Using $\pi K$ to understand heavy meson decays

Alessandro Pilloni
Joint Physics Analysis Center

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Outline

- The exotic XYZ states
- The $Z(4430)$
- Helicity and covariant formalism
- Playing with fits
- Conclusions
$K\pi$ interactions

The major source of data for $K\pi$ scattering is the LASS experiment

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Experience with the
Large Aperture Superconducting
Solenoid Spectrometer

Bill Dunwoody (SLAC Group B emeritus)

The major source of data for $K\pi$ scattering is the LASS experiment
although several people believe it is the PDG with photon beam.
A host of unexpected resonances have appeared decaying mostly into charmonium + light
Hardly reconciled with usual charmonium interpretation

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Exotic XYZ landscape

Esposito, AP, Polosa, Phys.Rept. 668
Several of them have been observed in $B \to (c\bar{c}\,\pi)K$ or $B \to (c\bar{c}\,\pi\pi)K$
The $Z(4430)$

In 2007, Belle claimed the observation of an exotic charged charmoniumlike state in the $B^+ \to (\psi'\pi^-)K^+$ decay

Belle, PRL100, 142001; PRD80, 031104; PRD88, 074026

BaBar did not confirm the observation

BaBar PRD79, 112001
The Z(4430)

LHCb presented the full 4D analysis estimating a 14σ significance for an exotic Z

The structure of $K^*$ is very rich

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Mass (MeV/$c^2$)</th>
<th>$\Gamma$ (MeV/$c^2$)</th>
<th>$J^P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^*(800)^0$</td>
<td>682±29</td>
<td>547±24</td>
<td>0$^+$</td>
</tr>
<tr>
<td>$K^*(892)^0$</td>
<td>895.81±0.19</td>
<td>47.4±0.6</td>
<td>1$^-$</td>
</tr>
<tr>
<td>$K^*(1410)^0$</td>
<td>1414±15</td>
<td>232±21</td>
<td>1$^-$</td>
</tr>
<tr>
<td>$K_0^*(1430)^0$</td>
<td>1425±50</td>
<td>270±80</td>
<td>0$^+$</td>
</tr>
<tr>
<td>$K_2^*(1430)^0$</td>
<td>1432.4±1.3</td>
<td>109±5</td>
<td>2$^+$</td>
</tr>
<tr>
<td>$K^*(1680)^0$</td>
<td>1717±27</td>
<td>322±110</td>
<td>1$^-$</td>
</tr>
<tr>
<td>$K_3^*(1780)^0$</td>
<td>1776±7</td>
<td>159±21</td>
<td>3$^-$</td>
</tr>
</tbody>
</table>

LHCb, PRL112, 222002

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The $Z(4430)$

A model-independent analysis confirmed the need for an exotic component

\[ I^G J^{PC} = 1^+ 1^- \]

\[ M = 4475 \pm 7^{+15}_{-25} \text{ MeV} \]

\[ \Gamma = 172 \pm 13^{+37}_{-34} \text{ MeV} \]

Caveat: the analysis is based on the expansion of the Dalitz plot in Legendre momenta, each one having contributions from infinite partial waves
**S-Matrix principles**

These are constraints the amplitudes have to satisfy, but do not fix the dynamics.

Resonances (QCD states) are poles in the unphysical Riemann sheets.

\[ A(s, t) = \sum_l A_l(s) P_l(z_s) \]

**Analyticity**

\[ A_l(s) = \lim_{\epsilon \to 0} A_l(s + i\epsilon) \]

These are constraints the amplitudes have to satisfy, but do not fix the dynamics.

Resonances (QCD states) are poles in the unphysical Riemann sheets.

See talk by B. Kubis

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These are constraints the amplitudes have to satisfy, but do not fix the dynamics.

Resonances (QCD states) are poles in the unphysical Riemann sheets.

See talk by B. Kubis.
Recipes to build an amplitude

M. Mikhasenko, AP, J. Nys et al. (JPAC), arXiv:1712.02815

The literature abounds with discussions on the optimal approach to construct the amplitudes for the hadronic reactions

- **Helicity formalism**

  Jacob, Wick, Annals Phys. 7, 404 (1959)

- **Covariant tensor formalisms**

  Chung, PRD48, 1225 (1993)
  Chung, Friedrich, PRD78, 074027 (2008)
  Anisovich, Sarantsev, EPJA30, 427 (2006)

The common lore is that the former one is nonrelativistic, especially when expressed in terms of LS couplings and the latter takes into account the proper relativistic corrections.
How helicity formalism works

- Helicity formalism enforces the constraints about rotational invariance
- It allows us to fix the angular dependence of the amplitude
- What about energy dependence?

Example: $B \rightarrow \psi K^* \rightarrow \pi K$

\[
\mathcal{M}^{K^*}_{\Delta \lambda \mu} \equiv \sum_n \sum_{\lambda_{K^*}} \sum_{\lambda_{\psi}} \mathcal{H}_{\lambda_{K^*}, \lambda_{\psi}}^{B \rightarrow K^*_{n} \psi} \delta_{\lambda_{K^*}, \lambda_{\psi}}
\]

\[
\mathcal{H}^{K^* \rightarrow K \pi}_{n} D_{\lambda_{K^*}, 0} J_{K^*_{n}} (\phi_{K}, \theta_{K^*}, 0)^* \\
R_{K^*_{n}} (m_{K \pi}) D_{\lambda_{\psi}, \Delta \lambda_{\mu}}^{1} (\phi_{\mu}, \theta_{\psi}, 0)^*
\]

Each set of angles is defined in a different reference frame
How tensor formalism works

The method is based on the construction of explicitly covariant expressions.

- To describe the decay $a \rightarrow bc$, we first consider the polarization tensor of each particle, $\varepsilon_{\mu_1...\mu_i}^j(p_i)$
- We combine the polarizations of $b$ and $c$ into a “total spin” tensor $S_{\mu_1...\mu_S}(\varepsilon_b, \varepsilon_c)$
- Using the decay momentum, we build a tensor $L_{\mu_1...\mu_L}(p_{bc})$ to represent the orbital angular momentum of the $bc$ system, orthogonal to the total momentum of $p_a$
- We contract $S$ and $L$ with the polarization of $a$

Tensor $\times R_X(m)$ which contain resonances and form factors
What do we know?

- Energy dependence is not constrained by symmetry
- Still, there are some known properties one can enforce

\[ R_X(m) = B'_{L_X} \left( \frac{p}{M_{\Lambda^0_b}} \right)^{L_X} \]
\[ \text{BW}(m|M_0X, \Gamma_{0X}) B'_{L_X} \left( \frac{q}{M_0X} \right)^{L_X} \]

- Kinematical singularities: e.g. barrier factors (known)
- Left hand singularities (need model, e.g. Blatt-Weisskopf)
- Right hand singularities = resonant content (Breit Wigner, K-matrix...)

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Kinematics

- Kinematical singularities appear because of the spin of the external particle involved
- We can write the most general covariant parametrization of the amplitude as tensor of external polarizations $\otimes$ scalar amplitudes
- Scalar amplitudes must be kinematical singularities free
- They can be matched to the helicity amplitudes
- We can get the minimal energy dependent factor
- Any other additional energy factor would be model-dependent
$B \rightarrow \psi \pi K$

To consider the effect of spin, let’s consider $B \rightarrow \psi \pi K$

We focus on the parity violating amplitude for the $K^*$ isobars, scattering kinematics

\[
p = \text{incoming 3-momentum in the COM} = \frac{\lambda_{12}^{1/2}}{2\sqrt{s}}
\]

\[
= \sqrt{[s - (m_1 + m_2)^2][s - (m_1 - m_2)^2]} \]

\[
= \frac{2\sqrt{s}}{2\sqrt{s}}
\]

