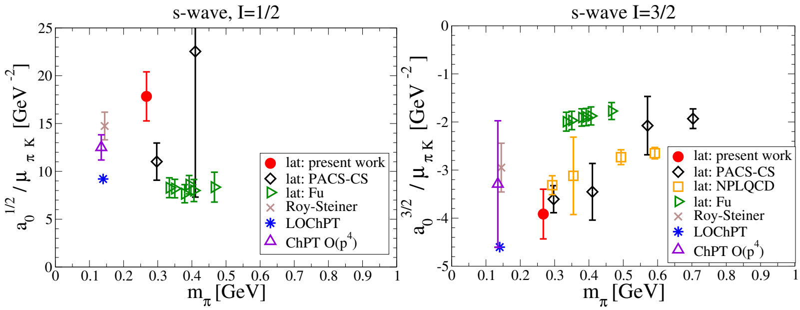
A determination of the pion-kaon scattering length from 2+1 flavor lattice QCD

Daniel Mohler

Newport News,
Feb 15th, 2018

Scattering: From exploratory to precision



Lang, Leskovec, DM, Prelovsek, PRD 86 054508 (2012)

- Much progress in the meantime
  - Formalism is now mature (coupled 2-particle channels well understood)
  - Lattice ensembles at light pion masses/ with several lattice spacings
- Exploratory resonance studies are much improved
  - See talk by Jo Dudek
  - See talk by Colin Morningstar
- For physics motivation see talks by Jose Pelaez, Luka Leskovec, . . .

(Further) previous dynamic lattice results

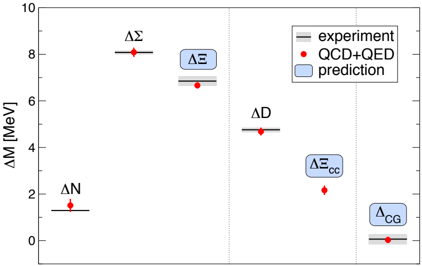
- 2+1 flavor Domain wall valence on staggered sea
  NPLQCD, Beane *et al.* PRD 74 114503 (2006)
  - Uses calculation of $K^+\pi^+$ scattering lengths + SU(3) ChiPT
  - Calculation at $a \approx 0.125$ fm and $290 \leq M_\pi \leq 600$ MeV
- Determination through scalar semileptonic decay form factor $F_0(q^2)$
  Flynn, Nieves, PRD 75 074024 (2007)
  - Lattice data from C. Dawson et al. PRD 74 114502 (2006)
  - $293 \leq M_\pi \leq 408$ MeV at lattice spacing $a = 0.12$ fm
- 2+1 flavor Wilson determination by PACS-CS
  PACS-CS, Sasaki *et al.* *PRD 89 054502 (2014)*
  - $170 \leq M_\pi \leq 710$ MeV at lattice spacing $a \approx 0.09$ fm
  - Fits (to $\pi\pi$, $K\pi$, and KK channels) with lattice version of $O(p^4)$ ChiPT
  - Authors comment that discretization effects should be checked
- Determination using staggered fermions by Fu
  Ziwen Fu, PRD 85 074501 (2012)
  - $330 \leq M_\pi \leq 460$ MeV at lattice spacing $a = 0.15$ fm
  - Not universally accepted due to use of staggered fermions at a single lattice spacing
- More comprehensive comparison → Jacobo Ruiz de Elvira's talk

Outline

1. Motivation
2. Introduction
   - Recent progress in Lattice QCD
   - Systematic uncertainties
   - Simulation landscape
3. Hadron-hadron scattering on CLS ensembles
   - Kaon-pion scattering lengths in $I = \frac{1}{2}$ and $I = \frac{3}{2}$
   - Towards coupled-channel results for the $\Lambda(1405)$

Recent progress in Lattice QCD

- Dynamical simulations with 2+1(+1) flavors of sea quarks
- Simulations at physical pion (light-quark) masses
- Isospin splitting and QCD+QED simulations
- Improved heavy quark actions for charm



