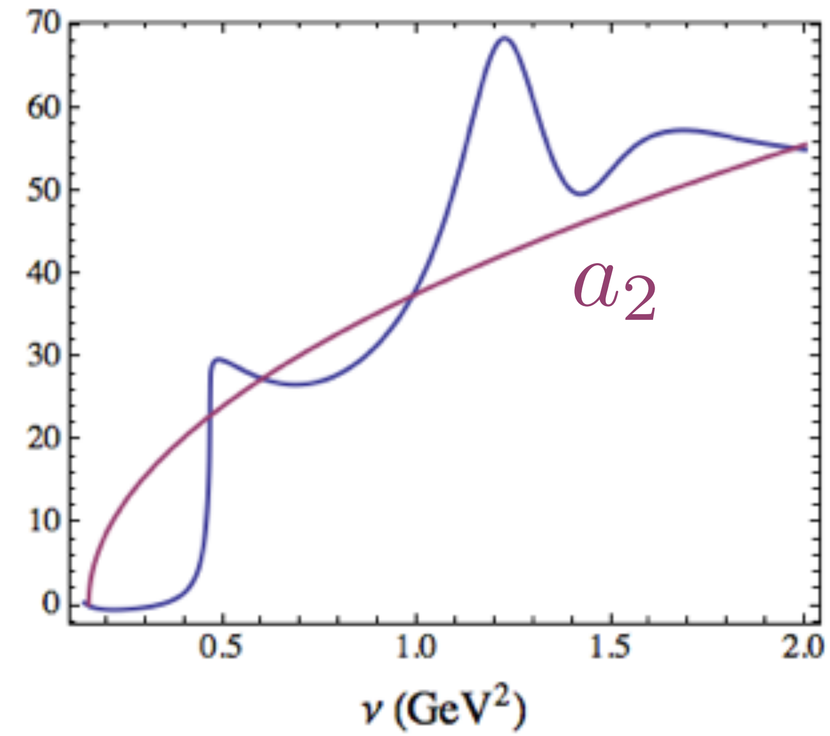

S-wave from Phys. Rev. D95 407 (2017)
(courtesy of J. Ruiz de Elvira)

no P-wave

D-wave is a Breit-Wigner with A2(1320)

Forward and Backward Sum Rules

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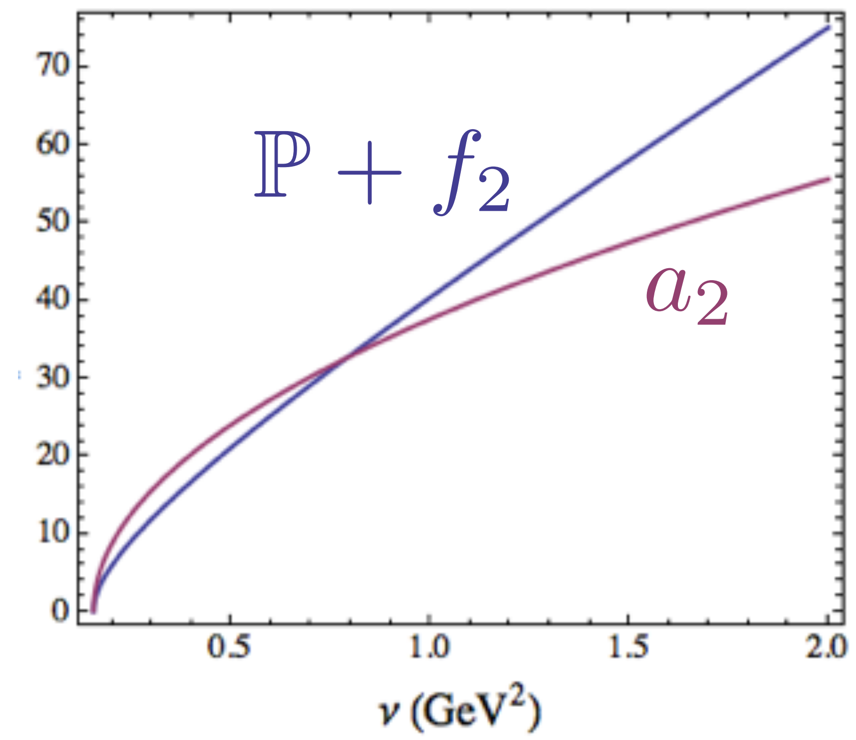

Backward:

$$\int_{\nu_0}^{\Lambda} \text{Im } A^{\pi\eta\to\pi\eta}(\nu, u=0)d\nu = \beta_a\frac{\Lambda^{\alpha_a+1}}{\alpha_a+1}$$

Combine and PW projection:

$$3\int_{\nu_0}^{\Lambda} \text{Im } t_1(\nu)P_1(z)d\nu = \beta_{\mathbb{P}}\frac{\Lambda^{\alpha_{\mathbb{P}}+1}}{\alpha_{\mathbb{P}}+1} + \beta_f\frac{\Lambda^{\alpha_f+1}}{\alpha_f+1} - \beta_a\frac{\Lambda^{\alpha_a+1}}{\alpha_a+1} + \mathcal{O}(m_\eta^2 - m_\pi^2)$$

