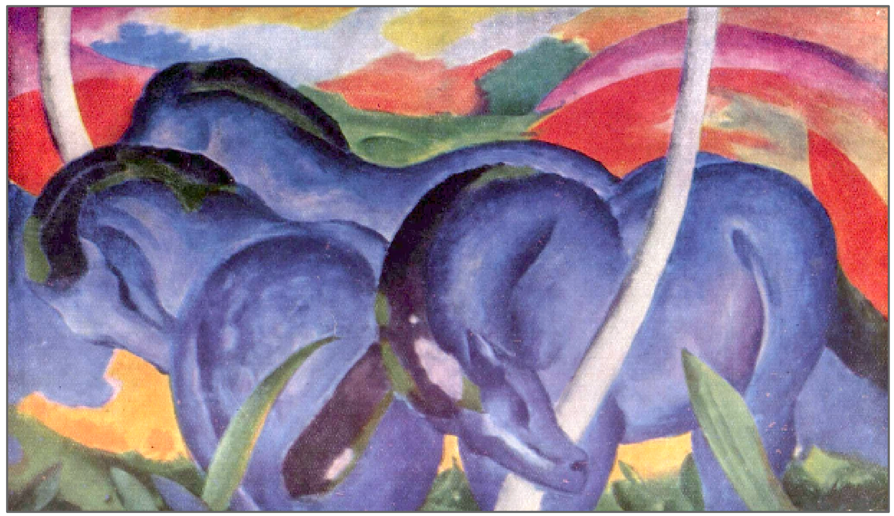
THREE-BODY INTERACTION IN ISOBAR FORMALISM



Maxim Mai
The George Washington University

[MM, Hu, Döring, Pilloni, Szczepaniak EPJ A53 (2017)]
[MM, Döring EPJ A53 (2017)]

THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

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INTRODUCTION

QCD at low energies → mass generation & confinement

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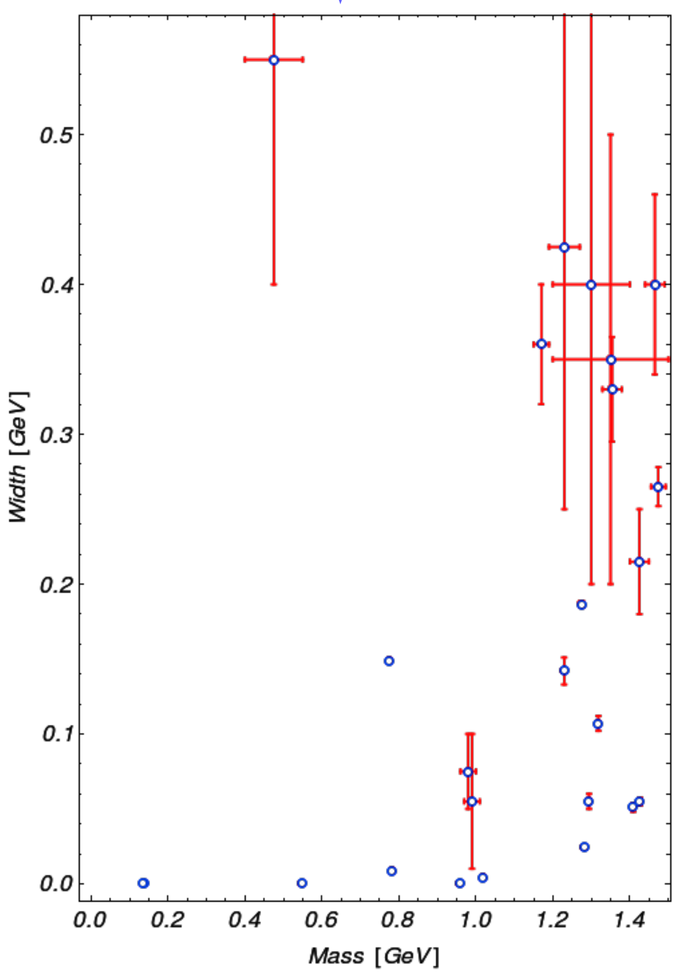
Non-perturbative dynamics → rich spectrum of excited states



QCD at low energies → mass generation & confinement

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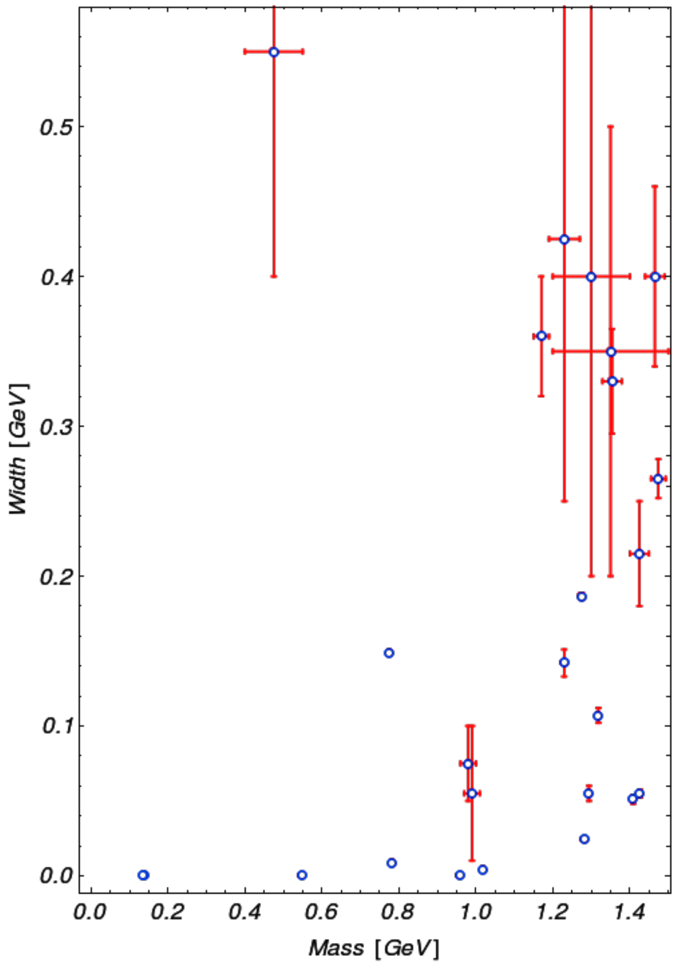
Q1: how many are there?

→ missing resonance problem

Q2: what are they?

– quark-antiquark
– gluons
– hadron-hardon dynamics



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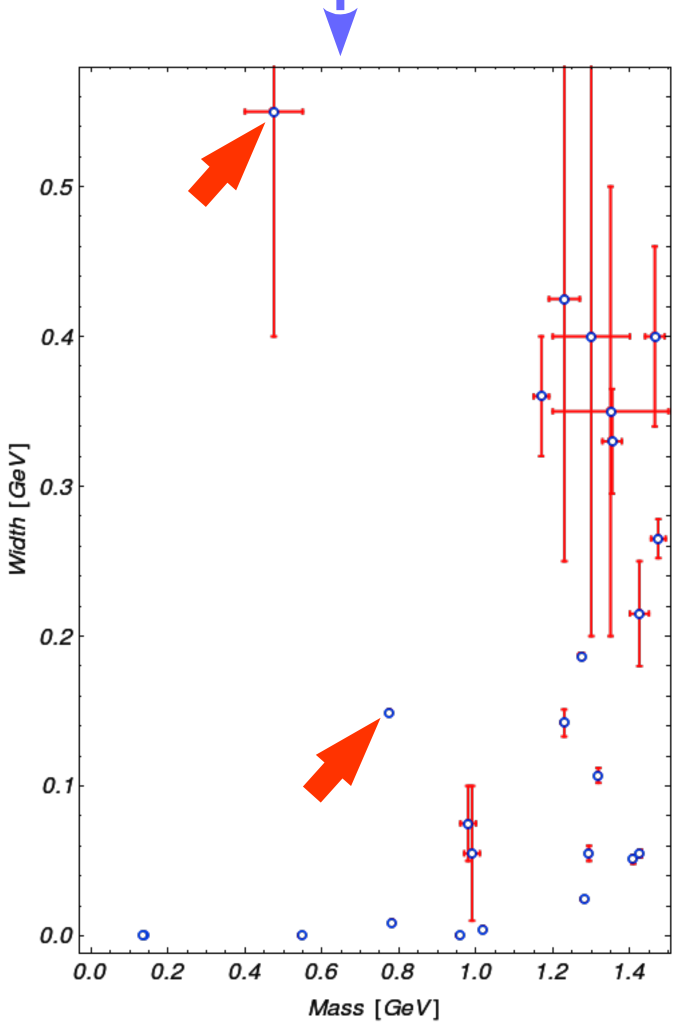
EXAMPLES:

– ***f0(500), ϱ(770), κ(800)...***
couple dominantly to $2\pi, K\pi, ...$

→ talk by Jose R. Pelaez



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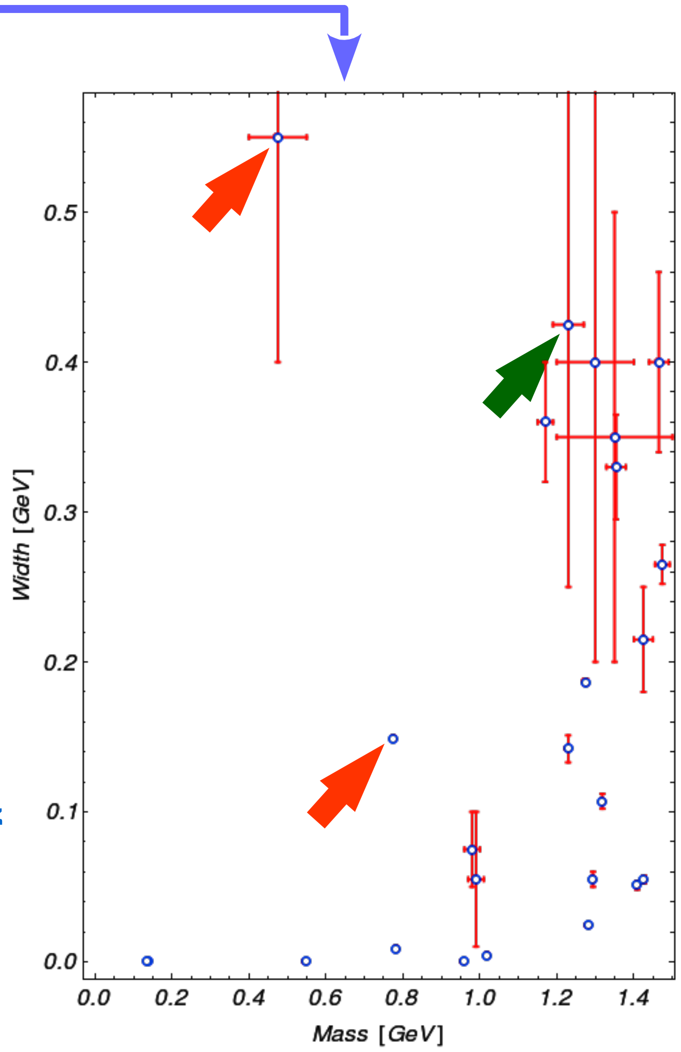

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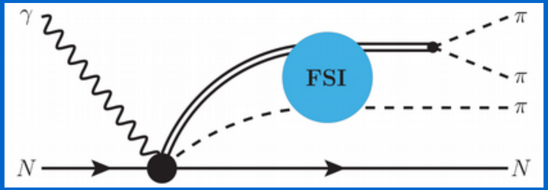
– ***a1(1260), K1(1400)***
couple dominantly to $3\pi, K\pi\pi, \ldots$

Experiment

– Search for QCD exotics @ GlueX

* a1(1260)



– KL Beam @ GlueX

* *K*(892)* signature in $KN \rightarrow K\pi N$

* *$K\pi\pi$ channels(?)*

– Further applications:

* Roper puzzle ($\pi\pi N$)

* X(3872)

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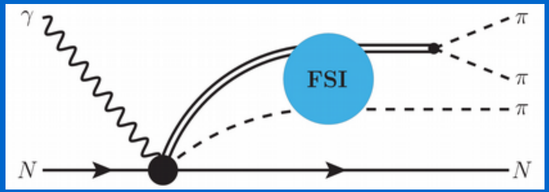

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Lattice QCD

Ab-initio numerical calculations

– Euclidean ST

– finite lattice spacing

– finite volume effects

→ 2-body **Q**uantization **C**ondition

[Lüscher (1986)]

→ talk by Morningstar

→ 3-body **QC** not **yet** established

[Rusetsky, Polejaeva, Sharpe, Hansen, Briceno, Davoudi, Guo MM, Doring, ...]