---

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$B \to \psi \pi K$

To consider the effect of spin, let's consider $B \to \psi \pi K$. We focus on the parity violating amplitude for the $K^*$ isobars, scattering kinematics.

$$q = \text{outgoing 3-momentum in the COM} = \frac{\lambda_{34}^{1/2}}{2\sqrt{s}}$$

$$= \frac{\sqrt{s - (m_3 + m_4)^2} \sqrt{[s - (m_3 - m_4)^2]}}{2\sqrt{s}}$$
$B \to \psi \pi K$

To consider the effect of spin, let’s consider $B \to \psi \pi K$

We focus on the parity violating amplitude for the $K^*$ isobars, scattering kinematics

\[
\begin{align*}
\psi(1^-) & \quad \pi(0^-) \\
K^*(0^+,1^- \ldots) & \\
B(0^+) & \quad K(0^-)
\end{align*}
\]

$z_s = \text{cosine of the scatt. angle in the COM}$

\[
= \frac{s(t-u) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{\lambda_{12}^{1/2} \lambda_{34}^{1/2}} = \frac{\text{polynomial}}{\lambda_{12}^{1/2} \lambda_{34}^{1/2}}
\]
Helicity amplitudes

\[ A_\lambda = \frac{1}{4\pi} \sum_{j=|\lambda|}^{\infty} (2j + 1) A_{\lambda}^j(s) \, d_{\lambda_0}^j(z_s) \]

\[ d_{\lambda_0}^j(z_s) = \hat{d}_{\lambda_0}^j(z_s) \xi_{\lambda_0}(z_s), \quad \xi_{\lambda_0}(z_s) = \left( \sqrt{1 - z_s^2} \right)^\lambda \]

\( \hat{d}_{\lambda_0}^j(z_s) \) is a polynomial of order \( j - |\lambda| \) in \( z_s \),

The kinematical singularities of \( A_{\lambda}^j(s) \) can be isolated by writing

\[ A_0^j = \frac{m_1}{p\sqrt{s}} \, (pq)^j \, \hat{A}_0^j \quad \text{for } j \geq 1, \]

\[ A_{\pm}^j = q \, (pq)^{j-1} \, \hat{A}_{\pm}^j \quad \text{for } j \geq 1, \]

\[ A_0^0 = \frac{p\sqrt{s}}{m_1} \, \hat{A}_0^0 \quad \text{for } j = 0, \]
Identify covariants

Two helicity couplings $\rightarrow$ two independent covariant structures

**Important**: we are not imposing any intermediate isobar

\[
A_\lambda(s, t) = \varepsilon_\mu(\lambda, p_1) \left[(p_3 - p_4)^\mu - \frac{m_3^2 - m_4^2}{s}(p_3 + p_4)^\mu\right] C(s, t) \\
+ \varepsilon_\mu(\lambda, p_1)(p_3 + p_4)^\mu B(s, t)
\]

\[
C(s, t) = \frac{1}{4\pi \sqrt{2}} \sum_{j>0} (2j + 1)(pq)^{j-1} \hat{A}_+^j(s) \hat{d}_{10}^j(z_s)
\]

\[
B(s, t) = \frac{1}{4\pi} \hat{A}_0^0 + \frac{1}{4\pi} \frac{4m_1^2}{\lambda_{12}} \sum_{j>0} (2j + 1)(pq)^j \left[\hat{A}_0^j(s)\hat{d}_{00}^j(z_s) + \frac{s + m_1^2 - m_2^2}{\sqrt{2}m_1^2} \hat{A}_+^j(s) z_s \hat{d}_{10}^j(z_s)\right]
\]

Everything looks fine **but** the $\lambda_{12}$ in the denominator

The brackets must vanish at $\lambda_{12} = 0 \Rightarrow s = s_\pm = (m_1 \pm m_2)^2$,

