Nucleon Structure & GPDs

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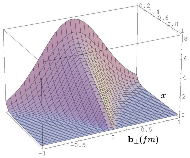
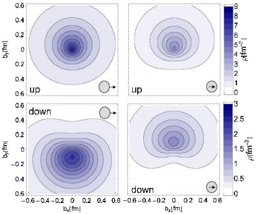
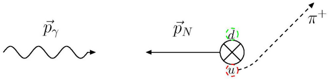
- GPDs: Motivation
  - impact parameter dependent PDFs
  - angular momentum sum rule
  - physics of form factors
- DVCS $\overset{Q^2\, evol.}{\rightsquigarrow}$ GPDs
- GPDs for $x = \xi$
- torque in DIS
- Summary

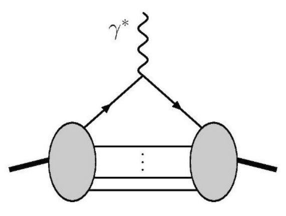






form factor

- electron hits nucleon & nucleon remains intact

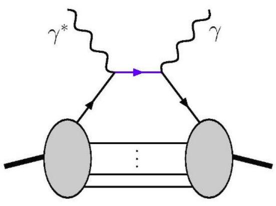


- study amplitude that nucleon remains intact as function of momentum transfer $\rightarrow F(q^2)$
- $F(q^2) = \int dx GPD(x, \xi, q^2)$

↪ GPDs provide momentum disected form factors

Compton scattering

- electron hits nucleon, nucleon remains intact & photon gets emitted



- study both energy & q^2 dependence

↪ additional information about momentum fraction x of active quark

↪ generalized parton distributions $GPD(x, \xi, q^2)$

What can we learn from GPDs?

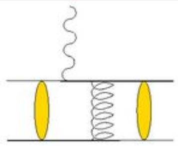
Quark (Orbital) Angular Momentum

X.Ji, 1996:

$$J_q = \frac{1}{2}\int dx\, x\,[H_q(x,\xi,0) + E_q(x,\xi,0)]$$

Physics of large Q^2 Form Factors

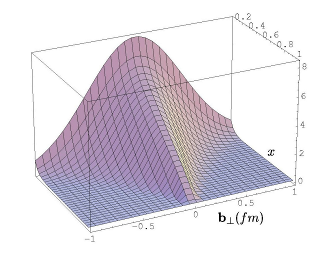
- Feynman: form factor at large Q^2 dominated by high-x quarks
- pQCD mechanism (SJB+...): dominated by typical valence quarks accompanied by hard gluon exchange



Transverse Imaging

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

$$-\frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

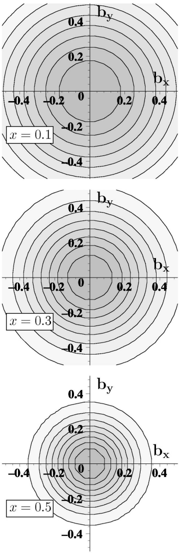


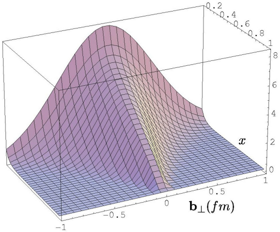
GPDs

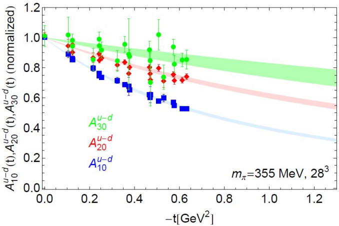
$$F_1(Q^2) = \int dx H(x,\xi,Q^2)$$

$$F_2(Q^2) = \int dx E(x,\xi,Q^2)$$

$q(x, \mathbf{b}_\perp)$ for unpol. p



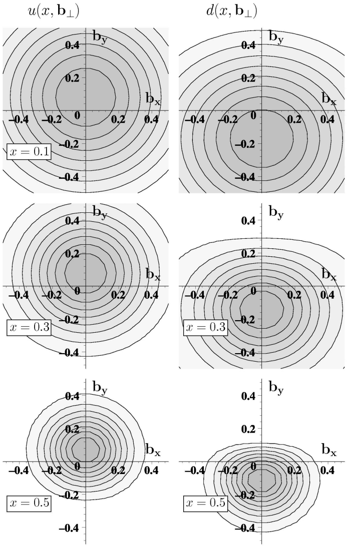


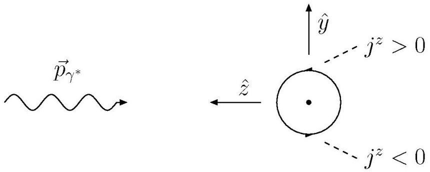


unpolarized proton

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$
- x = momentum fraction of the quark
- $\vec{b} = \perp$ distance of quark from $\perp$ center of momentum
- small x: large 'meson cloud'
- larger x: compact 'valence core'
- $x \to 1$: active quark becomes center of momentum

$\hookrightarrow$ $\vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$





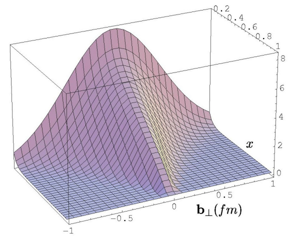
proton 'polarized in $+\hat{x}$ direction'

no axial symmetry!

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

$$-\frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

Physics: relevant density in DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from j^3



nucleon size at small x

→ J.Qiu

nucleon size at large x

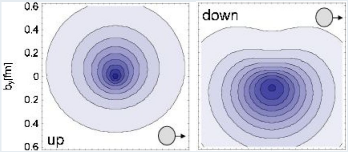
MB & G.A.Miller (2006): 'mathematical model'

$$H(x,0,Q^2) = (1-x)^3 \exp(-\mu^2(1-x)^2 Q^2)$$

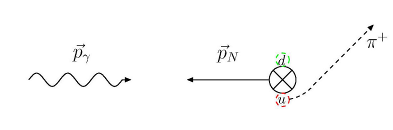
$$\hookrightarrow F_1(Q^2) = \int dx H(x,0,Q^2) \sim \frac{1}{Q^4}$$

large Q^2 behaviour dominated by large x regime → Feynman

Chromodynamic Lensing

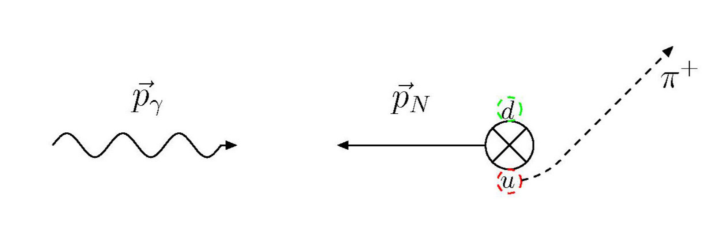


- qualitative connection between deformation and SSA
- one gluon exchange: SSA ↔ ⊥ density-dentity correlations



LHC physics

Frankfurt, Strikman, Weiss: ⊥ spatial distributions → hard parton-parton processes at LHC example: $\gamma p \to \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive final state interaction (FSI) deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction $\to$ chromodynamic lensing

$\Rightarrow$ $\kappa_p\,,\kappa_n \quad \longleftrightarrow \quad$ sign of SSA!!!!!!!! (MB,2004)

- confirmed by Hermes (& Compass) data

Angular Momentum Carried by Quarks

- $L_x = yp_z - zp_y$

$\hookrightarrow$ $J_x = \int d^3r \left[yT^{0z} - zT^{0y}\right]$

- if state invariant under rotations about $\hat{x}$ axis then $\langle yp_z\rangle = -\langle zp_y\rangle$

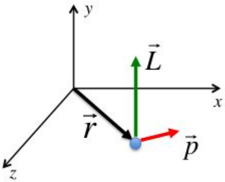
$\hookrightarrow$ $\langle L_x\rangle = 2\langle yp_z\rangle \rightarrow J_z = 2\int d^3r\, yT^{0z}$

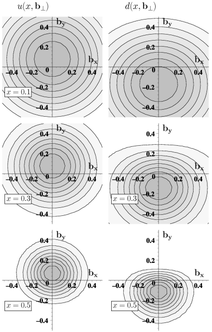
- GPDs provide simultaneous information about p_z & $\mathbf{b}_\perp$

$\hookrightarrow$ use quark GPDs to determine angular momentum carried by quarks

$\hookrightarrow$ $J_q^x = \frac{1}{2}\int dx\, x\left[H(x,0,0) + E(x,0,0)\right]$ (X.Ji, 1996)

- partonic interpretation in terms of 3D distribution (MB,2001,2005)





Total (Spin+Orbital) Quark Angular Momentum

$$J_q^x = L_q^x + S_q^x = \int d^3r \left[y T_q^{0z}(\vec{r}) - z T_q^{0y}(\vec{r}) \right]$$

- $T_q^{\mu\nu}(\vec{r})$ energy momentum tensor $(T_q^{\mu\nu}(\vec{r}) = T_q^{\nu\mu}(\vec{r}))$
- $T_q^{0i}(\vec{r})$ momentum density $[P_q^i = \int d^3r T_q^{0i}(\vec{r})\ ]$
- think: $(\vec{r} \times \vec{p})^x = yp^z - zp^y$

relate to impact parameter dependent quark distributions $q_\psi(x, \mathbf{r}_\perp)$:

Consider spherically symmetric wave packet with nucleon polarized in $+\hat{x}$ direction

- eigenstate under rotations about x-axis

$\hookrightarrow$ both terms in J_q^x equal:
$J_q^x = 2\int d^3r\, yT_q^{0z}(\vec{r}) = \int d^3r\, y \left[T_q^{0z}(\vec{r}) + T_q^{z0}(\vec{r})\right]$

- $\int d^3r\, yT_q^{00}(\vec{r}) = 0 = \int d^3r\, yT_q^{zz}(\vec{r})$

$$\Rightarrow \quad J_q^x = \int d^3r\, yT_q^{++}(\vec{r}) \quad \text{with} \quad T^{++} \equiv T^{00} + T^{0z} + T^{z0} + T^{zz}$$

relate to impact parameter dependent quark distributions $q_\psi(x,\mathbf{r}_\perp)$:

Consider spherically symmetric wave packet with nucleon polarized in $+\hat{x}$ direction

- eigenstate under rotations about x-axis

$\hookrightarrow$ both terms in J^x_q equal:
$J^x_q = 2\int d^3r\, yT^{0z}_q(\vec{r}) = \int d^3r\, y\left[T^{0z}_q(\vec{r}) + T^{z0}_q(\vec{r})\right]$

- $\int d^3r\, yT^{00}_q(\vec{r}) = 0 = \int d^3r\, yT^{zz}_q(\vec{r})$

$$\Rightarrow \quad J^x_q = \int d^3r\, yT^{++}_q(\vec{r}) \quad \text{with} \quad T^{++} \equiv T^{00} + T^{0z} + T^{z0} + T^{zz}$$

- $\int dx\, xq(x,\mathbf{r}_\perp) = \frac{1}{2m_N}\int dr^z T^{++}(\vec{r})$
(note: here x is momentum fraction and not r^x)

$\hookrightarrow$ $\langle\psi|J^x_q|\psi\rangle = m_N \int dx \int d^2b_\perp x b^y q_\psi(x,\mathbf{b}_\perp)$

distribution in delocalized wave packet (pol. in $+\hat{x}$ direction)

$$q_\psi(x,\mathbf{b}_\perp) = \int d^2r_\perp q(x,b_\perp - r_\perp)\left(|\psi(\mathbf{r}_\perp)|^2 - \frac{1}{2M}\frac{\partial}{\partial r_y}|\psi(\mathbf{r}_\perp)|^2\right) \text{ with}$$

$$q(x,\mathbf{b}_\perp) = \int\frac{d^2\Delta_\perp}{(2\pi)^2}H_q(x,0,-\Delta_\perp^2)e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp} - \frac{1}{2M}\frac{\partial}{\partial b_y}\int\frac{d^2\Delta_\perp}{(2\pi)^2}E_q(x,0,-\Delta_\perp^2)e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

two contributions to $\perp$ shift

- intrinsic shift relative to center of momentum $\mathbf{R}_\perp$
- overall shift of $\mathbf{R}_\perp$ for $\perp$ polarized nucleon

insert into $\langle\psi|J_q^x|\psi\rangle = \int dx \int d^2b_\perp q_\psi(x,\mathbf{b}_\perp)$ MB, PRD72, 094020 (2005)

$\langle\psi|J_q^x|\psi\rangle = \frac{1}{2}\int dx\, x\left[H_q(x,0,0) + E_q(x,0,0)\right]$ (here: derived for $\vec{p} = \vec{0}$ only!)

- X.Ji (1996): rotational invariance $\Rightarrow$ apply to all components of $\vec{J}$
- result for J_q^z also applies to $p_z \neq 0$
- partonic interpretation ($\perp$ shift) exists only for $\perp$ components of $\vec{J}_q$!

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gauge invariance

- matrix element of $T_q^{++} = \bar{q}\gamma^+ i\partial^+ q$ in $A^+ = 0$ gauge same as that of $\bar{q}\gamma^+ \left(i\partial^+ - gA^+\right) q$ in any gauge

$\hookrightarrow$ identify $\frac{1}{2}\int dx\, x\,[H(x,0,0) + E(x,0,0)]$ with J_q in decomposition where $\vec{L}_q = \int d^3x \langle P,S| q^\dagger(\vec{x})\left(\vec{x}\times i\vec{D}\right)q(\vec{x})|P,S\rangle$

Angular Momentum Carried by Quarks

insert into $\langle\psi|J_q^x|\psi\rangle = \int dx \int d^2b_\perp q_\psi(x,\mathbf{b}_\perp)$ MB, PRD72, 094020 (2005)

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caution!

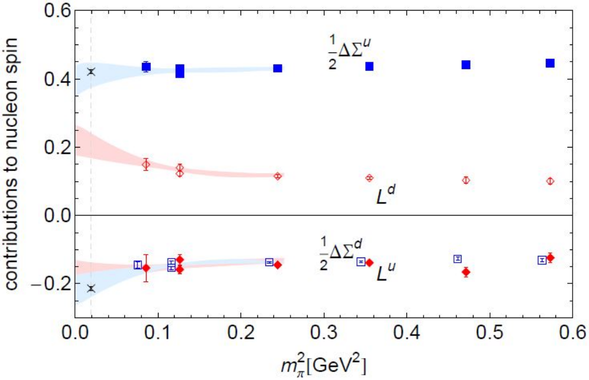
- made heavily use of rotational invariance

$\hookrightarrow$ itentification $\langle\psi|J_q^x|\psi\rangle = \frac{1}{2}\int dx\, x\left[H(x,0,0)+E(x,0,0)\right]$ does not apply to unintegrated quantities

  - $\int d^2\Delta_\perp e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp} \frac{x}{2}\left[H(x,0,-\Delta_\perp^2)+E(x,0,-\Delta_\perp^2)\right]$ not equal to $J^z(\mathbf{b})_\perp$
  - $J_q(x) \equiv \frac{x}{2}\left[H_q(x,0,0)+E_q(x,0,-\Delta_\perp^2)\right]$ not x-distribution of angular momentum $J_q^z(x)$ in long. pol. target (only x-distribution of 'half of J_q')

regardless whether one takes gauge covariant definition or not

lattice: LHPC



- no disconnected quark loops
- chiral extrapolation

$$J^q = \frac{1}{2}\int dx\, x\, [H(x,0,0) + E(x,0,0)] \qquad L^q = J^q - \frac{1}{2}\Delta\Sigma^q$$

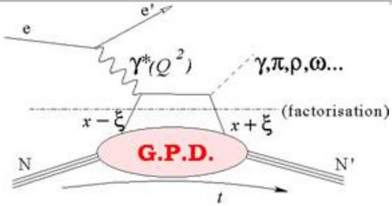
$L^u + L^d \approx 0$

signs of L^q counter-intuitive

$\mathcal{A}_{DVCS} \overset{?}{\rightsquigarrow} GPDs$

interesting GPD physics:

- $J_q = \int_0^1 dx x \left[H(x,\xi,0) + E(x,\xi,0)\right]$ requires $GPDs(x,\xi,0)$ for (common) fixed ξ for all x
- $\perp$ imaging requires $GPDs(x, \xi = 0, t)$



- ξ longitudinal momentum transfer on the target $\xi = \frac{p^{+\prime} - p^+}{p^{+\prime} + p^+}$
- x (average) momentum fraction of the active quark $x = \frac{k^{+\prime} + p^+}{p^{+\prime} + p^+}$

$\Im \mathcal{A}_{DVCS}(\xi, t) \longrightarrow GPD(\xi, \xi, t)$

- only sensitive to 'diagonal' $x = \xi$
- limited ξ range

$\Re \mathcal{A}_{DVCS}(\xi, t) \longrightarrow \int_{-1}^1 dx \frac{GPD(x,\xi,t)}{x - \xi}$

- limited ξ range
- most sensitive to $x \approx \xi$
- some sensitivity to $x \neq \xi$, but

Polynomiality/Dispersion Relations (GPV/AT DI)

$$\Re \mathcal{A}(\xi, t, Q^2) = \int_{-1}^1 dx \frac{H(x, \xi, t, Q^2)}{x - \xi} = \int_{-1}^1 dx \frac{H(x, x, t, Q^2)}{x - \xi} + \Delta(t, Q^2)$$

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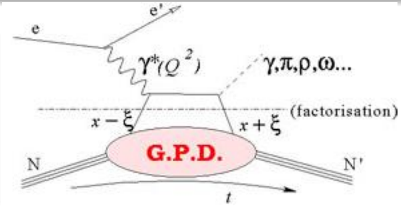

$\Im\mathcal{A}_{DVCS}(\xi,t) \longrightarrow GPD(\xi,\xi,t)$

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- Can 'condense' all information contained in contained in $\mathcal{A}_{DVCS}$ (fixed Q^2) into $GPD(x,x,t,Q^2)$ & $\Delta(t,Q^2)$
- if two models both satisfy polynomiality and are equal for $x = \xi$ (but not for $x \neq \xi$) and have same $\Delta(t,Q^2)$ then DVCS at fixed Q^2 cannot distinguish between the two models

$\mathcal{A}_{DVCS} \overset{?}{\rightsquigarrow} GPDs$

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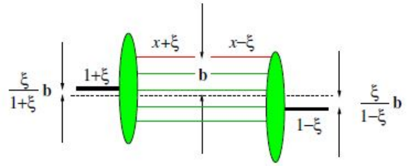
need Evolution!

$$\mu^2 \frac{d}{d\mu^2} H^{q(-)}(x,\xi,t) = \int_{-1}^{1} dx' \frac{1}{|\xi|} V_{\rm NS}\left(\frac{x}{\xi},\frac{x'}{\xi}\right) H^{q(-)}(x',\xi,t)$$

- Q^2 evolution changes x distribution in a known way for fixed ξ

$\hookrightarrow$ measure Q^2 dependence to disentangle x vs. ξ dependence

$\perp$ imaging: $\xi \neq 0$



- center of momentum of hadron not 'conserved' when $\xi \neq 0$,

$\hookrightarrow$ distance of active quark to COM not conserved

- $\perp$ position of each parton is conserved, and so is (any ξ)

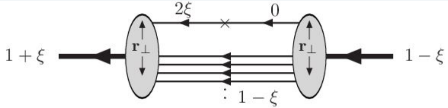
$\hookrightarrow$ distance $\mathbf{r}_\perp$ of active quark to spectators (any ξ)

- variable conjugate to $\Delta_\perp$ is $\frac{1-x}{1-\xi}\mathbf{r}_\perp$

$\perp$ imaging: $\xi = 0$

- probabilistic interpretation
- variable conjugate to $\Delta_\perp$ is $\mathbf{b}_\perp \equiv (1-x)\mathbf{r}_\perp$ distance to COM of hadron

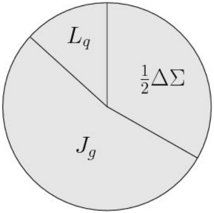
$\perp$ imaging ($x = \xi$)



- no probabilistic interpretation
- variable conjugate to $\mathbf{\Delta}_\perp$ is $\mathbf{r}_\perp$ distance to COM of spectators

$\hookrightarrow$ expect no dramatic ξ-dependence of $\Delta_\perp^2$-slope (size of system)

Ji decomposition



'pizza tre stagioni'

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

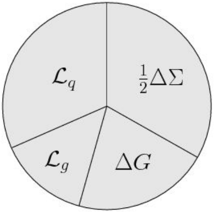
$$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P,S| \, q^\dagger(\vec{x})\Sigma^3 q(\vec{x}) \, |P,S\rangle$$

$$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) |P,S\rangle$$

$$J_g = \int d^3x \, \langle P,S| \left[\vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]^3 |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Jaffe decomposition



'pizza quattro stagioni'

light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P,S| \, \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) |P,S\rangle$$

$$\Delta G = \varepsilon^{+-ij} \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+i} A^j \, |P,S\rangle$$

$$\mathcal{L}_g = 2\int d^3r \langle P,S| \mathrm{Tr} F^{+j} \left(\vec{x} \times i\vec{\partial}\right)^z A^j |P,S\rangle$$

Ji decomposition

$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$

$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P,S|\, q^\dagger(\vec{x})\Sigma^3 q(\vec{x})\, |P,S\rangle$

$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) |P,S\rangle$

$J_g = \int d^3x \, \langle P,S|\, \left[\vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]^3 |P,S\rangle$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Jaffe decomposition

light-cone framework & gauge $A^+ = 0$

$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$

$\mathcal{L}_q = \int d^3r \langle P,S|\, \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) |P,S\rangle$

$\Delta G = \varepsilon^{+-ij} \int d^3r \, \langle P,S|\, \mathrm{Tr} F^{+i} A^j \, |P,S\rangle$

$\mathcal{L}_g = 2\int d^3r \langle P,S| \mathrm{Tr} F^{+j} \left(\vec{x} \times i\vec{\partial}\right)^z A^j |P,S\rangle$

- GPDs $\longrightarrow L^q$
- Δq evolution and/or $\vec{p}\overleftarrow{p} \longrightarrow \Delta G \longrightarrow \mathcal{L} \equiv \sum_{i\in q,g} \mathcal{L}^i$
- $L^q \neq \mathcal{L}^q$
- $\mathcal{L}^q - L^q = ?$
  - can we calculate/predict the difference?
  - what does it represent?

straight line ($\to$Ji)

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple ($\to$ Jaffe-Manohar)

$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$

difference $\mathcal{L}^q - L^q$

$$\mathcal{L}^q - L^q = -g\int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+[\vec{x}\times(\mathbf{A}(\infty,\mathbf{x}_\perp) - \mathbf{A}(\vec{x}))]^z q(\vec{x})|P,S\rangle$$
$$\mathbf{A}_\perp(\infty,\mathbf{x}_\perp) - \mathbf{A}_\perp(\vec{x}) = \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)$$

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x$$

Torque along the trajectory of q

$$T^z = \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

Change in OAM

$$\Delta L^z = \int_{x^-}^{\infty} dr^- \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

straight line (→Ji)

$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

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$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

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$$\mathbf{A}_\perp(\infty,\mathbf{x}_\perp) - \mathbf{A}_\perp(\vec{x}) = \int_{x^-}^\infty dr^- F^{+\perp}(r^-,\mathbf{x}_\perp)$$

color Lorentz Force acting on ejected quark (MB: arXiv:08103589)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x$$

Torque along the trajectory of q

$$T^z = \left[\vec{x}\times\left(\vec{E} - \hat{\vec{z}}\times\vec{B}\right)\right]^z$$

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$$L_q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{D}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$

light-cone staple (→ Jaffe-Manohar)

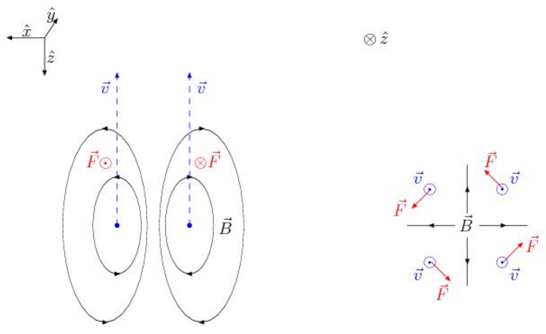
$$\mathcal{L}^q = \int d^3x \langle P,S|\bar{q}(\vec{x})\gamma^+\left(\vec{x}\times i\vec{\mathcal{D}}\right)^z q(\vec{x})|P,S\rangle$$

- $i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$

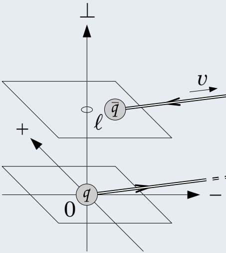
difference $\mathcal{L}^q - L^q$

$\mathcal{L}^q - L^q$ = change in OAM as the quark leaves the nucleon

example: torque in magnetic dipole field



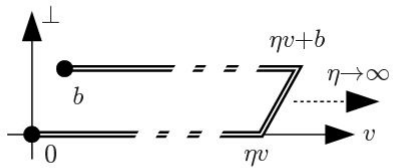
challenge



- TMDs/Wigner functions relevant for SIDIS require (near) light-like Wilson lines
- on Euclidean lattice, all distances are space-like

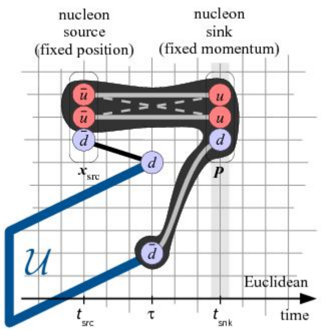
TMDs in lattice QCD

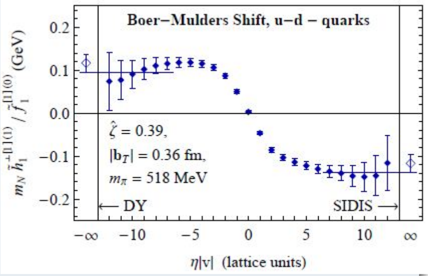
B. Musch, P. Hägler, M. Engelhardt



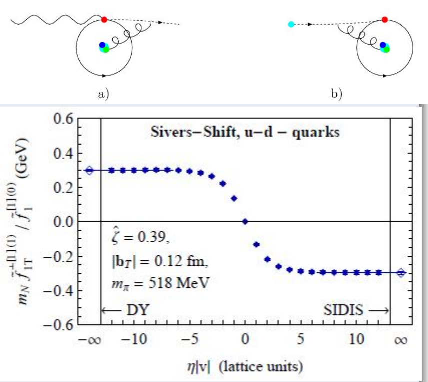
- calculate space-like staple-shaped Wilson line pointing in $\hat{z}$ direction; length $L \to \infty$
- momentum projected nucleon sources/sinks
- remove IR divergences by considering appropriate ratios

↪ extrapolate/evolve to $P_z \to \infty$

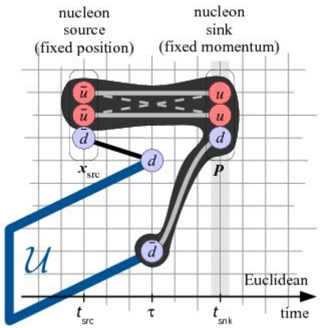


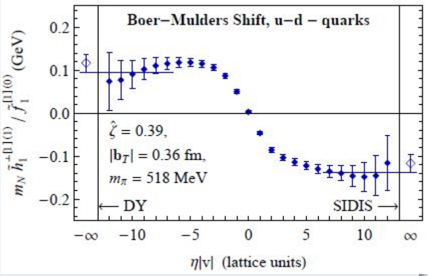


$f^{\perp}_{1T,SIDIS} = -f^{\perp}_{1T,DY}$ (Collins)



$f^{\perp}_{1T}(x, \mathbf{k}_\perp)$ is $\mathbf{k}_\perp$-odd term in quark-spin averaged momentum distribution in $\perp$ polarized target





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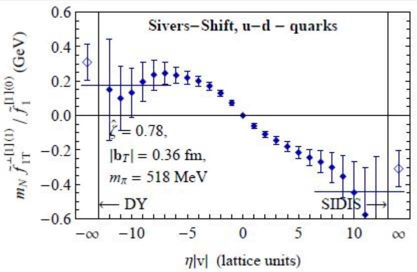
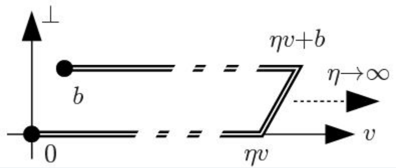


$f^{\perp}_{1T}(x, \mathbf{k}_\perp)$ is $\mathbf{k}_\perp$-odd term in quark-spin averaged momentum distribution in $\perp$ polarized target

TMDs in lattice QCD

B. Musch, P. Hägler, M. Engelhardt



- calculate space-like staple-shaped Wilson line pointing in $\hat{z}$ direction; length $L \to \infty$
- momentum projected nucleon sources/sinks
- remove IR divergences by considering appropriate ratios

↪ extrapolate/evolve to $P_z \to \infty$

next: Orbital Angular Momentum

- same operator as for TMDs, only nonforward matrix elements:
  - momentum transfer provides position space information ($\to \mathbf{r}_\perp \times \mathbf{k}_\perp$)
  - staple with long side in $\hat{z}$ direction
  - (large) nucleon momentum in $\hat{z}$ direction
  - small momentum transfer in $\hat{y}$ direction

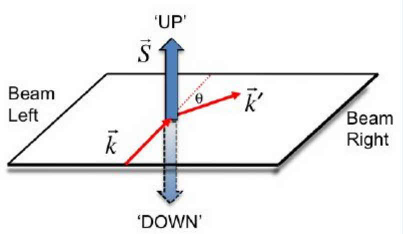
↪ generalized TMD F_{14} (Metz et al.)

- quark OAM
- renormalization same as $f_{1T}^{\perp}$

↪ study ratios...

inclusive SSA

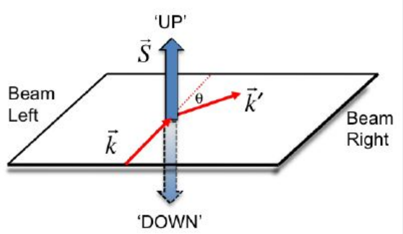
Hall A: inclusively scattered (summed over all final states) electrons off $\perp$ polarized target show left-right asymmetry relatve to target spin



for n target (i.e. ${}^3He$), the e^- is preferentially deflected to beam-right

inclusive SSA

Hall A: inclusively scattered (summed over all final states) electrons off $\perp$ polarized target show left-right asymmetry relatve to target spin



for n target (i.e. ^3He), the e^- is preferentially deflected to beam-right

Metz et al. arXiv:1209.3138

- beam e^- approx. at same $\perp$ position as struck quark

$\hookrightarrow$ $\langle \mathbf{k}_{\perp e} \rangle \sim \sum_q (-e) F^q_{FT}$ with

$$F^q_{FT} \equiv \int dr^- \langle P, S | \bar{q}(0) \gamma^+ e F^{+\perp}_{QED}(0, r^-) q(0) | P, S \rangle$$

Metz et al. arXiv:1209.3138

- beam e^- approx. at same $\perp$ position as struck quark

$\hookrightarrow$ $\langle \mathbf{k}_{\perp e} \rangle \sim \sum_q (-e) F_{FT}^q$ with

$$F_{FT}^q \equiv \int dr^- \langle P,S|\bar{q}(0)\gamma^+ e F_{QED}^{+\perp}(0,r^-) q(0)|P,S\rangle$$

model (Metz et al. arXiv:1209.3138)

- if FSI in QCD is caused by one-gluon exchange between valence quarks then

$$F_{FT}^q = 2 \frac{e_q' \alpha}{\frac{4}{3}\alpha_s} F_{QCD}^q$$

- e_s' charge of spectators, e.g. $e_q' = \frac{1}{3}$ for u quarks in proton

Metz et al.

- multiply by charge of spectator flavor, e.g. $e'_q = \frac{1}{3}$ for u in p
- divide by $\frac{4}{3}\alpha_s$

$$F_{FT}^{u/p}(x,x) = -\frac{\alpha_{em}}{6\pi C_F \alpha_s M} g T_F^{u/p}(x,x)$$

$$F_{FT}^{d/p}(x,x) = -\frac{2\alpha_{em}}{3\pi C_F \alpha_s M} g T_F^{d/p}(x,x)$$

$$F_{FT}^{u/n}(x,x) = \frac{\alpha_{em}}{3\pi C_F \alpha_s M} g T_F^{d/p}(x,x)$$

$$F_{FT}^{d/n}(x,x) = -\frac{\alpha_{em}}{6\pi C_F \alpha_s M} g T_F^{u,p}(x,x).$$

violates $\frac{2}{3} F_{FT}^{u/N} - \frac{1}{3} F_{FT}^{d/N} \stackrel{!}{=} 0$

MB

- multiply by charge of spectator flavor, e.g. $e'_q = -\frac{1}{3}$ for u in p
- divide by $\frac{2}{3}\alpha_s$ for u in p

$$F_{FT}^{u/p}(x,x) = \frac{\alpha_{em}}{3\pi C_F \alpha_s M} g T_F^{u/p}(x,x)$$

$$F_{FT}^{d/p}(x,x) = -\frac{2\alpha_{em}}{3\pi C_F \alpha_s M} g T_F^{d/p}(x,x)$$

$$F_{FT}^{u/n}(x,x) = \frac{\alpha_{em}}{3\pi C_F \alpha_s M} g T_F^{d/p}(x,x)$$

$$F_{FT}^{d/n}(x,x) = -\frac{2\alpha_{em}}{3\pi C_F \alpha_s M} g T_F^{u,p}(x,x).$$

satisfies $\frac{2}{3} F_{FT}^{u/N} - \frac{1}{3} F_{FT}^{d/N} \stackrel{!}{=} 0$

assume $T_F^{q/p}$ saturate sum rule $\sum_{q=u,d} \int dx T_F^{q/p}(x,x) = 0$.

Metz et al. vs. MB

Metz et al.

$$F_{FT}^{u/p}(x,x) = -\frac{\alpha_{em}}{6\pi C_F \alpha_s M} gT_F^{u/p}(x,x)$$

$$F_{FT}^{d/p}(x,x) = -\frac{2\alpha_{em}}{3\pi C_F \alpha_s M} gT_F^{d/p}(x,x)$$

$$F_{FT}^{u/n}(x,x) = \frac{\alpha_{em}}{3\pi C_F \alpha_s M} gT_F^{d/p}(x,x)$$

$$F_{FT}^{d/n}(x,x) = -\frac{\alpha_{em}}{6\pi C_F \alpha_s M} gT_F^{u,p}(x,x).$$

violates $\frac{2}{3}F_{FT}^{u/N} - \frac{1}{3}F_{FT}^{d/N} \stackrel{!}{=} 0$

- $\sigma_{UT}^p \propto \frac{2\alpha_{em} g}{3\pi C_F \alpha_s}\left(-T_F^{u/p} - T_F^{d/p}\right)$
- $\sigma_{UT}^n \propto \frac{2\alpha_{em} g}{3\pi C_F \alpha_s}\left(2T_F^{d/p} - \frac{1}{4}T_F^{u/p}\right)$
- $\sigma_{UT}^p \ll \sigma_{UT}^n$

MB

$$F_{FT}^{u/p}(x,x) = \frac{\alpha_{em}}{3\pi C_F \alpha_s M} gT_F^{u/p}(x,x)$$

$$F_{FT}^{d/p}(x,x) = -\frac{2\alpha_{em}}{3\pi C_F \alpha_s M} gT_F^{d/p}(x,x)$$

$$F_{FT}^{u/n}(x,x) = \frac{\alpha_{em}}{3\pi C_F \alpha_s M} gT_F^{d/p}(x,x)$$

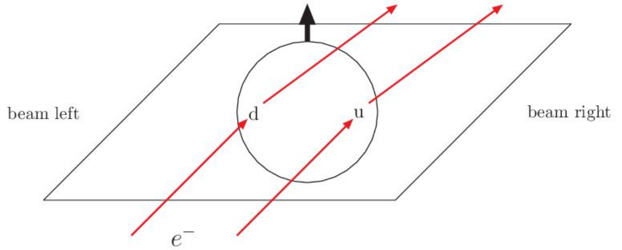
$$F_{FT}^{d/n}(x,x) = -\frac{2\alpha_{em}}{3\pi C_F \alpha_s M} gT_F^{u,p}(x,x).$$

satisfies $\frac{2}{3}F_{FT}^{u/N} - \frac{1}{3}F_{FT}^{d/N} \stackrel{!}{=} 0$

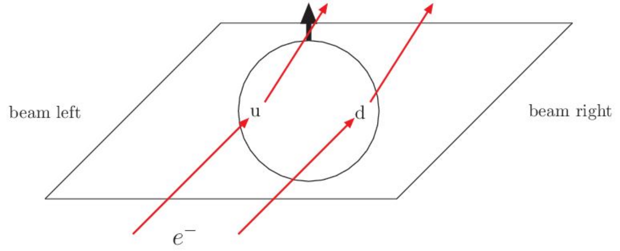
- $\sigma_{UT}^p \propto \frac{2\alpha_{em} g}{3\pi C_F \alpha_s}\left(2T_F^{u/p} - T_F^{d/p}\right)$
- $\sigma_{UT}^n \propto \frac{2\alpha_{em} g}{3\pi C_F \alpha_s}\left(2T_F^{d/p} - T_F^{u/p}\right)$
- $\sigma_{UT}^p \approx -\sigma_{UT}^n$

- inclusive SSA $\sigma_{UT} \propto 4F_{FT}^u + F_{FT}^d$
- assume $T_F^d \approx -T_F^u$

neutron target



proton target



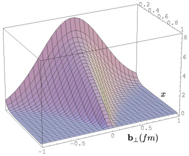
- GPDs → impact parameter dependent PDFs

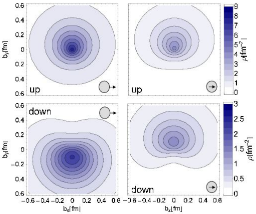
↪ parton interpretation for Ji-relation

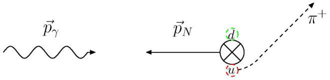
- (in principle) $GPD(x, \xi, Q^2)$ from QCD evolution of $GPD(\xi, \xi, Q^2)$

↪ EIC (need wide Q^2 range)

- Ji vs. Jaffe-Manohan quark OAM: interpretation of $\mathcal{L}_q - L_q$ as change in OAM of ejected quark







- L_q matrix element of

$$q^\dagger \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q = \bar{q}\gamma^0 \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$$

- $\mathcal{L}_q^z$ matrix element of $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\bar{q}\gamma^+ \left[\vec{r} \times i\vec{\partial}\right]^z q\Big|_{A^+=0}$$

- (for $\vec{p} = 0$) matrix element of $\bar{q}\gamma^z \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$ vanishes (parity!)

$\hookrightarrow$ L_q identical to matrix element of $\bar{q}\gamma^+ \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$ (nucleon at rest)

$\hookrightarrow$ even in light-cone gauge, L_q^z and $\mathcal{L}_q^z$ still differ by matrix element of $q^\dagger \left(\vec{r} \times g\vec{A}\right)^z q\Big|_{A^+=0} = q^\dagger \left(r^x g A^y - r^y g A^x\right) q\Big|_{A^+=0}$

- how significant is that difference?

What is Orbital Angular Momentum

first: QED without electrons

- apply $\vec{a}\times(\vec{b}\times\vec{c}) = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{b}(\vec{a}\cdot\vec{c})$ to $\vec{E}\times\left(\vec{\nabla}\times\vec{A}\right)$

$$\begin{aligned}\vec{J} &= \int d^3r\,\vec{x}\times\left(\vec{E}\times\vec{B}\right) = \int d^3r\,\vec{x}\times\left[\vec{E}\times\left(\vec{\nabla}\times\vec{A}\right)\right]\\ &= \int d^3r\left[E^j\left(\vec{x}\times\vec{\nabla}\right)A^j - \vec{x}\times(\vec{E}\cdot\vec{\nabla})\vec{A}\right]\end{aligned}$$

- integrate by parts (drop surface term)

$$\vec{J} = \int d^3r\left[E^j\left(\vec{x}\times\vec{\nabla}\right)A^j + \left(\vec{x}\times\vec{A}\right)\vec{\nabla}\cdot\vec{E} + \vec{E}\times\vec{A}\right]$$

- drop 2^{nd} term (eq. of motion $\vec{\nabla}\cdot\vec{E}=0$), yielding $\vec{J} = \vec{L} + \vec{S}$ with

$$\vec{L} = \int d^3r\,E^j\left(\vec{x}\times\vec{\nabla}\right)A^j \qquad \vec{S} = \int d^3r\,\vec{E}\times\vec{A}$$

- note: $\vec{L}$ and $\vec{S}$ not separately gauge invariant as written, but can be made so ($\rightarrow$ nonlocal)

QED with electrons

$$\vec{J}_\gamma = \int d^3r\, \vec{r} \times \left(\vec{E} \times \vec{B}\right) = \int d^3r\, \vec{r} \times \left[\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right)\right]$$
$$= \int d^3r \left[E^j \left(\vec{r} \times \vec{\nabla}\right) A^j - \vec{r} \times (\vec{E} \cdot \vec{\nabla})\vec{A}\right]$$
$$= \int d^3r \left[E^j \left(\vec{r} \times \vec{\nabla}\right) A^j + \left(\vec{r} \times \vec{A}\right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}\right]$$

- replace 2^{nd} term (eq. of motion $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

$$\vec{J}_\gamma = \int d^3r \left[\psi^\dagger \vec{r} \times e\vec{A}\psi + E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \vec{E} \times \vec{A}\right]$$

- $\psi^\dagger \vec{r} \times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger \vec{r} \times (\vec{p} - e\vec{A})\psi$

↪ decomposing $\vec{J}_\gamma$ into spin and orbital also shuffles angular momentum from photons to electrons!

- can also be done for only part of $\vec{A} \rightarrow$ Chen/Goldman, