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IPN
INSTITUT DE PHYSIQUE NUCLÉAIRE
ORSAY

From πK amplitudes to πK form factors (and back)

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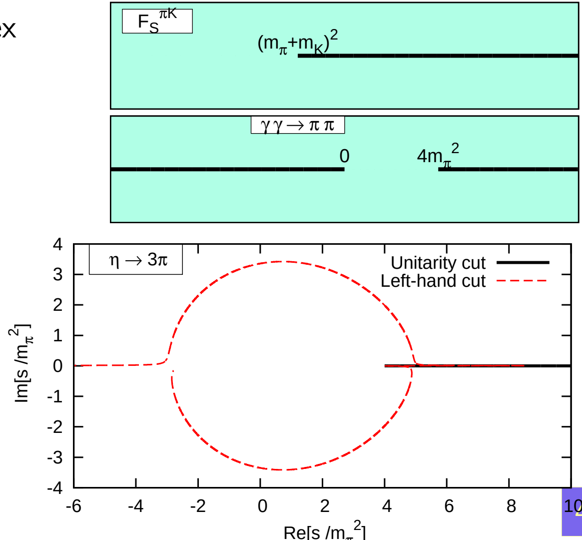
Outline:

- Introduction
- $\pi\pi$ scalar form factor
- πK: $J = 0$ amplitude and scalar form factor
- πK: $J = 1$ amplitude and vector form factor
- Conclusions

Introduction

- Light mesons π, K are stable particles in QCD
- meson-meson scattering amplitudes contain fundamental info. on resonances (as complex poles)
- In re-scattering or FSI: same resonance poles but different residues [e.g. κ resonance more visible in $D \to K\pi\pi$]
- Experimental status:
  - → Before 1985: many dedicated studies of meson-meson amplitudes ($\pi\pi \to \pi\pi$, $\pi K \to \pi K$, $\pi\pi \to K\overline{K}$)
  - → After 1985: mostly FSI data (B, D , J/ψ, $\tau \cdots$ decays)

- Relation between FSI and scattering: Analyticity/Unitarity properties of QCD[Goldberger, Thirring PR 95,1612 (1954)]
- Various complex plane cuts:



- Treatment of FSI easy if scattering is elastic [Omnès, Nuov.Cim. 8,316 (1958)]

$$f_0^{K\pi}(s) = \exp\left(\frac{s}{\pi}\int_{(m_K+m_\pi)^2}^{\infty} \frac{\delta_0^{1/2}(s')}{s'(s'-s)}\, ds'\right)$$

(up to polynomial)

→ (Aizu-Fermi)-Watson theorem holds

- Accounting for inelasticity extends region where FSI can be treated

$\pi\pi$ Scalar form factor

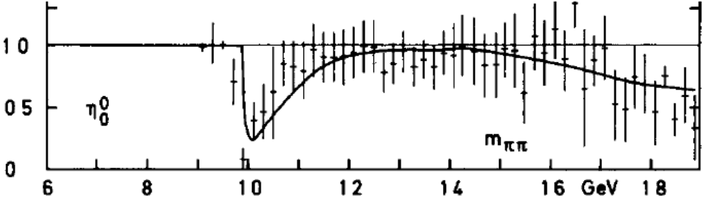
- Main ideas for dispersive construction of $\pi\pi$ scalar form factors[Donoghue, Gasser, Leutwyler NP B343,341 (1990)]

$$\langle\pi\pi|\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}|0\rangle, \quad \langle\pi\pi|\bar{s}s|0\rangle, \quad \langle\pi\pi|\alpha_s G^{\mu\nu}G_{\mu\nu}|0\rangle$$

(Motivation: $H\to\pi\pi$ amplitude if $m_H \lesssim 1$ GeV)

→ Assume 2-channel unitarity for $J=0$ $\pi\pi$ amplitude

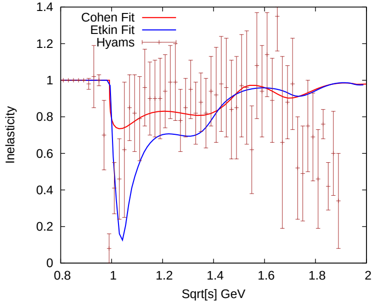
- Support for this picture: $\pi\pi$ inelasticity measurement [Hyams et al., NP B64,134 (1973)]

