Kaon-pion scattering from lattice QCD

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Pion-Kaon Interactions Workshop
JLab, Newport News, VA

February 14, 2018

Overview

- q.m. resonance in a box
- two-particle Luscher formalism
  - resonance information from finite-volume energies
- two-particle energies in lattice QCD
- use of the K-matrix and the box B matrix
- recent results

Collaborators

- people involved in this work:

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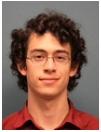

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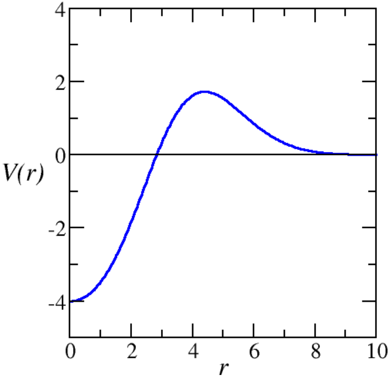
- thanks to NSF XSEDE:
  - Stampede at TACC
  - Comet at SDSC

Resonances in a box: an example

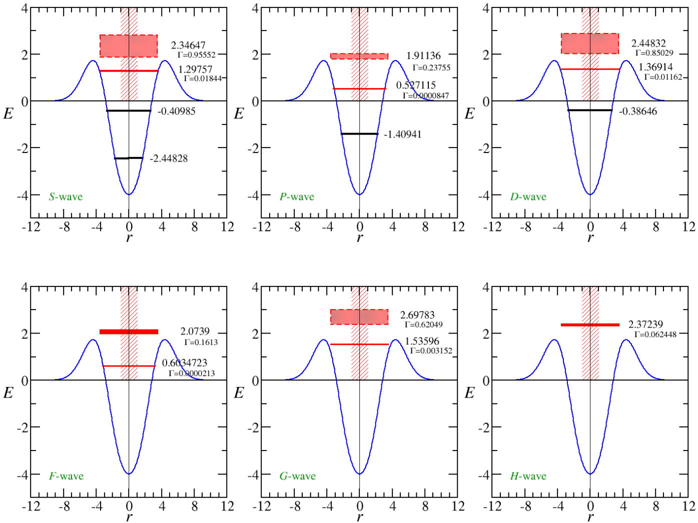
- consider a simple quantum mechanical example
- Hamiltonian

$$H = \frac{1}{2}\boldsymbol{p}^2 + V(r), \qquad V(r) = \left(-4 + \frac{1}{16}r^4\right) e^{-r^2/8}$$