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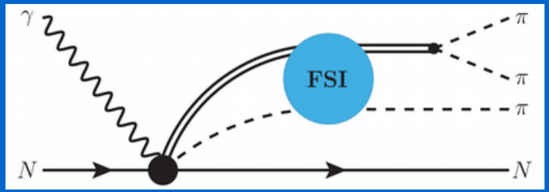

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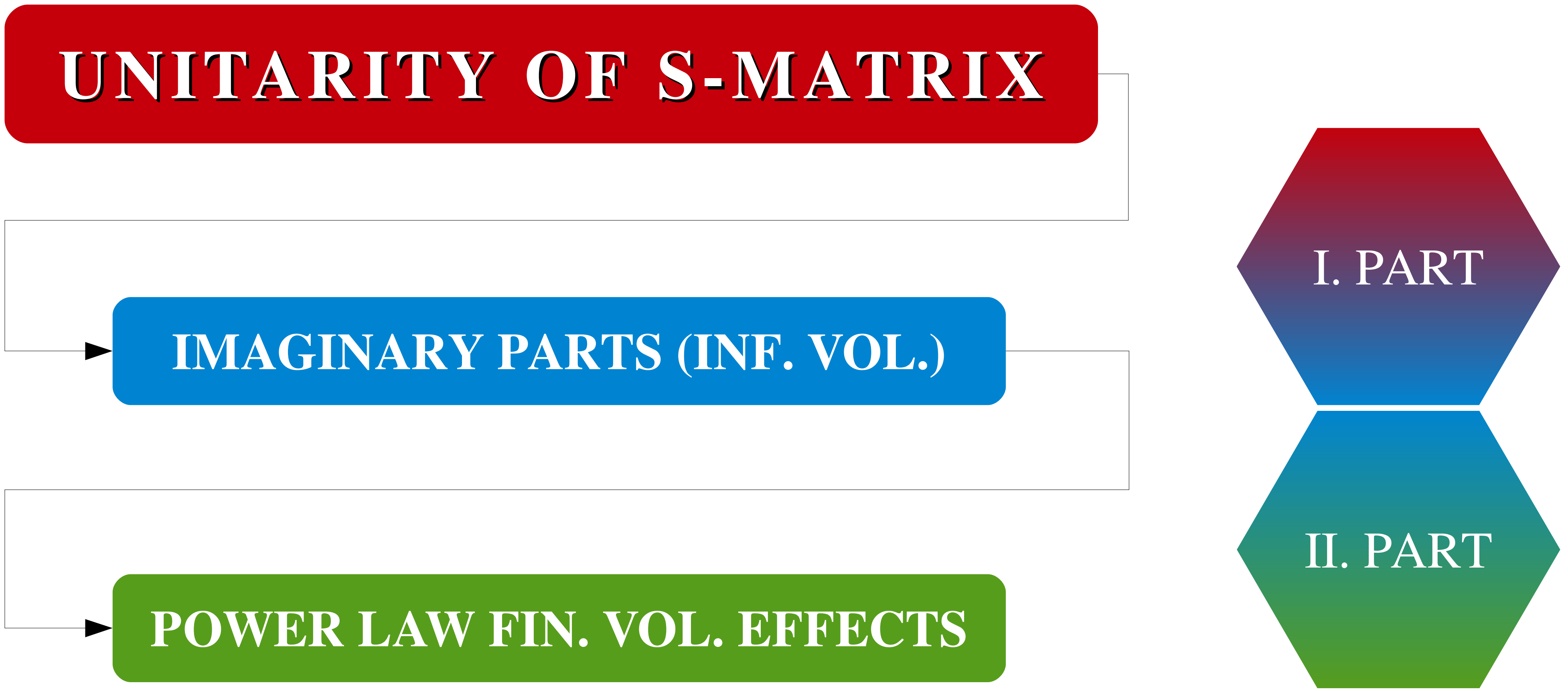
THIS TALK: 3-BODY SCATTERING AMPLITUDE IN ISOBAR-FORMULATION


UNITARITY OF S-MATRIX
IMAGINARY PARTS (INF. VOL.)
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II. PART

I. PART

3→3 SCATTERING AMPLITUDE IN INFINITE VOLUME

[MM, Hu, Döring, Pilloni, Szczepaniak EPJ A53 (2017)]

T-MATRIX

- **3 asymptotic states (scalar particles of equal mass (m))**

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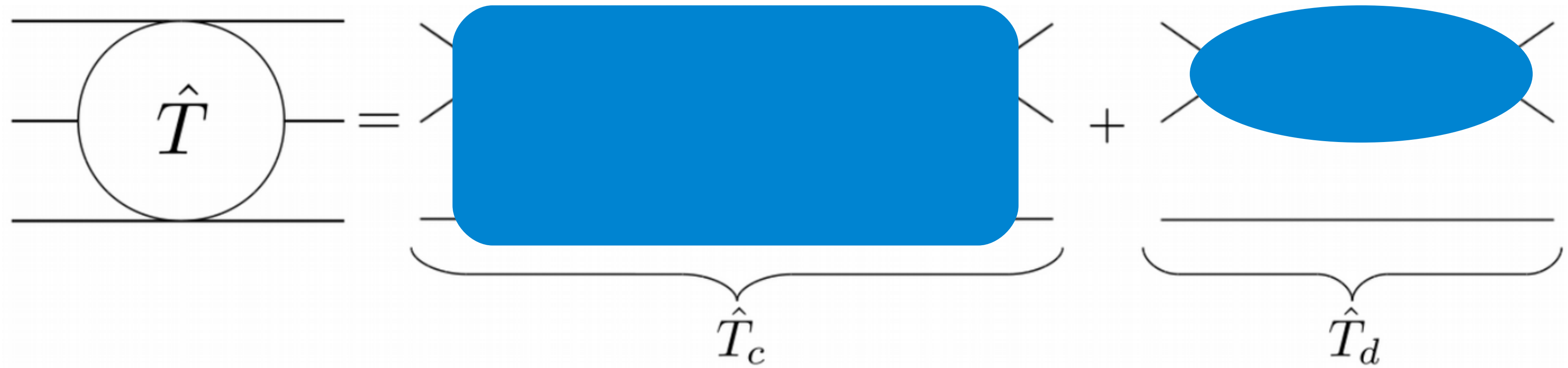
- ***Connectedness structure*** **of matrix elements: (all permutations considered)**



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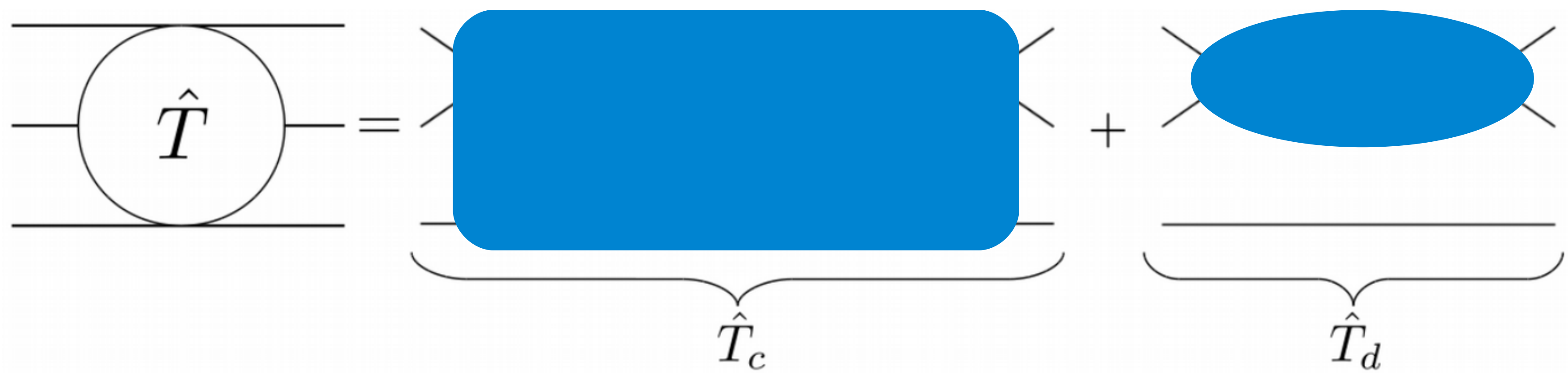

- **isobar-parametrization of two-body amplitude** **[Bedaque, Griesshammer (1999)]**

  → "*isobars*"~ ***S(M_inv)*** for definite QN & correct r.h.-singularities

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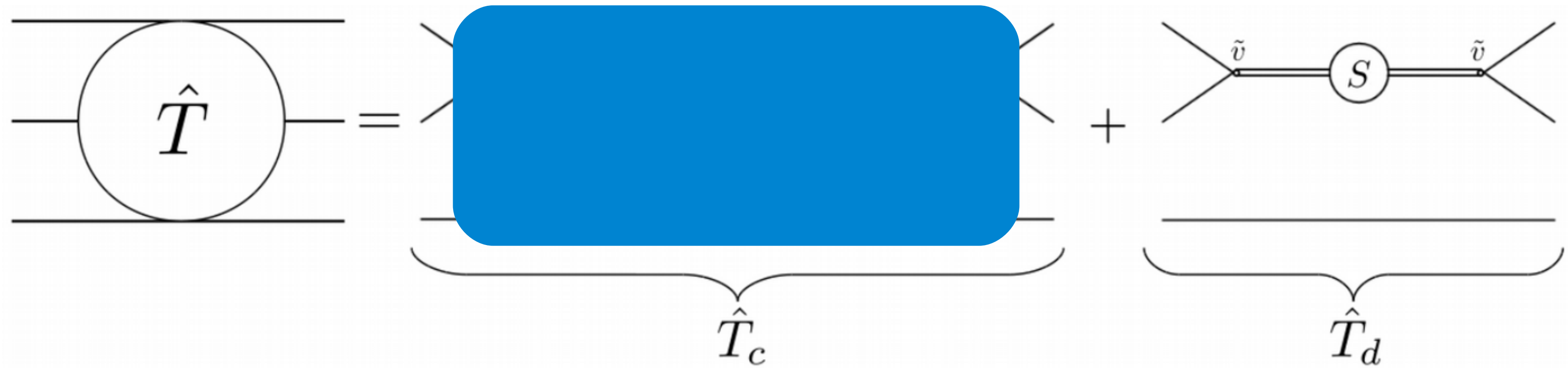

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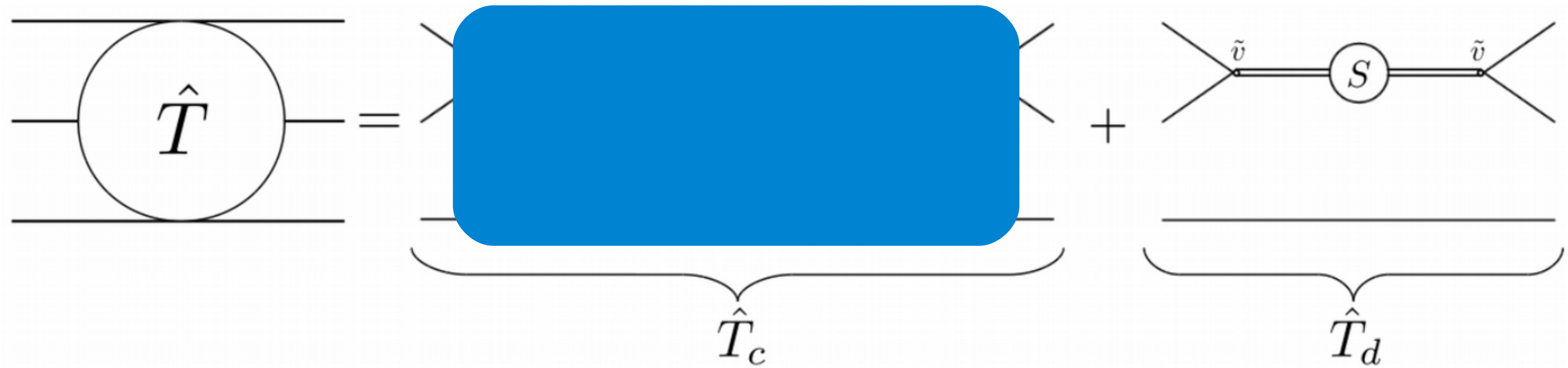

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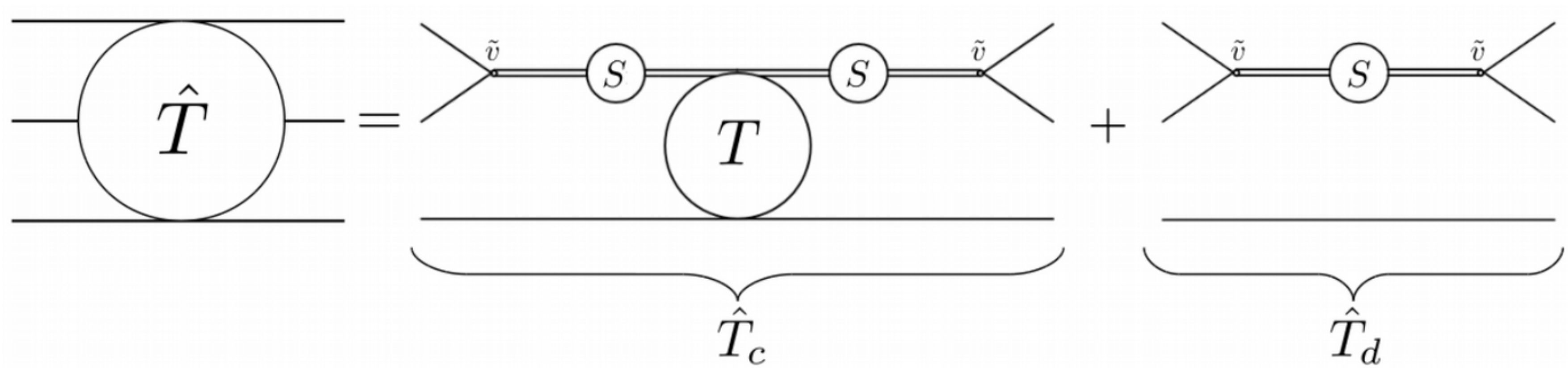

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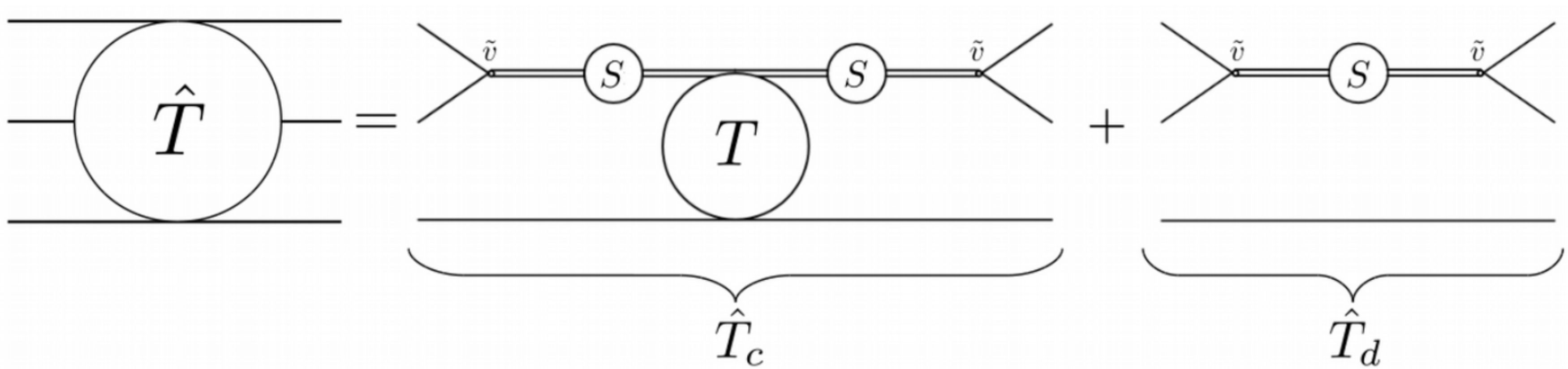

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- Connected part: due to isobar-spectator interaction → ***$T(q_{in}, q_{out}; s)$***

→ 3 unknown functions

→ 8 kinematic variables

UNITARITY

3-body Unitarity (normalization condition ↔ phase space integral)

$$\langle q_1, q_2, q_3 | (\hat{T} - \hat{T}^\dagger) | p_1, p_2, p_3 \rangle \quad = \quad i \int_P \langle q_1, q_2, q_3 | \hat{T}^\dagger | k_1, k_2, k_3 \rangle \langle k_1, k_2, k_3 | \hat{T} | p_1, p_2, p_3 \rangle$$