$\hat{A}_+^j$ and $\hat{A}_0^j$ cannot be independent
General expression and comparison

\[
\hat{A}_+^j = \langle j - 1, 0; 1, 1 | j, 1 \rangle g_j(s) + f_j(s)
\]

\[
\hat{A}_0^j = \langle j - 1, 0; 1, 0 | j, 0 \rangle \frac{s + m_1^2 - m_2^2}{2m_1^2} g_j'(s) + f_j'(s)
\]

\[g_j(s_{\pm}) = g_j'(s_{\pm}), \text{ and } f_j(s), f_j'(s) \sim O(s - s_{\pm})\]

All these four functions are free of kinematic singularity.

Comparison with tensor formalisms \((j = 1)\)

\[g_1 = g_1' = \frac{4\pi}{3} g_S, \quad f_1 = \frac{2\pi \lambda_{12}}{3s} g_D, \quad f_1' = -\frac{4\pi \lambda_{12}}{3s} \frac{s + m_1^2 - m_2^2}{m_1^2} g_D.\]

If the \(g_S, g_D\) are the usual Breit-Wigner, the \(g', f'\) are fine.

There is no unique recipe to build the right amplitude, but one can ensure the right singularities to be respected.
General expression and comparison

We consider the example of two intermediate $K^*(892)$ and $K^*(1410)$
We set $g_S(s) = 0$ and $g_D(s) = \text{sum of Breit-Wigner}$
For the plot on the right we multiply the amplitudes by the Blatt-Weisskopf barrier factors
Playing with fits

Disclaimer: The following fits are just qualitative, based on fits on 2D Dalitz plots. This is to estimate the model dependence.

Using the JPAC model & adding higher spin resonances reproduces a peak similar to the $Z(4430)$.
Model dependence and physics

Understanding the model dependence is mandatory: models with similar fit qualities can lead to dramatically different physical interpretations.

E.g. $e^+ e^- \rightarrow J/\psi \pi\pi$ and the $Z_c(3900)$

AP et al. (JPAC), PLB772, 200
Conclusions

- The knowledge of $K\pi$ interactions is not only important *per se*, but it can help constrain the XYZ sector

- Several states observed in B decays involving $K\pi$ or $K\pi\pi$ and charmonium
  - $X(3872)$, $Z(4430)$, $Z(4200)$, $Z(4050)$, $Z(4250)$

- Despite the high significance, the systematic due to the model dependence is often not estimated

- Approaches *à la* Khuri-Treiman allow for the direct implementation of partial waves into these analyses

- More «exotic» descriptions based on dual models can also give a complementary insights on the behavior of the Dalitz plots

Thank you!
BACKUP
Pole hunting

Bound states on the real axis 1st sheet
Not-so-bound (virtual) states on the real axis 2nd sheet
Pole hunting

More complicated structure when more thresholds arise: two sheets for each new threshold

III sheet: usual resonances

IV sheet: cusps (virtual states)

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Tetraquark: the $c\bar{c}s\bar{s}$ states

Good description of the spectrum but one has to assume the axial assignment for the $X(4274)$ to be incorrect (two unresolved states with $0^{++}$ and $2^{++}$)

Maiani, Polosa and Riquer, PRD 94, 054026

Much narrower than LHCb! Look for prompt!
Joint Physics Analysis Center

- Joint effort between theorists and experimentalists to work together to make the best use of the next generation of very precise data taken at JLab and in the world
- Created in 2013 by JLab & IU agreement
- It is engaged in education of further generations of hadron physics practitioners

- Effective Field Theories
- Analyticity+Unitarity
- Dispersion Relations
- Regge Theory
- Insight on QCD dynamics
- Fundamental parameters
  - Resonances, exotic states

- Experiments
  - CLAS, GlueX, BESIII, COMPASS,
  - LHCb, BaBar, Belle II, KLOE, MAMI
  - Lattice
Strategy

• We fit the following invariant mass distributions:
  • BESIII PRL110, 252001 $J/\psi \pi^+, J/\psi \pi^-, \pi^+\pi^-$ at $E_{CM} = 4.26$ GeV
  • BESIII PRL110, 252001 $J/\psi \pi^0$ at $E_{CM} = 4.23, 4.26, 4.36$ GeV
  • BESIII PRD92, 092006 $D^0 D^*+, \overline{D}^*0 D^+$ (double tag) at $E_{CM} = 4.23, 4.26$ GeV
  • BESIII PRL115, 222002 $D^0 D^*0, \overline{D}^*0 D^0$ at $E_{CM} = 4.23, 4.26$ GeV
  • BESIII PRL112, 022001 $D^0 D^*+, \overline{D}^*0 D^+$ (single tag) at $E_{CM} = 4.26$ GeV
  • Belle PRL110, 252002 $J/\psi \pi^\pm$ at $E_{CM} = 4.26$ GeV
  • CLEO-c data PLB727, 366 $J/\psi \pi^\pm, J/\psi \pi^0$ at $E_{CM} = 4.17$ GeV

• Published data are not efficiency/acceptance corrected, → we are not able to give the absolute normalization of the amplitudes

• No given dependence on $E_{CM}$ is assumed – the couplings at different $E_{CM}$ are independent parameters

AP et al. (JPAC), arXiv:1612.06490
Strategy

- **Reducible (incoherent) backgrounds are pretty flat** and do not influence the analysis, except the peaking background in $D^0D^*, D^*D^0$ (subtracted)

- Some information about **angular distributions** has been published, but it’s **not constraining** enough → we do not include in the fit

- Because of that, **we approximate all the particles to be scalar** – this affects the value of couplings, which are not normalized anyway – but not the position of singularities. This also limits the number of free parameters
Lineshapes at 4260

Figure 7: Interplay of scattering amplitude poles and triangle singularity to reconstruct the peak. We focus on the $J/\psi\pi$ channel, at $E_{CM} = 4.26$ GeV. The red curve is the $t_{12}$ scattering amplitude, the green curve is the $c_1 + H(s, D_1) + +H(s, D_0)$ term in Eq. (9), and the blue curve is the product of the two. The upper plots show the magnitudes of these terms, the lower plots the phases. The middle row shows the contributions to the unitarized term due to the $D_1$ (dashed) and the $D_0$ (dotted). Only for $D_1$ the singularity is close enough to the physical region to generate a large peak. (a) The pole on the III sheet generates a narrow Breit-Wigner-like peak. The contribution of the triangle is not particularly relevant. (b) The sharp cusp in the scattering amplitude is due to the IV sheet pole close by; the triangle contributes to make the peak sharper. (c) The scattering amplitude has a small cusp due to the threshold factor, and the triangle is needed to make it sharp enough to fit the data.
Lineshapes at 4230

Figure 8: Same as Figure 7, but for $E_{CM} = 4.23$ GeV.

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Statistical analysis

Toy experiments according to the different hypotheses, to estimate the relative rejection of various scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>III+tr.</th>
<th>IV+tr.</th>
<th>tr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>III</td>
<td>1.5σ (1.5σ)</td>
<td>1.5σ (2.7σ)</td>
<td>“2.4σ” (“1.4σ”)</td>
</tr>
<tr>
<td>III+tr.</td>
<td>–</td>
<td>1.5σ (3.1σ)</td>
<td>“2.6σ” (“1.3σ”)</td>
</tr>
<tr>
<td>IV+tr.</td>
<td></td>
<td></td>
<td>“2.1σ” (“0.9σ”)</td>
</tr>
</tbody>
</table>

Not conclusive at this stage
Searching for resonances in $\eta\pi$

- The $\eta\pi$ system is one of the golden modes for hunting hybrid mesons
- We build the partial waves amplitude according to the $N/D$ method