BMW Collaboration, Borsanyi et al. Science 347 1452 (2015)

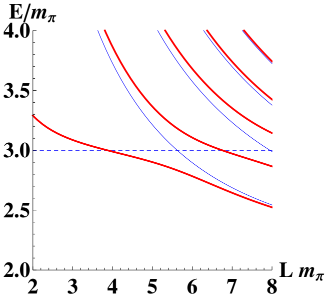
Progress from an old idea: Lüscher's finite-volume method

M. Lüscher Commun. Math. Phys. 105 (1986) 153;
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

Basic observation: Finite volume, multi-particle energies are shifted with regard to the free energy levels due to the interaction

$$E = E(p_1) + E(p_2) + \Delta_E$$

- Energy shifts encode scattering amplitude(s)
- Original method: Elastic scattering in the rest-frame in multiple spatial volumes L^3
- Coupled 2-hadron channels well understood
- $2 \leftrightarrow 1$ and $2 \leftrightarrow 2$ transitions well understood (example $\pi\pi \to \pi\gamma^*$)
- significant progress for 3-particle scattering
  $\to$ see talks by Akaki Rusetsky, Maxim Mai



Fully systematic calculation vs. exploratory study

Important lattice systematics from

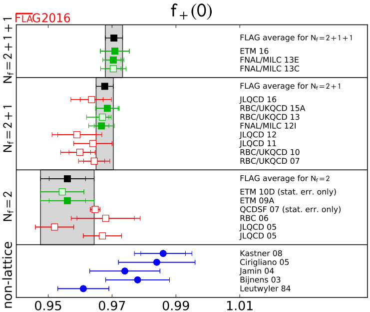
- Taking the *continuum limit*: $a(g, m) \to 0$
- taking the *infinite volume limit*: $L \to \infty$
- Calculation at (or extrapolation to) the physical pion mass

So far: Mostly *exploratory* results

- Should be compared only qualitatively to experiment
- Provide an outlook on future Lattice QCD results

Example for fully systematic results:

- Flavor physics results listed in the FLAG review http://itpwiki.unibe.ch/flag/



Fully systematic calculation vs. exploratory study

Important lattice systematics from

- Taking the *continuum limit*: $a(g, m) \to 0$
- Exploit power law finite volume effects (keeping exponential effects small)
- Calculation at (or extrapolation to) the physical pion mass

So far: Mostly *exploratory* results

- Should be compared only qualitatively to experiment
- Provide an outlook on future Lattice QCD results

Example for fully systematic results:

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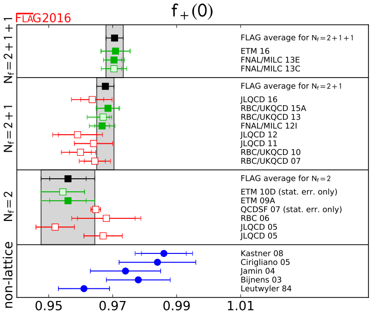
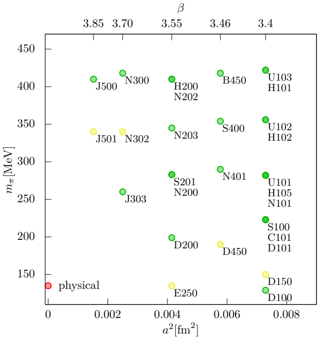


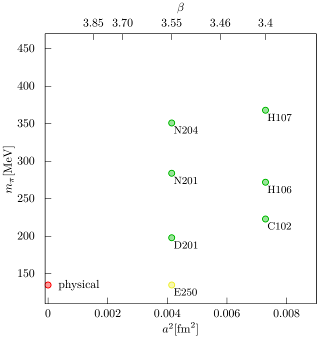
CLS 2+1 flavor ensembles: Overview

Bruno *et al.* JHEP 1502 043 (2015); Bali *et al.* PRD 94 074501 (2016)

$Tr(M) = \text{const.}$



$m_s = \text{const.}$



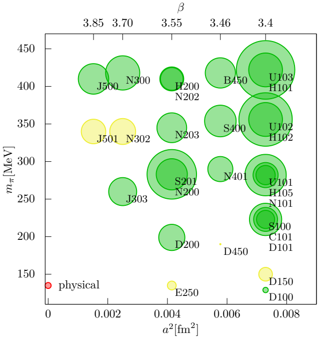
plots by Jakob Simeth, RQCD

- Ensembles at 5 lattice spacings and with a range of $M_\pi \leq 420$MeV
- Ensembles to control (or exploit) finite volume effects

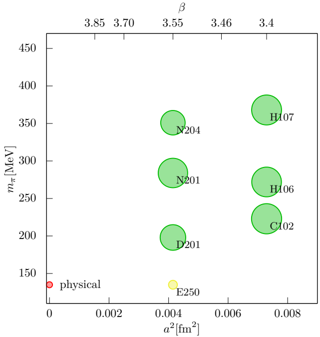
CLS 2+1 flavor ensembles: Statistics – area $\propto$ MDU

`Bruno et al. JHEP 1502 043 (2015); Bali et al. PRD 94 074501 (2016)`

$Tr(M) = \text{const.}$



$m_s = \text{const.}$



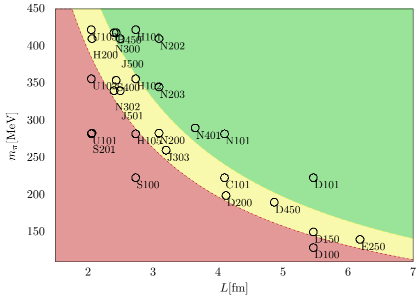
`plots by Jakob Simeth, RQCD`

- >4000 MDU for many ensembles
  Typically save 1 configuration every 4 MDU
- target statistics chosen considering largest τ_{int} (often YM action density)

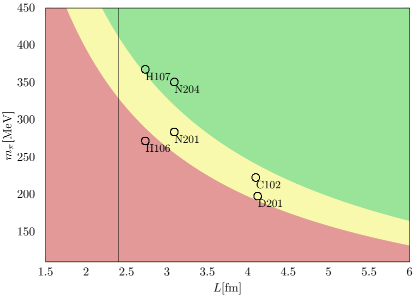
CLS 2+1 flavor ensembles: Volumes used

Bruno *et al.* JHEP 1502 043 (2015); Bali *et al.* PRD 94 074501 (2016)

$Tr(M) = \text{const.}$



$m_s = \text{const.}$



plots by Jakob Simeth, RQCD

- red: $m_\pi L \leq 4$; yellow: $4 \leq m_\pi L \leq 5$; green $5 \leq m_\pi L$
- Most ensembles with $m_\pi L \geq 4$
- Some smaller volumes to check finite size effects

Ensembles to be used

with Ben Hörz, John Bulava, Colin Morningstar

β	a [fm]	m_π [MeV]	sizes	Ensemble names
3.4	0.086	280	$24^3 \times 128$, $32^3 \times 96$, $48^3 \times 128$	U101, H105, N101
3.4	0.086	220	$48^3 \times 96$, $64^3 \times 128$	C101, D101
3.46	0.076	280	$48^3 \times 128$	N401
3.55	0.065	280	$32^3 \times 128$, $48^3 \times 128$	S201,N200
3.55	0.065	200	$64^3 \times 128$	D200
3.55	0.065	137	$96^3 \times 192$	E250
3.7	0.050	280	$64^3 \times 192$	J303

- Using most CLS 2+1 ensembles with $m_\pi \leq 280$ MeV
- PRELIMINARY data from ensembles not grayed out
- Distillation setup differs slightly ensemble by ensemble
- Perambulators and meson functions saved to disk

Ensembles to be used & current statistics

with Ben Hörz, John Bulava, Colin Morningstar

β	a [fm]	m_π [MeV]	size	Ensemble	statistics ($n_{cnfg} \times n_t$)
3.4	0.086	220	$64^3 \times 128$	D101	143×3
3.46	0.076	280	$48^3 \times 128$	N401	275×2
3.55	0.065	280	$48^3 \times 128$	N200	427×2
3.55	0.065	200	$64^3 \times 128$	D200	559×2
3.7	0.050	280	$64^3 \times 192$	J303	329×3

- Using most CLS 2+1 ensembles with $m_\pi \leq 280$ MeV
- PRELIMINARY data from ensembles not grayed out
- Distillation setup differs slightly ensemble by ensemble
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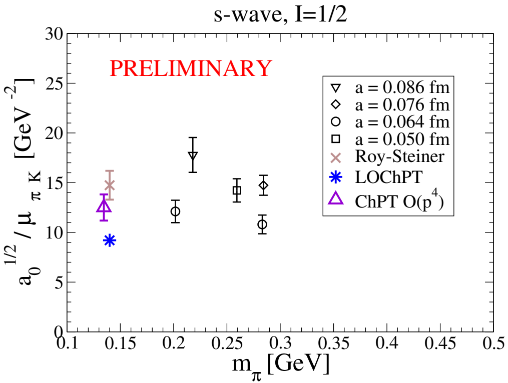
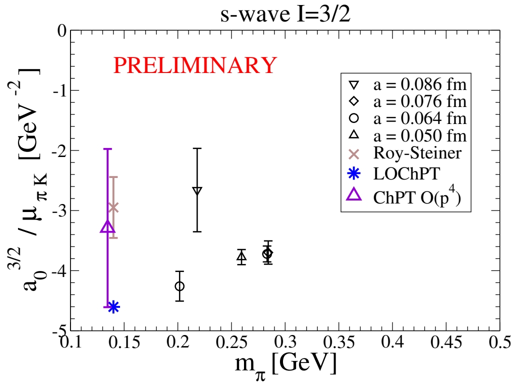
Preliminary results for $a_{\frac{1}{2}}$



- Naive extraction from the energy close to threshold (assumes single-channel, single wave)
- Uncertainty is statistical only
- Non-negligible uncertainty from scale, fitting choices, nonzero effective range, . . .

Preliminary results for $a_{\frac{3}{2}}$



- Naive extraction from the energy close to threshold (assumes single-channel, single wave)
- Uncertainty is statistical only
- Non-negligible uncertainty from scale, fitting choices, nonzero effective range, . . .

Exploratory study: $\Lambda(1405)$, $J^P = \frac{1}{2}^-$

- PDG (4 star resonance)

$$M_\Lambda = 1405^{+1.3}_{-1.0} MeV \qquad \Gamma_\Lambda = 50.5 \pm 2.0$$

- However
  - Unitarized χPT + Model input yields 2 poles with $\Re \approx 1400$MeV
  - CLAS observes different line shapes for $\Sigma^-\pi^+$, $\Sigma^+\pi^-$ and $\Sigma^0\pi^0$
    Interference between $I = 0$ and $I = 1$ amplitudes is the likely reason
  - Even the $\Sigma^0\pi^0$ is badly described by a single Breit-Wigner
  - CLAS data consistent with popular 2-pole picture
  - No satisfactory lattice results (although claims exist)
- Relevant channels: $\Sigma\pi$, $N\bar{K}$ (and maybe $\Lambda\eta$); simulation in isospin limit
- Goal: Explore coupled channel problem and extract scattering amplitudes from the low-lying energy spectrum

together with J. Bulava, M. Hansen, B. Hörz, C. Morningstar

Ensemble and group theory

Current data on CLS Ensemble N200

a [fm]	$T \times L^3$	m_π [MeV]	m_K [MeV]	$m_\pi L$	N_{cnfg}
0.0644	128×48^3	280	460	4.3	427

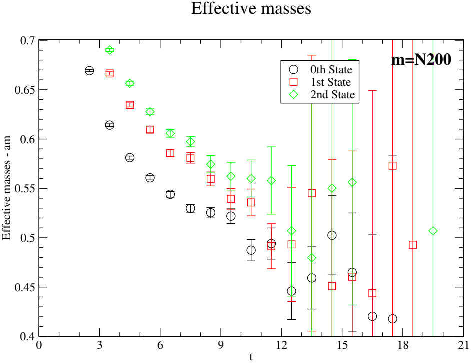
Lattice irreducible representations for a given J^P

`see Morningstar et al. arXiv:1303.6816`

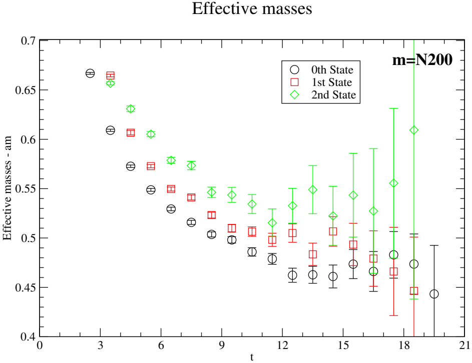
J^P	[000]	[00n]	[0nn]	[nnn]	
$\frac{1}{2}^+$	G_{1g}	G_1	G	G	$\Lambda, \Lambda(1600)$
$\frac{1}{2}^-$	G_{1u}	G_1	G	G	$\Lambda(1405), \Lambda(1670)$
$\frac{3}{2}^+$	H_g	G_1, G_2	$2G$	F_1, F_2, G	$\Lambda(1690)$
$\frac{3}{2}^-$	H_u	G_1, G_2	$2G$	F_1, F_2, G	$\Lambda(1520), \Lambda(1690)$

A first look at some data: Truncating the basis

3-quark interpolators alone

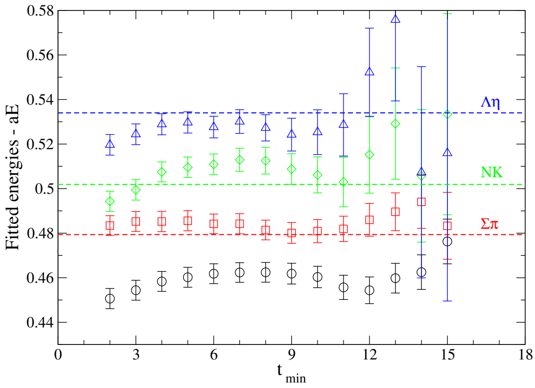


basis including $N\bar{K}$ and $\Sigma\pi$



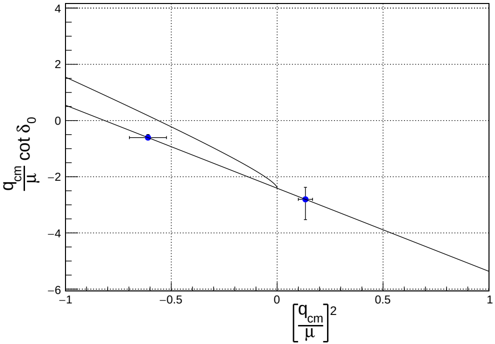
- Plot from an older dataset
- Basis including meson-meson states starts to yield usable plateaus
- Even a diverse 3-quark basis does not yield the correct spectrum!

Rest frame calculation: Fit stability



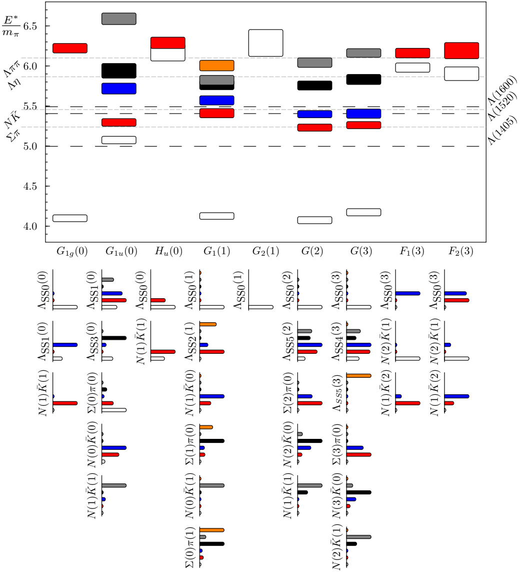
- Results for fits from t_{min} to 20
- Non-interacting levels indicated by their central value
- Correlated differences start to become significant