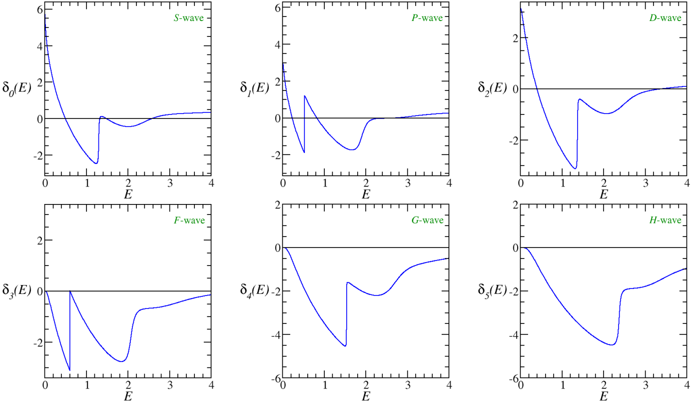
Spectrum of example Hamiltonian

- spectrum for $E < 4$ and $l = 0, 1, 2, 3, 4, 5$ of example system



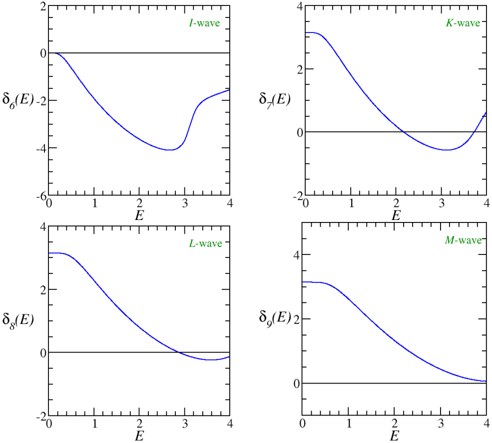
Scattering phase shifts

- scattering phase shifts for various partial waves



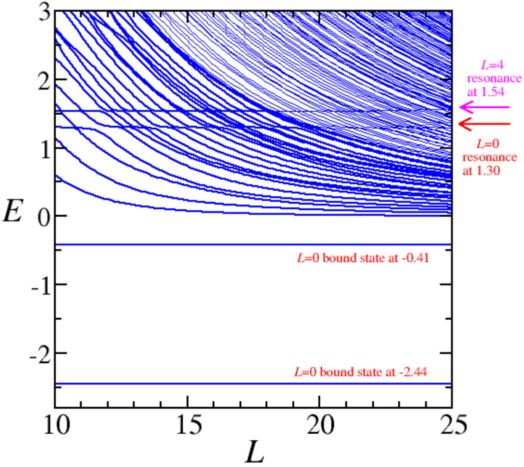
More scattering phase shifts

- scattering phase shifts for higher partial waves



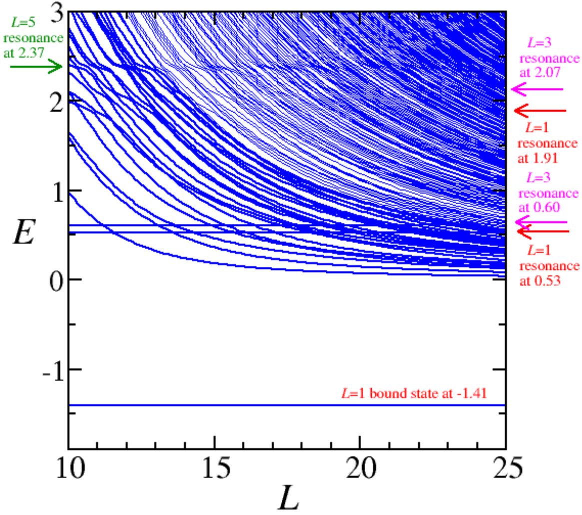
Spectrum in box: A_{1g} channel

- spectrum discrete in box, periodic b.c., momenta quantized
- stationary-state energies in A_{1g} channel shown below
- narrow resonance is avoided level crossing, broad resonances?



Spectrum in box: T_{1u} channel

- stationary-state energies in T_{1u} channel shown below

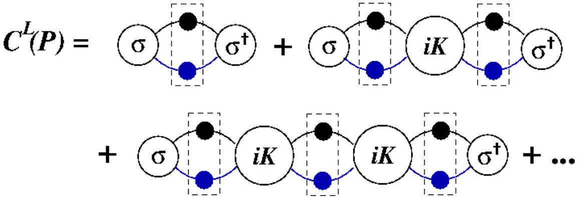


Scattering phase shifts in lattice QCD timeline

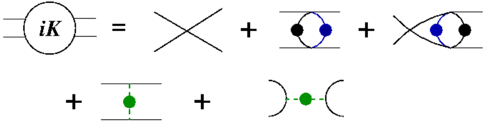
- DeWitt 1956: finite-volume energies related to scattering phase shifts
- Lüscher 1986: fields in a cubic box
- Rummukainen and Gottlieb 1995: nonzero total momenta
- Kim, Sachrajda, and Sharpe 2005: derivation reworked
- explosion of papers since then
- Briceno 2014: generalized to arbitrary spin, multiple channels

Two-particle correlator in finite-volume

- correlator of two-particle operator σ in finite volume



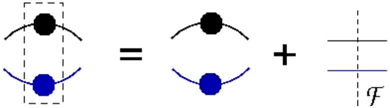
- Bethe-Salpeter kernel



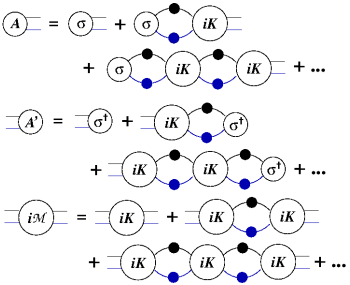
- $C^{\infty}(P)$ has branch cuts where two-particle thresholds begin
- momentum quantization in finite volume: cuts $\rightarrow$ series of poles
- C^{L} poles: two-particle energy spectrum of finite volume theory

Corrections from finite momentum sums

- finite-volume momentum sum is infinite-volume integral plus correction $\mathcal{F}$

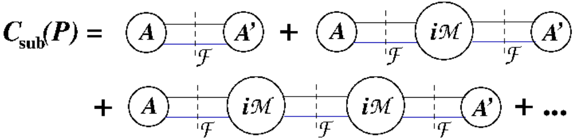


- define the following quantities: A, A', invariant scattering amplitude $i\mathcal{M}$



Quantization condition

- subtracted correlator $C_{\rm sub}(P) = C^L(P) - C^\infty(P)$ given by



- sum geometric series

$$C_{\rm sub}(P) = A\,\mathcal{F}(1 - i\mathcal{M}\mathcal{F})^{-1}\,A'.$$

- poles of $C_{\rm sub}(P)$ are poles of $C^L(P)$ from $\det(1 - i\mathcal{M}\mathcal{F}) = 0$
- key tool: for $g_c(\boldsymbol{p})$ spatially contained and regular

$$\frac{1}{L^3}\sum_{\boldsymbol{p}} g_c(\boldsymbol{p}) = \int \frac{d^3k}{(2\pi)^3}\, g_c(\mathbf{k}) + O(e^{-mL})$$

$$\frac{1}{L^3}\sum_{\boldsymbol{p}} \frac{g_c(\boldsymbol{p}^2)}{(\boldsymbol{p}^2 - a^2)} = \frac{1}{L^3}\sum_{\boldsymbol{p}} \frac{g_c(a^2)}{(\boldsymbol{p}^2 - a^2)} + \int \frac{d^3k}{(2\pi)^3} \frac{g_c(\boldsymbol{p}^2) - g(a^2)}{(\boldsymbol{p}^2 - a^2)} + O(e^{-mL})$$