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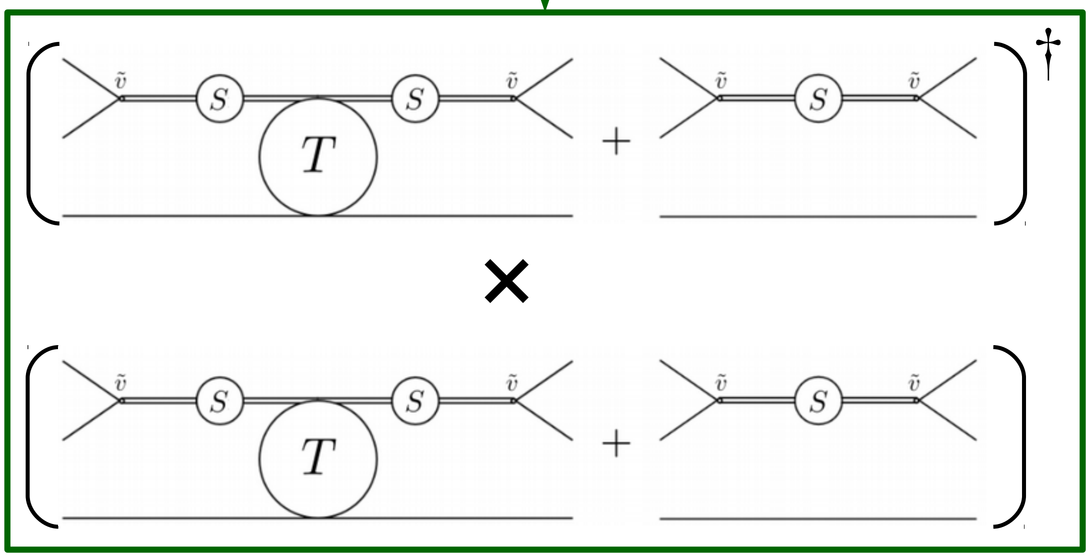

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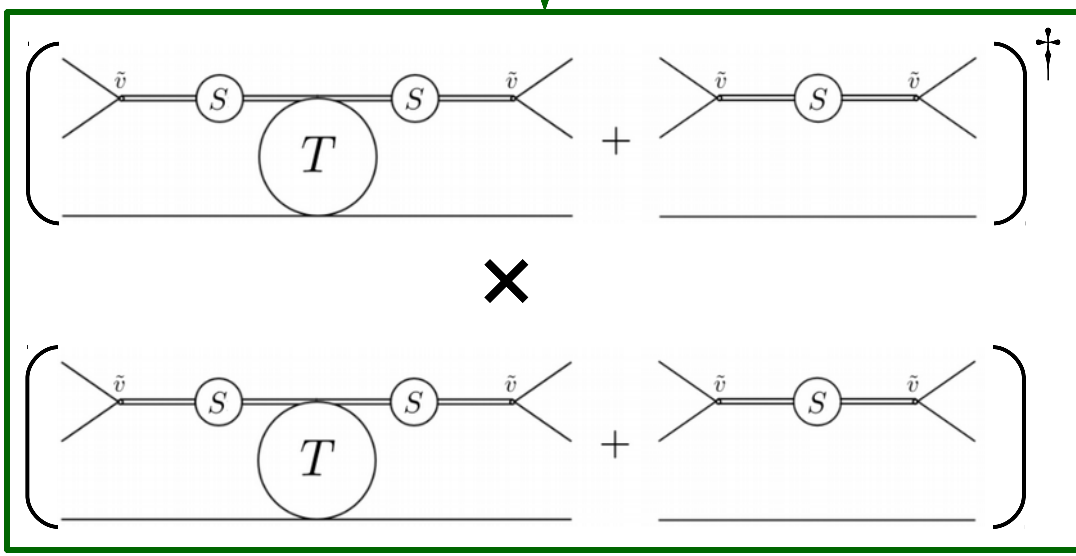

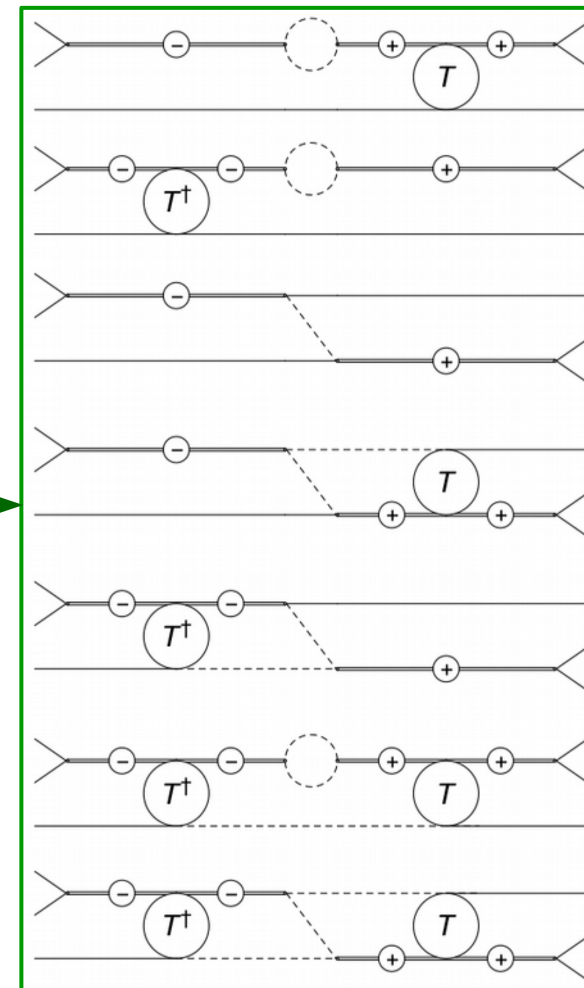

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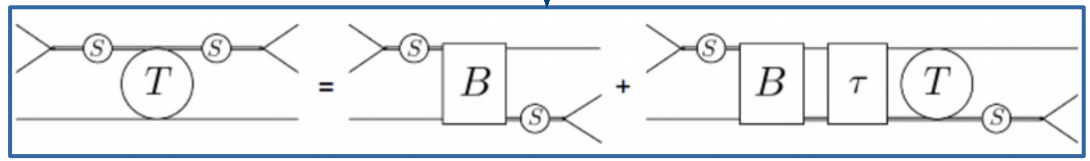

General Ansatz for the isobar-spectator interaction

→ B & τ are unknown!!!

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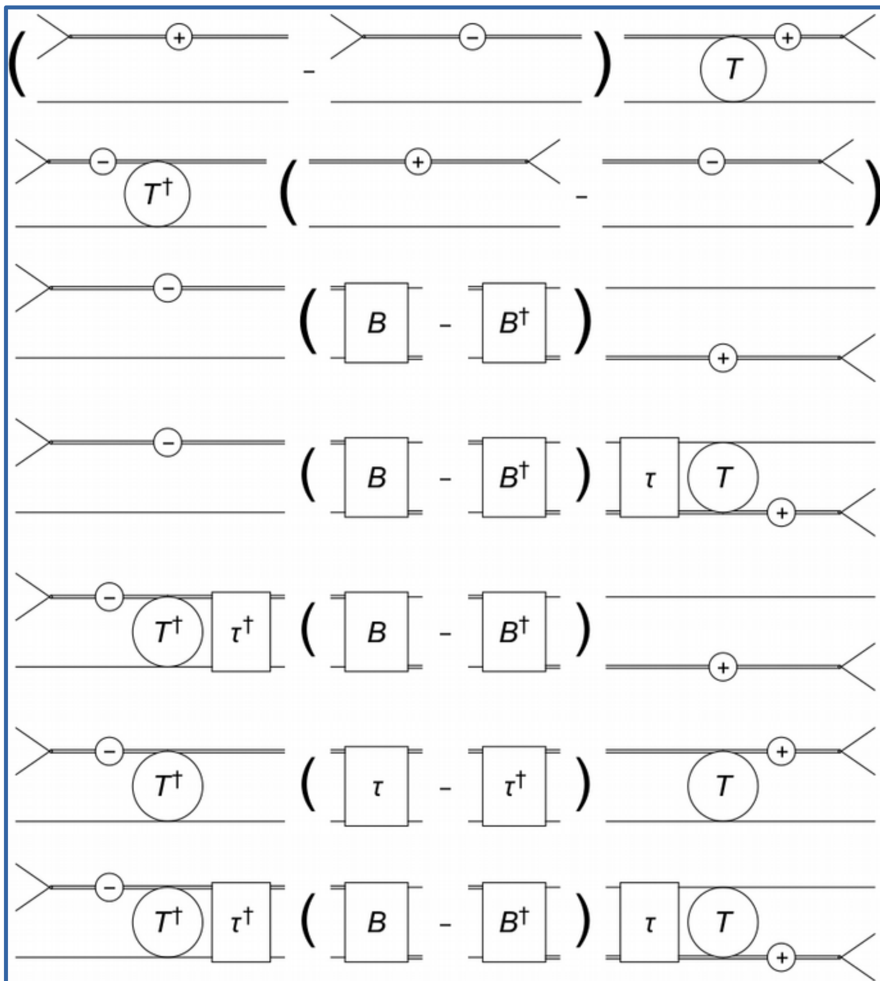

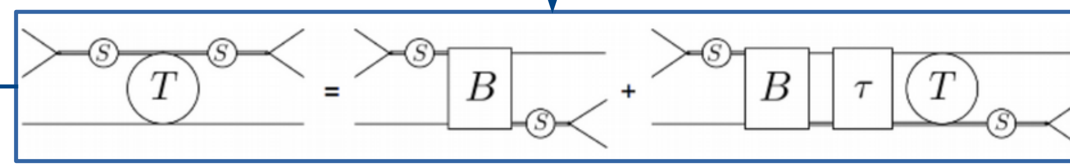

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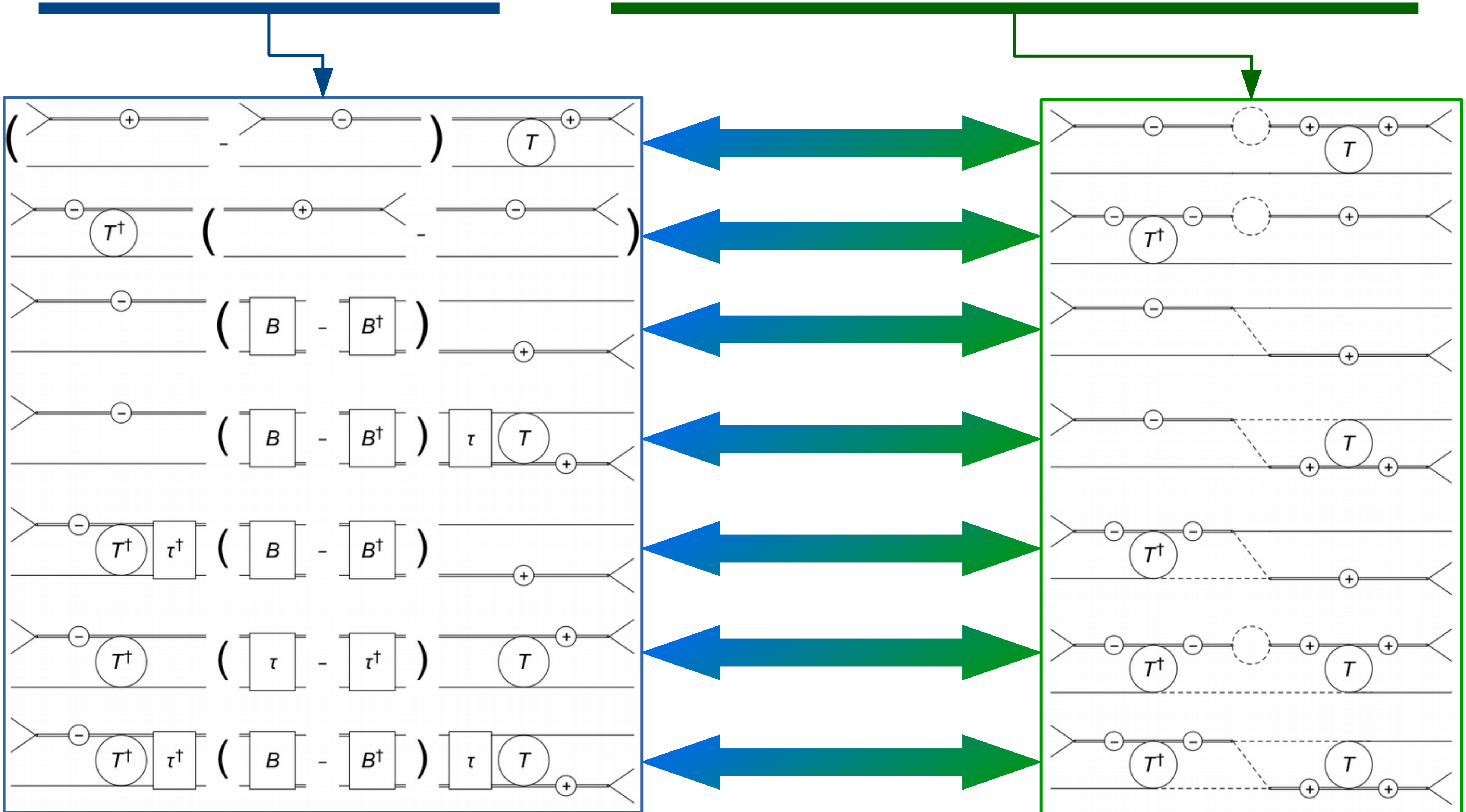
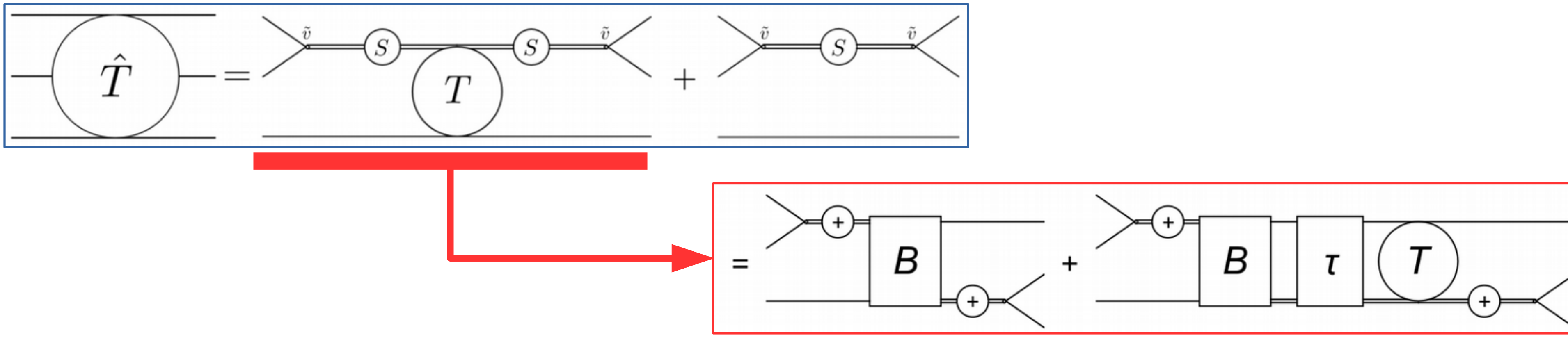
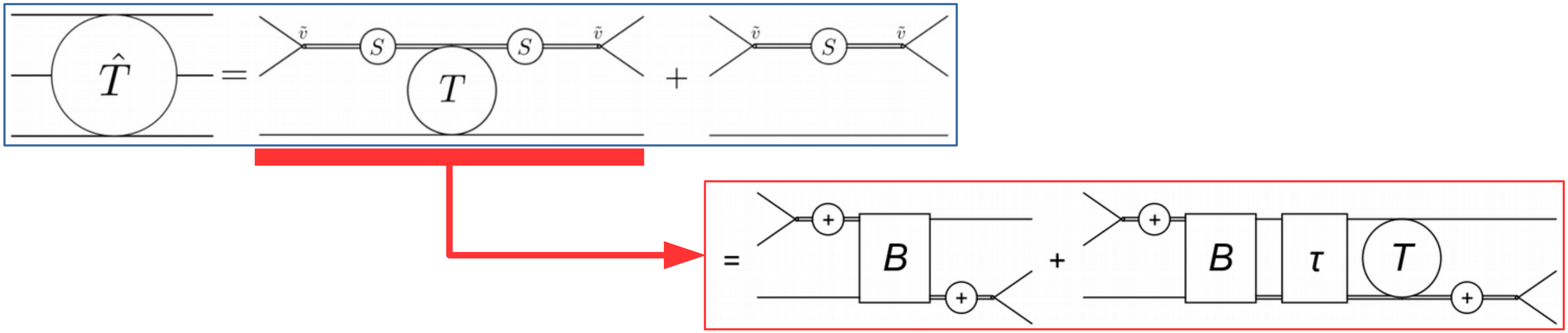