Forward and Backward Sum Rules

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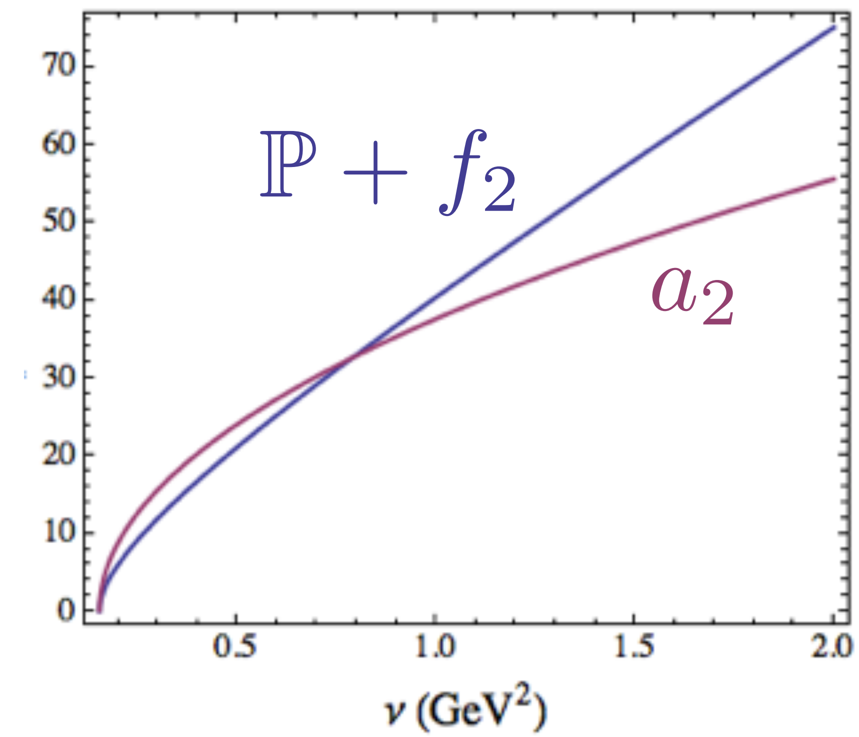
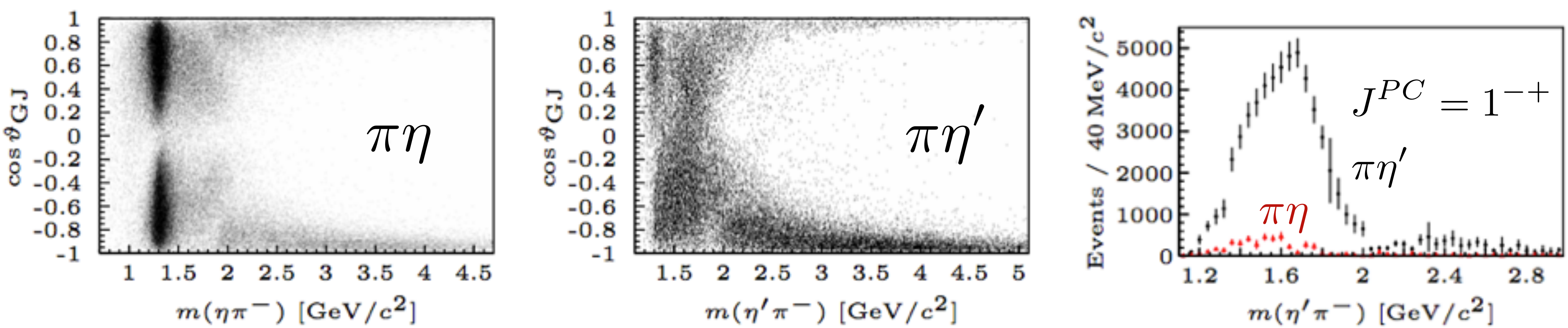
Backward:

$$\int_{\nu_0}^{\Lambda} \text{Im } A^{\pi\eta\to\pi\eta}(\nu, u=0)d\nu = \beta_a \frac{\Lambda^{\alpha_a+1}}{\alpha_a+1}$$



Combine and PW projection:

$$3\int_{\nu_0}^{\Lambda} \text{Im } t_1(\nu)P_1(z)d\nu = \beta_{\mathbb{P}} \frac{\Lambda^{\alpha_{\mathbb{P}}+1}}{\alpha_{\mathbb{P}}+1} + \beta_f \frac{\Lambda^{\alpha_f+1}}{\alpha_f+1} - \beta_a \frac{\Lambda^{\alpha_a+1}}{\alpha_a+1} + \mathcal{O}(m_\eta^2 - m_\pi^2)$$

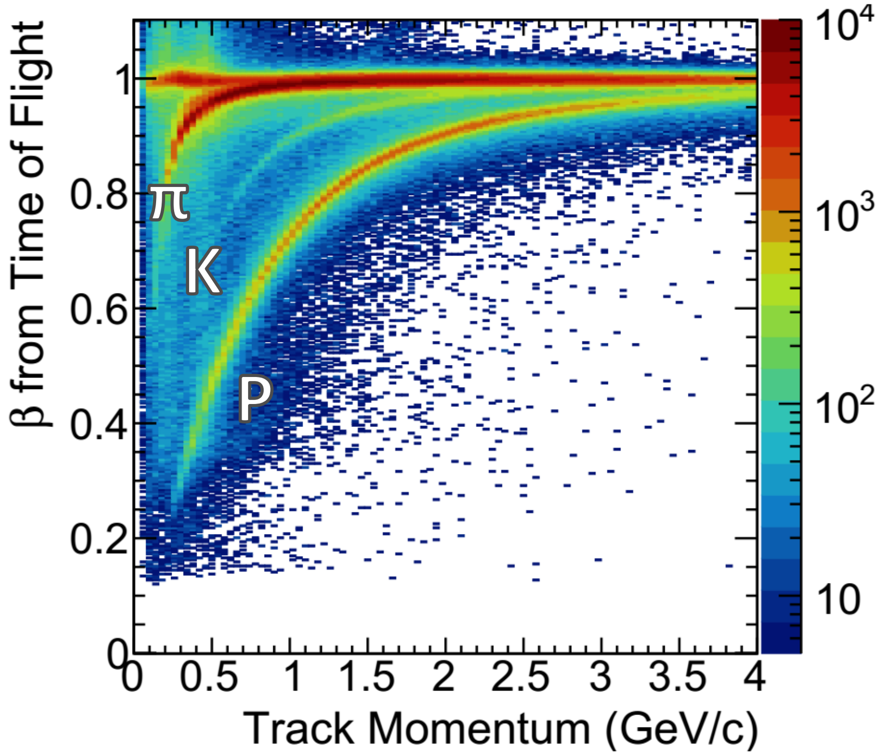


Summary

Dispersion relations constrains parametrizations

Data will be available in a large mass region

One can develop un-subtracted or moments dispersion relations



Example: high mass asymmetry generate exotic wave

$$3\int_{\nu_0}^{\Lambda} \mathrm{Im}\ t_1(\nu)P_1(z)d\nu = \beta_{\mathbb{P}}\frac{\Lambda^{\alpha_{\mathbb{P}}+1}}{\alpha_{\mathbb{P}}+1} + \beta_f\frac{\Lambda^{\alpha_f+1}}{\alpha_f+1} - \beta_a\frac{\Lambda^{\alpha_a+1}}{\alpha_a+1} + \mathcal{O}(m_\eta^2 - m_\pi^2)$$

Phase shifts (and inelasticities) of pseudoscalar scatterings are needed

JPAC Interactive Website

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National Science Foundation

JPAC acknowledges support from DOE and NSF

NEWS

Photoproduction:

1. High energy model for $\pi\Delta$ photoproduction beam asymmetry: (in construction)
2. High energy model for ρ, ω, ϕ photoproduction spin density matrix elements: (in construction)
3. High energy model for η' photoproduction beam asymmetry: $\gamma p \to \eta^{(\prime)} p$ page
4. High energy model for η photoproduction: $\gamma p \to \eta p$ page
5. High energy model for π^0 photoproduction: $\gamma p \to \pi^0 p$ page
6. High energy model for J/ψ photoproduction: $\gamma p \to J/\psi p$ page