Rest frame calculation: effective range approximation



- Cannot rule out the simplest scenario of one bound state below $\Sigma\pi$
- Makes no statement about more complicated scenarios or physical m_π

Adding moving frames: Pattern of energy levels



- With $Tr(M) = \text{const.}$, expect mild chiral extrapolation of *uds* states
- Our $G_1(1)$ data does not have a large enough basis (caution)
- The Λ ground state is seen where expected
- No indication of levels close to $\Lambda(1600)$ in G_{1g} irrep
- No indication of levels close to $\Lambda(1520)$ in any irrep
- Apparent absence of FV states should constrain models

Outlook: What to expect?

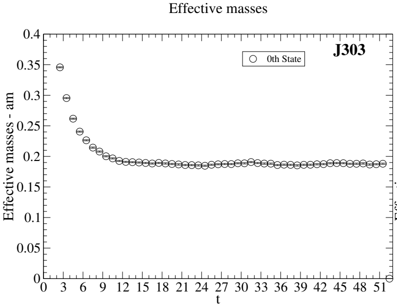
- Isospin $\frac{1}{2}$ and $\frac{3}{2}$ kaon-pion scattering lengths
  - Add further ensembles at coarsest lattice spacing (CLS H105, N101)
  - Add a data point at physical pion and kaon mass (CLS E250)
  - Add statistics on C101/D101 ?
  - Investigate effective range effects
    $\rightarrow$ See talk by C. Morningstar for methodology
  - Address various systematic uncertainties
- Coupled channel calculation for the $\Lambda(1405)$
  - Currently doubling N200 statistics ($\approx$ 400 configs $\rightarrow$ $\approx$ 800 configs)
  - Still enlarging the basis
    (include more 3-quark interpolators; include some $\Lambda\eta$)
  - We plan to run further pion mass, lattice spacing
  - Once we have bigger dataset:
    Check for consistency with model-inspired K-matrix parameterizations

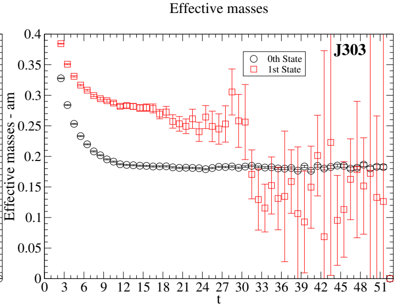
Thank you!

The purpose of computing is insight, not numbers

– Richard Hamming

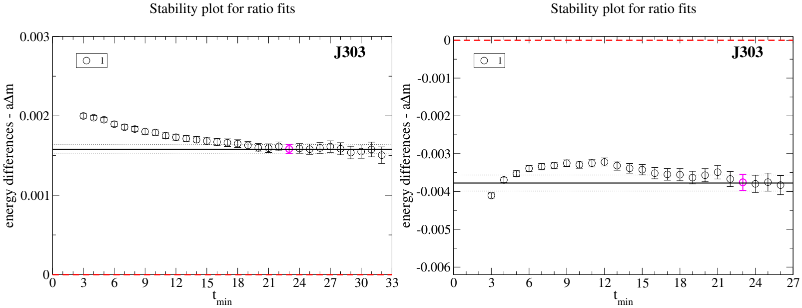
Example plateaus: J303





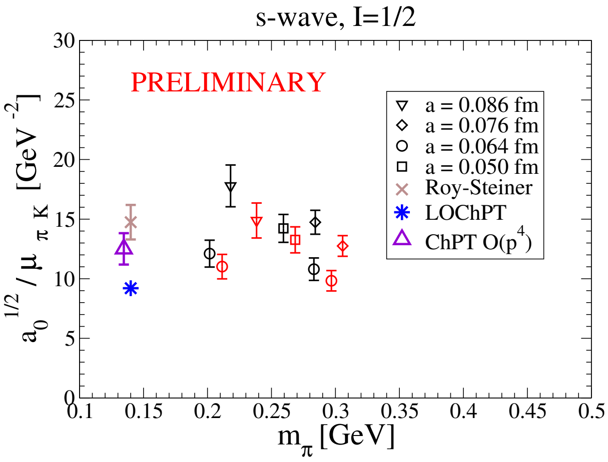
- Would benefit from optimized interpolators for pion/kaon

Fit systematics: Correlated differences vs. ratios



- Ratios from ratio fits agree with correlated energy differences at large time separation
- Significant fit systematics in some cases

Alternate chiral trajectory for $a_{\frac{1}{2}}$

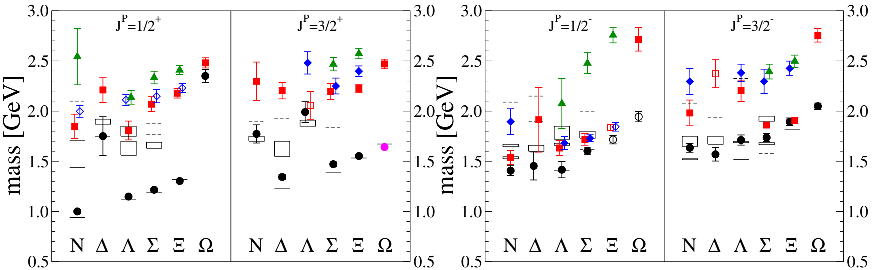


- Demonstrates importance of discretization effects

Results from 3-quark interpolating fields

- Many older results from 3-quark interpolators

  `Example:  Engel, Lang, DM, Schäfer, PRD 87 074504 (2013)`
  `Λ's in Engel et al.  PRD 87 034502 (2013)`



- OK at heavy quark masses where states are stable
- Typically no indications of close-by scattering thresholds
- Experience from meson sector: Spectrum not only incomplete but wrong!

Meson-baryon scattering – challenges

- Nucleon noise to signal
  - For the nucleon we have (argument by Parisi, Lepage)

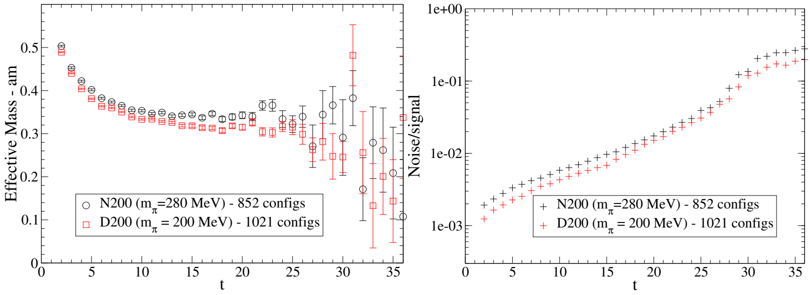
$$N\sigma^2_{N,\mathbf{p}=0} = \left\langle C_N(\mathbf{p}=0,t;m)^2\right\rangle - \langle C_N(\mathbf{p}=0,t;m)\rangle^2$$
$$\propto Z_{3\pi}\mathrm{e}^{-3m_\pi t} + Z_N^2\mathrm{e}^{-2m_N t}$$

  - The noise to signal ratio therefore degrades exponentially

$$\frac{\sigma_N(t)}{\langle C_N(t)\rangle} \simeq \frac{1}{\sqrt{N}}\mathrm{e}^{\left(m_N-\frac{3}{2}m_\pi\right)t}$$

- Contractions are more complicated
- Less cases where 3-particle scattering can be ignored (in a first step)

Nucleon noise/signal



Data from Tim Harris, Konstantin Ottnad, Mainz

- Slope in (most of) plateau region does not reach asymptotic value (given by $m_N - \frac{3}{2}m_\pi$)
- Suggests that in practice noise/signal scaling is not as severe
- Exponential growth qualitatively observed