Kinematics

- work in spatial L^3 volume with periodic b.c.
- total momentum $\boldsymbol{P} = (2\pi/L)\boldsymbol{d}$, where $\boldsymbol{d}$ vector of integers
- calculate lab-frame energy E of two-particle interacting state in lattice QCD
- boost to center-of-mass frame by defining:

$$E_{\rm cm} = \sqrt{E^2 - \boldsymbol{P}^2}, \qquad \gamma = \frac{E}{E_{\rm cm}},$$

- assume N_d channels
- particle masses m_{1a}, m_{2a} and spins s_{1a}, s_{2a} of particle 1 and 2
- for each channel, can calculate

$$\begin{aligned} \boldsymbol{q}^2_{{\rm cm},a} &= \frac{1}{4}E^2_{\rm cm} - \frac{1}{2}(m^2_{1a} + m^2_{2a}) + \frac{(m^2_{1a} - m^2_{2a})^2}{4E^2_{\rm cm}}, \\ u^2_a &= \frac{L^2 \boldsymbol{q}^2_{{\rm cm},a}}{(2\pi)^2}, \qquad \boldsymbol{s}_a = \left(1 + \frac{(m^2_{1a} - m^2_{2a})}{E^2_{\rm cm}}\right)\boldsymbol{d} \end{aligned}$$

Quantization condition re-expressed

- E related to S matrix (and phase shifts) by

$$\det[1 + F^{(\boldsymbol{P})}(S - 1)] = 0$$

- F matrix in $JLSa$ basis states given by

$$\langle J' m_{J'} L' S' a' | F^{(\boldsymbol{P})} | J m_J L S a\rangle = \delta_{a'a}\delta_{S'S}\ \frac{1}{2}\Big\{\delta_{J'J}\delta_{m_{J'} m_J}\delta_{L'L}$$
$$+\langle J' m_{J'} | L' m_{L'} S m_S\rangle \langle L m_L S m_S | J m_J\rangle W^{(\boldsymbol{P}a)}_{L' m_{L'};\ L m_L}\Big\}$$

- total ang mom J, J', orbital L, L', spin S, S', channels a, a'
- W given by

$$-iW^{(\boldsymbol{P}a)}_{L' m_{L'};\ L m_L} = \sum_{l=|L'-L|}^{L'+L} \sum_{m=-l}^{l} \frac{\mathcal{Z}_{lm}(\boldsymbol{s}_a, \gamma, u_a^2)}{\pi^{3/2}\gamma u_a^{l+1}} \sqrt{\frac{(2L'+1)(2l+1)}{(2L+1)}}$$
$$\times \langle L' 0, l 0 | L 0\rangle \langle L' m_{L'}, l m | L m_L\rangle.$$

- above expressions apply for both distinguishable and indistinguishable particles

RGL shifted zeta functions

- compute Rummukainen-Gottlieb-Lüscher (RGL) shifted zeta functions $\mathcal{Z}_{lm}$ using

$$\mathcal{Z}_{lm}(\boldsymbol{s},\gamma,u^2) = \sum_{\boldsymbol{n}\in\mathbb{Z}^3} \frac{\mathcal{Y}_{lm}(\boldsymbol{z})}{(\boldsymbol{z}^2-u^2)} e^{-\Lambda(\boldsymbol{z}^2-u^2)} + \delta_{l0}\frac{\gamma\pi}{\sqrt{\Lambda}} F_0(\Lambda u^2)$$

$$+\frac{i^l\gamma}{\Lambda^{l+1/2}}\int_0^1 dt\left(\frac{\pi}{t}\right)^{l+3/2} e^{\Lambda t u^2} \sum_{\substack{\boldsymbol{n}\in\mathbb{Z}^3\\ \boldsymbol{n}\neq 0}} e^{\pi i \boldsymbol{n}\cdot\boldsymbol{s}}\mathcal{Y}_{lm}(\mathbf{w})\ e^{-\pi^2\mathbf{w}^2/(t\Lambda)}$$

- where

$$\boldsymbol{z} = \boldsymbol{n} - \gamma^{-1}\left[\tfrac{1}{2} + (\gamma-1)s^{-2}\boldsymbol{n}\cdot\boldsymbol{s}\right]\boldsymbol{s},$$

$$\mathbf{w} = \boldsymbol{n} - (1-\gamma)s^{-2}\boldsymbol{s}\cdot\boldsymbol{n}\boldsymbol{s}, \qquad \mathcal{Y}_{lm}(\mathbf{x}) = |\mathbf{x}|^l\ Y_{lm}(\widehat{\mathbf{x}})$$

$$F_0(x) = -1 + \frac{1}{2}\int_0^1 dt\ \frac{e^{tx}-1}{t^{3/2}}$$

- choose $\Lambda \approx 1$ for convergence of the summation
- integral done Gauss-Legendre quadrature
- $F_0(x)$ given in terms of Dawson or erf function