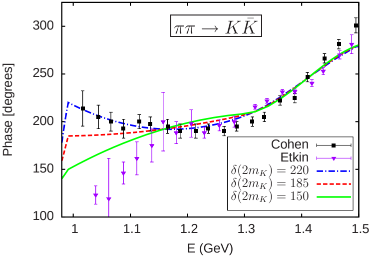


- Implied relation with $\pi\pi \to K\overline{K}$ amplitude:

$$2\left(\left(1-\frac{4m_\pi^2}{s}\right)\left(1-\frac{4m_K^2}{s}\right)\right)^{1/4} T_0^{\pi\pi\to K\overline{K}}(s) = \sqrt{1-\eta_0^2}\, e^{i(\delta_{\pi\pi}+\delta_{K\overline{K}})}$$

Experimental measurements: [Cohen et al., PR D22, 2595 (1980), Etkin et al, PR D25, 1786 (1982)]





→ Validity of 2-ch. model: $E \lesssim 1.5$ GeV.

- Form factors from 2×2 T-matrix.
  - → Put: $\vec{F}(s) \equiv \begin{pmatrix} F^{\pi\pi}(s) \\ F^{K\overline{K}}(s) \end{pmatrix}$,
  - → Unitarity for form factors:

$$\boxed{\mathrm{Im}[\vec{F}] = \mathrm{disc}[\vec{F}] = \mathbb{T}\,\Sigma\,\vec{F}^*}$$

  - → Dispersion relations:

$$\boxed{\vec{F}(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds'\,\frac{\mathbb{T}\,\Sigma\,\vec{F}^*(s')}{s'-s}}$$

No subtractions needed: $F_i(s)_{s\to\infty} \sim \frac{\alpha_s(s)}{s}$
[(Brodsky-Lepage)]

→ Number of independent solutions: (Noether) index

$$N = \frac{\sum \left(\delta_i(\infty) - \delta_i(s_{th})\right)}{\pi}$$

"Natural" value: $N = 2$

Two constraints e.g. $F^{\pi\pi}(0)$, $F^{K\overline{K}}(0)$ uniquely define the solutions

πK: $J = 0$ amplitude and scalar form factor

$J = 0, I = 1/2\ \pi K$ amplitude

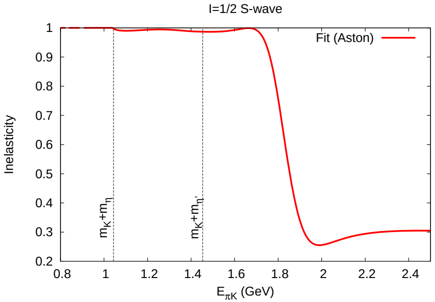
- Latest experimental data[Aston et al. (LASS coll.) NP B296, 493 (1988)]: $\pi^+ K^- \to \pi^+ K^-$

$$t_0^{LASS} = \frac{1}{2i}\left[\eta_0^{1/2} e^{2i\delta_0^{1/2}} + \frac{1}{2}\eta_0^{3/2} e^{2i\delta_0^{3/2}} - \frac{3}{2}\right]$$

Use also[Estabrooks et al.,NP B133, 490 (1978)]

→ Result for inelasticity:

→ Driven by $K_0^*(1950)$



- Two-channel model for amplitude and scalar form factor proposed [Jamin, Oller, Pich NP B587,331 (2000), NP B622,279 (2002)]
  - → Assumption: $K\eta'$ dominates inelasticity
  - → K-matrix type: resonance poles + ChPT component (also combining $1/N_c$, p^2 expansions)
  - → Scalar form factors then determined from MO equations + two conditions:

$$\begin{aligned} f_0^{K\pi}(0) &= 1 + O((m_s - m_u)^2) \text{ (Ademollo-Gatto)} \\ f_0^{K\pi}(m_K^2 - m_\pi^2) &= \frac{F_K}{F_\pi} + O(m_\pi^2) \text{ (Callan-Treiman)} \end{aligned}$$

NLO ChPT calc. [Gasser, Leutwyler NP B250, 517 (1985)]

- Note: πK scalar form factor is measurable:
  $\tau \to K\pi\nu_\tau$ or $K \to \pi e \nu_e, \pi\mu\nu_\mu$ amplitudes:

$$\sqrt{2}\langle K^+(p_K)|\bar{u}\gamma^\mu s|\pi^0(p_\pi)\rangle = f_+^{K\pi}(t)\,(p_K+p_\pi)^\mu + f_-^{K\pi}(t)\,(p_K-p_\pi)^\mu$$

with $t = (p_K - p_\pi)^2$. Introduce:

$$f_0^{K\pi}(t) = f_+^{K\pi}(t) + \frac{t}{m_K^2 - m_\pi^2} f_-^{K\pi}(t)$$

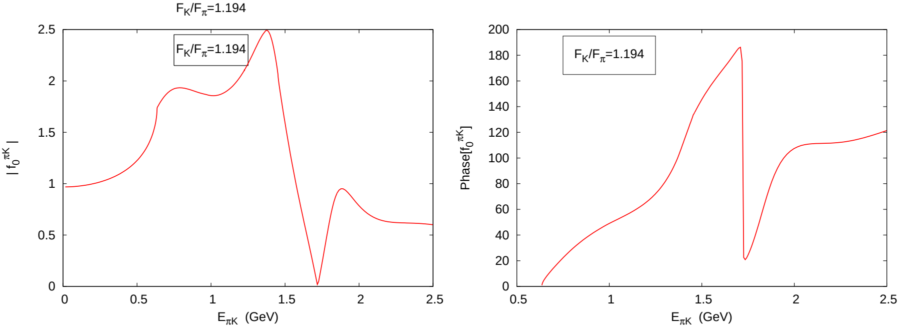
→ $f_0^{K\pi}(t)$: Scalar form factor ($K\pi$ scattering in $J = 0$)

→ $f_+^{K\pi}(t)$: Vector form factor ($K\pi$ scattering in $J = 1$)

- Prediction of two channel model: Using

$$f_0^{K\pi}(0) = 0.968(3), \qquad \frac{F_K}{F_\pi} = 1.194(5)$$

[FLAG, EPJ C77,112 (2017)] (LQCD 2+1 simulations)

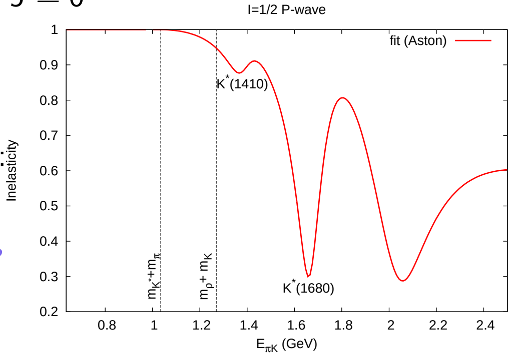


Presence of a (quasi) zero: some suppression of $K_0^*(1430)$ peak

πK: $J = 1$ amplitude and vector form factor

P-wave amplitude:

- Elastic range smaller than $J = 0$



- Leading inelastic channels: $K^*\pi$, $K\rho$ [Aston et al., NP B292, 693 (1987)]

Branching fractions:

	$K\pi$	$K^*\pi$	$K\rho$
$K^*(1410)$	6.6 ± 1	> 40	< 7
$K^*(1680)$	38.7 ± 2.5	$29.9^{+2.2}_{-4.7}$	$31.4^{+4.7}_{-2.1}$

- Amplitude model with 3 channels
  [B.M., EPJ C35, 401 (2008)]

  ① $K\pi$
  ② $K^*\pi$
  ③ $K\rho$

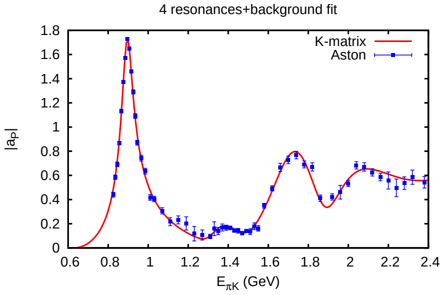
- K-matrix

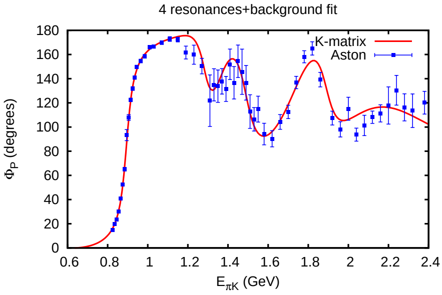
$$K_{ij} = \sum_R \frac{g_R^i g_R^j}{m_R^2 - s} + K_{ij}^{back}$$

Four resonances:
$K^*(892)$, $K^*(1410)$, $K^*(1680)$, $K^*(2300)$ (not in PDG)

Couplings:
$g_R^i(s)$: s-dependence from Chiral Lagrangian
$K^*(1410)$ suppressed coupl. to $K\rho$
$K^*(1680)$ couplings approx. equal

Fit to LASS P-wave data:





15 parameters, $\chi^2/d.o.f = 1.8$

Vector form factor from 3-channel T-matrix

- Need 3 values at $t=0$: $\begin{cases} H_1(0) = f_+^{K\pi}(0) = f_0^{K\pi}(0): \text{ known} \\ H_2(0) \\ H_3(0) \end{cases}$

  → In chiral limit (exact flav. symm.) + assuming VMD
  $\longrightarrow$ relation with ABJ anomaly

$$H_2(0) = -H_3(0) = \frac{\sqrt{2} N_c M_V}{16\pi^2 F_V F_\pi} \simeq 1.50 \text{ GeV}^{-1}$$

  → Allowing for linear flav. symm. breaking

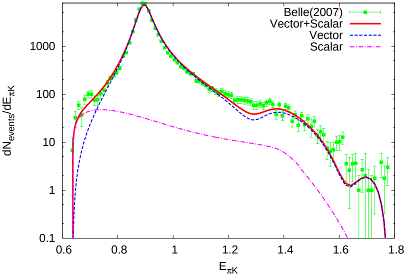
$$H_2(0) = 1.50\,(1+a), \quad H_3(0) = -1.50\,(1+b)$$

  we expect: $|a|, |b| \lesssim 30\%$

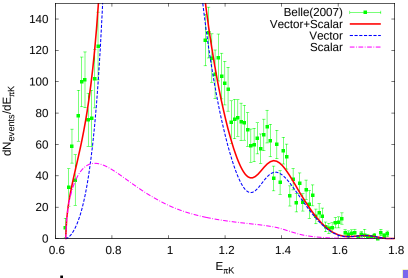
- Fit to $\boxed{\tau^- \to K_S \pi^- \nu_\tau}$ data [Belle, PL B654,65 (2007)]

2 params: $\left.\frac{\chi^2}{N}\right|_{Belle} = 8.6$

→ Log scale:



→ Linear scale:



→ Low energy OK (clear κ effect through scalar form factor).

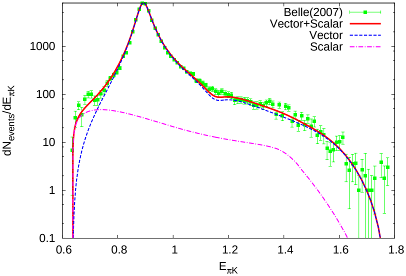
But: $K^*(1410)$ region not in agreement

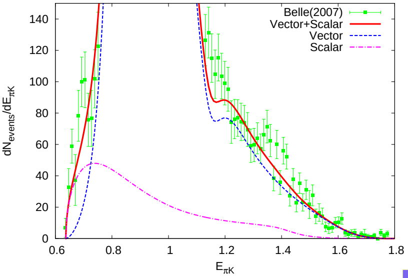
- Assume nevertheless that model is correct !
  - → Then, τ data can tell us something on πK phase shift
  - → Perform fit on combined LASS + τ data, varying K-matrix params + sym. breaking params: a, b
  - → LASS χ^2 weighted by 1/2:

$$\left.\frac{\chi^2}{N}\right|_{Belle} = 2.3 \qquad \left.\frac{\chi^2}{N}\right|_{LASS} = 3.6$$

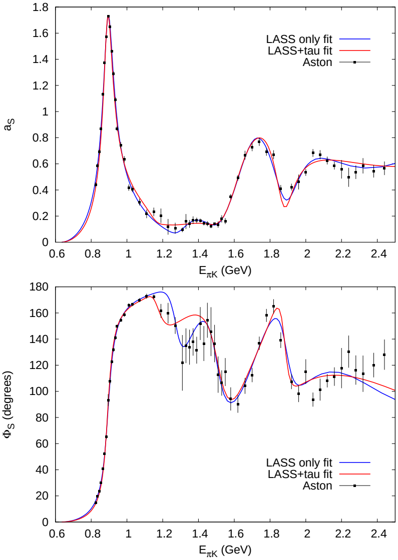
with $a = -0.15$, $b = -0.21$

- Comparison of combined fit with Belle:





- Comparison of combined fit with LASS:



Conclusions

- Reviewed $\pi\pi$, πK form factors beyond elastic region based on some assumptions

- πK scalar form factor plausible: more information on $\pi K \to \eta' K$, $K_0^*(1950) \to \eta' K$ would be essential

- πK vector form factor: seems to require some modif. of $J = 1$ phase-shift in $K^*(1410)$ region

$\tau \to K\pi\nu$: Improved statistics (x50) at Belle2

Measure distrib. as a function of πK energy $+\ \vec{p}_\pi$, $\vec{p}_K$ directions $\Rightarrow$ separate vector/scalar form factors determinations