3 → 3 scattering amplitude as a 3-dimensional integral equation



INTEGRAL EQUATION

$3 \to 3$ scattering amplitude as a 3-dimensional integral equation



Unitarity/matching

$$\text{Disc}\, B(u) = 2\pi i \, \frac{\delta\left(E_Q - \sqrt{m^2 + \mathbf{Q}^2}\right)}{2\sqrt{m^2 + \mathbf{Q}^2}} v^2$$

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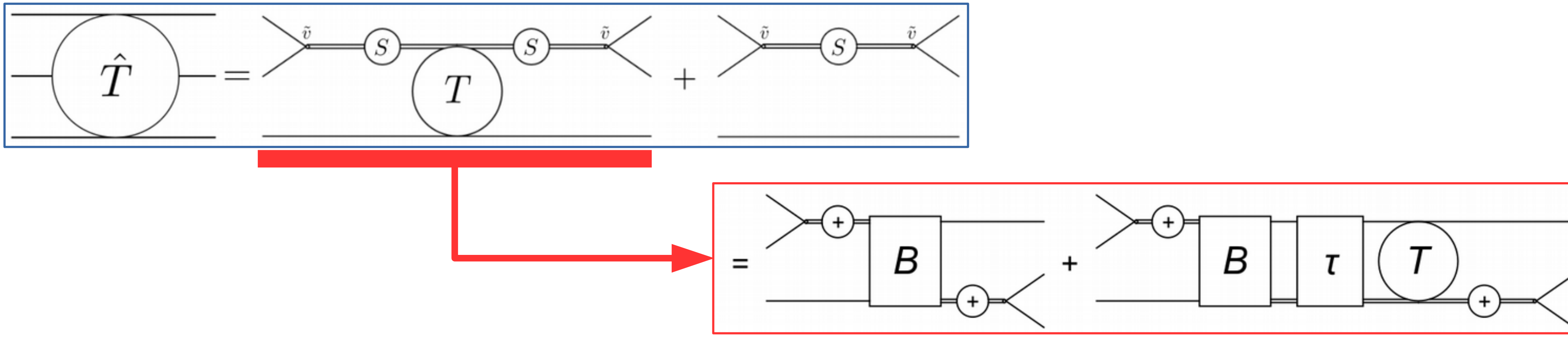

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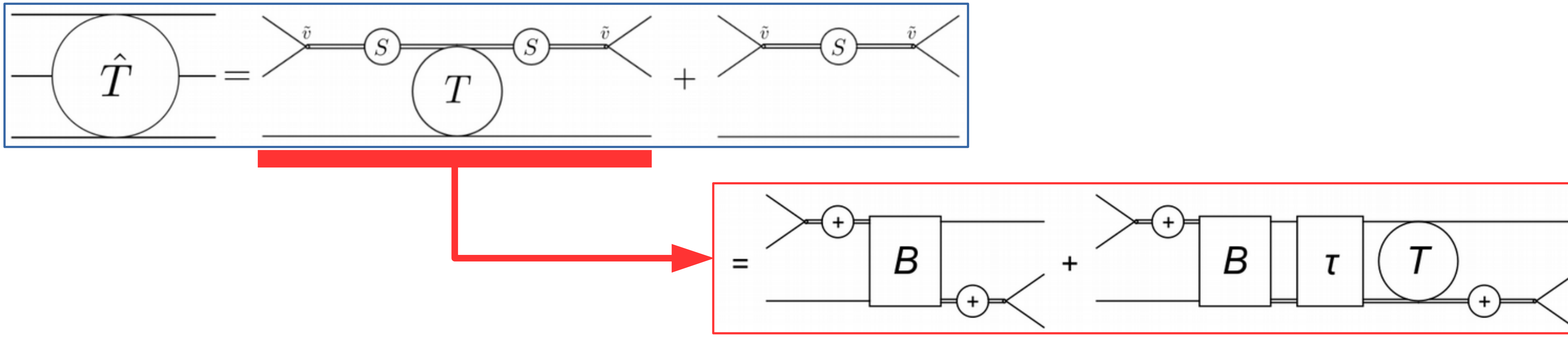

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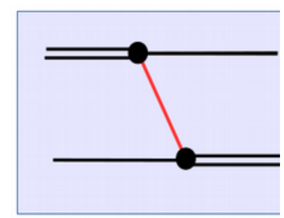

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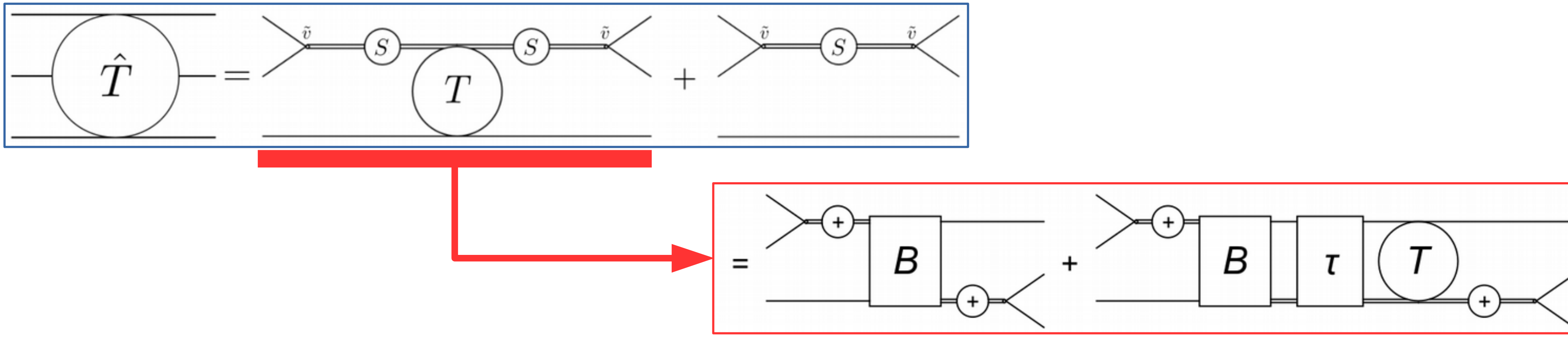

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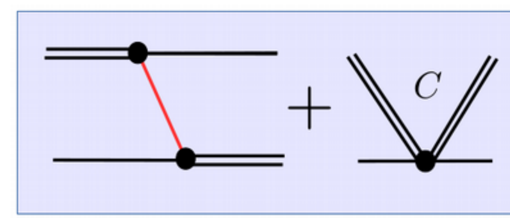

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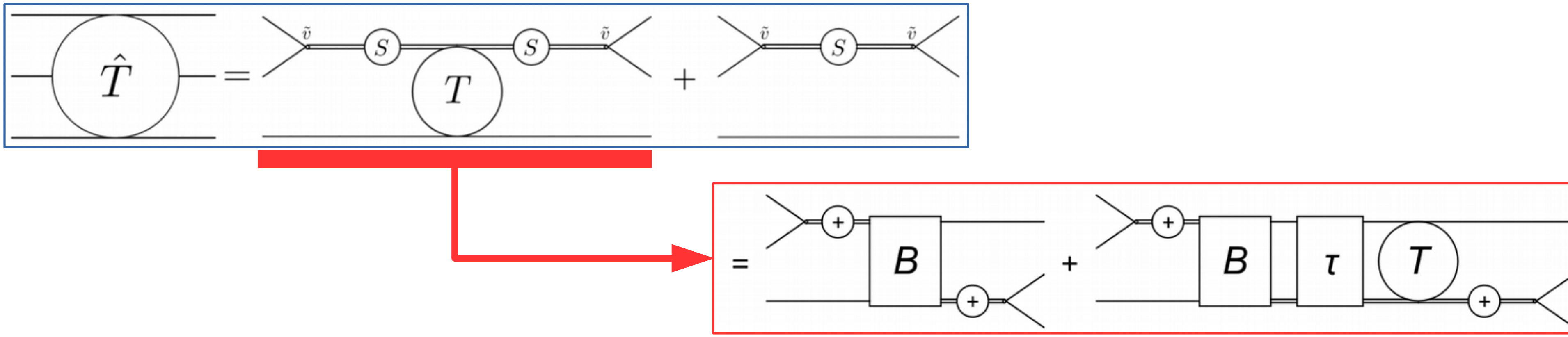

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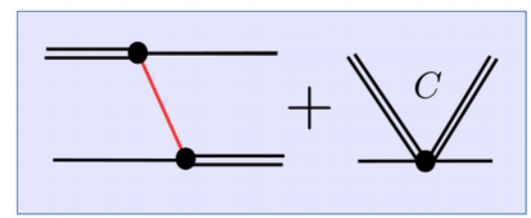

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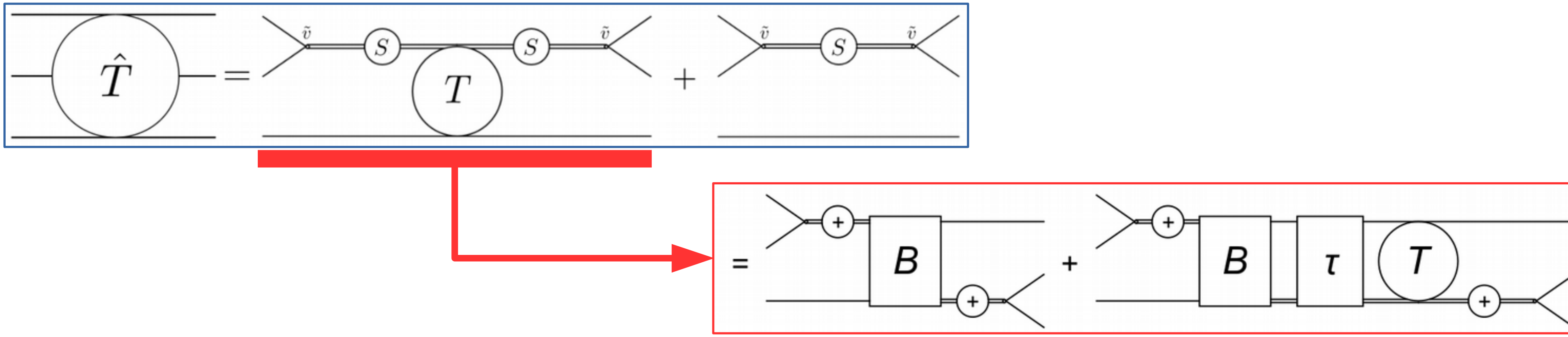

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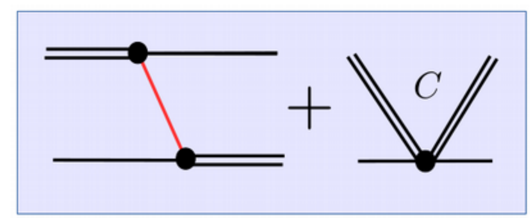

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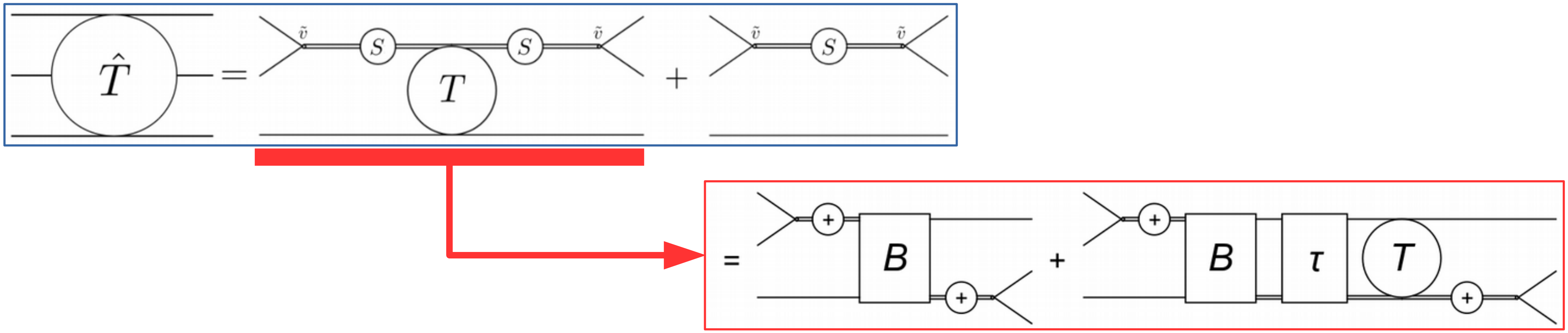

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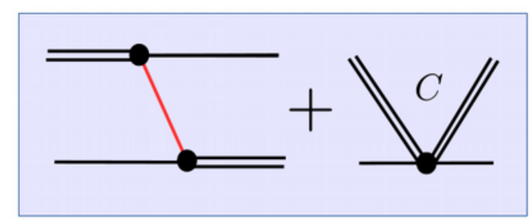