K matrix

- quantization condition relates single energy E to entire S-matrix
- cannot solve for S-matrix (except single channel, single wave)
- approximate S-matrix with functions depending on handful of fit parameters
- obtain estimates of fit parameters using many energies
- easier to parametrize Hermitian matrix than unitary matrix
- introduce K-matrix (Wigner 1946)

$$S = (1 + iK)(1 - iK)^{-1} = (1 - iK)^{-1}(1 + iK)$$

- Hermiticity of K-matrix ensures unitarity of S-matrix
- with time reversal invariance, K-matrix must be real and symmetric

K matrix

- rotational invariance implies

$$\langle J' m_{J'} L' S' a' |\ K\ | J m_J L S a\rangle = \delta_{J'J}\delta_{m_{J'} m_J}\ K^{(J)}_{L'S'a';\ LSa}(E)$$

where $K^{(J)}$ is real, symmetric, independent of m_J

- invariance under parity gives

$$K^{(J)}_{L'S'a';\ LSa}(E) = 0 \quad \text{when } \eta^{P\prime}_{1a'}\eta^{P}_{1a}\eta^{P\prime}_{2a'}\eta^{P}_{2a}(-1)^{L'+L} = -1,$$

where η^P_{ja} is intrinsic parity of particle j in channel a

- multichannel effective range expansion (Ross 1961)

$$K^{-1}_{L'S'a';\ LSa}(E) = q_{a'}^{-L'-\frac{1}{2}}\ \widehat{K}^{-1}_{L'S'a';\ LSa}(E_{\rm cm})\ q_a^{-L-\frac{1}{2}},$$

where $\widehat{K}^{-1}_{L'S'a';\ LSa}(E_{\rm cm})$ real, symmetric, analytic function of $E_{\rm cm}$

The "box matrix" B

- effective range expansion suggests writing

$$K^{-1}_{L'S'a';\,LSa}(E) = u_{a'}^{-L'-\frac{1}{2}}\ \widetilde{K}^{-1}_{L'S'a';\,LSa}(E_{\rm cm})\ u_a^{-L-\frac{1}{2}}$$

  since $\widetilde{K}^{-1}_{L'S'a';\,LSa}(E_{\rm cm})$ behaves smoothly with $E_{\rm cm}$

- quantization condition can be written

$$\det(1 - B^{(\boldsymbol{P})}\widetilde{K}) = \det(1 - \widetilde{K}B^{(\boldsymbol{P})}) = 0$$

- we define the box matrix by

$$\langle J'm_{J'}L'S'a'|\ B^{(\boldsymbol{P})}\ |Jm_JLSa\rangle = -i\delta_{a'a}\delta_{S'S}\ u_a^{L'+L+1}\ W^{(\boldsymbol{P}a)}_{L'm_{L'};\ Lm_L}$$
$$\times\langle J'm_{J'}|L'm_{L'}, Sm_S\rangle\langle Lm_L, Sm_S|Jm_J\rangle$$

- box matrix is Hermitian for u_a^2 real
- quantization condition can also be expressed as

$$\det(\widetilde{K}^{-1} - B^{(\boldsymbol{P})}) = 0$$

- these determinants are real

Block diagonalization

- quantization condition involves determinant of infinite matrix
- make practical by (a) transforming to a block-diagonal basis and (b) truncating in orbital angular momentum
- for symmetry operation G, define unitary matrix

$$\langle J'm_{J'}L'S'a'|\, Q^{(G)}\, |Jm_JLSa\rangle = \begin{cases} \delta_{J'J}\delta_{L'L}\delta_{S'S}\delta_{a'a}D^{(J)}_{m_{J'}m_J}(R), & (G=R), \\ \delta_{J'J}\delta_{m_{J'}m_J}\delta_{L'L}\delta_{S'S}\delta_{a'a}(-1)^L, & (G=I_s), \end{cases}$$

where $D^{(J)}_{m'm}(R)$ Wigner rotation matrices, R ordinary rotation, I_s spatial inversion

- can show that box matrix satisfies

$$B^{(G\boldsymbol{P})} = Q^{(G)}\, B^{(\boldsymbol{P})}\, Q^{(G)\dagger}.$$

- if G in little group of $\boldsymbol{P}$, then $G\boldsymbol{P}=\boldsymbol{P},\ G\boldsymbol{s}_a=\boldsymbol{s}_a$ and

$$[B^{(\boldsymbol{P})}, Q^{(G)}] = 0, \qquad (G \text{ in little group of } \boldsymbol{P}).$$

- can use eigenvectors of $Q^{(G)}$ to block diagonalize $B^{(\boldsymbol{P})}$

Block diagonalization (con't)

- block-diagonal basis

$$|\Lambda\lambda n JLSa\rangle = \sum_{m_J} c_{m_J}^{J(-1)^L;\,\Lambda\lambda n} |Jm_J LSa\rangle$$

- little group irrep Λ, irrep row λ, occurrence index n
- transformation coefficients depend on J and $(-1)^L$, not on S, a
- replaces m_J by (Λ, λ, n)
- group theoretical projections with Gram-Schmidt used to obtain coefficients
- use notation and irrep matrices from PRD 88, 014511 (2013)