INDIANA UNIVERSITY

Interactive webpage: http://www.indiana.edu/~jpac/index.html

Simulation

Beam energy in the lab frame (target rest frame):

E_γ in GeV 9.00

Natural exchanges (vector exchanges): [show/hide]

ρ	$g_{\rho\eta\gamma}$:	0.479	$g_{\rho\eta'\gamma}$:	0.401
	$g_{1\rho}$:	13.49	$b_{1\rho}$:	0.00
	$\gamma^{\rho}_{1,1}$:	0.00	$\gamma^{\rho}_{1,2}$:	0.00
ω	$g_{\omega\eta\gamma}$:	0.136	$g_{\omega\eta'\gamma}$:	0.127
	$g_{1\omega}$:	0.00	$b_{1\omega}$:	0.00
	$\gamma^{\omega}_{1,1}$:	0.00	$\gamma^{\omega}_{1,2}$:	0.00
ϕ	$g_{\phi\eta\gamma}$:	0.210	$g_{\phi\eta'\gamma}$:	0.217
	$g_{1\phi}$:	0.00	$b_{1\phi}$:	0.00
	$\gamma^{\phi}_{1,1}$:	0.00	$\gamma^{\phi}_{1,2}$:	0.00

Unnatural exchanges (axial exchanges):[show/hide]

Unnatural exchanges (pseudo-tensor exchanges):[show/hide]

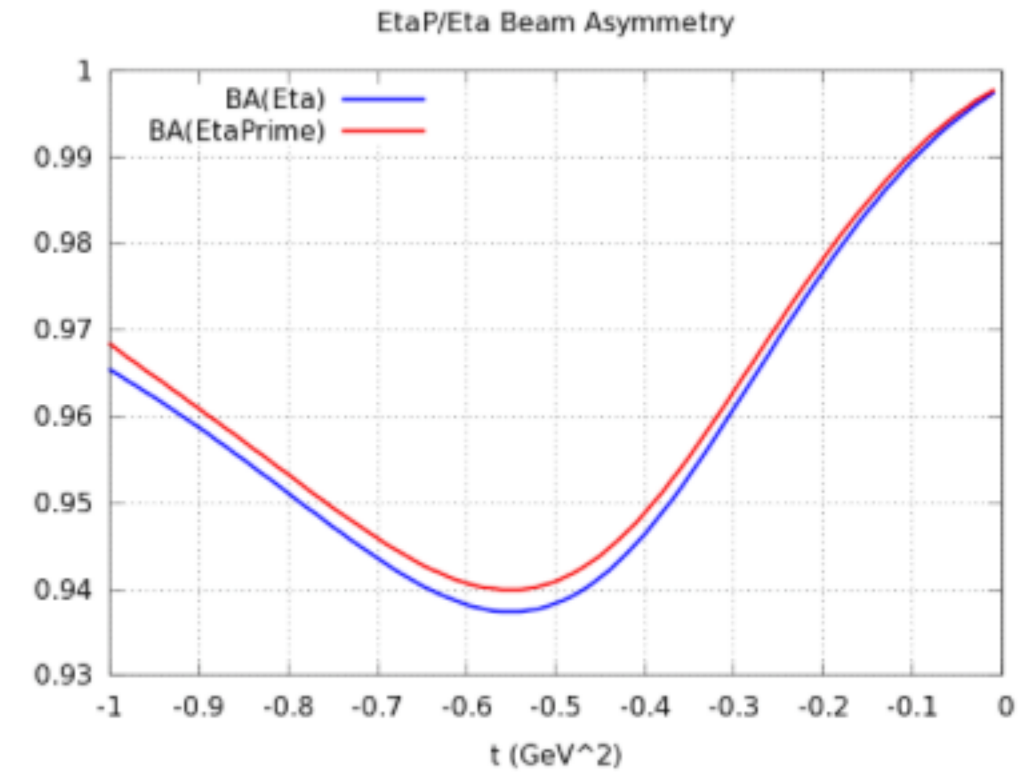
Resources

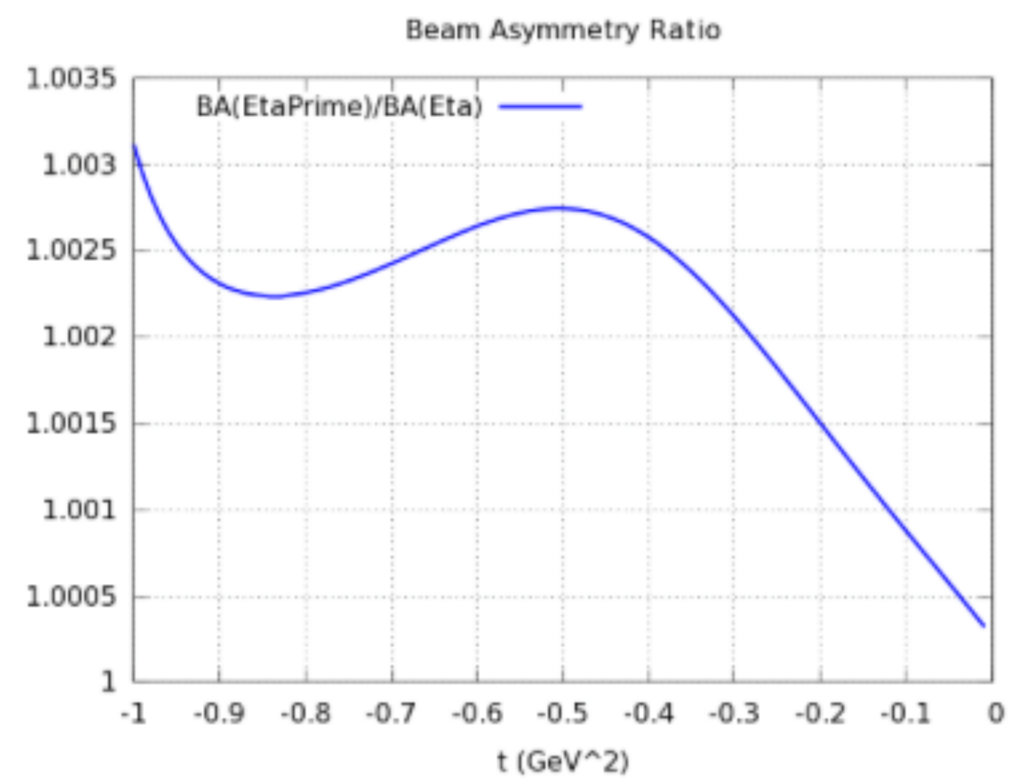
- **Publications:** [Mat17a]
- **C/C++:** C/C++ file
- **Input file:** param.txt , EtaBA.txt .
- **Output files:** EtaP-BA.txt .
- **Contact person:** Vincent Mathieu
- **Last update:** May 2017

Format of the input and output files: [show/hide]

Results

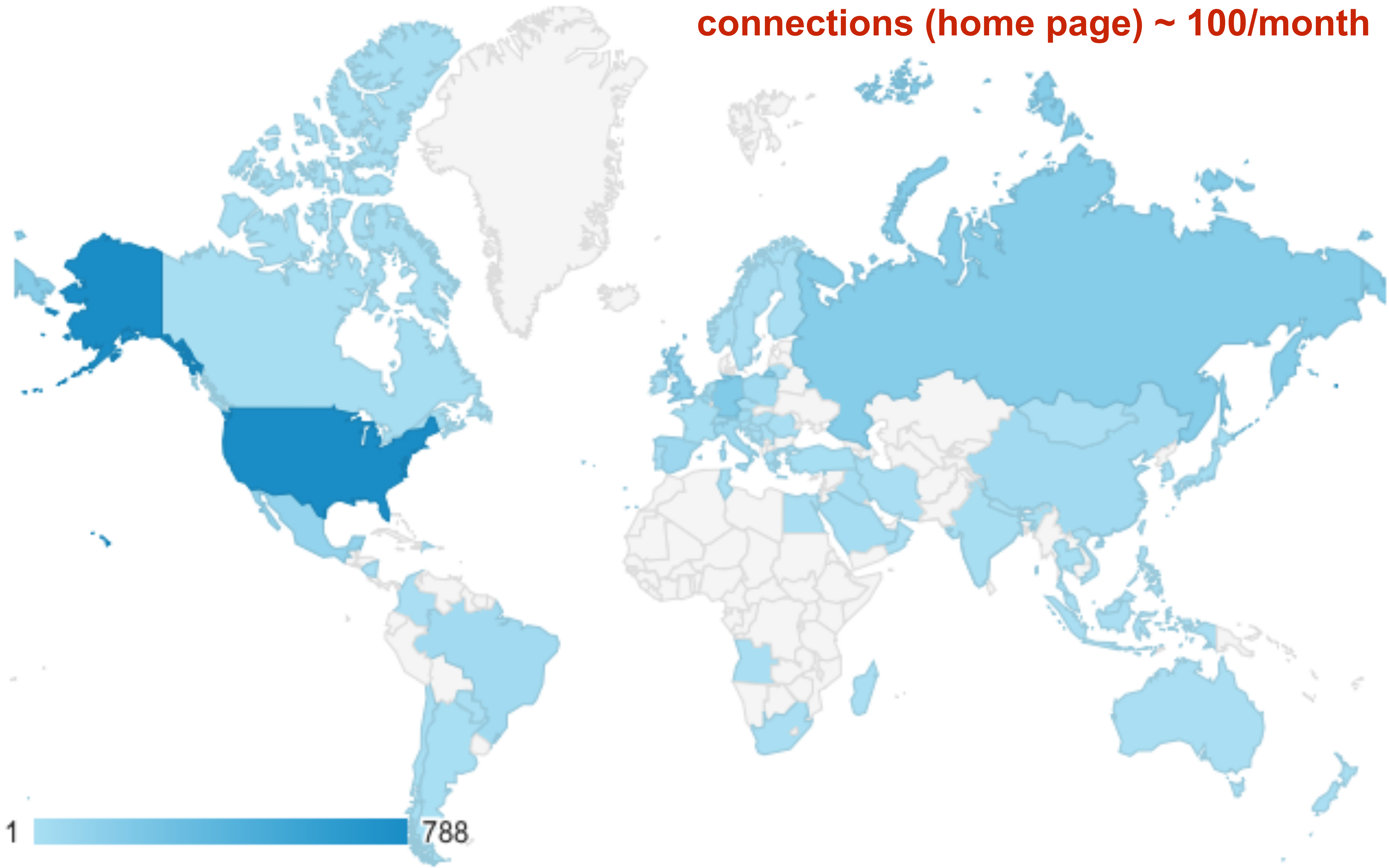
Download the output file: EtaP-BA.txt
Download the plots: BA.png , kVA.png , ratio.png