A. Jackura, et al. (JPAC & COMPASS), 1707.02848

The denominator $D(s)$ contains all the Final State Interactions constrained by unitarity → universal
The numerator $n(s)$ depends on the exchanges → process-dependent, smooth
Searching for resonances in $\eta\pi$

The denominator $D(s)$ contains all the FSI constrained by unitarity $\rightarrow$ universal

$$D(s)_{ij} = (K^{-1})_{ij}(s) - \frac{s}{\pi} \int_{s_i}^{\infty} \frac{\rho_i(s') N_{ij}(s')}{s'(s' - s)} ds'$$

$$K_{ij}(s) = \sum_R \frac{g_i^R g_j^R}{M_R^2 - s}$$ Standard K matrix, with usual trick for vanishing determinant

The numerator $n(s)$ depends on the exchanges $\rightarrow$ process-dependent, smooth

$$\rho_i(s) N_{ij}(s) = \frac{\lambda^{(2l+1)/2} (s, m_{\pi}^2, m_{\eta}^2)}{(s + \Lambda)^7}$$
Searching for resonances in $\eta\pi$

Smooth «background»

Precise determination of pole position
Searching for resonances in $\eta\pi$

- The coupled channel analysis involving the $\eta\pi$ and $\eta'\pi$ for $P$- and $D$-wave is ongoing

\[ a_2(1320) \]
\[ a'_2(1700) \]

- The extension to the GlueX production mechanism and kinematics is also ongoing
- Same $D(s)$, different numerator

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Dynamical movie \(Z^+(4430)\)

- Since this is still a \(3 \leftrightarrow \bar{3}\) color interaction, just use the Cornell potential:

\[
V(r) = -\frac{4\alpha_s}{3} \frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m_{cq}^2} \left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma r^2} S_{cq} \cdot S_{\bar{cq}},
\]

- Use that the kinetic energy released in \(B^0 \rightarrow K^- Z^+(4430)\) converts into potential energy until the diquarks come to rest.
- Hadronization most effective at this point (WKB turning point)

\[
r_Z = 1.16 \text{ fm}, \langle r_{\psi(2S)} \rangle = 0.80 \text{ fm}, \langle r_{J/\psi} \rangle = 0.39 \text{ fm}
\]

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\[
\frac{B(Z^+(4430) \rightarrow \psi(2S)\pi^+)}{B(Z^+(4430) \rightarrow J/\psi \pi^+)} \sim 72
\]

(> 10 exp.)
Example: The charged $Z_c(3900)$

A charged charmonium-like resonance has been claimed by BESIII in 2013.

$e^+e^- \rightarrow Z_c(3900)^+\pi^- \rightarrow J/\psi \, \pi^+\pi^- \text{ and } (DD^*)^+\pi^-
\quad M = 3888.7 \pm 3.4 \, \text{MeV, } \Gamma = 35 \pm 7 \, \text{MeV}$

Such a state would require a minimal $4q$ content and would be manifestly exotic.
Amplitude analysis for $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to different singularities $\rightarrow$ different natures

Triangle rescattering, logarithmic branching point

$Y \to \pi \bar{D}$

$D_1(2420)$

$D_0(2400)$

$D_1(2420)$

$(anti)bound state, II/IV sheet pole

(«molecule»)

Tornqvist, Z.Phys. C61, 525
Swanson, Phys.Rept. 429
Hanhart et al. PRL111, 132003

Resonance, III sheet pole

(«compact state»)

Maiani et al., PRD71, 014028
Faccini et al., PRD87, 111102
Esposito et al., Phys.Rept. 668

AP et al. (JPAC), PLB772, 200

$Z_c(3900)$?