Box and $\widetilde{K}$ matrices in block diagonal basis

- in block-diagonal basis, box matrix has form

$$\langle \Lambda'\lambda' n' J' L' S' a' |\ B^{(\boldsymbol{P})}\ | \Lambda\lambda n J L S a\rangle = \delta_{\Lambda'\Lambda}\delta_{\lambda'\lambda}\delta_{S'S}\delta_{a'a}\ B^{(\boldsymbol{P}\Lambda_B S a)}_{J'L'n';\ JLn}(E)$$

- $\widetilde{K}$-matrix for $(-1)^{L+L'} = 1$ has form

$$\langle \Lambda'\lambda' n' J' L' S' a' |\ \widetilde{K}\ | \Lambda\lambda n J L S a\rangle = \delta_{\Lambda'\Lambda}\delta_{\lambda'\lambda}\delta_{n'n}\delta_{J'J}\ \mathcal{K}^{(J)}_{L'S'a';\ LSa}(E_{\rm cm})$$

- $(-1)^{L+L'} = 1 \Rightarrow \eta^{P\prime}_{1a'}\eta^{P\prime}_{2a'} = \eta^{P}_{1a}\eta^{P}_{2a}$, always applies in QCD
- Λ is irrep for K-matrix, need Λ_B for box matrix
- when $\eta^P_{1a}\eta^P_{2a} = 1$, then $\Lambda_B = \Lambda$

$\boldsymbol{d}$	LG	Λ_B relationship to Λ when $\eta^P_{1a}\eta^P_{2a} = -1$
$(0,0,0)$	O_h	Subscript $g \leftrightarrow u$
$(0,0,n)$	C_{4v}	$A_1 \leftrightarrow A_2;\ B_1 \leftrightarrow B_2$; E, G_1, G_2 stay same
$(0,n,n)$	C_{2v}	$A_1 \leftrightarrow A_2;\ B_1 \leftrightarrow B_2$; G stays same
(n,n,n)	C_{3v}	$A_1 \leftrightarrow A_2;\ F_1 \leftrightarrow F_2$; E, G stay same

- see PRD 88, 014511 (2013) for notation

K matrix parametrizations

- $\widetilde{K}$ matrix in block diagonal basis

$$\begin{aligned}\langle \Lambda'\lambda' n' J' L' S' a'|\ \widetilde{K}\ |\Lambda\lambda n J L S a\rangle &= \delta_{\Lambda'\Lambda}\delta_{\lambda'\lambda}\delta_{n'n}\delta_{J'J}\ \mathcal{K}^{(J)}_{L'S'a';\ LSa}(E_{\rm cm})\\ \langle \Lambda'\lambda' n' J' L' S' a'|\ \widetilde{K}^{-1}\ |\Lambda\lambda n J L S a\rangle &= \delta_{\Lambda'\Lambda}\delta_{\lambda'\lambda}\delta_{n'n}\delta_{J'J}\ \mathcal{K}^{(J)-1}_{L'S'a';\ LSa}(E_{\rm cm})\end{aligned}$$

- common parametrization

$$\mathcal{K}^{(J)-1}_{\alpha\beta}(E_{\rm cm}) = \sum_{k=0}^{N_{\alpha\beta}} c^{(Jk)}_{\alpha\beta} E^k_{\rm cm}$$

- α, β compound indices for (L, S, a)
- another common parametrization

$$\mathcal{K}^{(J)}_{\alpha\beta}(E_{\rm cm}) = \sum_p \frac{g^{(Jp)}_\alpha g^{(Jp)}_\beta}{E^2_{\rm cm} - m^2_{Jp}} + \sum_k d^{(Jk)}_{\alpha\beta} E^k_{\rm cm},$$

- Lorentz invariant form using $E_{\rm cm} = \sqrt{s}$
- Mandelstam variable $s = (p_1 + p_2)^2$, with p_j four-momentum of particle j

Building blocks for single-hadron operators

- building blocks: covariantly-displaced LapH-smeared quark fields
- stout links $\widetilde{U}_j(x)$
- Laplacian-Heaviside (LapH) smeared quark fields

$$\widetilde{\psi}_{a\alpha}(x) = \mathcal{S}_{ab}(x,y)\ \psi_{b\alpha}(y), \qquad \mathcal{S} = \Theta\left(\sigma_s^2 + \widetilde{\Delta}\right)$$

- 3d gauge-covariant Laplacian $\widetilde{\Delta}$ in terms of $\widetilde{U}$
- displaced quark fields:

$$q^A_{a\alpha j} = D^{(j)}\widetilde{\psi}^{(A)}_{a\alpha}, \qquad \overline{q}^A_{a\alpha j} = \widetilde{\overline{\psi}}^{(A)}_{a\alpha}\gamma_4\, D^{(j)\dagger}$$

- displacement $D^{(j)}$ is product of smeared links:

$$D^{(j)}(x,x') = \widetilde{U}_{j_1}(x)\ \widetilde{U}_{j_2}(x+d_2)\ \widetilde{U}_{j_3}(x+d_3)\dots\widetilde{U}_{j_p}(x+d_p)\delta_{x',\ x+d_{p+1}}$$