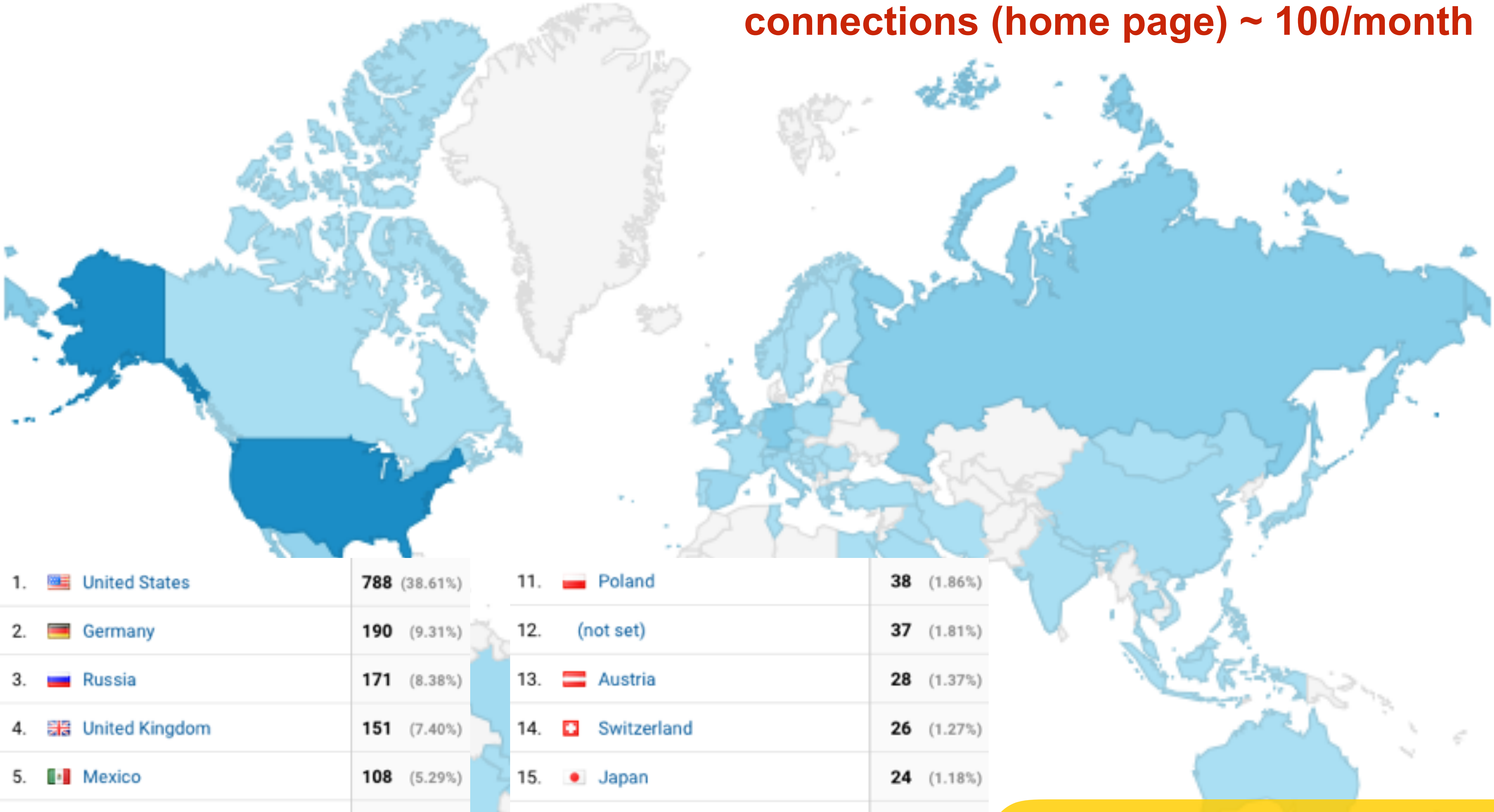
JPAC Interactive Website

connections (home page) ~ 100/month



JPAC Interactive Website

connections (home page) ~ 100/month



1.	United States	788	(38.61%)
2.	Germany	190	(9.31%)
3.	Russia	171	(8.38%)
4.	United Kingdom	151	(7.40%)
5.	Mexico	108	(5.29%)
6.	Belgium	84	(4.12%)
7.	Spain	67	(3.28%)
8.	Italy	56	(2.74%)
9.	Brazil	40	(1.96%)
10.	China	39	(1.91%)
11.	Poland	38	(1.86%)
12.	(not set)	37	(1.81%)
13.	Austria	28	(1.37%)
14.	Switzerland	26	(1.27%)
15.	Japan	24	(1.18%)
16.	Saudi Arabia	19	(0.93%)
17.	Iraq	17	(0.83%)
18.	France	13	(0.64%)
19.	Netherlands	13	(0.64%)
20.	Canada	11	(0.54%)

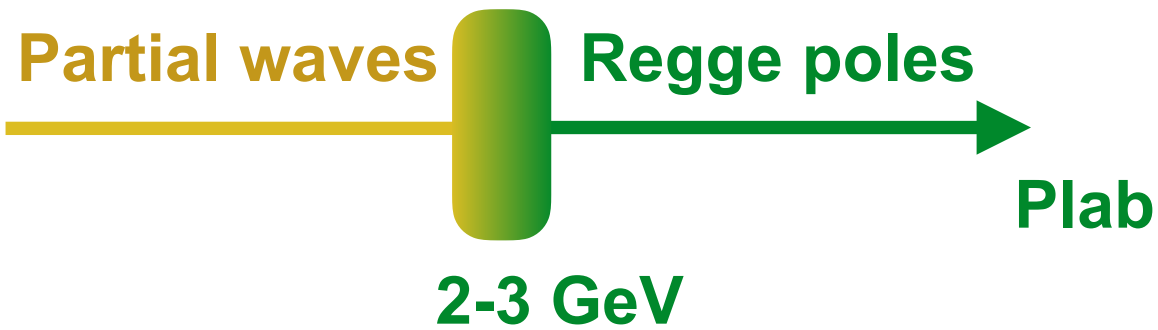
$\pi N \to \pi N$

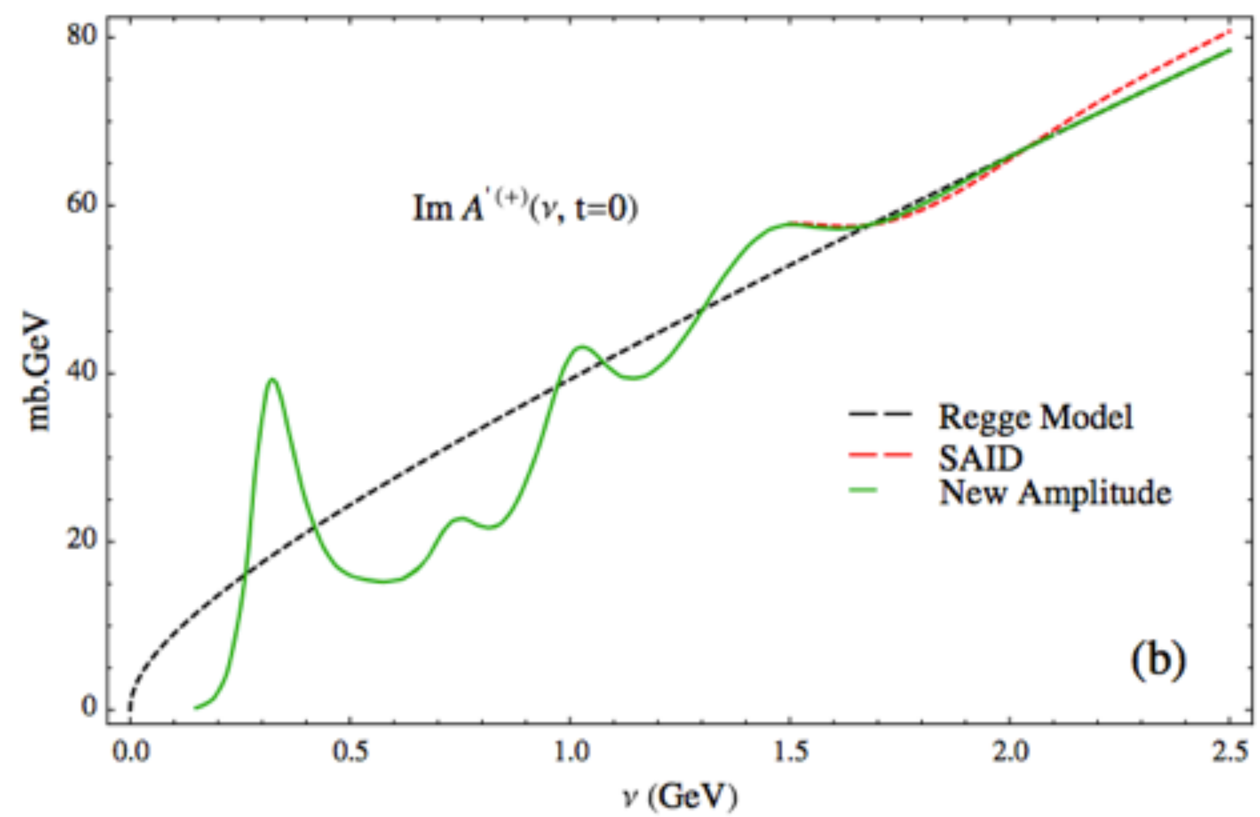
This page has been accessed 2015 times.
Designed by Vincent Mathieu