$\sigma, f_0(980)$
Amplitude model

\[ f_i(s, t, u) = 16\pi \sum_{l=0}^{L_{\text{max}}} (2l + 1) \left( a_{l,i}^{(s)}(s) P_l(z_s) + a_{l,i}^{(t)}(t) P_l(z_t) + a_{l,i}^{(u)}(u) P_l(z_u) \right) \]

Khuri-Treiman

\[ f_{0,i}(s) = \frac{1}{32\pi} \int_{-1}^{1} dz_s f_i(s, t(s, z_s), u(s, z_s)) = a_{0,i}^{(s)} + \frac{1}{32\pi} \int_{-1}^{1} dz_s \left( a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) \equiv a_{0,i}^{(s)} + b_{0,i}(s) \]

\[ f_{l,i}(s) = \frac{1}{32\pi} \int_{-1}^{1} dz_s P_l(z_s) \left( a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) \equiv b_{l,i}(s) \quad \text{for } l > 0. \]

\[ f_{0,i}(s) = b_{0,i}(s) + \sum_j t_{ij}(s) \frac{1}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' - s}, \]

\[ f_i(s, t, u) = 16\pi \left[ a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left( c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' (s' - s)} \right) \right], \]
Logarithmic branch points due to exchanges in the cross channels can simulate a resonant behavior, only in very special kinematical conditions (Coleman and Norton, Nuovo Cim. 38, 438). However, this effect cancels in Dalitz projections, no peaks (Schmid, Phys.Rev. 154, 1363).

...but the cancellation can be spread in different channels, you might still see peaks in other channels only!
Testing scenarios

- **We approximate all the particles to be scalar** – this affects the value of couplings, which are not normalized anyway – but not the position of singularities. This also limits the number of free parameters.

\[
f_i(s, t, u) = 16\pi \left[ a^{(t)}_{0,i}(t) + a^{(u)}_{0,i}(u) + \sum_j t_{ij}(s) \left( c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s'(s' - s)} \right) \right],
\]

The scattering matrix is parametrized as \((t^{-1})_{ij} = K_{ij} - i \rho_i \delta_{ij}\)

Four different scenarios considered:

- **III**: the K matrix is \(\frac{g_i g_j}{M^2 - s'}\), this generates a pole in the closest unphysical sheet the rescattering integral is set to zero
- **III+tr.**: same, but with the correct value of the rescattering integral
- **IV+tr.**: the K matrix is constant, this generates a pole in the IV sheet
- **tr.**: same, but the pole is pushed far away by adding a penalty in the \(\chi^2\)
Singularities and lineshapes

Different lineshapes according to different singularities

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Fit: III

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Fit: III+tr.

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A. Pilloni – Using $\pi K$ to understand heavy meson decays
Fit: tr.

\begin{align*}
E_{\text{CM}} &= 4.26 \text{ GeV} \\
E_{\text{CM}} &= 4.23 \text{ GeV}
\end{align*}
Naive loglikelihood ratio test give a $\sim 4\sigma$ significance of the scenario III+tr. over IV+tr., looking at plots it looks too much – better using some more solid test
Pole extraction

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<td>III</td>
<td>1.5σ (1.5σ)</td>
<td>1.5σ (2.7σ)</td>
<td>“2.4σ” (&quot;1.4σ&quot;)</td>
</tr>
<tr>
<td>III+tr.</td>
<td>–</td>
<td>1.5σ (3.1σ)</td>
<td>“2.6σ” (&quot;1.3σ&quot;)</td>
</tr>
<tr>
<td>IV+tr.</td>
<td>–</td>
<td>–</td>
<td>“2.1σ” (&quot;0.9σ&quot;)</td>
</tr>
</tbody>
</table>

Not conclusive at this stage

<table>
<thead>
<tr>
<th>Scenario</th>
<th>III</th>
<th>III+tr.</th>
<th>IV+tr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>M (MeV)</td>
<td>3893.2^{+5.5}_{-7.7}</td>
<td>3905^{+11}_{-9}</td>
<td>3900^{+140}_{-90}</td>
</tr>
<tr>
<td>Γ (MeV)</td>
<td>48^{+19}_{-14}</td>
<td>85^{+45}_{-26}</td>
<td>240^{+230}_{-130}</td>
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