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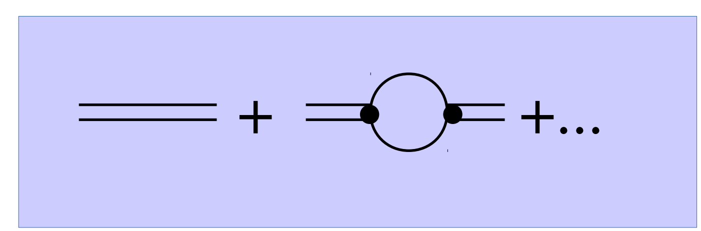


THE ONLY IMAGINARY PARTS REQUIRED BY 3b UNITARITY

UNITARITY OF S-MATRIX

- ✔ IMAGINARY PARTS (INF. VOL.)
- POWER LAW FIN. VOL. EFFECTS

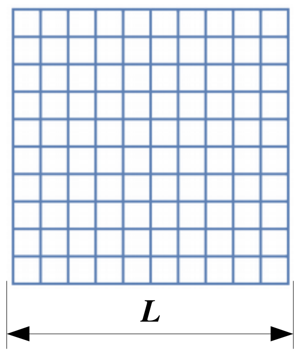
I. PART ✔

II. PART

[MM & Döring EPJ A53 (2017)]

LATTICE QCD SETUP

Lattice QCD calculations are performed in finite volume



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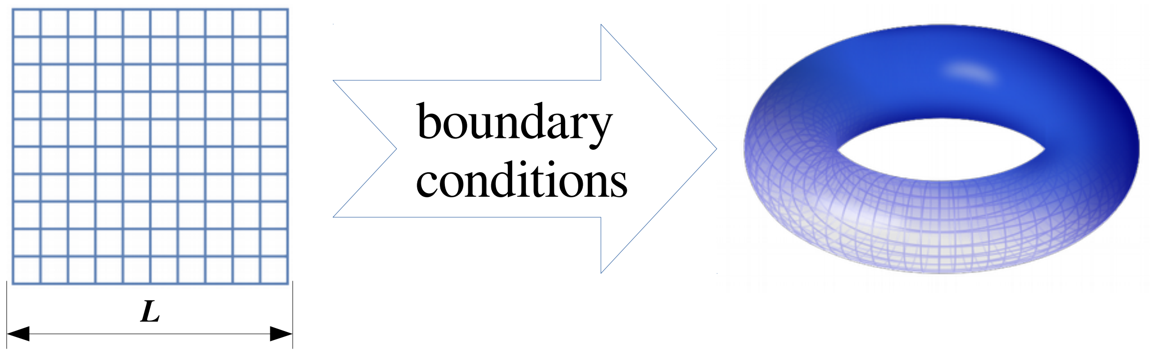

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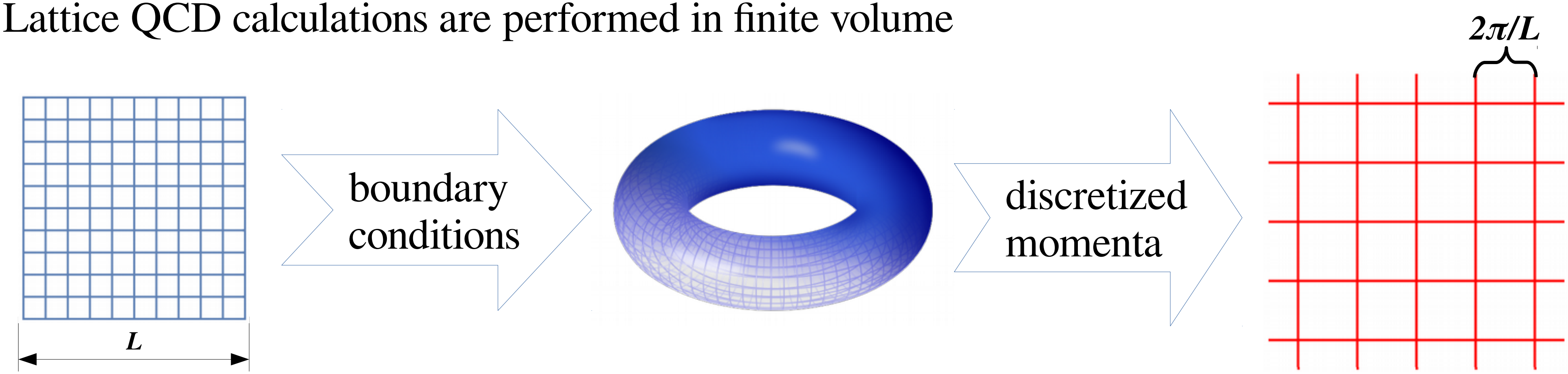
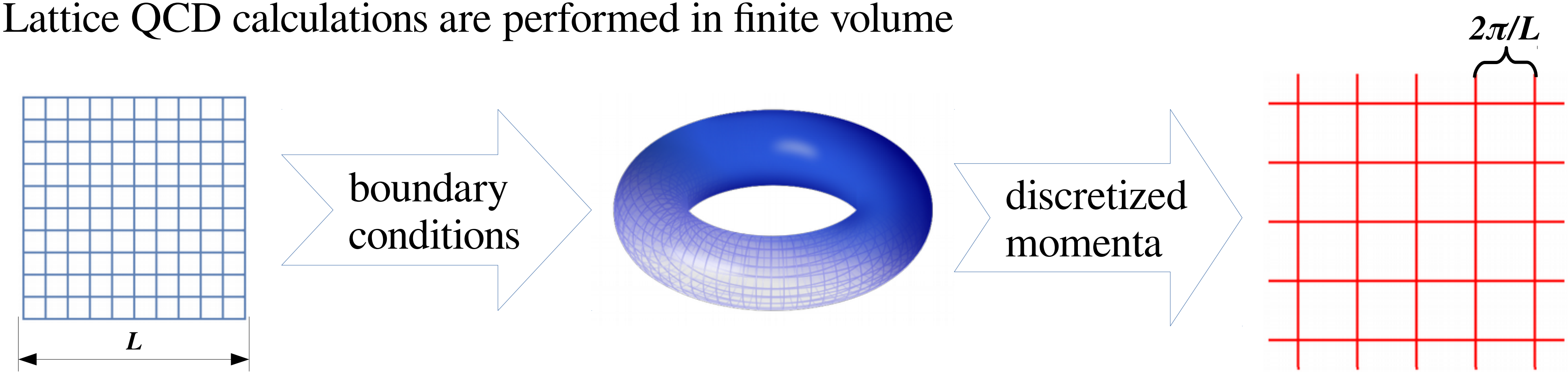


momenta & spectra are discretized

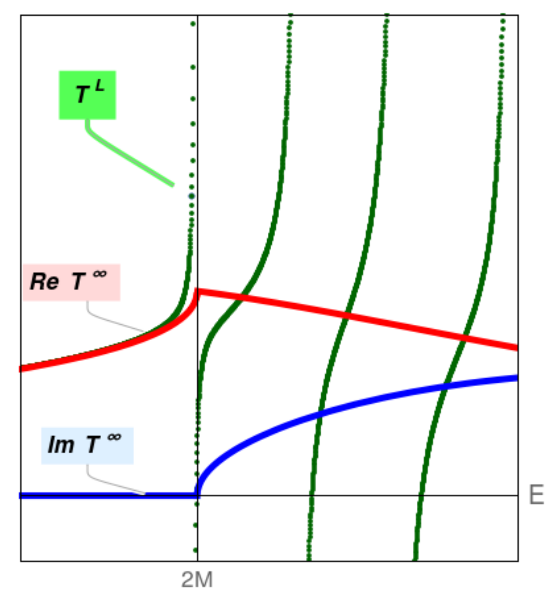
LATTICE QCD SETUP

Lattice QCD calculations are performed in finite volume



momenta & spectra are discretized

– LSZ formalism relates Greens fct. & S-matrix



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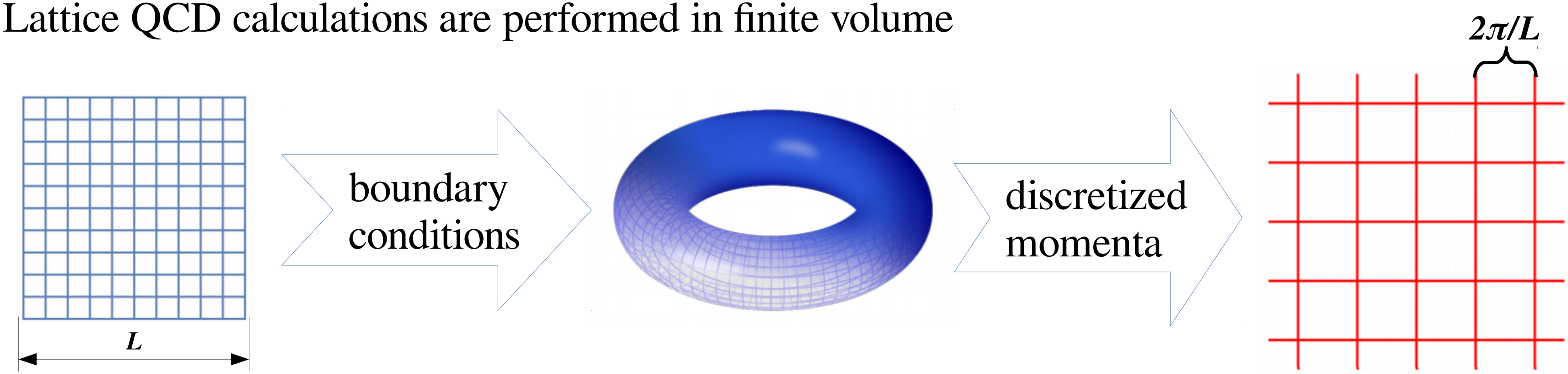

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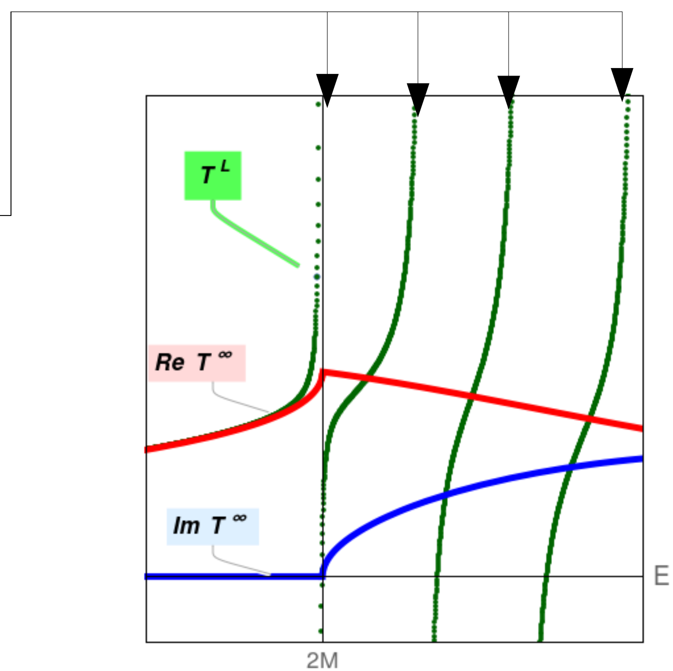
→ ***T(E*) = ∞*** ↔ ***E**** ∈ *Energy-Eigenvalues*



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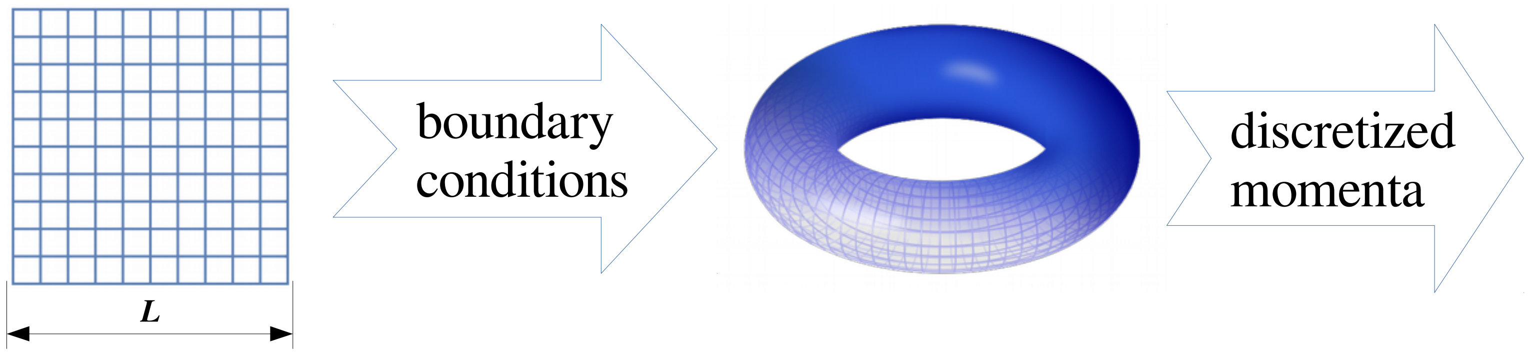

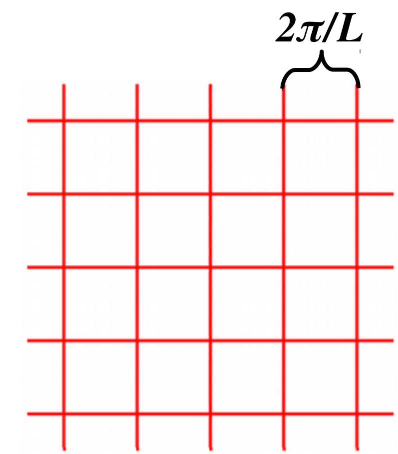

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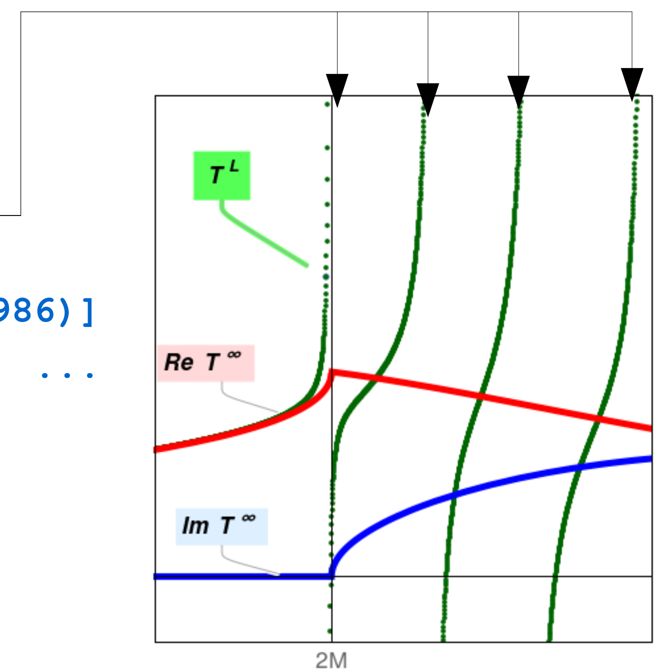
  → $T(E^*) = \infty \quad \leftrightarrow \quad E^* \in$ *Energy-Eigenvalues*

- well established in 2-body

[Lüscher (1986)]

...



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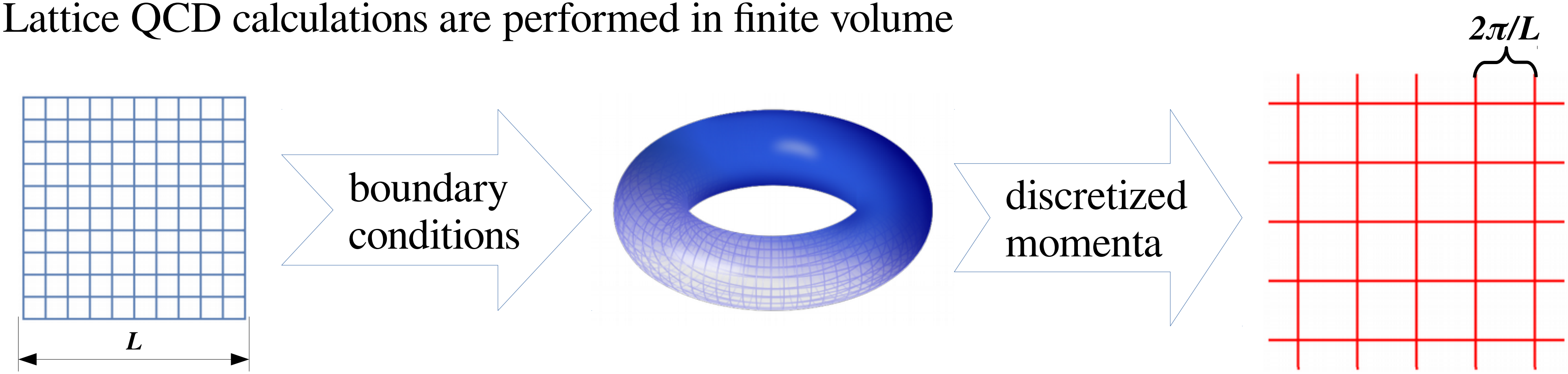

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→ $T(E^*) = \infty \quad \leftrightarrow \quad E^* \in$ *Energy-Eigenvalues*

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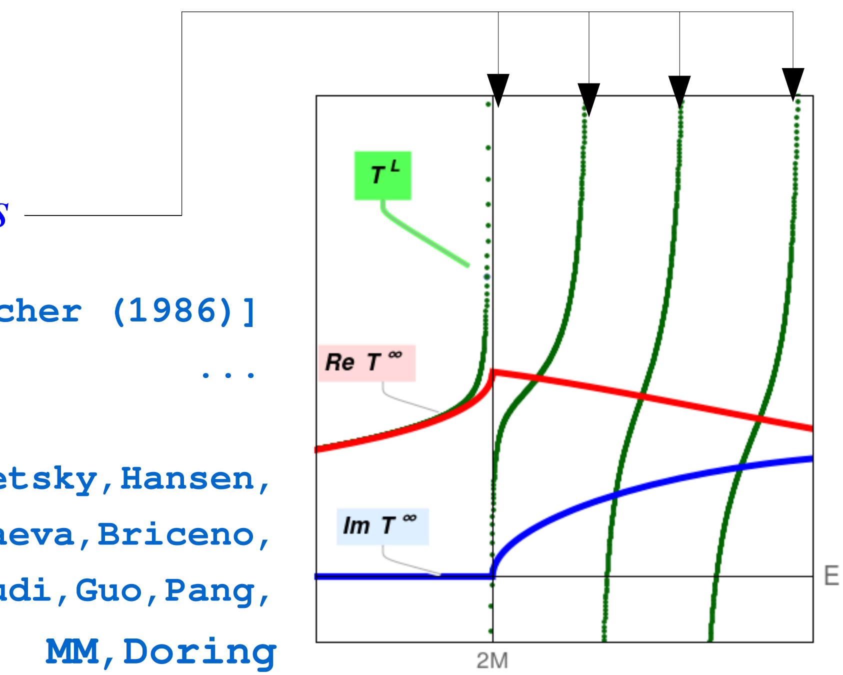
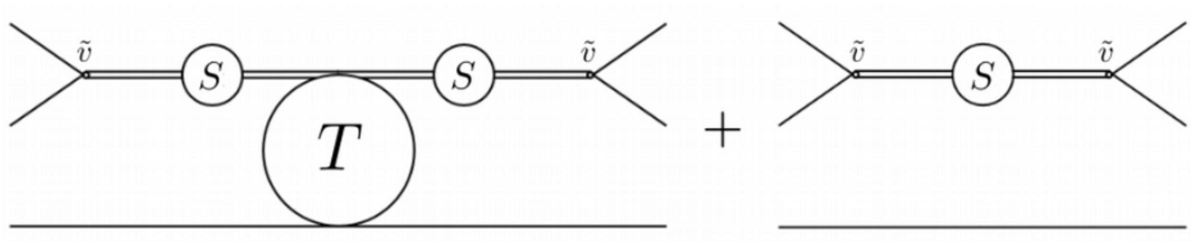
– 3-body analog under investigation Sharpe, Rusetsky, Hansen, Polejaeva, Briceno, Davoudi, Guo, Pang, **MM, Doring**



DISCRETIZATION

Discretize 3b-scattering amplitude → 3b Quantization Condition



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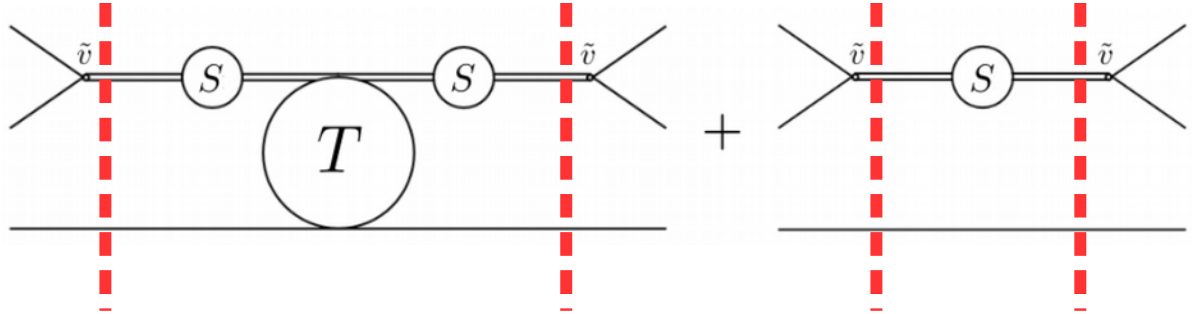
Discretize 3b-scattering amplitude → 3b Quantization Condition

- *v* is cut-free



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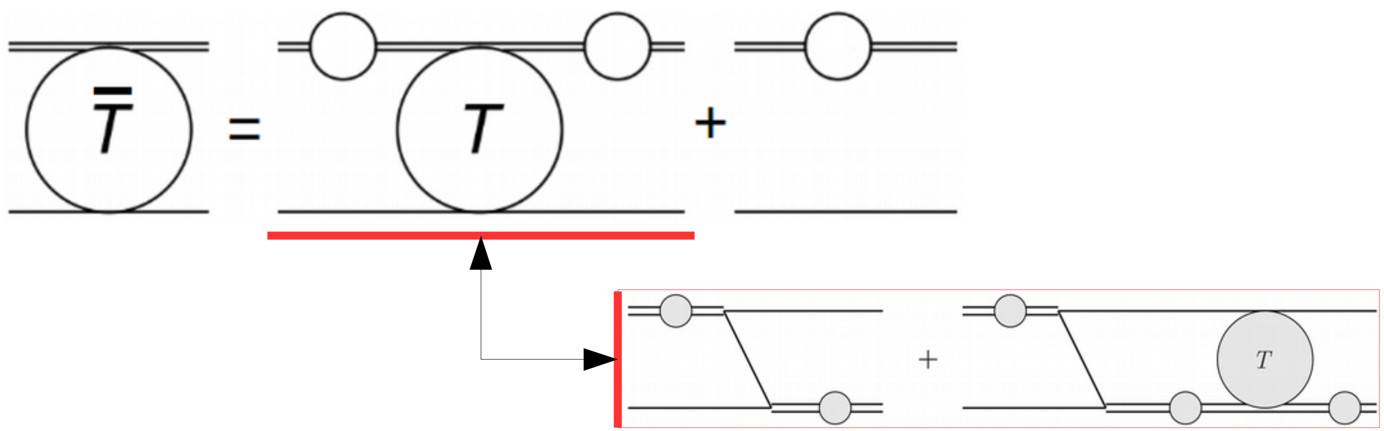

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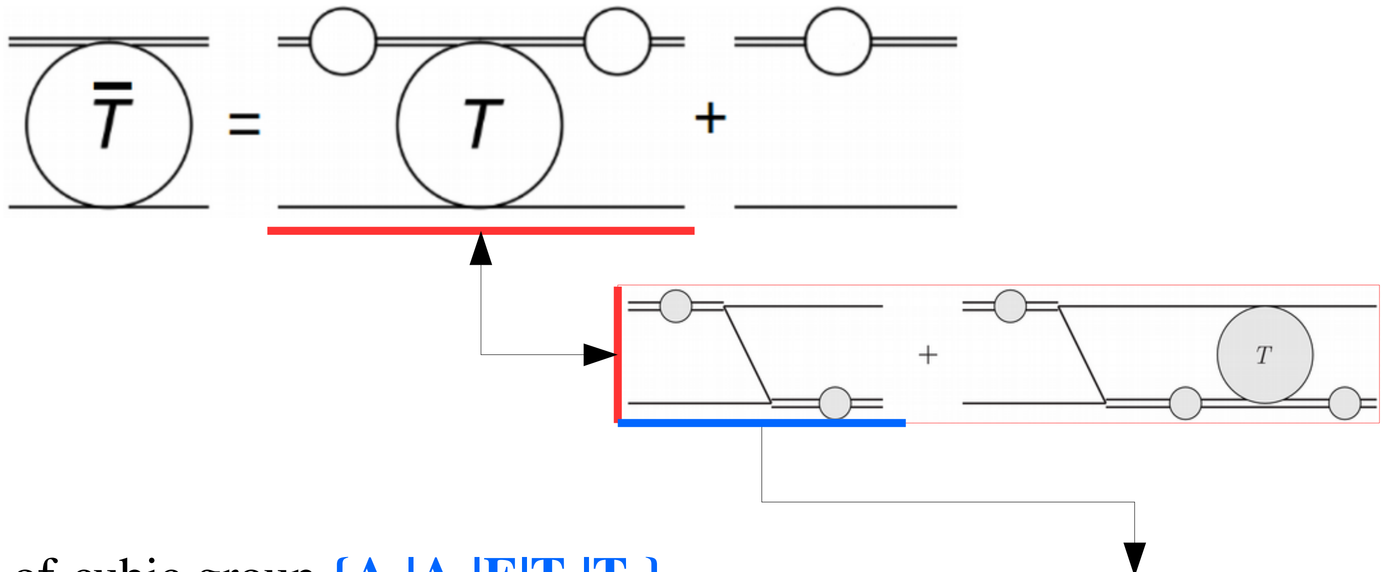
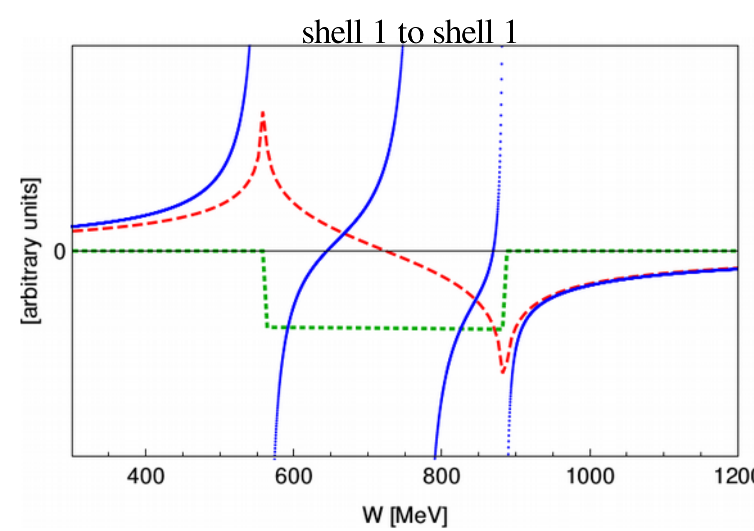


- Project to irreps of cubic group $\{A_1|A_2|E|T_1|T_2\}$
  - reduce dimensionality
  - B (ope potential) is singular!

S-wave infinite volume vs. A_1^+ finite volume

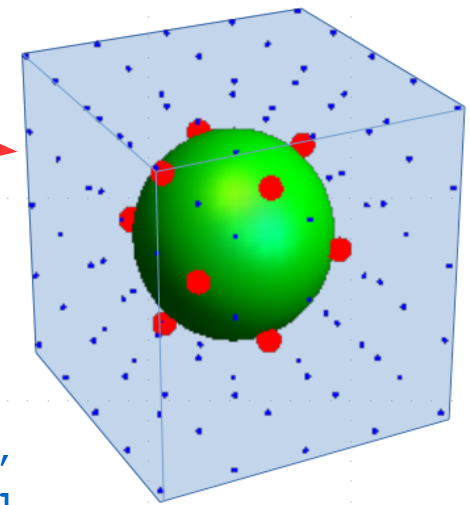


PROJECTION TO IRREPS

1) Separation of variables

– shells = sets of points related by $\boldsymbol{O_h}$

– *inf. vol. analog:* radii and angles



[Döring,Hammer,MM,Pang, Rusetsky,Wu(2018)]

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2) Find the ONB of functions on each shell

– $f^s(\hat{\mathbf{p}}_j) = \sqrt{4\pi} \sum_{\Gamma\alpha} \sum_{u} f_u^{\Gamma\alpha s} \chi_u^{\Gamma\alpha s}(\hat{\mathbf{p}}_j)$

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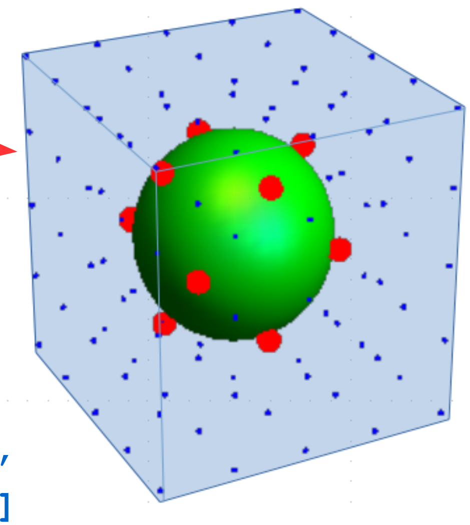

[Döring,Hammer,MM,Pang,
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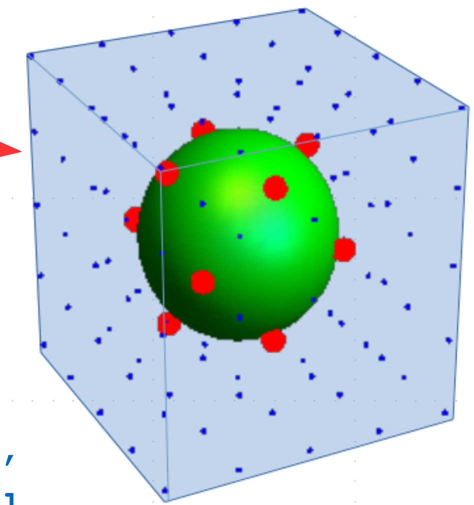

[Döring,Hammer,MM,Pang, Rusetsky,Wu(2018)]

Projection of 3-body-Quantization-Condition = FINAL RESULT

$$\mathrm{Det}\left(\mathbf{B}_{\mathbf{uu'}}^{\mathbf{\Gamma ss'}}(\mathbf{W^2}) + \frac{\mathbf{2E_s\,L^3}}{\vartheta(\mathbf{s})}\tau_{\mathbf{s}}(\mathbf{W^2})^{-1}\delta_{\mathbf{ss'}}\delta_{\mathbf{uu'}}\right) = \mathbf{0}$$

[MM,Döring]

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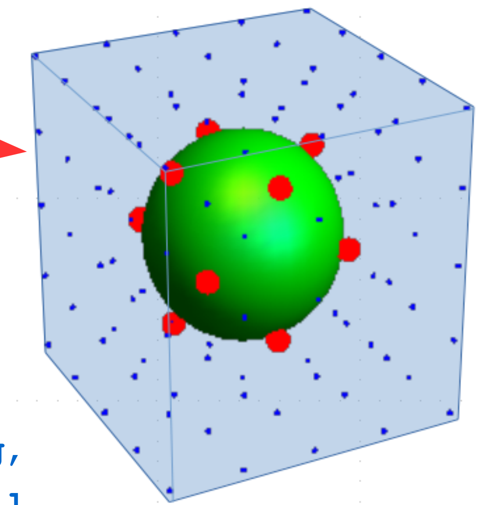

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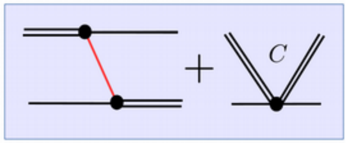
[MM,Döring]



W – total energy

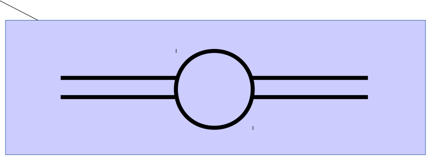
s/s' - shell index

u/u' - basis index

ϑ – multiplicity

L – lattice volume

Es – 1p. energy



NUMERICAL EXAMPLE

$$\mathrm{Det}\left(\mathbf{B}^{\Gamma\mathbf{s}\mathbf{s}'}_{\mathbf{u}\mathbf{u}'}(\mathbf{W}^2)+\frac{\mathbf{2E_s\,L^3}}{\vartheta(\mathbf{s})}\tau_{\mathbf{s}}(\mathbf{W}^2)^{-1}\delta_{\mathbf{s}\mathbf{s}'}\delta_{\mathbf{u}\mathbf{u}'}\right)=\mathbf{0}$$

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- **3 particles in finite volume:** ***m=138 MeV, L=3 fm***

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  - vertex(Isobar→2 stable particles)
  - subtraction constant (~mass)

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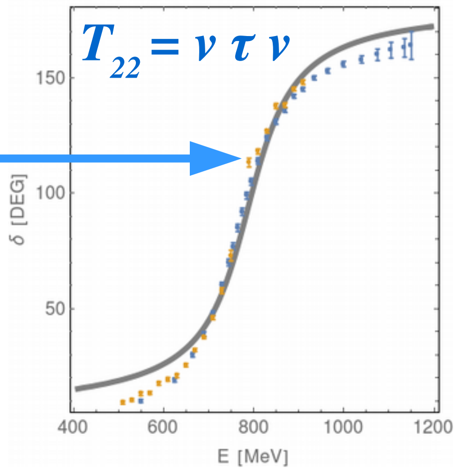

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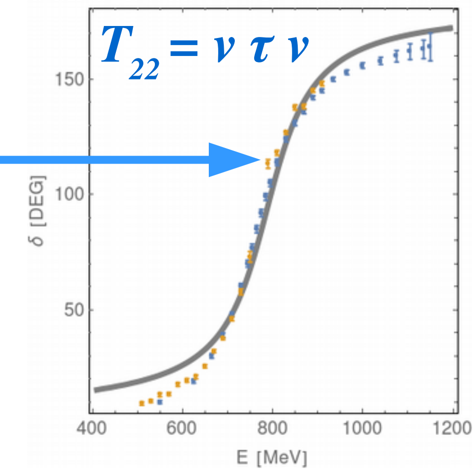
  - **→ prediction of 3body energy-eigenlevels**



NUMERICAL EXAMPLE

$$\mathrm{Det}\left(B^{\Gamma ss'}_{uu'}(W^2) + \frac{2E_s L^3}{\vartheta(s)}\tau_s(W^2)^{-1}\delta_{ss'}\delta_{uu'}\right) = 0$$

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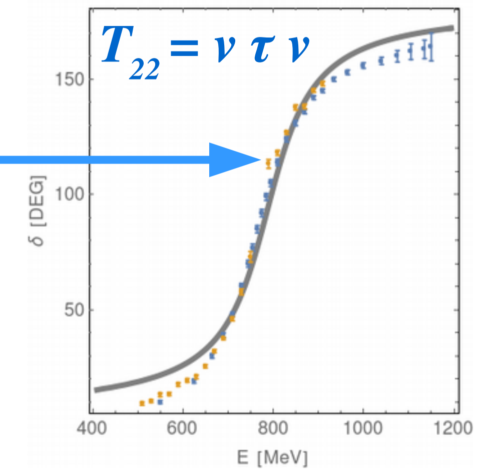

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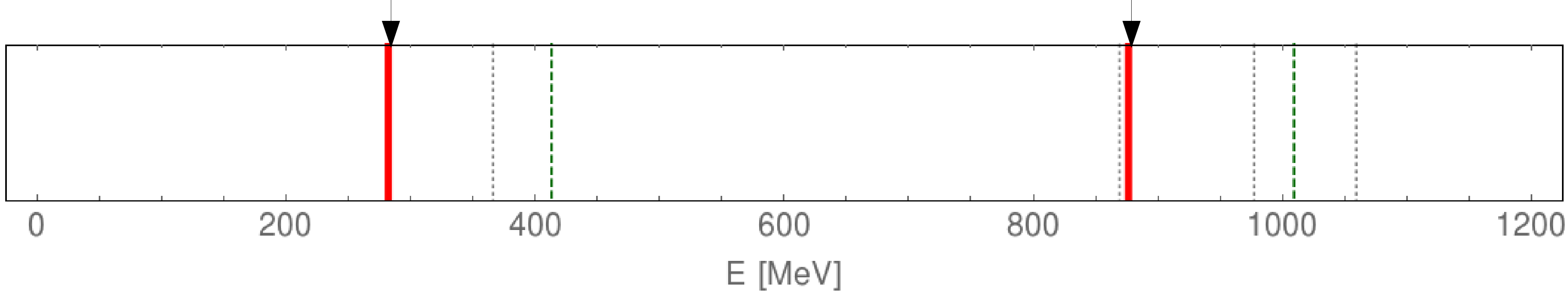

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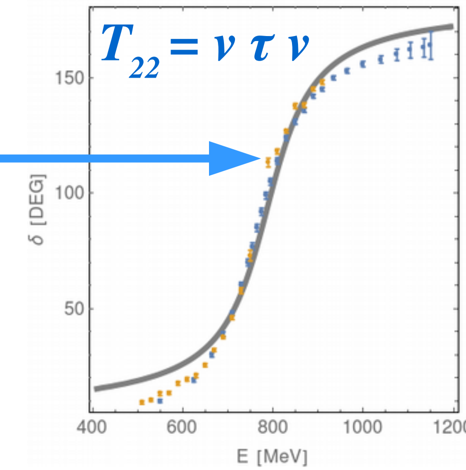
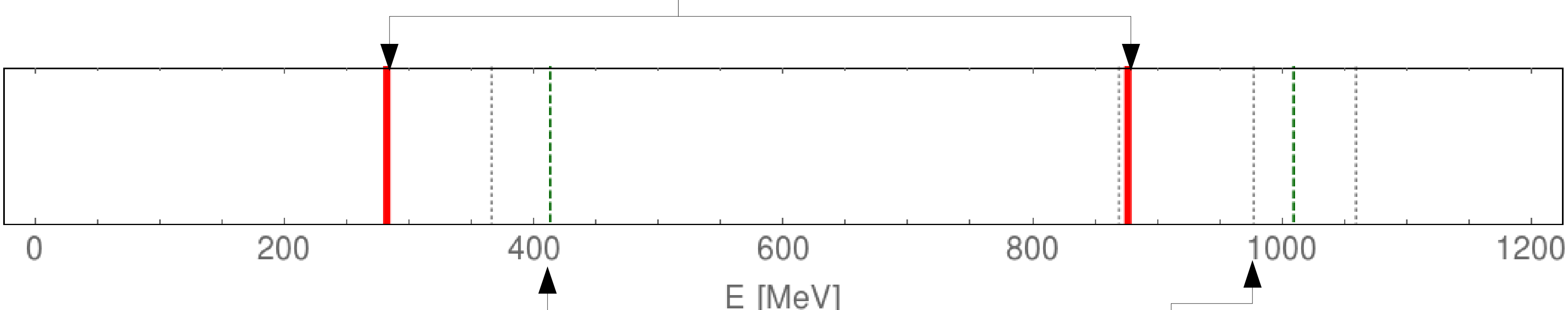




unphysical lvls cancel out (exact proof available)

SUMMARY/OUTLOOK

3-body scattering amplitude derived from 2&3 body Unitarity

– interaction kernel = one-particle-exchange

– flexible parametrization: real contributions can be added to the kernel

TBD: analysis of physical systems

3-body Quantization Condition in fin. vol. derived

– cancellations of unphysical poles revealed

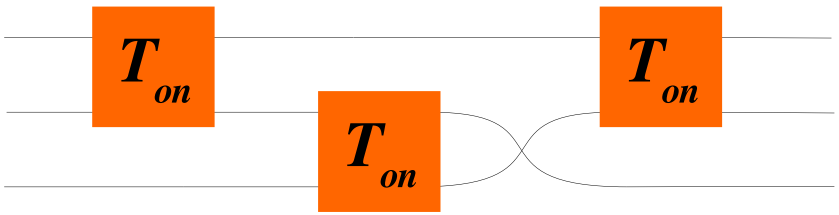
– projection to irreps done

– technical feasibility on a numerical example

– the only approximation = number of isobars

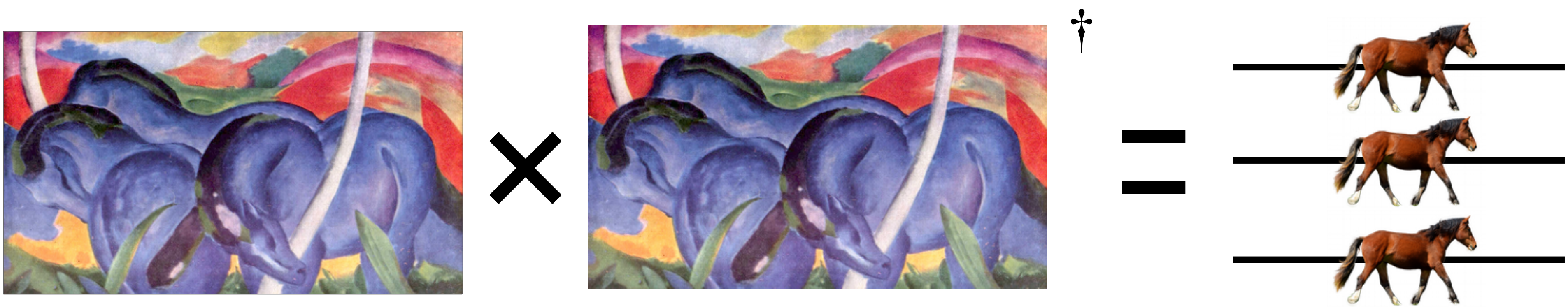
TBD: multiple channels

TBD: inclusion of isospin & angular momentum



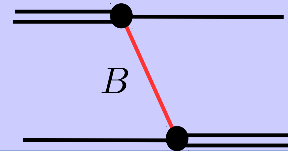
"power of Unitarity"

THANK YOU!

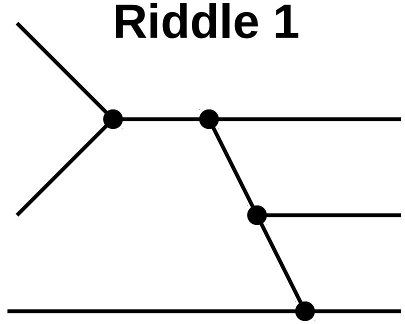


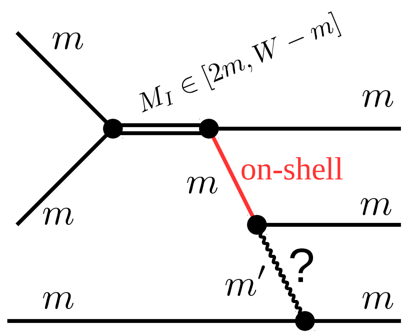
SPARES

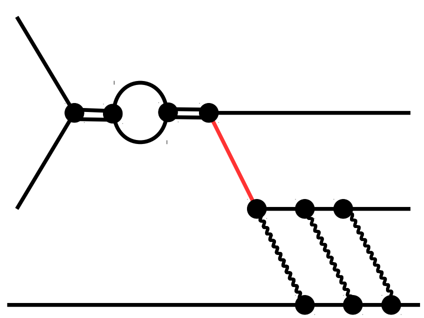
The Power of Unitarity

Question: Does  provide full imaginary part of all possible 3→ 3 transitions?

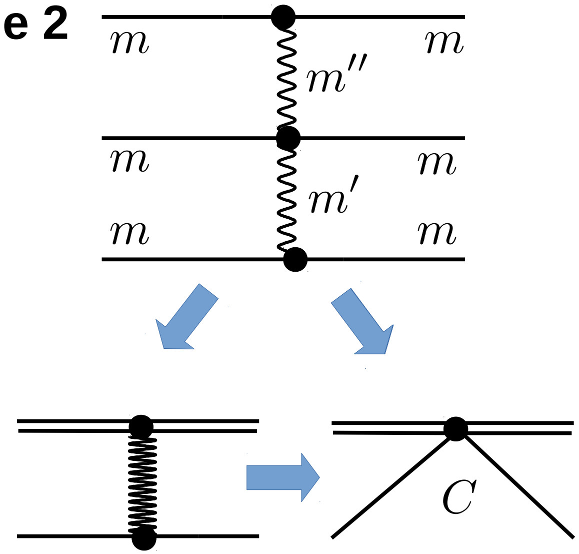
Riddle 1



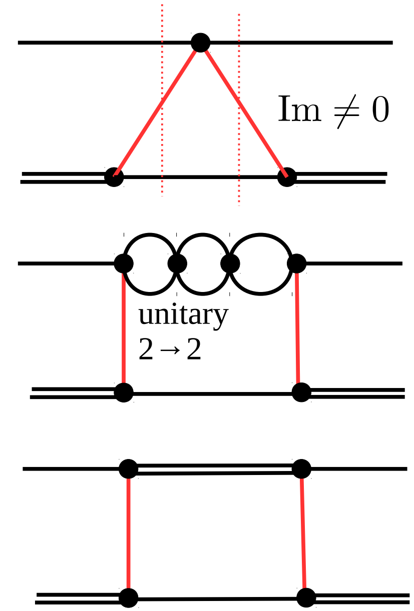


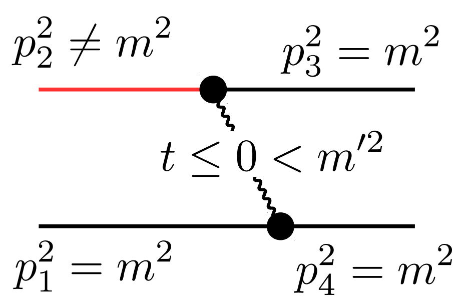


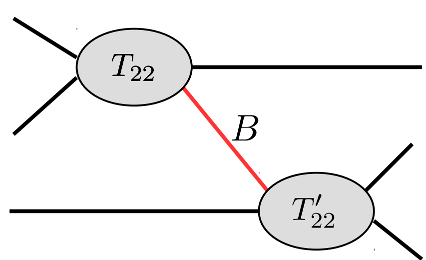
Riddle 2

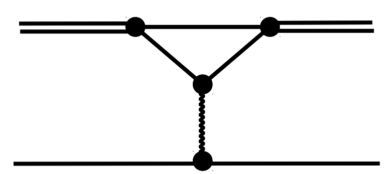


Riddle 3

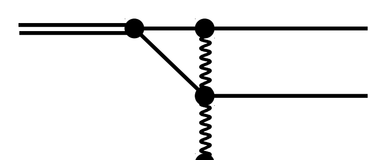








Riddle 4



Riddle 5

- Projection of T

$$T^{ss'}(\hat{\mathbf{p}}_j, \hat{\mathbf{p}}_{j'}) = 4\pi \sum_{\Gamma\alpha} \sum_{uu'} \chi_u^{\Gamma\alpha s}(\hat{\mathbf{p}}_j) T_{uu'}^{\Gamma ss'} \chi_{u'}^{\Gamma\alpha s'}(\hat{\mathbf{p}}_{j'}) \,,$$

$$T_{uu'}^{\Gamma ss'} = \frac{4\pi}{\vartheta(s)\vartheta(s')} \sum_{j=1}^{\vartheta(s)} \sum_{j'=1}^{\vartheta(s')} \chi_u^{\Gamma\alpha s}(\hat{\mathbf{p}}_j) T^{ss'}(\hat{\mathbf{p}}_j, \hat{\mathbf{p}}_{j'}) \chi_{u'}^{\Gamma\alpha s'}(\hat{\mathbf{p}}_{j'})$$

QUANTIZATION CONDITION

Cancellations:

→ fin. vol. normalization of δ-distribution!

$$\bar{T}^{A_1^+}_{nm}(s) = \tau_n(s) T^{A_1^+}_{nm}(s) \tau_m(s) - 2E_n \tau_n(s) \frac{L^3}{\vartheta(n)} \delta_{nm}$$

$$T^{A_1^+}_{nm}(s) = B^{A_1^+}_{nm}(s) - \frac{1}{L^3} \sum_{x \in sets_8} \vartheta(x) B^{A_1^+}_{nx}(s) \frac{\tau_x(s)}{2E_x} T^{A_1^+}_{xm}(s)$$

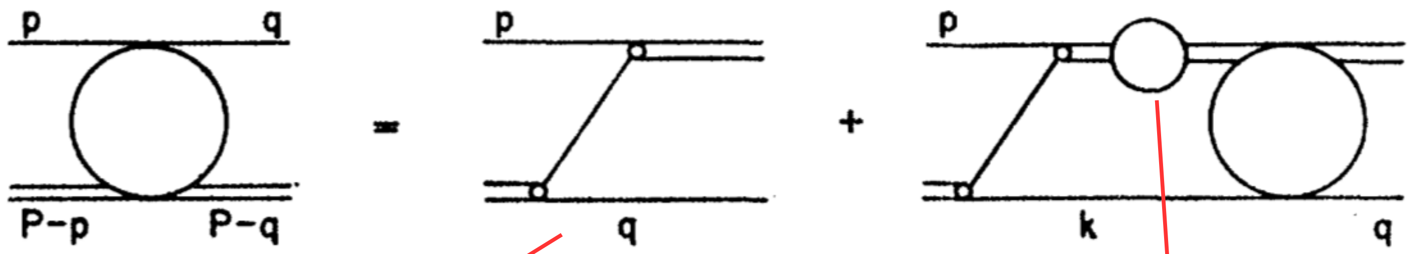
$B^{A_1^+}$ singular at $W^+ = E_m + E_n + E(\boldsymbol{q}_{nj} + \boldsymbol{p}_{mi})$

τ_m^{-1} singular at $W^{\pm\pm} = E_m \pm E((2\pi/L)\boldsymbol{y}) \pm E((2\pi/L)\boldsymbol{y} + \boldsymbol{p}_{mi})$ for $\boldsymbol{y} \in \mathbb{Z}^3$

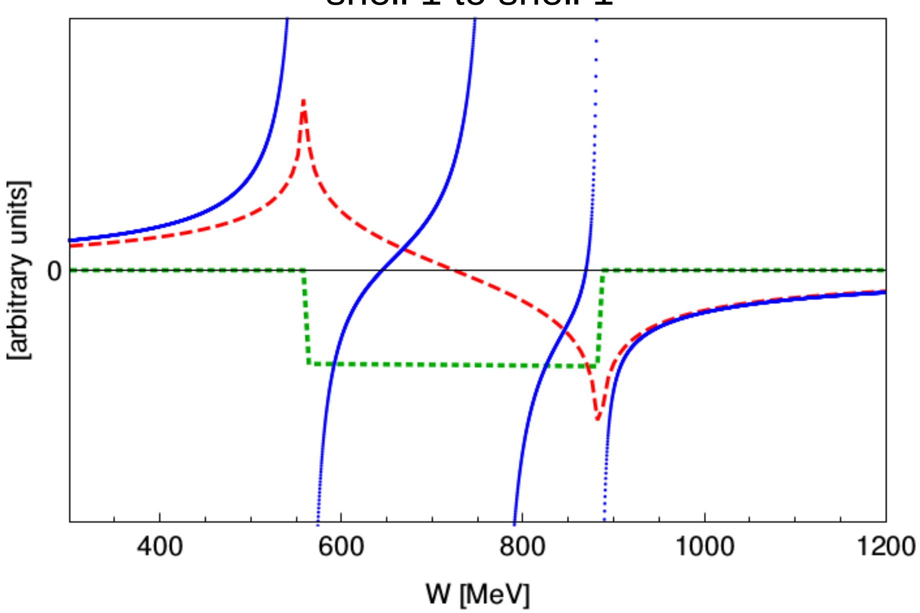
– when isobar-momenta are discretized in the 3-body cms momenta

$$\tau = \sigma(k) - M_0^2 - \frac{1}{(2\pi)^3} \int d^3\boldsymbol{\ell} \frac{\lambda^2}{2E_\ell(\sigma(k) - 4E_\ell^2 + i\epsilon)}$$

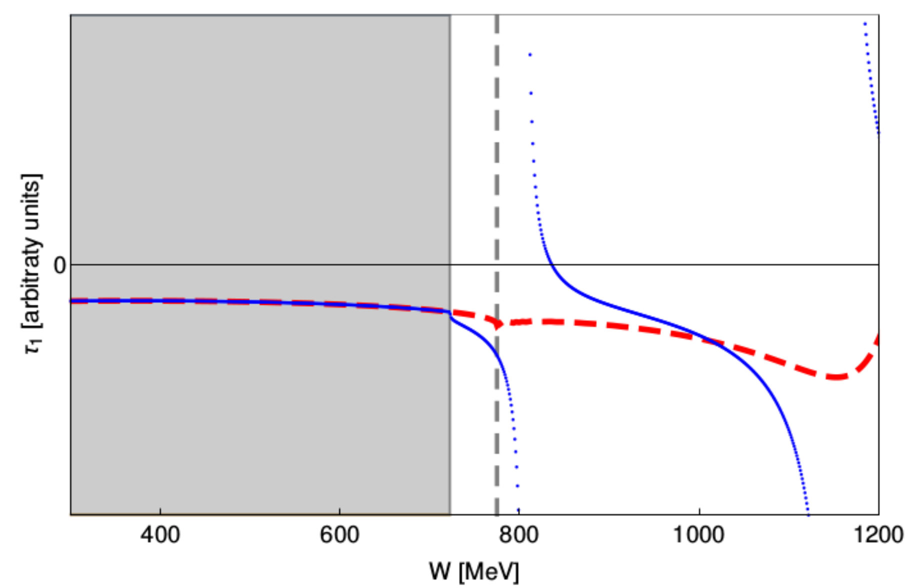
Power-law finite-volume effects dictated by three-body unitarity



S-wave infinite volume vs. A_1^+ finite volume shell 1 to shell 1

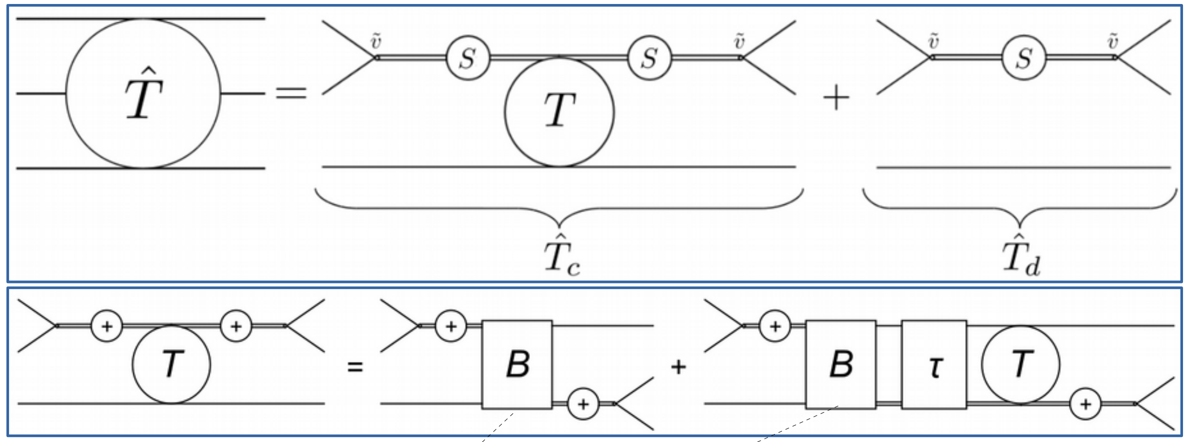


Tower of boosted 2→2 amplitudes
to implement 3-body quantization condition



SCATTERING AMPLITUDE

$3 \to 3$ scattering amplitude is a 3-dimensional integral equation



– Imaginary parts of ***B, S*** are fixed by **unitarity/matching**

– For simplicity ***υ=λ*** (full relations available)

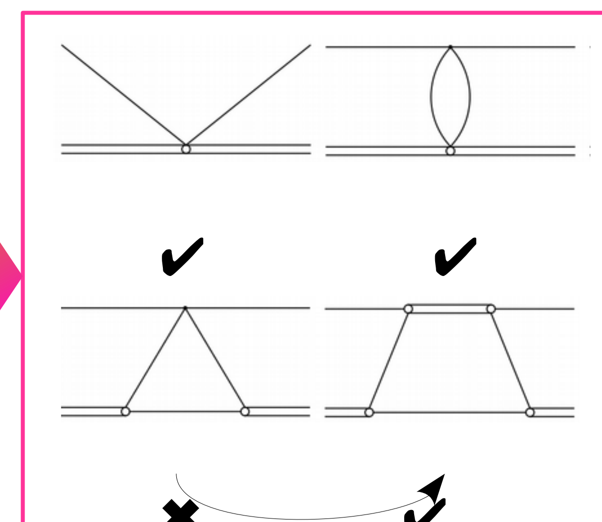
$$\text{Disc}\, B(u) = 2\pi i \lambda^2 \frac{\delta\left(E_Q - \sqrt{m^2 + \mathbf{Q}^2}\right)}{2\sqrt{m^2 + \mathbf{Q}^2}}$$

- un-subtracted dispersion relation

$$\langle q|B(s)|p\rangle = -\frac{\lambda^2}{2\sqrt{m^2+\mathbf{Q}^2}\left(E_Q-\sqrt{m^2+\mathbf{Q}^2}+i\epsilon\right)}$$

- one-**π** exchange in TOPT → ***R E S U L T !***

?



Unitarity & Matching

- 3-body Unitarity (normalization condition ↔ phase space integral)

$$\langle q_1,q_2,q_3|(\hat{T}-\hat{T}^\dagger)|p_1,p_2,p_3\rangle = i\int_P \langle q_1,q_2,q_3|\hat{T}^\dagger|k_1,k_2,k_3\rangle\langle k_1,k_2,k_3|\hat{T}|p_1,p_2,p_3\rangle$$