Backup Slides

Checking Analyticity

Match low energy (PW) and high energy (Regge) imaginary parts



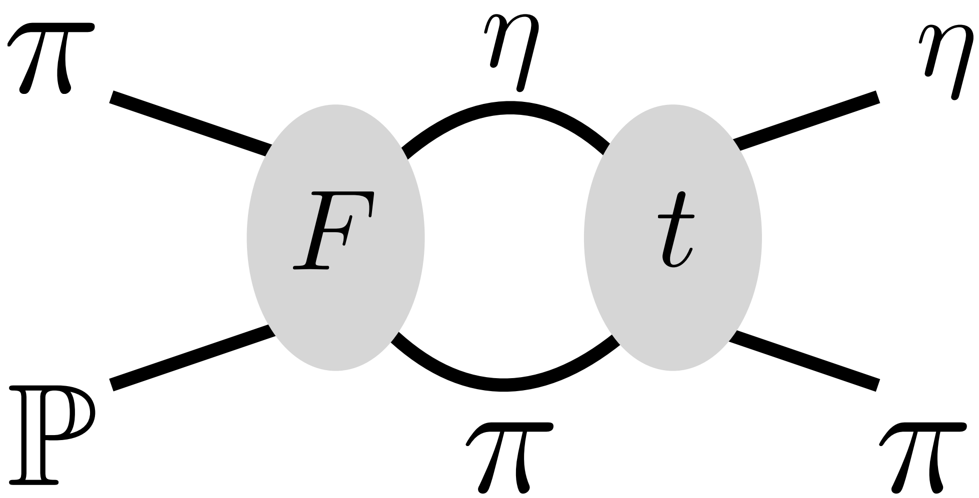


Reconstruct the real part from the dispersion relation

$$A(\nu,t) = \frac{2}{\pi}\int_{\nu_0}^{\infty} \frac{\mathrm{Im}\,A(\nu',t)}{\nu'^2-\nu^2}\nu' d\nu'$$

VM et al (JPAC) PRD92 arXiv:1506.01764

Inelastic Scattering



$$\text{Im } F(s) = \rho(s) t^*(s) F(s)$$

$$F(s) = F_R(s) + F_L(s)$$

F_R has only right hand cut

F_L has only left hand cut

Omnes solution (for known F_L)

$$F(s) = F_L(s) + t(s) \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\rho(s') F_L(s')}{s' - s} ds'$$

Check: $$\text{Im } F(s) = 0 + \rho t^*(s) t(s) \int_{s_0}^{\infty} \frac{\rho(s') F_L(s')}{s' - s} ds' + t^*(s) F_L(s)$$

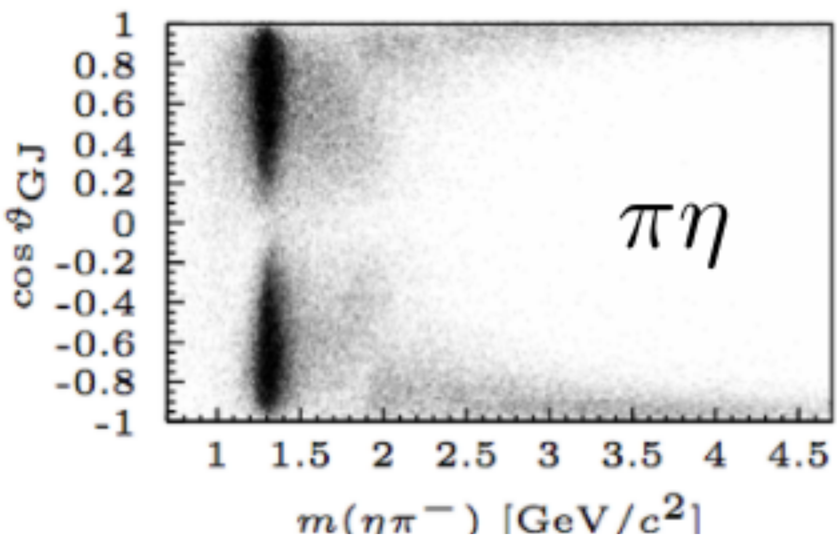
$$\text{Im } (AB) = [\text{Im } A] B + A^* \text{Im } B$$

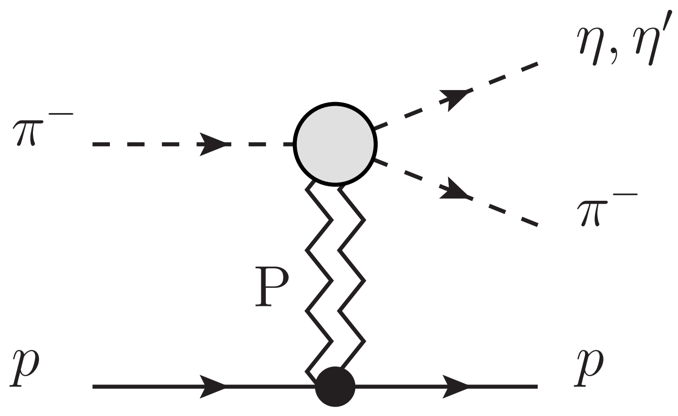
We know how to parametrize t. We need to fit or to model F_L

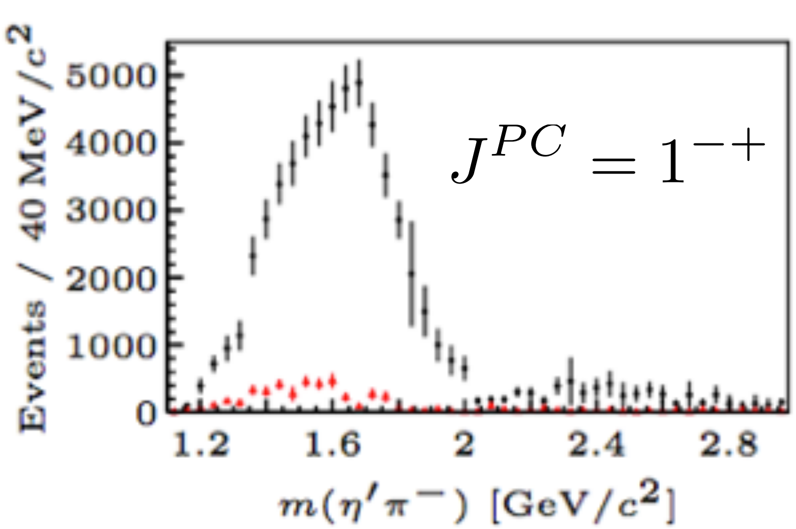
Eta-Pi @COMPASS

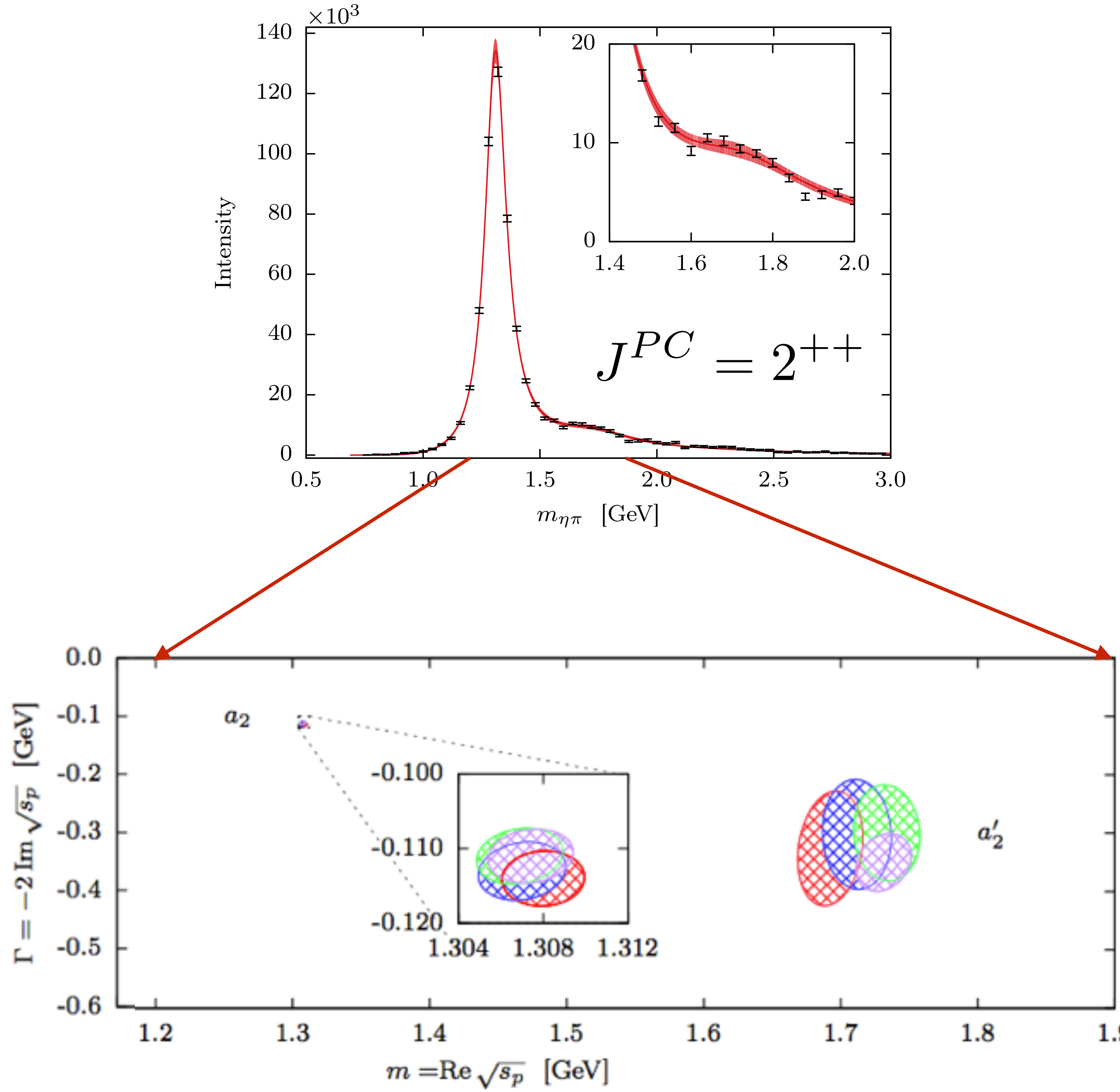
A. Jackura et al (JPAC) and COMPASS, arXiv:1707.02848

COMPASS PLB740 (2015)





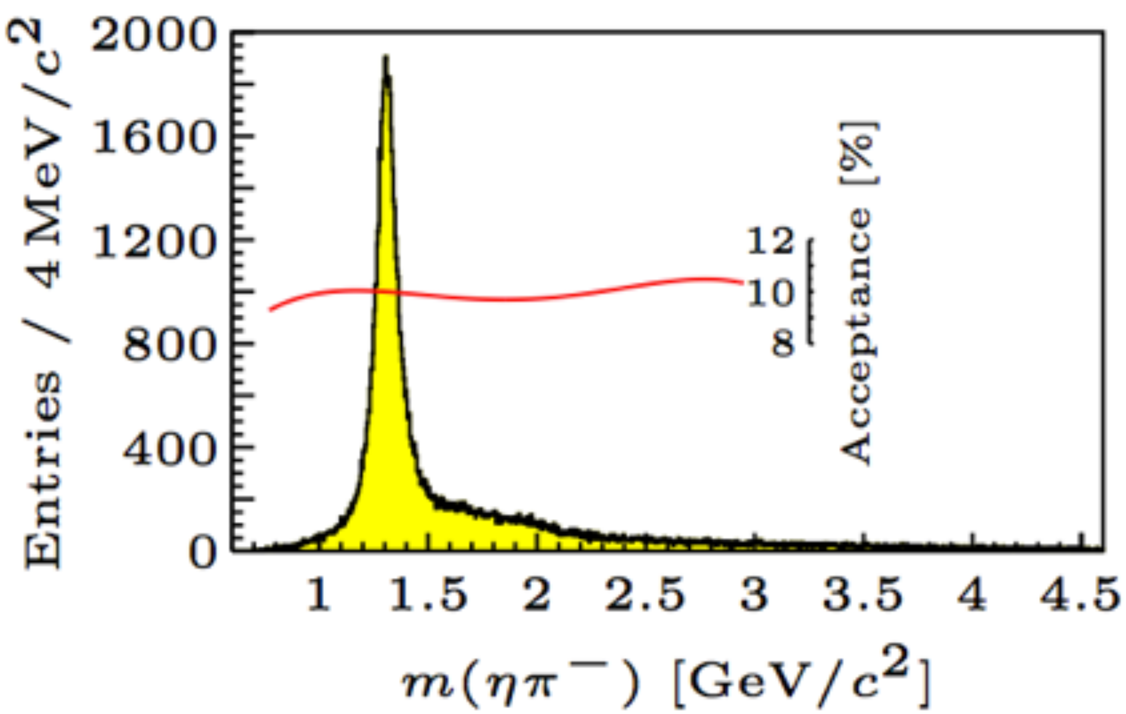


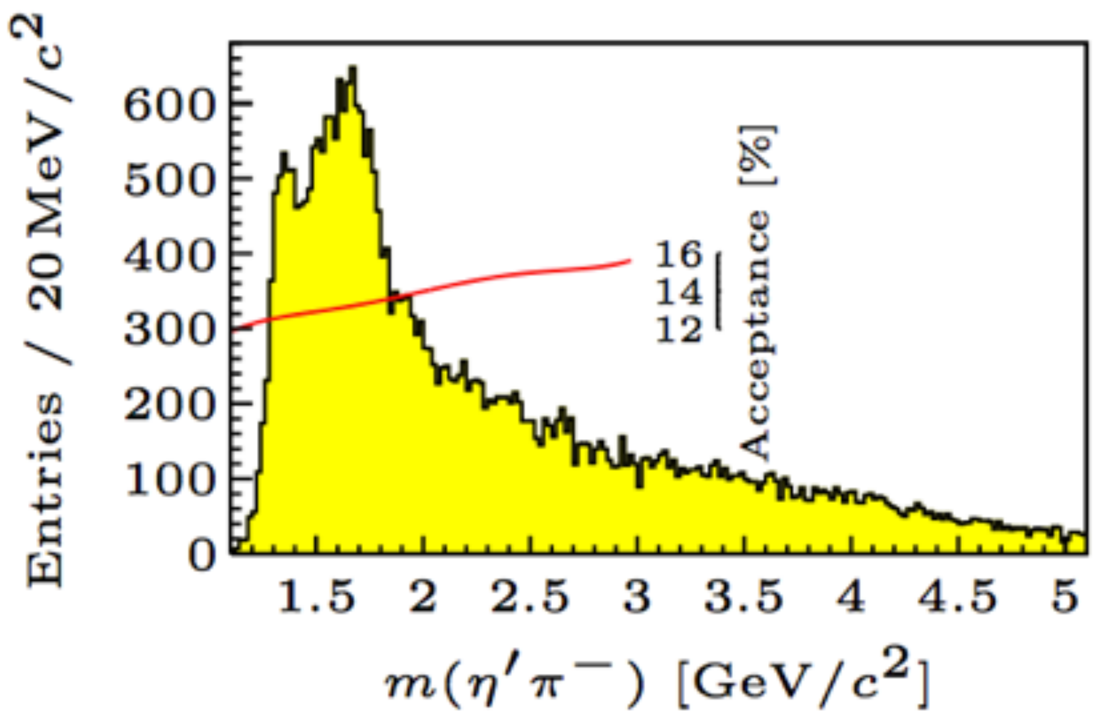


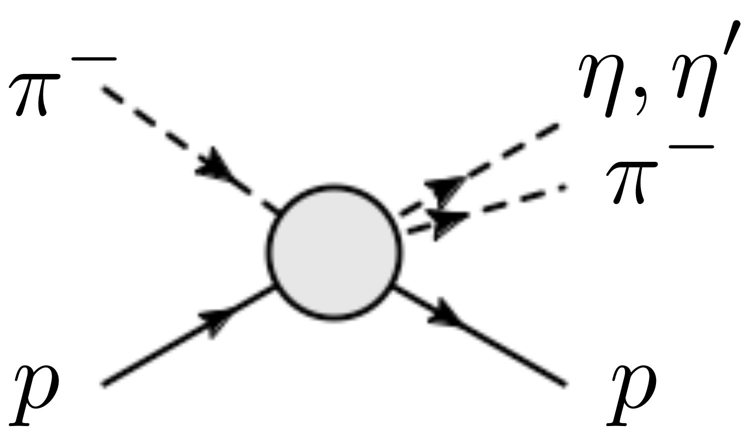
Eta-Pi @COMPASS

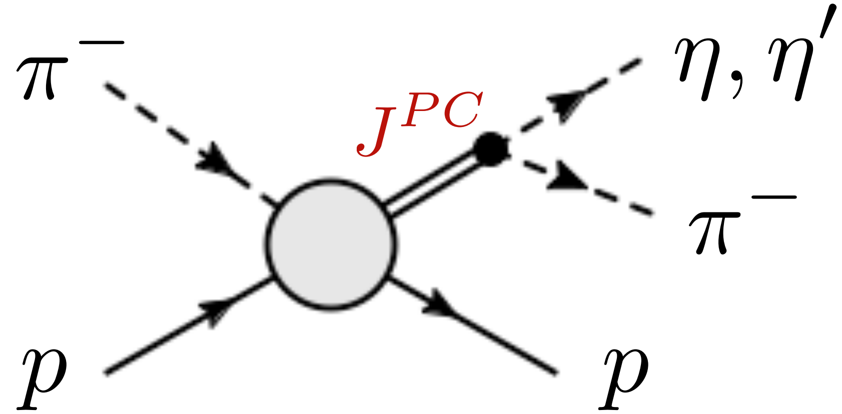
High energy beam: $p_{\mathrm{lab}} = 190\ \mathrm{GeV}$

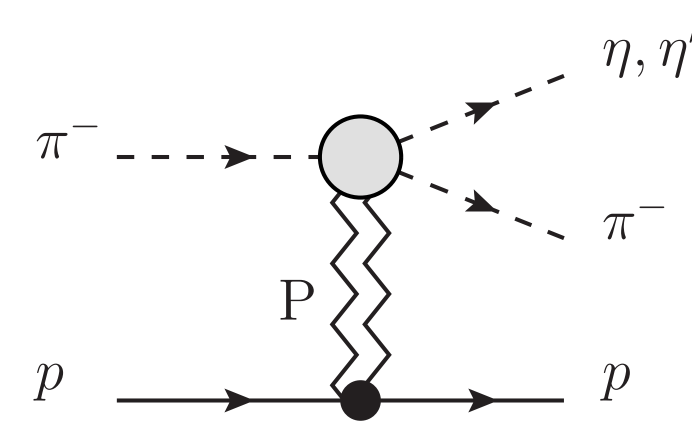
final state: $\underbrace{\pi^-\pi^+\pi^-\gamma\gamma}_{\eta \text{ or } \eta'}$ **with** $\gamma\gamma : \pi^0 \text{ or } \eta$











COMPASS Phys. Lett. B740 (2015)