

Dalitz plot analysis of three-body charmonium decays at BaBar

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On behalf of the BaBar Collaboration

Outline

- Dalitz plot analysis of three-body η_c decays in two-photon interactions.
- Measurement of the $I=1/2$ $K\pi$ \mathcal{S} -wave amplitude from a Dalitz plot analysis of $\eta_c \rightarrow K\bar{K}\pi$ decays.
- Dalitz plot analysis of $J/\psi \rightarrow K_S^0 K^\pm \pi^\mp$ three-body hadronic decay.

Pion-Kaon Interactions Workshop,

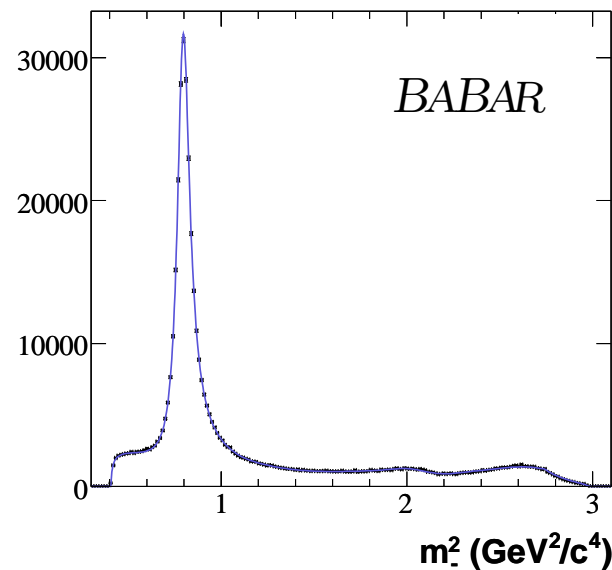
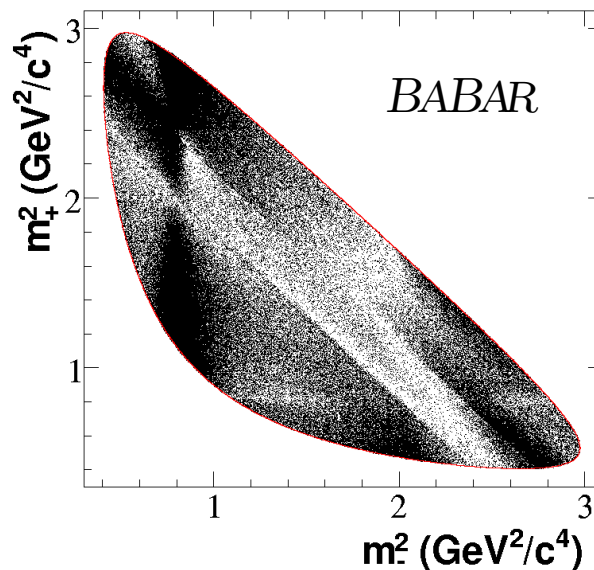
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Thomas Jefferson National Accelerator Facility, Newport News, VA

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The problem of the $K\pi$ S -wave. (I)

- An accurate description of the $K\pi$ S -wave is of basic importance for many important physics topics.
- Examples are: Measurement of γ , study and search for CP violations in heavy flavors decays through Dalitz plot analyses of 3-body or 4-body decays.
- $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ Dalitz plot and $K_S^0 \pi^-$ squared mass projection.

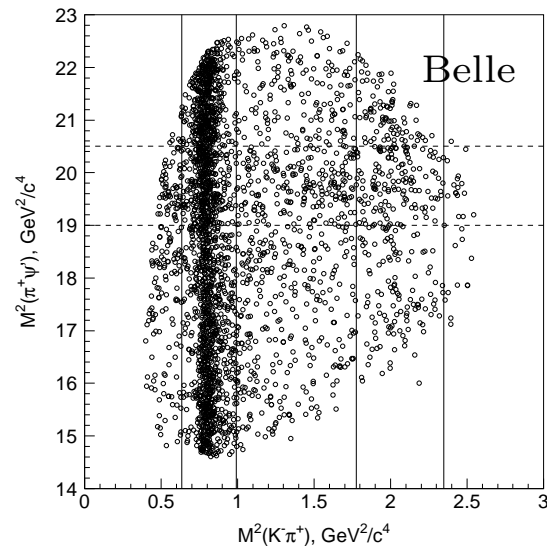
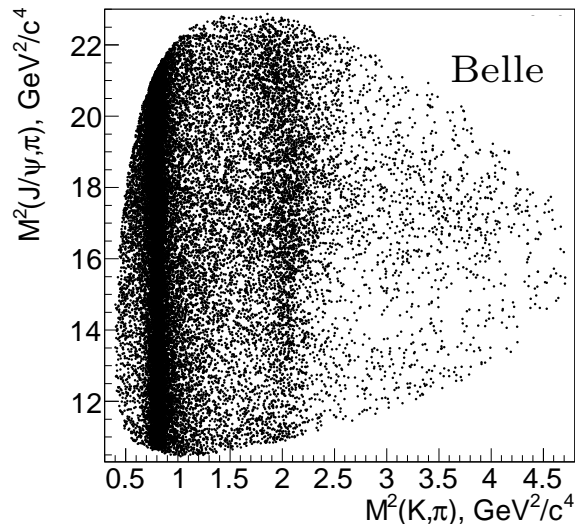
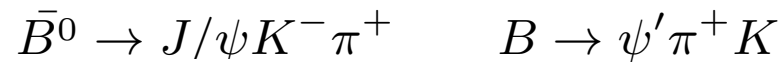


- Very high statistics are currently available.
- Strong resonance production along the $K\pi$ axis which needs to be correctly described. (from arXiv:0804.2089, BaBar: Phys.Rev.D78:034023,2008)

The problem of the $K\pi$ S -wave. (II)

- Recent evidences/observations of exotic states in the decays of heavy flavors.
- The observation of the Z particles in $B \rightarrow \psi/\psi' K\pi$ is strongly correlated with an accurate description of the Dalitz plot.
- In this description we also need a full understanding of the K^* resonances and in particular of the $K\pi$ S -wave.
- Data from Belle.

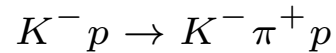
(arXiv:0905.2869, Phys.Rev.D80:031104,2009),(arXiv:1408.6457, Phys. Rev. D 90, 112009 (2014))



- Strong resonance production along the $K\pi$ axis.

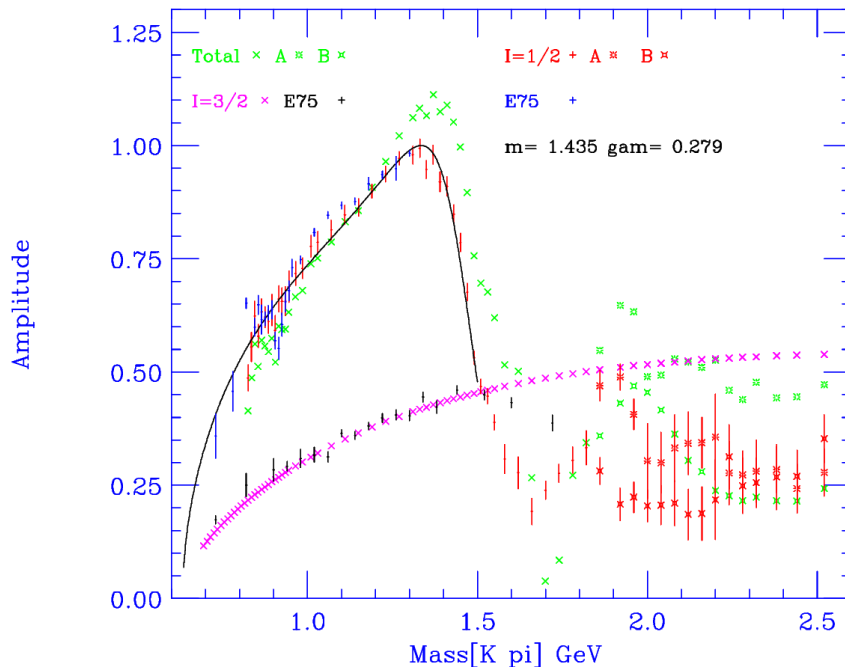
The $K\pi$ S -wave from LASS

- The best measurement of the $K\pi$ S -wave comes from the LASS (Nucl. Phys. B296, 493 (1988))

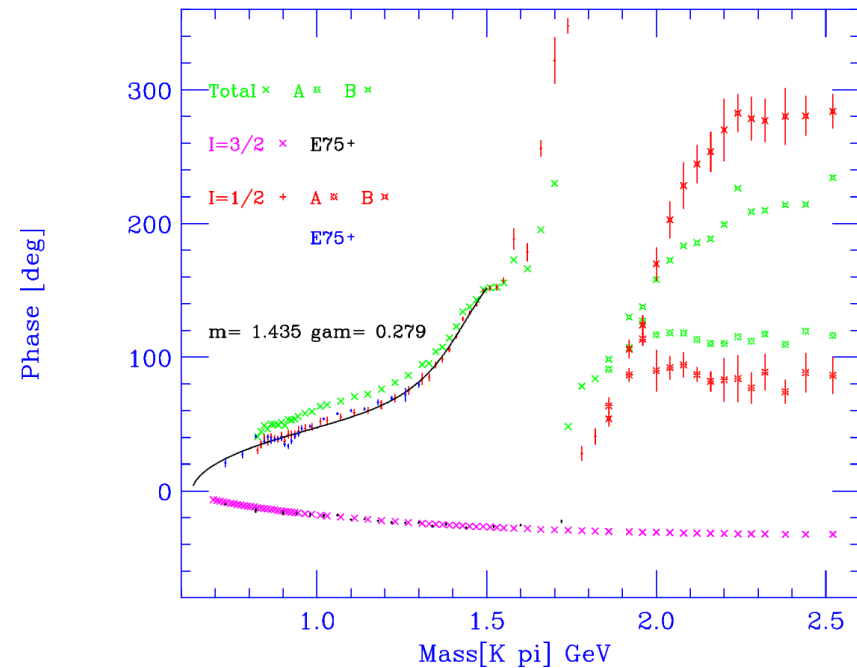


- The $K\pi$ S -wave is described by a coherent sum of a scattering length and a relativistic Breit-Wigner.
- However the LASS PWA is affected by a two-fold ambiguity for $m(K\pi) > 1.9$ GeV.
- This final state is also affected by the presence of an $I = 3/2$ background.