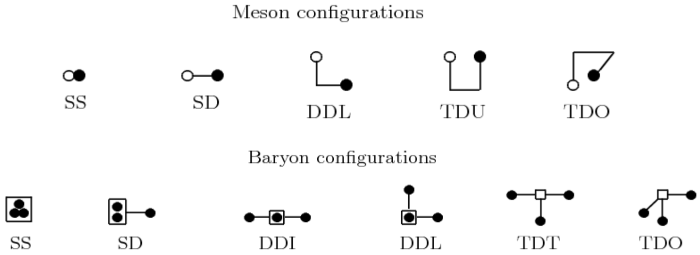
- to good approximation, LapH smearing operator is

$$\mathcal{S} = V_s V_s^\dagger$$

  - columns of matrix V_s are eigenvectors of $\widetilde{\Delta}$

Extended operators for single hadrons

- quark displacements build up orbital, radial structure



$$\overline{\Phi}^{AB}_{\alpha\beta}(\boldsymbol{p},t) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot(\mathbf{x}+\frac{1}{2}(\boldsymbol{d}_\alpha+\boldsymbol{d}_\beta))}\delta_{ab}\ \overline{q}^B_{b\beta}(\boldsymbol{x},t)\ q^A_{a\alpha}(\boldsymbol{x},t)$$

$$\overline{\Phi}^{ABC}_{\alpha\beta\gamma}(\boldsymbol{p},t) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\mathbf{x}}\varepsilon_{abc}\ \overline{q}^C_{c\gamma}(\boldsymbol{x},t)\ \overline{q}^B_{b\beta}(\boldsymbol{x},t)\ \overline{q}^A_{a\alpha}(\boldsymbol{x},t)$$

- group-theory projections onto irreps of lattice symmetry group

$$\overline{M}_l(t) = c^{(l)*}_{\alpha\beta}\ \overline{\Phi}^{AB}_{\alpha\beta}(t) \qquad \overline{B}_l(t) = c^{(l)*}_{\alpha\beta\gamma}\ \overline{\Phi}^{ABC}_{\alpha\beta\gamma}(t)$$

- definite momentum $\boldsymbol{p}$, irreps of little group of $\boldsymbol{p}$

Two-hadron operators

- our approach: superposition of products of single-hadron operators of definite momenta

$$c^{I_{3a}I_{3b}}_{\boldsymbol{p}_a\lambda_a;\,\boldsymbol{p}_b\lambda_b}\; B^{I_aI_{3a}S_a}_{\boldsymbol{p}_a\Lambda_a\lambda_a i_a}\; B^{I_bI_{3b}S_b}_{\boldsymbol{p}_b\Lambda_b\lambda_b i_b}$$

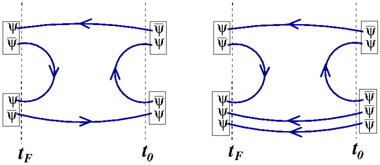
- fixed total momentum $\boldsymbol{p} = \boldsymbol{p}_a + \boldsymbol{p}_b$, fixed $\Lambda_a, i_a, \Lambda_b, i_b$
- group-theory projections onto little group of $\boldsymbol{p}$ and isospin irreps
- restrict attention to certain classes of momentum directions
  - on axis $\pm\widehat{\boldsymbol{x}},\ \pm\widehat{\boldsymbol{y}},\ \pm\widehat{\boldsymbol{z}}$
  - planar diagonal $\pm\widehat{\boldsymbol{x}}\pm\widehat{\boldsymbol{y}},\ \pm\widehat{\boldsymbol{x}}\pm\widehat{\boldsymbol{z}},\ \pm\widehat{\boldsymbol{y}}\pm\widehat{\boldsymbol{z}}$
  - cubic diagonal $\pm\widehat{\boldsymbol{x}}\pm\widehat{\boldsymbol{y}}\pm\widehat{\boldsymbol{z}}$
- crucial to know and fix all phases of single-hadron operators for all momenta
  - each class, choose reference direction $\boldsymbol{p}_{\rm ref}$
  - each $\boldsymbol{p}$, select one reference rotation $R^{\boldsymbol{p}}_{\rm ref}$ that transforms $\boldsymbol{p}_{\rm ref}$ into $\boldsymbol{p}$
- efficient creating large numbers of two-hadron operators
- generalizes to three, four, ... hadron operators

Quark propagation

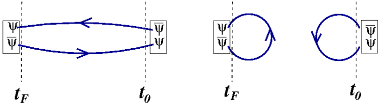
- quark propagator is inverse K^{-1} of Dirac matrix
  - rows/columns involve lattice site, spin, color
  - very large $N_{\rm tot} \times N_{\rm tot}$ matrix for each flavor
  $$N_{\rm tot} = N_{\rm site} N_{\rm spin} N_{\rm color}$$
  - for $32^3 \times 256$ lattice, $N_{\rm tot} \sim 101$ million
- not feasible to compute (or store) all elements of K^{-1}
- solve linear systems $Kx = y$ for source vectors y
- translation invariance can drastically reduce number of source vectors y needed
- multi-hadron operators and isoscalar mesons require large number of source vectors y

Quark line diagrams

- temporal correlations involving our two-hadron operators need
  - slice-to-slice quark lines (from all spatial sites on a time slice to all spatial sites on another time slice)
  - sink-to-sink quark lines



- isoscalar mesons also require sink-to-sink quark lines



- solution: the stochastic LapH method!
  - (expensive alternative: distillation)

Decay width of ρ

- applied to $I = 1$ $\rho \to \pi\pi$ system NPB 910, 842 (2016)
- included $L = 1, 3, 5$ partial waves in NPB 924, 477 (2017)
- large $32^3 \times 256$ anisotropic lattice, $m_\pi \approx 240$ MeV
- fit forms (first ever inclusion of $L = 5$ in lattice QCD):

$$(\widetilde{K}^{-1})_{11} = \frac{6\pi E_{\rm cm}}{g^2 m_\pi}\left(\frac{m_\rho^2}{m_\pi^2} - \frac{E_{\rm cm}^2}{m_\pi^2}\right)$$

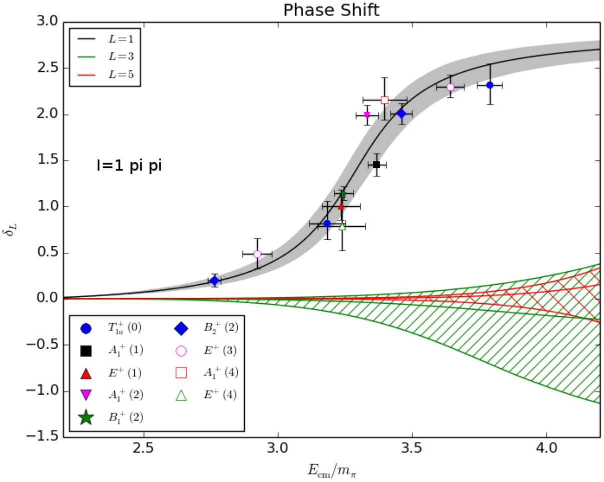
$$(\widetilde{K}^{-1})_{33} = \frac{1}{m_\pi^7 a_3} \qquad (\widetilde{K}^{-1})_{55} = \frac{1}{m_\pi^{11} a_5}$$

- results

$$\frac{m_\rho}{m_\pi} = 3.349(25),\ g = 5.97(27),\ m_\pi^7 a_3 = -0.00021(100),$$
$$m_\pi^{11} a_5 = -0.00006(24),\ \chi^2/\text{dof} = 1.15$$

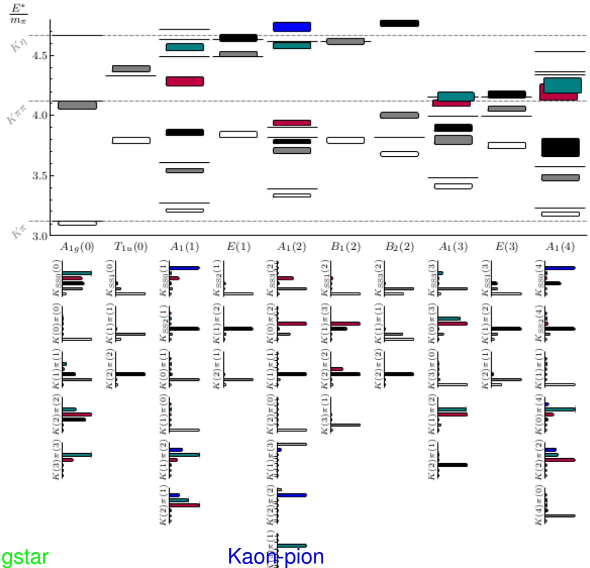
Decay of ρ

- plot of phase shifts



$K\pi$ energies in finite volume

- finite volume energies $32^3 \times 256$ lattice, $m_\pi \approx 240$ MeV



Decay of $K^*(892)$

- studied $K^*(892)$
- included $L = 0, 1, 2$ partial waves
- large $32^3 \times 256$ anisotropic lattice, $m_\pi \approx 240$ MeV
- fit forms

$$(\widetilde{K}^{-1})_{11} = \frac{6\pi E_{\rm cm}}{g^2_{K^*\pi\pi} m_\pi}\left(\frac{m^2_{K^*}}{m^2_\pi} - \frac{E^2_{\rm cm}}{m^2_\pi}\right) \qquad (\widetilde{K}^{-1})_{22} = \frac{-1}{m^5_\pi a_2}$$