LASS K pi S wave Amplitude



LASS K pi S wave Phase

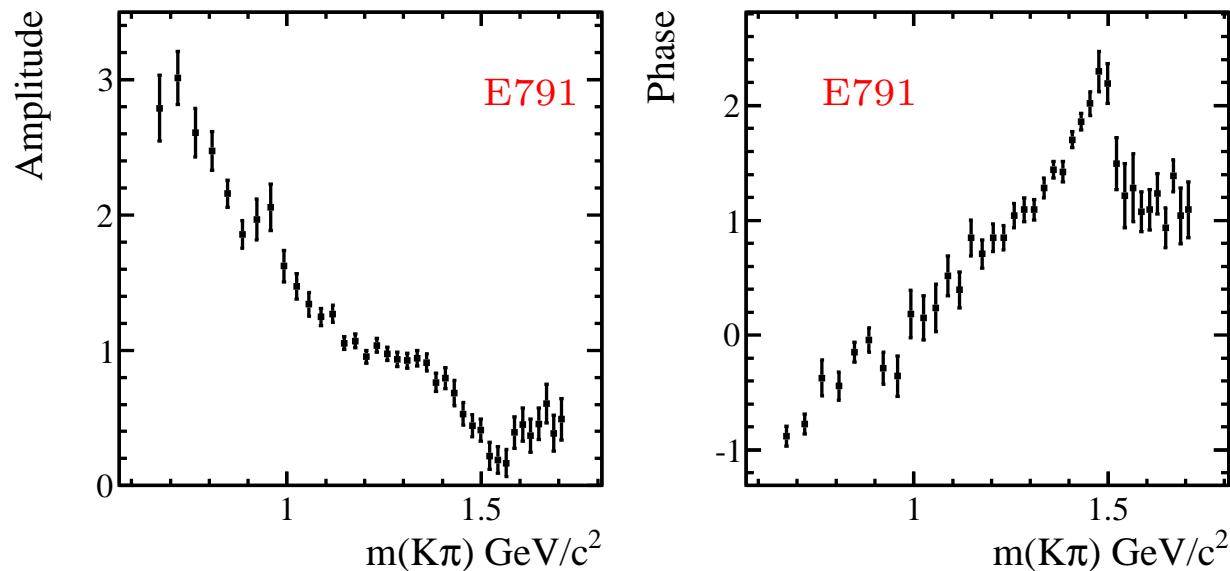


The $K\pi$ S -wave from D^+ decays

- Other measurements of the $K\pi$ S -wave come from the Dalitz plot analysis of:

$$D^+ \rightarrow K^- \pi^+ \pi^+$$

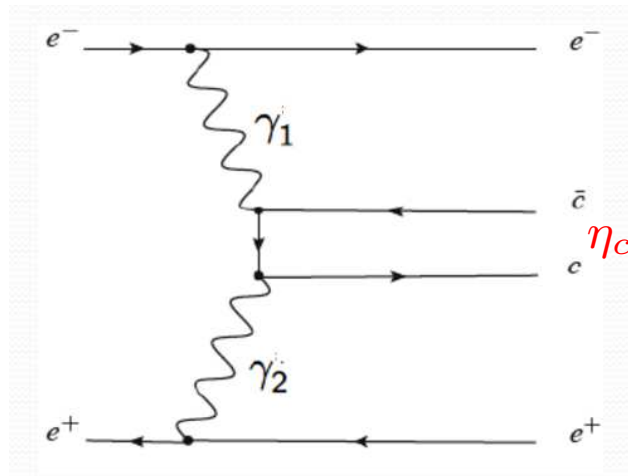
- The Model Independent Partial Wave method was introduced for the first time (E791, arXiv:hep-ex/0506040v2, Phys. Rev. D 73, 032004).
- The $K\pi$ S -wave amplitude and phase was measured up to a mass of 1.5 GeV.



- Also in this case the final state is affected by the presence of a not well known contribution from $I=3/2$.

The $K\pi$ S -wave from η_c decays

- Charmonium decays can be used to obtain new information on light meson spectroscopy.
- In e^+e^- interactions, samples of charmonium decays can be obtained using different processes.
- In two-photon interactions we select events in which the e^+ and e^- beam particles are scattered at small angles and remain undetected.
- Only resonances with $J^{PC} = 0^{\pm+}, 2^{\pm+}, 3^{++}, 4^{\pm+} \dots$ can be produced.



Use of η_c decays produced in two-photon interactions

- In these BaBar analyses we make use of the following final states.

$$\gamma\gamma \rightarrow K_S^0 K^+ \pi^- \text{ (*)},$$

$$\gamma\gamma \rightarrow K^+ K^- \pi^0,$$

$$\begin{aligned} \gamma\gamma \rightarrow K^+ K^- \eta \\ \rightarrow \gamma\gamma \\ \rightarrow \pi^+ \pi^- \pi^0 \end{aligned}$$

- We find that the η_c three-body hadronic decays proceed almost entirely through:

$$\eta_c \rightarrow \text{pseudoscalar} + \text{scalar}$$

- Therefore three body decays of the η_c are a unique window to study the properties of the scalar mesons.

(*) Charge conjugation is implied through all this work.

Phys.Rev. D89 (2014) no.11, 112004, Phys.Rev. D93 (2016) 012005

Example of selection: $\gamma\gamma \rightarrow K_S^0 K^+ \pi^-$

- Select events having only four tracks.
- p_T : transverse momentum of the $K_S^0 K^+ \pi^-$ system with respect to the beam axis.

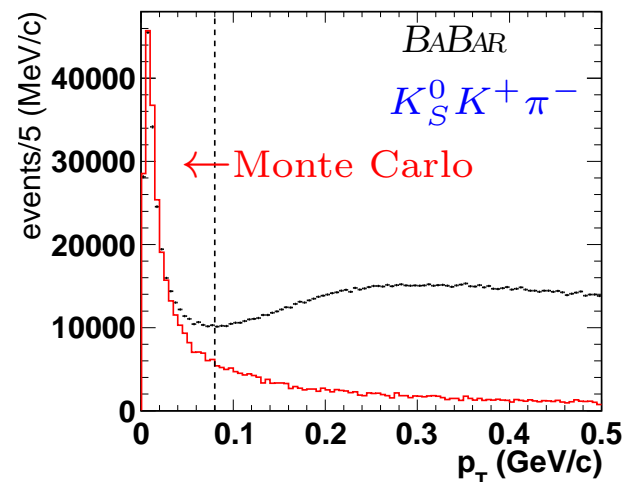
□ The signal at low p_T indicates the presence of two-photon events. We require $p_T < 0.08 \text{ GeV}/c$.

□ We define M_{rec}^2 as:

$$M_{\text{rec}}^2 \equiv (p_{e^+e^-} - p_{\text{rec}})^2$$

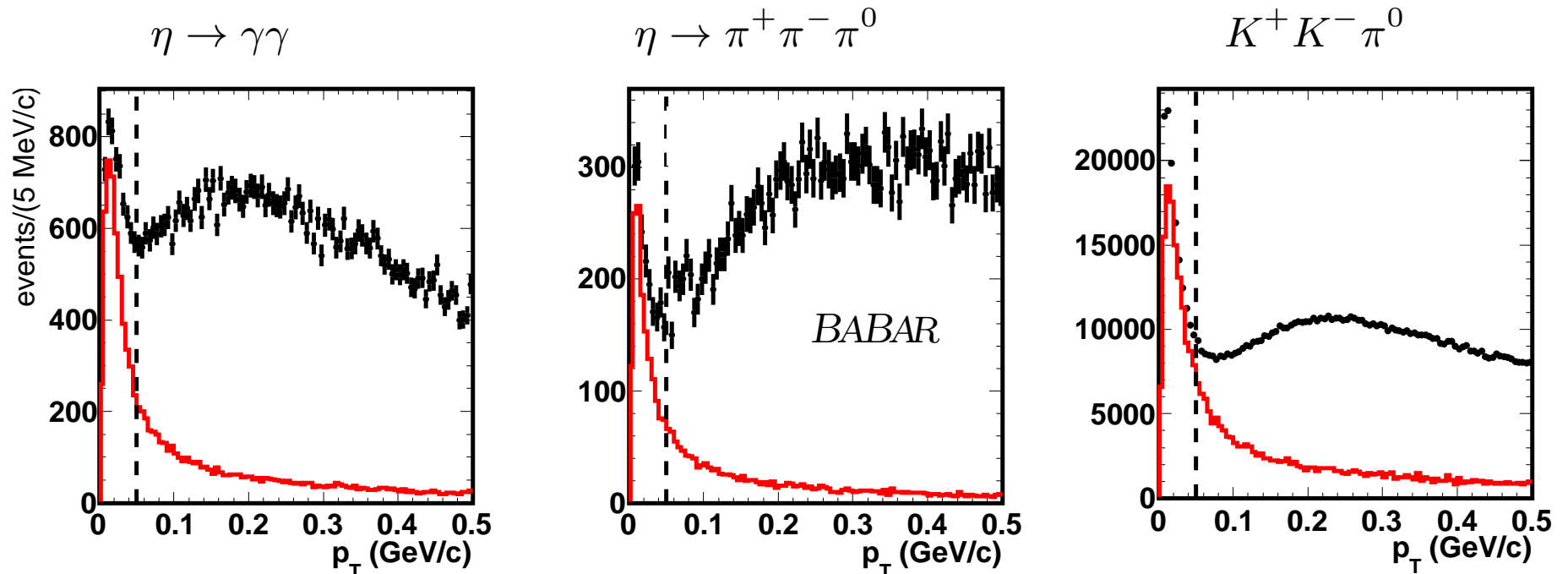
□ $p_{e^+e^-}$ is the four-momentum of the initial state and p_{rec} is the four-momentum of the $K_S^0 K^+ \pi^-$ system.

□ We remove ISR events by requiring $M_{\text{rec}}^2 > 10 \text{ GeV}^2/c^4$.



p_T distribution for η_c decays with γ 's

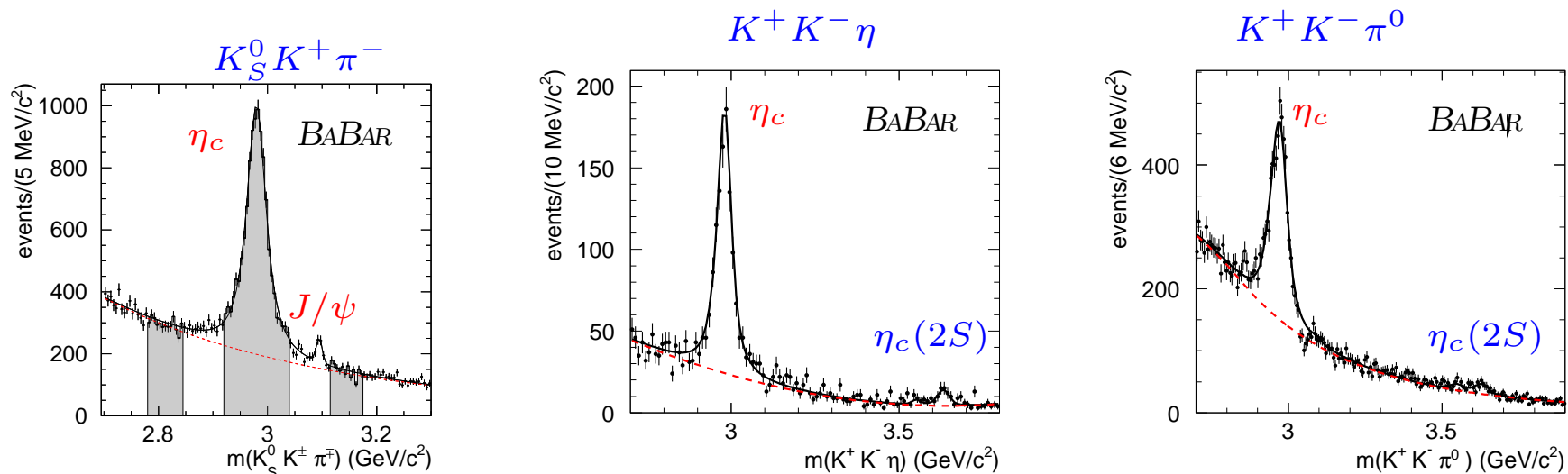
- For each final state we select events having the exact number of expected charged tracks.
- Due to soft photons background we allow the presence of extra low energy γ 's.
- We select two-photon events by requiring the conservation of the transverse momentum p_T . We require $p_T < 0.05$ GeV/c
- p_T distributions for the three reactions.



- In red are MC simulations, in black are the data.

Mass spectra in the η_c region

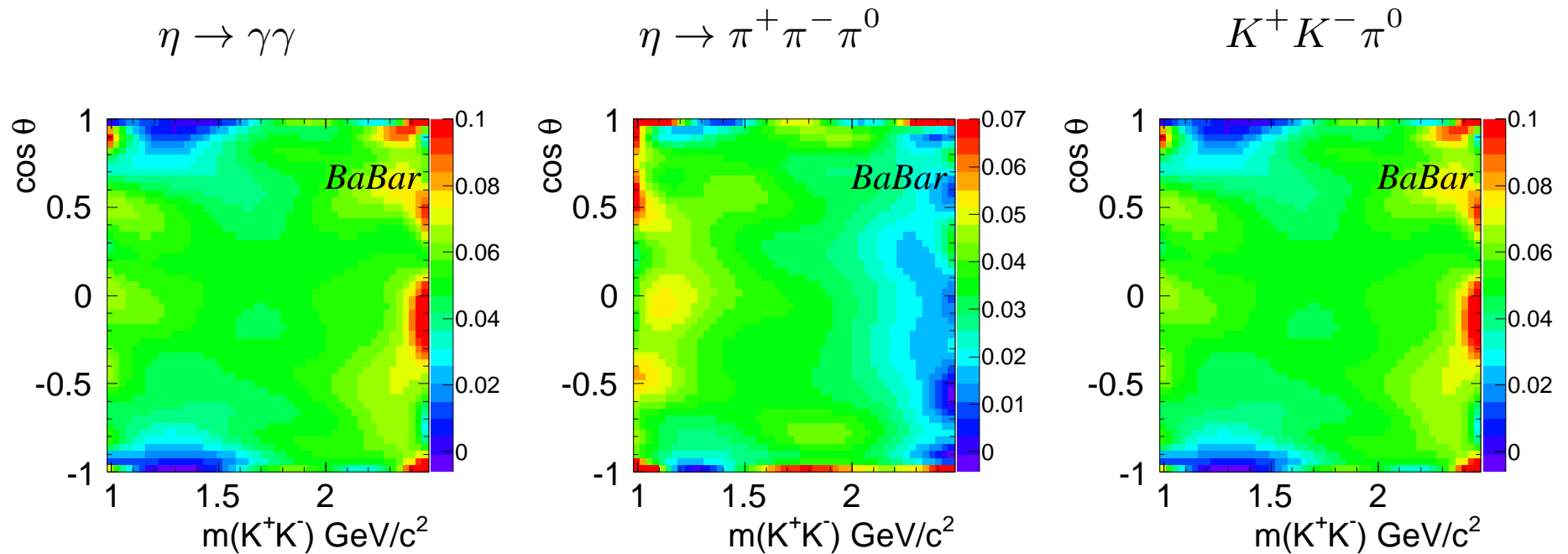
- Strong η_c signals and evidence for $\eta_c(2S)$.
- Small J/ψ signal from residual ISR background.
- $\eta_c \rightarrow K_S^0 K^+ \pi^-$, 12849 evts with $(64.3 \pm 0.4)\%$ purity.
- $\eta_c \rightarrow K^+ K^- \pi^0$, 6494 evts with $(55.2 \pm 0.6)\%$ purity.
- $\eta_c \rightarrow K^+ K^- \eta$, 1161 evts with $(76.1 \pm 1.3)\%$ purity.



- $\text{Purity} = \text{Signal} / (\text{Signal} + \text{Background})$
- Charmonium signals fitted using Breit-Wigner functions convoluted with the resolution functions.

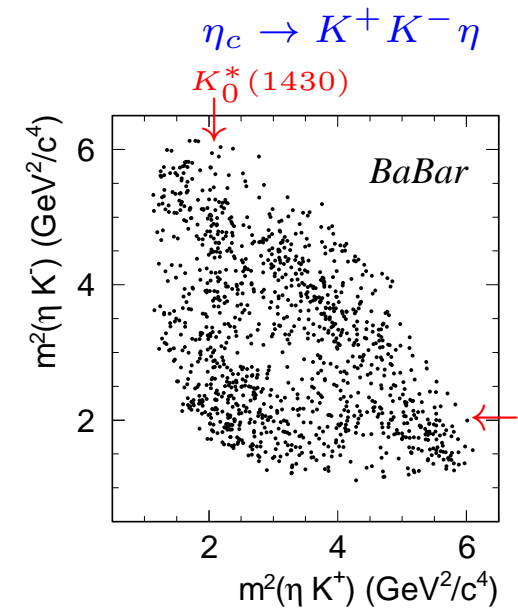
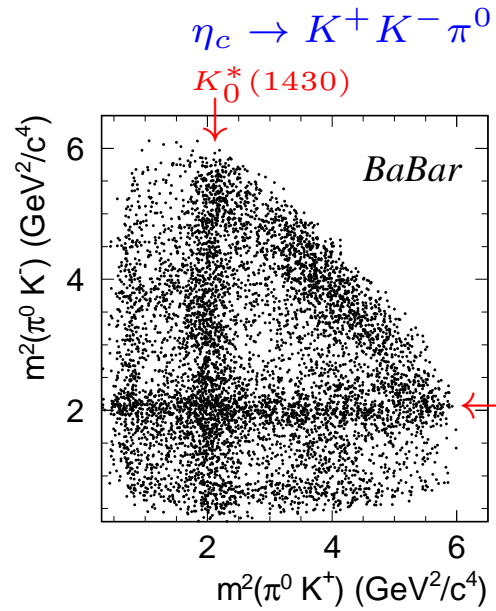
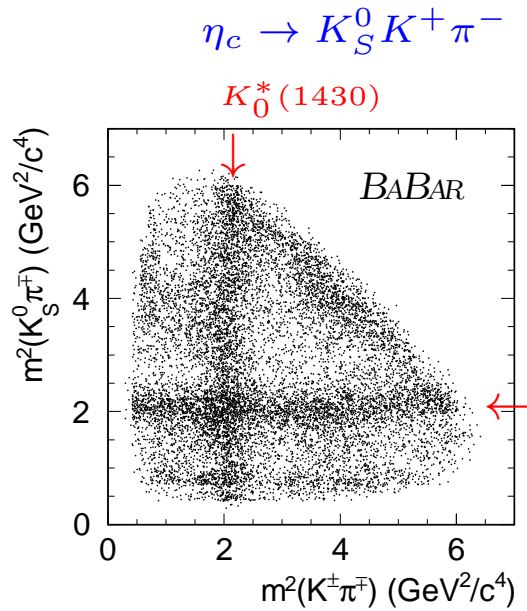
Efficiency.

- Fitted detection efficiency in the $\cos\theta$ vs. $m(K^+K^-)$ plane, where θ is the K^+ helicity angle.
- Efficiency distributions for the three reactions in the η_c mass region.



- Efficiency fitted using Legendre polynomials moments.
- Some efficiency loss due to low momentum kaons or π^0 .

Dalitz plots.



- Dominated by the presence of scalar mesons.
- In particular, strong contribution from $K_0^*(1430)$ in the three Dalitz plots.

Branching fractions.

□ We compute the ratios of the branching fractions for η_c decay to the $K^+K^-\eta$ final state compared to the respective branching fractions to the $K^+K^-\pi^0$ final state.

$$\mathcal{R} = \frac{\mathcal{B}(\eta_c \rightarrow K^+K^-\eta)}{\mathcal{B}(\eta_c \rightarrow K^+K^-\pi^0)} = \frac{N_{K^+K^-\eta}}{N_{K^+K^-\pi^0}} \frac{\epsilon_{K^+K^-\pi^0}}{\epsilon_{K^+K^-\eta}} \frac{1}{\mathcal{B}_\eta}$$

□ Presence of non-negligible backgrounds in the η_c signals, which have different distributions in the Dalitz plot

□ We perform a sideband subtraction by assigning a weight $w = 1/\epsilon(m, \cos\theta)$ to events in the signal region and a negative weight $w = -f/\epsilon(m, \cos\theta)$ to events in the sideband regions, where f is a normalization factor.

□ We obtain:

$$\mathcal{R}(\eta_c) = \frac{\mathcal{B}(\eta_c \rightarrow K^+K^-\eta)}{\mathcal{B}(\eta_c \rightarrow K^+K^-\pi^0)} = 0.571 \pm 0.025 \pm 0.051$$

□ Consistent with the BESIII measurement of 0.46 ± 0.23 (6.7 ± 3.2 events for $\eta_c \rightarrow K^+K^-\eta$) (Phys.Rev. D **86**, 092009 (2012)).

Dalitz plot analysis. Isobar Model

- Unbinned Maximum Likelihood fit.
- Amplitudes parametrized as in a standard *pseudoscalar* → *three pseudoscalars* Dalitz analysis (D. Asner, Review of Particle Physics”, Phys. Lett. B 592, 1 (2004)).
- Full interference allowed among the amplitudes.
- No evidence for interference between signal and background. Therefore the sidebands fitted using the incoherent sum of resonance amplitudes.
- Background in the signal region estimated interpolating the sidebands.
- A Non-Resonant contribution (*NR*) is included in the fit.
- The fit quality is tested by performing a 2-dimensional χ^2 test that is constructed by dividing the Dalitz plot in N_{cells} cells and computing:

$$\chi^2 = \sum_{i=1}^{N_{cells}} (N_{obs}^i - N_{exp}^i)^2 / N_{exp}^i$$

where N_{obs}^i and N_{exp}^i are event yields from data and normalized simulation, respectively.

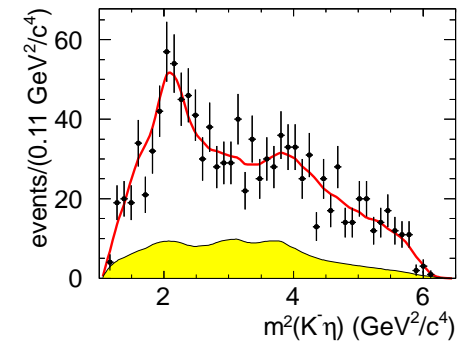
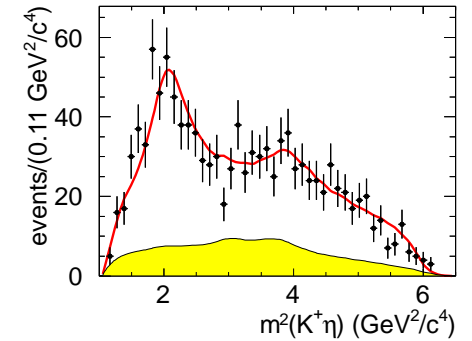
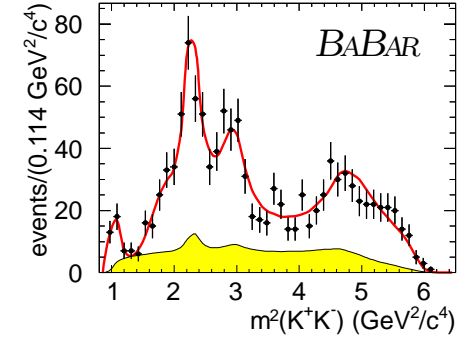
Denoting by n the number of free parameters in the fit, we label $\nu = N_{cells} - n$.

$\eta_c \rightarrow \eta K^+ K^-$ Dalitz plot analysis.

- Results from the Dalitz analysis and fit projections.
- Charge conjugated amplitudes symmetrized.

Final state	Fraction %	Phase (radians)
$f_0(1500)\eta$	$23.7 \pm 7.0 \pm 1.8$	0.
$f_0(1710)\eta$	$8.9 \pm 3.2 \pm 0.4$	$2.2 \pm 0.3 \pm 0.1$
$f_0(2200)\eta$	$11.2 \pm 2.8 \pm 0.5$	$2.1 \pm 0.3 \pm 0.1$
$f_0(1350)\eta$	$5.0 \pm 3.7 \pm 0.5$	$0.9 \pm 0.2 \pm 0.1$
$f_0(980)\eta$	$10.4 \pm 3.0 \pm 0.5$	$-0.3 \pm 0.3 \pm 0.1$
$f_2'(1525)\eta$	$7.3 \pm 3.8 \pm 0.4$	$1.0 \pm 0.1 \pm 0.1$
$K_0^*(1430)^+ K^-$	$16.4 \pm 4.2 \pm 1.0$	$2.3 \pm 0.2 \pm 0.1$
$K_0^*(1950)^+ K^-$	$2.1 \pm 1.3 \pm 0.2$	$-0.2 \pm 0.4 \pm 0.1$
NR	$15.5 \pm 6.9 \pm 1.0$	$-1.2 \pm 0.4 \pm 0.1$
Sum	$100.0 \pm 11.2 \pm 2.5$	
χ^2/ν	$87/65$	

- Largest amplitudes are $f_0(1500)\eta$ and $K_0^*(1430)K$.
- First observation of $K_0^*(1430) \rightarrow \eta K$.

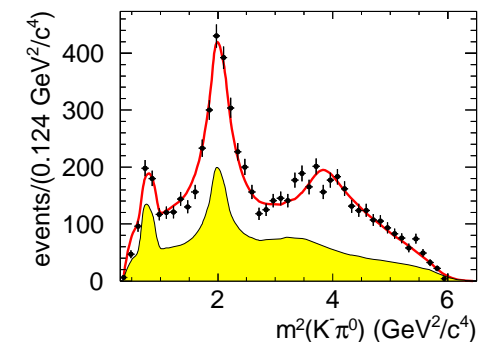
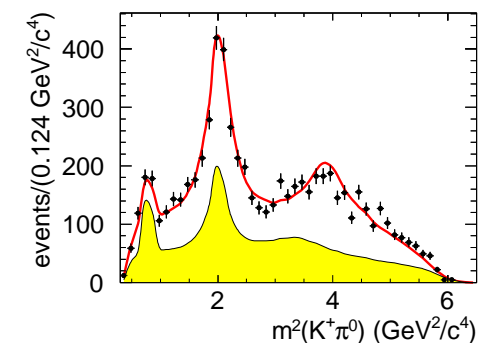
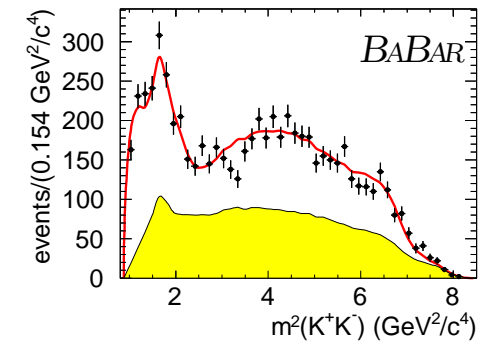


$\eta_c \rightarrow \pi^0 K^+ K^-$ Dalitz analysis.

□ Results from the Dalitz analysis and fit projections.

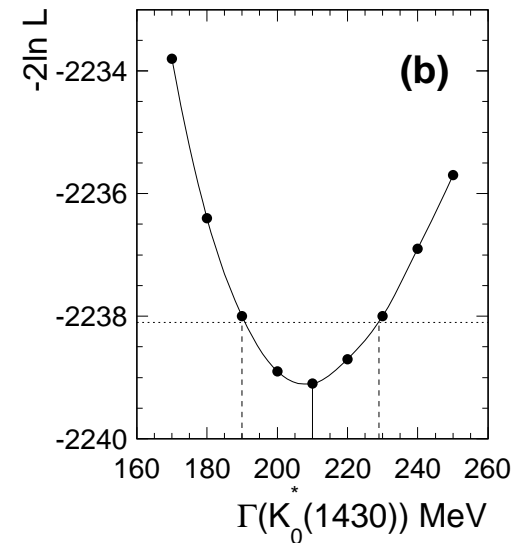
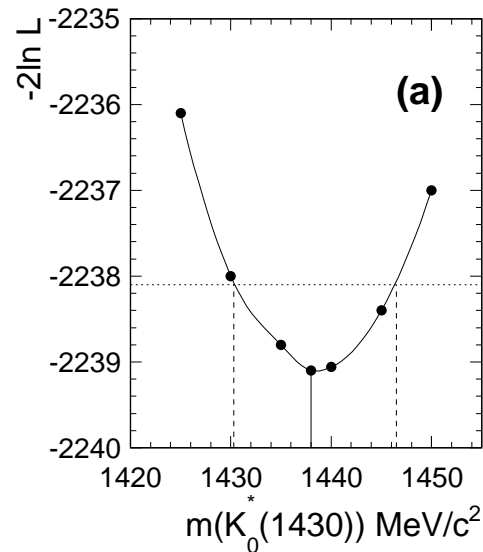
Final state	Fraction %			Phase (radians)
$K_0^*(1430)^+ K^-$	$33.8 \pm 1.9 \pm 0.4$			0.
$K_0^*(1950)^+ K^-$	$6.7 \pm 1.0 \pm 0.3$			$-0.67 \pm 0.07 \pm 0.03$
$K_2^*(1430)^+ K^-$	$6.8 \pm 1.4 \pm 0.3$			$-1.67 \pm 0.07 \pm 0.03$
$a_0(980)\pi^0$	$1.9 \pm 0.1 \pm 0.2$			$0.38 \pm 0.24 \pm 0.02$
$a_0(1450)\pi^0$	$10.0 \pm 2.4 \pm 0.8$			$-2.4 \pm 0.05 \pm 0.03$
$a_2(1320)\pi^0$	$2.1 \pm 0.1 \pm 0.2$			$0.77 \pm 0.20 \pm 0.04$
<i>NR</i>	$24.4 \pm 2.5 \pm 0.6$			$1.49 \pm 0.07 \pm 0.03$
Sum	$85.8 \pm 3.6 \pm 1.2$			
χ^2/ν	$212/130$			

- Largest amplitudes are $K_0^*(1430)K$ and $a_0(1450)\pi^0$.
- $K^*(892)K$ amplitude consistent with zero.
- Spin-one resonances are consistent with originating entirely from background.
- The Isobar Model does not fit very well the data.



The $K_0^*(1430)$ parameters.

□ In the $\eta_c \rightarrow \pi^0 K^+ K^-$ Dalitz plot analysis we scan the likelihood as a function of the $K_0^*(1430)$ mass and width.



□ We obtain:

$$m(K_0^*(1430)) = 1438 \pm 8 \pm 4 \text{ MeV}/c^2$$

$$\Gamma(K_0^*(1430)) = 210 \pm 20 \pm 12 \text{ MeV}$$

$K_0^*(1430)$ branching fraction.

- First observation of $K_0^*(1430) \rightarrow K\eta$.
- The observation of $K_0^*(1430)$ in both $K\eta$ and $K\pi^0$ decay modes allows a measurement of the relative branching fraction.
- The Dalitz plot analysis of $\eta_c \rightarrow K^+K^-\eta$ decay gives a total $K_0^*(1430)^+K^-$ contribution of

$$f_{\eta K} = 0.164 \pm 0.042 \pm 0.010$$

- The Dalitz plot analysis of the $\eta_c \rightarrow K^+K^-\pi^0$ decay mode gives a total $K_0^*(1430)^+K^-$ contribution of

$$f_{\pi^0 K} = 0.338 \pm 0.019 \pm 0.004$$

- Using the measurement of $\mathcal{R}(\eta_c)$, we obtain the $K_0^*(1430)$ branching ratio

$$\frac{\mathcal{B}(K_0^*(1430) \rightarrow \eta K)}{\mathcal{B}(K_0^*(1430) \rightarrow \pi K)} = \mathcal{R}(\eta_c) \frac{f_{\eta K}}{f_{\pi K}} = 0.092 \pm 0.025^{+0.010}_{-0.025}$$

where $f_{\pi K}$ denotes $f_{\pi^0 K}$ after correcting for the $K^0\pi$ decay mode.

- Asymmetric systematic uncertainty due to the modeling of the NR contribution.

$K_0^*(1430)$ branching fraction.

□ LASS experiment has also studied the reaction:

$$K^- p \rightarrow K^- \eta p$$

where $\eta \rightarrow \pi^+ \pi^- \pi^0$

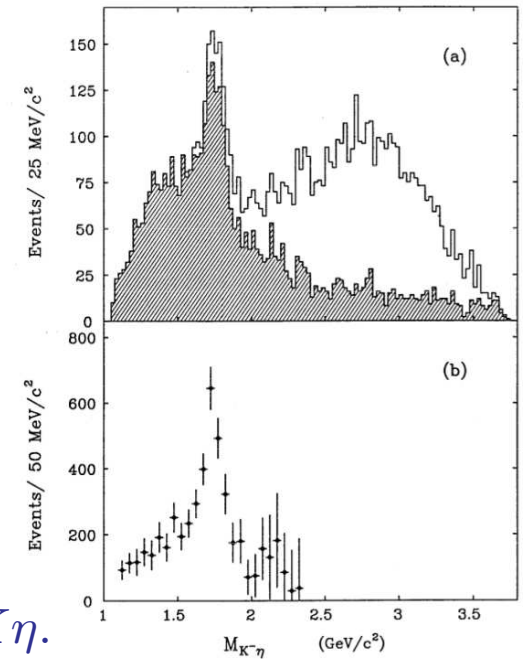
□ They require $m(\eta p) > 2$ GeV, $m(K^- p) > 1.85$ GeV and keep the shaded region.

□ No evidence is found of $K_0^*(1430)$ or $K_2^*(1430)$ decays to $K\eta$.

□ However, from PDG:

$$\Gamma(K_0^*(1430) \rightarrow K\pi) / \Gamma(K_0^*(1430)) = 0.93 \pm 0.04 \pm 0.09$$

□ Not in conflict with the presence of a small branching fraction for the $K\eta$ decay mode.



Model Independent Partial Wave Analysis (MIPWA)

□ We perform a model Independent Partial Wave Analysis of the decays $\eta_c \rightarrow K_S^0 K^+ \pi^-$ and $\eta_c \rightarrow K^+ K^- \pi^0$ (Phys. Rev. D **73**, 032004 (2006)).

□ The $K\pi$ \mathcal{S} -wave (A_1) is taken as the reference amplitude.

$$A = A_1 + c_2 A_2 e^{i\phi_2} + c_3 A_3 e^{i\phi_3} + \dots$$

□ The $K\pi$ mass spectrum is divided into 30 equally spaced mass intervals 60 MeV wide and for each bin we add to the fit two new free parameters, the amplitude and the phase of the $K\pi$ \mathcal{S} -wave (constant inside the bin).

□ We also fix the A_1 amplitude to 1.0 and its phase to $\pi/2$ in an arbitrary interval of the mass spectrum (bin 14 which corresponds to a mass of 1.45 GeV/ c^2).

□ The number of additional free parameters is therefore 58.

Model Independent Partial Wave Analysis (MIPWA)

□ Interference between the two $K\pi$ modes is determined by Isospin conservation.

□ For $\eta_c \rightarrow K_S^0 K^+ \pi^-$:

$$A_{S-wave} = \frac{1}{\sqrt{2}} (a_j^{K^+ \pi^-} e^{i\phi_j^{K^+ \pi^-}} + a_j^{\bar{K}^0 \pi^-} e^{i\phi_j^{\bar{K}^0 \pi^-}})$$

where $a^{K^+ \pi^-}(m) = a^{\bar{K}^0 \pi^-}(m)$ and $\phi^{K^+ \pi^-}(m) = \phi^{\bar{K}^0 \pi^-}(m)$

□ For $\eta_c \rightarrow K^+ K^- \pi^0$:

$$A_{S-wave} = \frac{1}{\sqrt{2}} (a_j^{K^+ \pi^0} e^{i\phi_j^{K^+ \pi^0}} + a_j^{K^- \pi^0} e^{i\phi_j^{K^- \pi^0}})$$

where $a^{K^+ \pi^0}(m) = a^{K^- \pi^0}(m)$ and $\phi^{K^+ \pi^0}(m) = \phi^{K^- \pi^0}(m)$

□ The $K_2^*(1420)$, $a_0(980)$, $a_0(1400)$, $a_2(1310)$, ... contributions are modeled as relativistic Breit-Wigner functions multiplied by the corresponding angular functions.

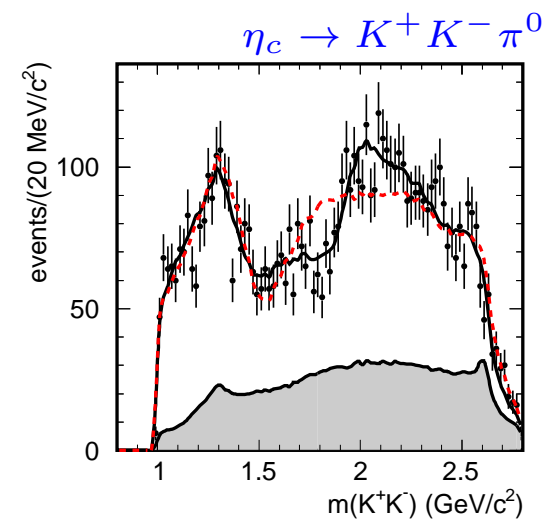
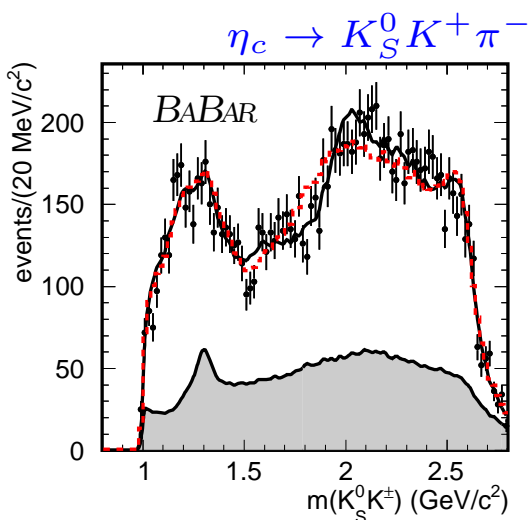
□ Backgrounds are fitted separately and interpolated into the η_c signal regions.

An additional $a_0(1950)$ resonance

- The fits improves when an additional high mass $a_0(1950) \rightarrow K\bar{K}$ $I=1$ resonance is included with free parameters in both η_c decay modes.
- The fits return the following parameters:

Final state	Mass (MeV/ c^2)	Width (MeV)
$\eta_c \rightarrow K_S^0 K^\pm \pi^\mp$	$1949 \pm 32 \pm 76$	$265 \pm 36 \pm 110$
$\eta_c \rightarrow K^+ K^- \pi^0$	$1927 \pm 15 \pm 23$	$274 \pm 28 \pm 30$
Weighted mean	$1931 \pm 14 \pm 22$	$271 \pm 22 \pm 29$

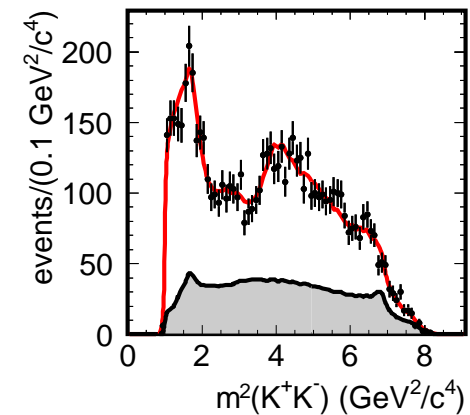
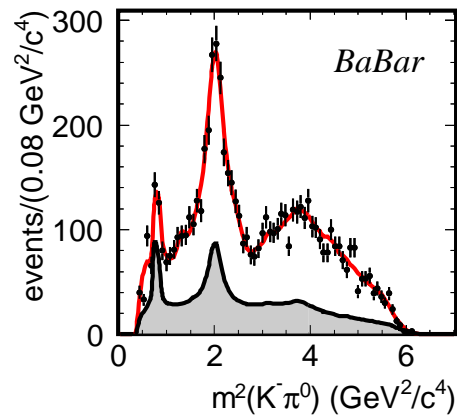
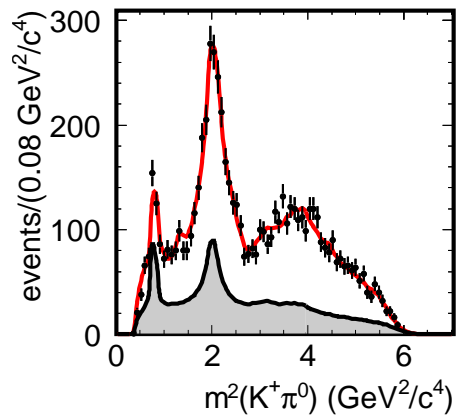
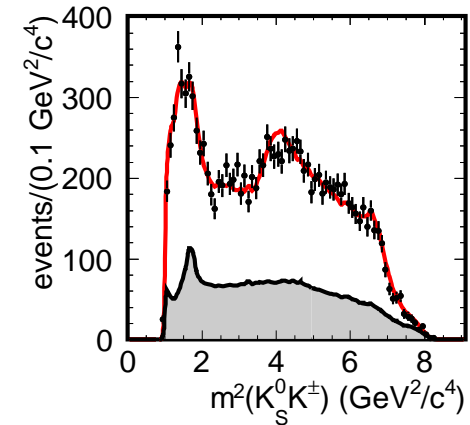
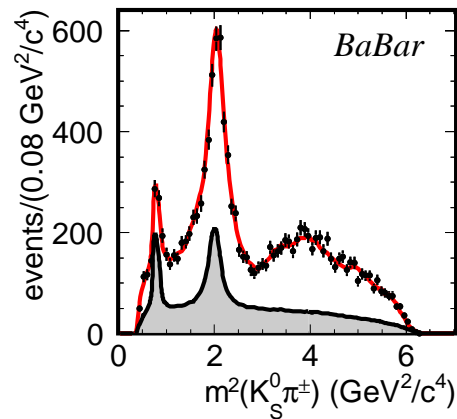
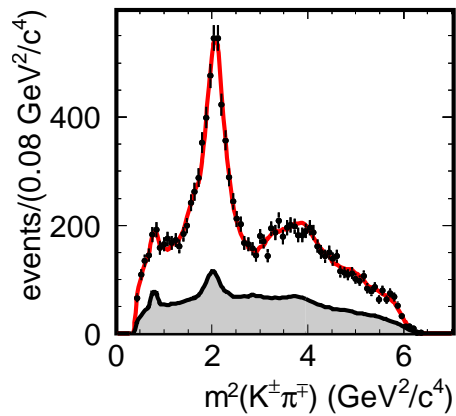
red line: no $a_0(1950)$



- Statistical significances for the $a_0(1950)$ effect (including systematics) are 2.5σ for $\eta_c \rightarrow K_S^0 K^+ \pi^-$ and 4.2σ for $\eta_c \rightarrow K^+ K^- \pi^0$.

Dalitz plots mass projections

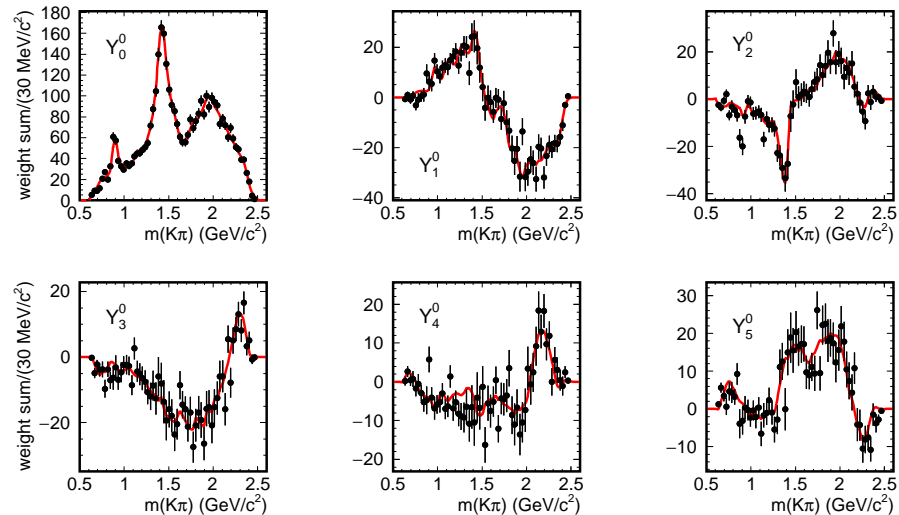
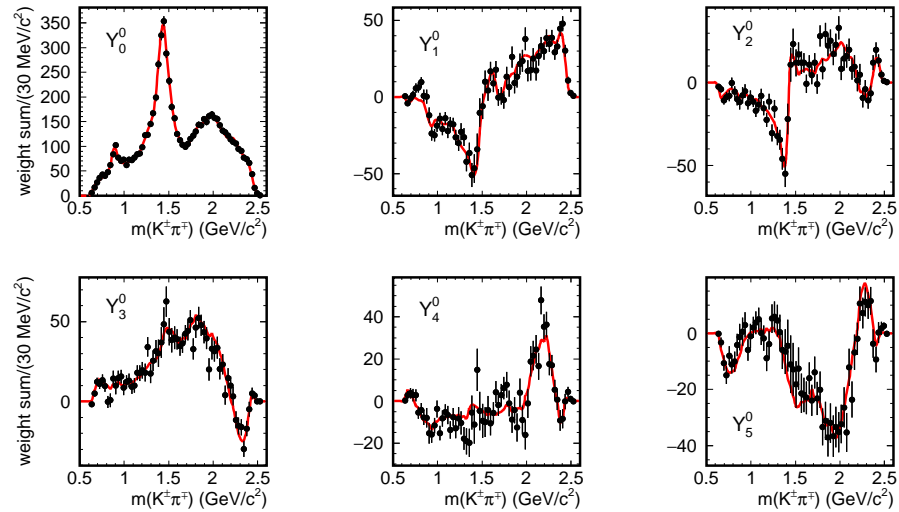
- Dalitz plot projections with fit results for $\eta_c \rightarrow K_S^0 K^+ \pi^-$ (top) and $\eta_c \rightarrow K^+ K^- \pi^0$ (bottom)



- Shaded is contribution from the interpolated background.
- $K^*(892)$ contributions entirely from background.

Legendre polynomial moments

- Weight the $K\pi$ mass spectra by Legendre polynomial moments for $\eta_c \rightarrow K_S^0 K^+ \pi^-$ (top) and $\eta_c \rightarrow K^+ K^- \pi^0$ (bottom)



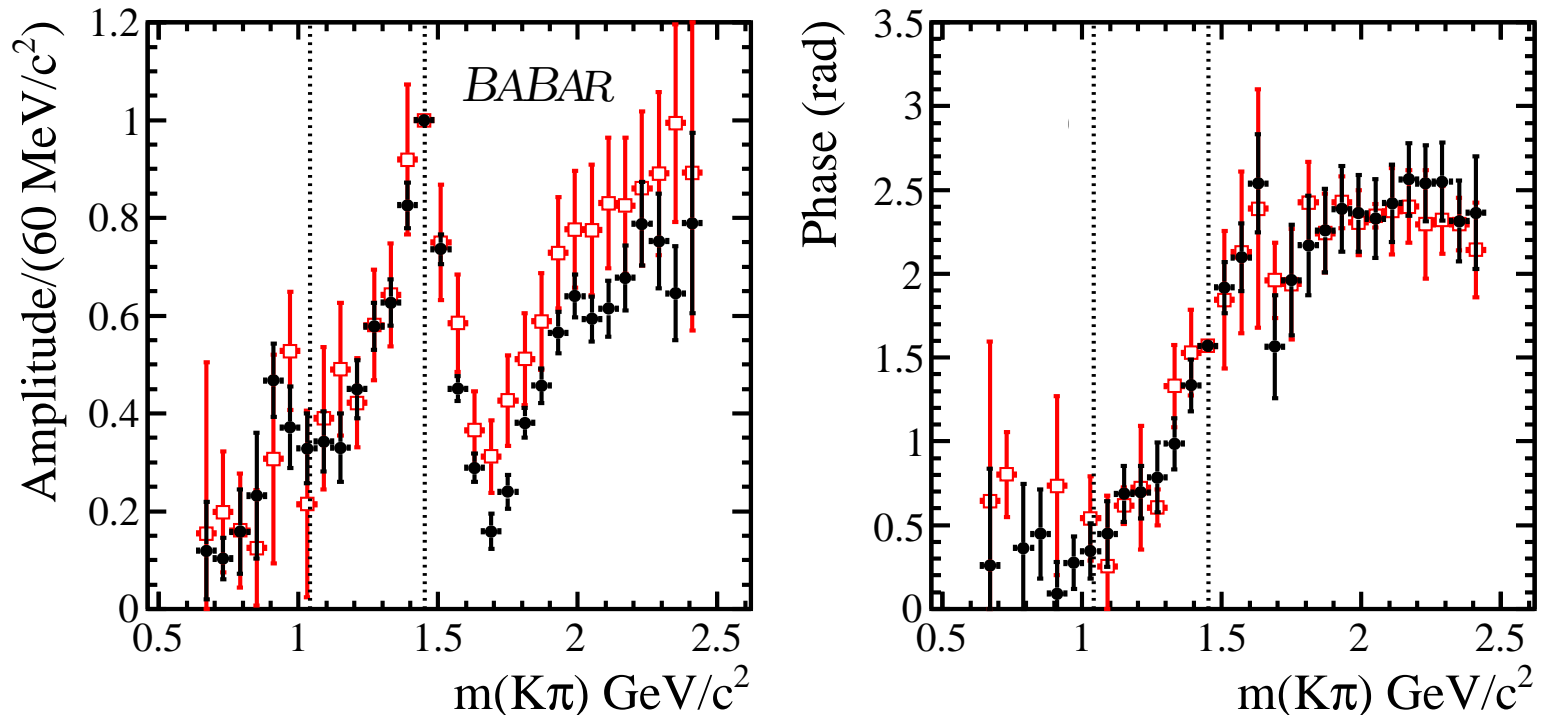
Fit fractions from the MIPWA. Comparison with the Isobar Model

Amplitude	$\eta_c \rightarrow \mathbf{K}_S^0 \mathbf{K}^+ \pi^-$ Fraction (%)	$\eta_c \rightarrow \mathbf{K}^+ \mathbf{K}^- \pi^0$ Fraction (%)
$(K\pi \mathcal{S}\text{-wave}) K$	$107.3 \pm 2.6 \pm 17.9$	$125.5 \pm 2.4 \pm 4.2$
$a_0(1950)\pi$	$3.1 \pm 0.4 \pm 1.2$	$4.4 \pm 0.8 \pm 0.7$
$K_2^*(1430)^0 K$	$4.7 \pm 0.9 \pm 1.4$	$3.0 \pm 0.8 \pm 4.4$
$+a_0(980), a_0(1450), a_0(1950)$		
$+a_2(1320), K_2^*(1430)$		
χ_2/N_{cells}	301/254=1.17	283.2/233=1.22
Isobar Model		
$(K_0^*(1430)K)+$	73.6 ± 3.7	63.6 ± 5.6
$(K_0^*(1950)K)+$		
<i>Nonresonant</i>		
$+a_0(980), a_0(1450), a_0(1950)$		
$+a_2(1320), K_2^*(1430)$		
χ_2/N_{cells}	467/256=1.82	383/233=1.63

- For MIPWA, good agreement between the two η_c decay modes.
- $(K\pi \mathcal{S}\text{-wave})K$ amplitude dominant with small contributions from $K_2^*(1430)^0 K$ and $a_0(1950)\pi$.
- Good description of the data with MIPWA.
- Worse description of the data with the Isobar Model.

The $I=1/2$ $K\pi$ S-wave

- Fitted amplitude and phase. Average systematic uncertainty is 16%.
- Red, open squares: $\eta_c \rightarrow K^+ K^- \pi^0$. Black, filled squares: $\eta_c \rightarrow K_S^0 K^+ \pi^-$.
- Clear $K_0^*(1430)$ resonance and corresponding phase motion.
- At high mass broad $K_0^*(1950)$ contribution.



- Dashed lines are $K\eta$ and $K\eta'$ thresholds.
- Good agreement between the two η_c decay modes.

Comparison with the LASS and E791 experiments

□ Black is $\eta_c \rightarrow K_S^0 K^+ \pi^-$.

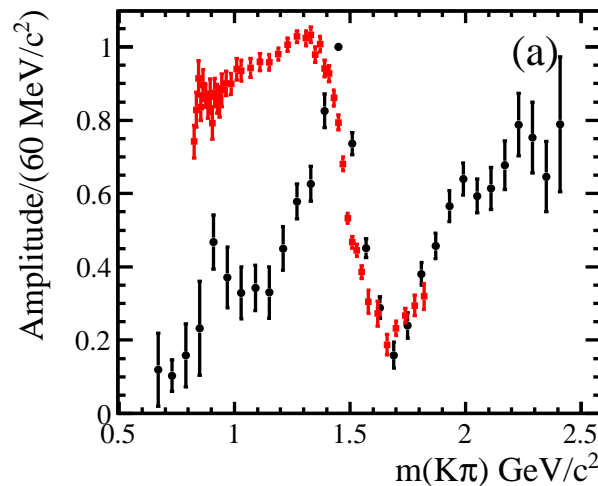
□ Normalization is arbitrary.

□ LASS analysis has two solutions above 1.9 GeV.

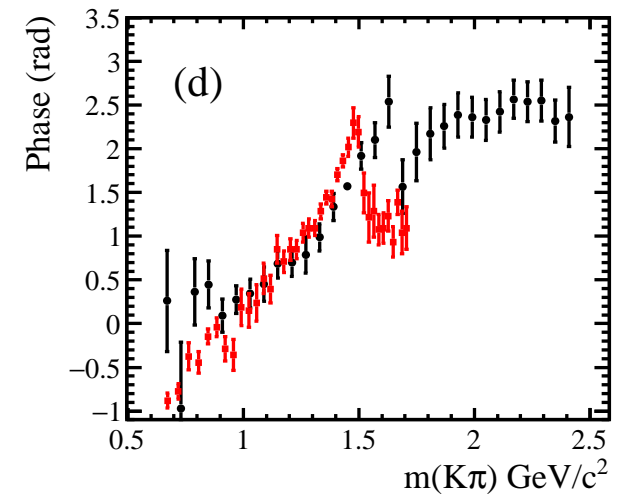
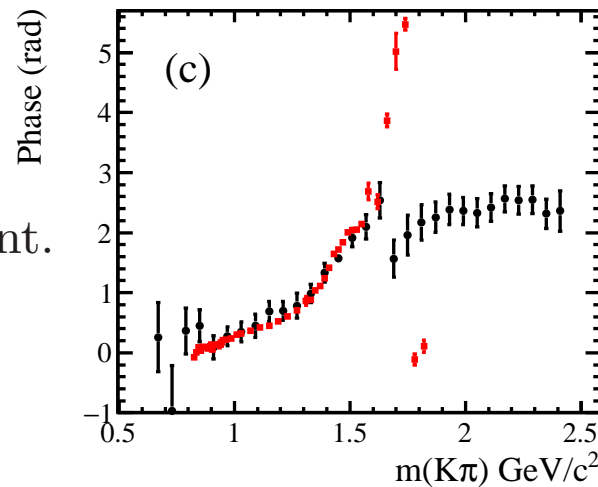
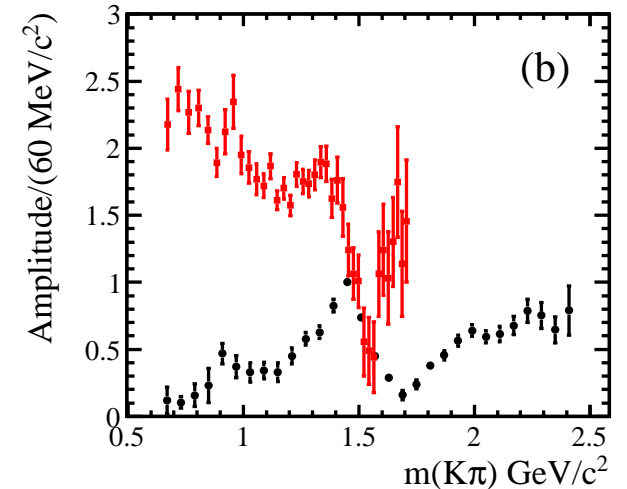
□ Phases before the $K\eta'$ threshold are similar, as expected from Watson theorem.

□ Amplitudes are very different.

LASS($K^- p$)



E791($D^+ \rightarrow K^- \pi^+ \pi^+$)

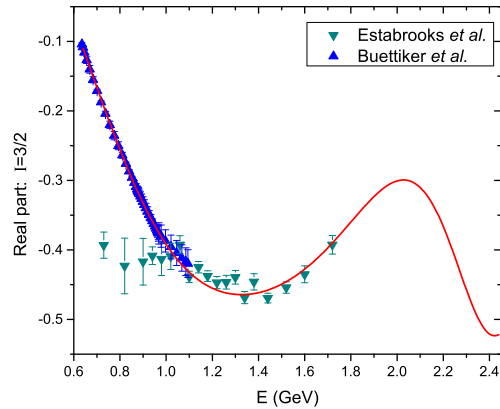


(LASS: Nucl. Phys. B **296**, 493 (1988)), (E791: Phys. Rev. D **73**, 032004 (2006)), (K.M. Watson, Phys. Rev. **88**, 1163 (1952))

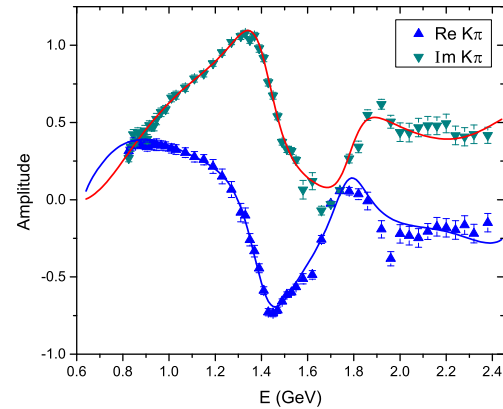
Overall fit of LASS and η_c data.

□ Preliminary K-matrix fit. (A. Palano, M. Pennington, arXiv:1701.04881)

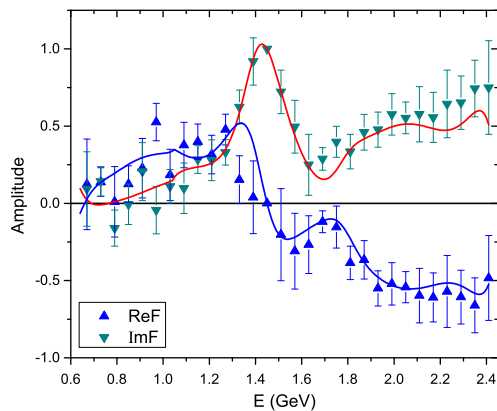
Elastic $K^- \pi^- \rightarrow K^- \pi^-$



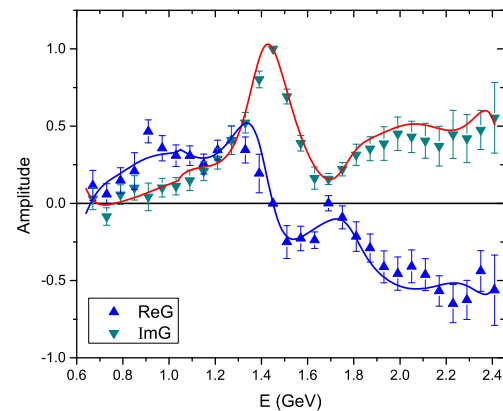
LASS : $K^- \pi^+ \rightarrow K^- \pi^+$



BABAR $\eta_c \rightarrow K^+ K^- \pi^0$



BABAR $\eta_c \rightarrow K_S^0 K^+ \pi^-$



- Data fitted in terms of Real and Imaginary parts of the complex amplitudes.
- Solution A for the LASS data.
- Curves are fit results. Red: Imaginary, Blue: Real.

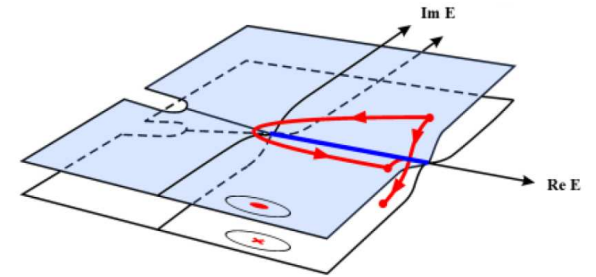
Overall K-matrix fit of LASS and η_c data.

Measured pole positions.

Pole 1	$E_{P1} = 659 - i302 \text{ MeV}$	on Sheet II
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Pole 2	$E_{P2} = 1409 - i128 \text{ MeV}$	on Sheet III
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Pole 3	$E_{P3} = 1768 - i107 \text{ MeV}$	on Sheet III
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□ Pole 1 is identified with the κ , the pole position of which was found to be at $[(658 \pm 7) - i(278 \pm 13)] \text{ MeV}$, in the dispersive analysis of (arXiv:0310283, Eur.Phys.J. C33, 409 (2004)).

□ Pole 2 is identified with $K_0^*(1430)$, to be compared with $[(1438 \pm 8 \pm 4) - i(105 \pm 20 \pm 12)] \text{ MeV}$ using the Breit-Wigner form.

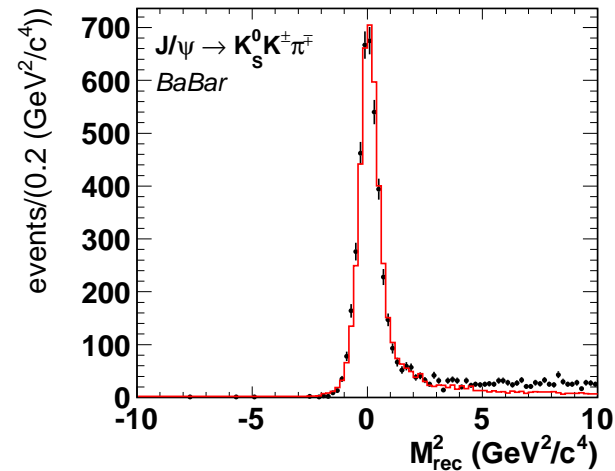
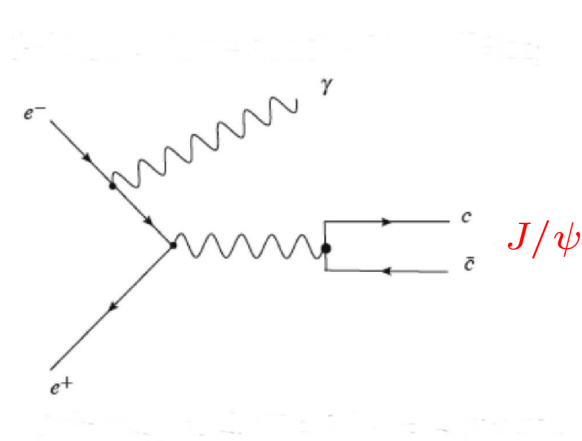
□ Pole 3 may be identified with the $K_0^*(1950)$ with a pole mass closer to that of the reanalysis of the LASS by Anisovich (Phys. Lett.B413, 137 (1997)) with a pole at $E = (1820 \pm 20) - i(125 \pm 50) \text{ MeV}$.

□ For pole 2, the $K_0^*(1430)$, we have a ratio of $K\eta$ to $K\pi$ decay of 0.05 consistent with the branching ratio of $(0.092 \pm 0.025_{-0.025}^{+0.010})$ determined from the Dalitz plot analysis of $\eta_c \rightarrow K^+ K^- \eta / \pi^0$ decays.

Dalitz plot analysis of $J/\psi \rightarrow K_S^0 K^\pm \pi^\mp$

- We use of the Initial State Radiation (ISR) process to obtain clean J/ψ samples.
- We reconstruct events having a (mostly undetected) fast forward γ_{ISR} .

$$e^+ e^- \rightarrow \gamma_{ISR} K_S^0 K^\pm \pi^\mp,$$



- Only $J^{PC} = 1^{--}$ states can be produced. We compute:

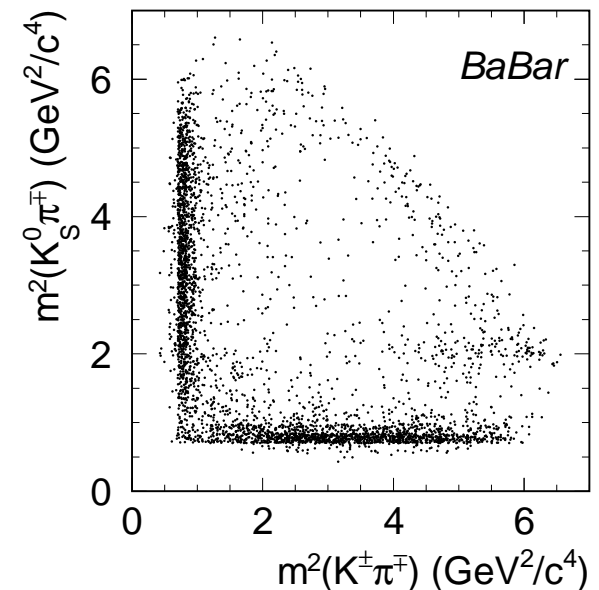
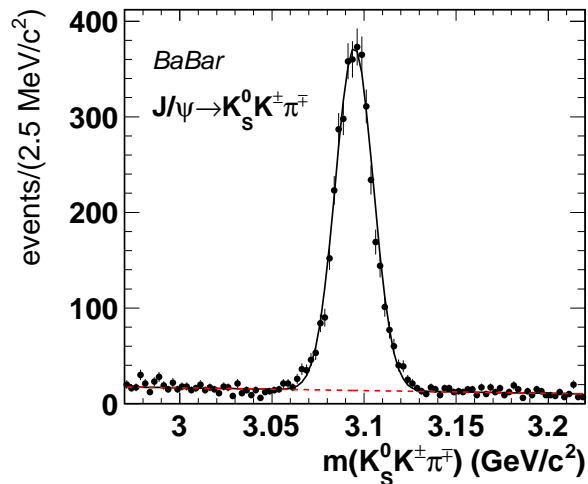
$$M_{\text{rec}}^2 \equiv (p_{e^-} + p_{e^+} - p_K - p_{K_S^0} - p_\pi)^2$$

- This quantity should peak near zero for ISR events.
- Plot of M_{rec}^2 in the J/ψ signal region, In red is Monte Carlo simulation.

(Phys.Rev. D95 (2017) 072007, arXiv:1702.01551)

$J/\psi \rightarrow K_S^0 K^\pm \pi^\mp$ Dalitz plot analysis

- We select events in the ISR region by requiring ($|M_{\text{rec}}^2| < 1.5 \text{ GeV}^2/c^4$) and obtain 3694 ± 64 events with $(93.1 \pm 0.4 \%)$ purity.
- Dalitz plot analysis performed using Isobar Model using Zemach tensors;
C. Zemach, Phys Rev. **133**, B1201 (1964), C. Dionisi et. al., Nucl. Phys. **B169**, 1 (1980).
- $J/\psi \rightarrow K_S^0 K^\pm \pi^\mp$ Dalitz plot analysis performed here for the first time.
- J/ψ signal and Dalitz plot: dominated by K^* bands



$J/\psi \rightarrow K_S^0 K^\pm \pi^\mp$ Dalitz plot analysis

- Significant improvement by leaving free the $K^*(892)$ mass and width parameters.

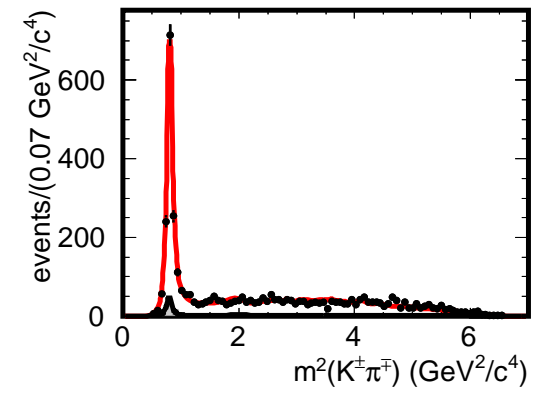
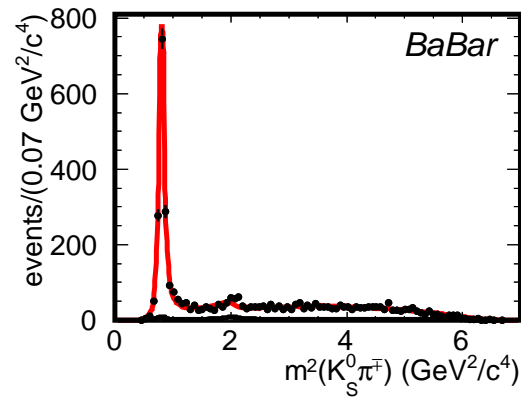
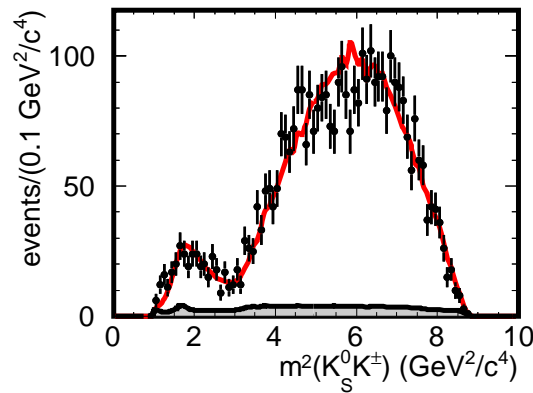
$$m(K^*(892)^+) = 895.6 \pm 0.8 \text{ MeV}/c^2, \Gamma(K^*(892)^+) = 43.6 \pm 1.3 \text{ MeV}$$

$$m(K^*(892)^0) = 898.1 \pm 1.0 \text{ MeV}/c^2, \Gamma(K^*(892)^0) = 52.6 \pm 1.7 \text{ MeV}$$

- The $K^*(892)^+$ measured parameters in good agreement with those measured in τ lepton decays.

Final state	fraction (%)	phase (radians)
$K^*(892)\bar{K}$	$90.5 \pm 0.9 \pm 3.8$	0.
$\rho(1450)^\pm \pi^\mp$	$6.3 \pm 0.8 \pm 0.6$	$-3.25 \pm 0.13 \pm 0.21$
$K_1^*(1410)\bar{K}$	$1.5 \pm 0.5 \pm 0.9$	$1.42 \pm 0.31 \pm 0.35$
$K_2^*(1430)\bar{K}$	$7.1 \pm 1.3 \pm 1.2$	$-2.54 \pm 0.12 \pm 0.12$
Total	105.3 ± 3.1	
χ^2/ν	$274/217 = 1.26$	

- Dalitz plot projections:



Summary

- We show results on the Dalitz plot analyses of $\eta_c \rightarrow K_S^0 K^+ \pi^-$, $\eta_c \rightarrow K^+ K^- \pi^0$ and $\eta_c \rightarrow K^+ K^- \eta$ produced in two-photon interactions.
- We observe for the first time the decay $K_0^*(1430) \rightarrow K \eta$ and measure its branching fraction.
- We extract for the first time the $I=1/2$ $K\pi$ \mathcal{S} -wave amplitude and phase using the MIPWA method up to a mass of $2.5 \text{ GeV}/c^2$. We find a very different amplitude with respect to that measured by previous experiments in different processes.
- A K-matrix formalism is able to obtain a good description of the $I=1/2$ $K\pi$ \mathcal{S} -wave.
- We show results on the Dalitz plot analysis of $J/\psi \rightarrow K_S^0 K^\pm \pi^\mp$ produced in Initial State Radiation events using the Isobar model.

$\eta_c \rightarrow \eta K^+ K^-$ Dalitz plot analysis.

- Weight the mass spectra by Legendre polynomial moments.