- S-wave forms tried:

$$\begin{aligned}
(\widetilde{K}^{-1})^{\rm lin}_{00} &= a_{\rm l} + b_{\rm l} E_{\rm cm}, \\
(\widetilde{K}^{-1})^{\rm quad}_{00} &= a_{\rm q} + b_{\rm q} E^2_{\rm cm}, \\
(\widetilde{K}^{-1})^{\rm ERE}_{00} &= \frac{-1}{m_\pi a_0} + \frac{m_\pi r_0}{2}\frac{q^2_{\rm cm}}{m^2_\pi}, \\
(\widetilde{K}^{-1})^{\rm BW}_{00} &= \left(\frac{m^2_{K^*_0}}{m^2_\pi} - \frac{E^2_{\rm cm}}{m^2_\pi}\right)\frac{6\pi m_\pi E_{\rm cm}}{g^2_{K^*_0\pi\pi} m^2_{K^*_0}}
\end{aligned}$$

K-matrix fits

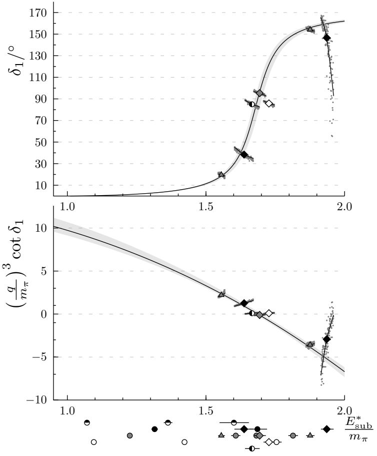
- summary of fit results

Fit	s-wave par.	m_{K^*}/m_π	$g_{K^*K\pi}$	$m_\pi a_0$	$\chi^2/\mathrm{d.o.f.}$
(1a,1b)	LIN	$3.819(20)$	$5.54(25)$	$-0.333(31)$	(1.04,–)
2	LIN	$3.810(18)$	$5.30(19)$	$-0.349(25)$	1.49
3	QUAD	$3.810(18)$	$5.31(19)$	$-0.350(25)$	1.47
4	ERE	$3.809(17)$	$5.31(20)$	$-0.351(24)$	1.47
5	BW	$3.808(18)$	$5.33(20)$	$-0.353(25)$	1.42
6	BW	$3.810(17)$	$5.33(20)$	$-0.354(25)$	1.50

- $q\bar{q}$ operators in $A_{1g}(0)$ channel overlap many eigenvectors
- better energy resolution needed for $K_0^*(800)$ determination (future work)
- from NLO effective range parametrization find $m_R/m_\pi = 4.66(13) - 0.87(18)i$ (consistent with BW fit)

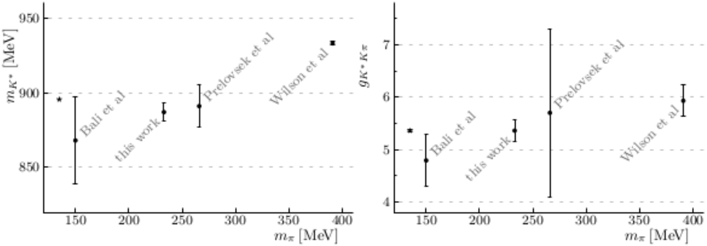
Decay of $K^*(892)$

- plot of P-wave and S-wave phase shift
- included $L = 0, 1, 2$ partial waves
- large $32^3 \times 256$ anisotropic lattice, $m_\pi \approx 240$ MeV
- κ fit: quadratic



Comparison to other works

- comparison of our m_{K^*} and $g_{K^*K\pi}$ to other works

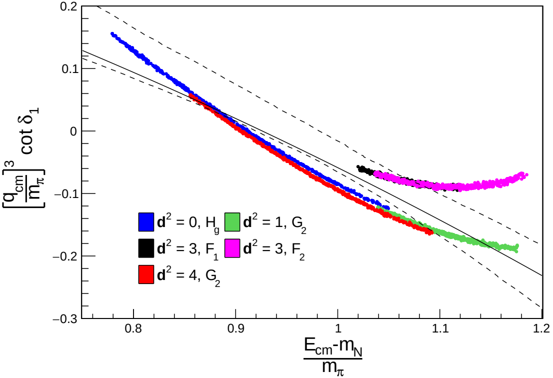


Future work

- $32^3 \times 256$ lattice run was not optimized for $K\pi$
- larger 48^3 and 64^3 lattices should allow better reconstruction of phase shifts
- runs with 96^3 lattice at physical point in progress!

Decay of Δ

- included $L = 1$ wave only (for now) PRD **97**, 014506 (2018)
- large $48^3 \times 128$ isotropic lattice, $m_\pi \approx 280$ MeV, $a \sim 0.076$ fm
- with student Christian Walther Andersen (U. Southern Denmark)
- Breit-Wigner fit gives $g_{\Delta N\pi} = 19.0(4.7)$ in agreement with experiment ~ 16.9



Conclusion

- two-particle Luscher formalism
  - K-matrix from finite-volume energies
- use of the K-matrix and the box B matrix
- successful results depend on time-slice to time-slice quark propagators needed for temporal correlators involving two-hadron operators
  - Stochastic LapH method!
- results for $K\pi$ scattering on $32^3 \times 256$ lattice (3.7 fm) at $m_\pi \approx 240$ MeV
- included $L = 0, 1, 2$ partial waves
- future work in larger volumes and at physical point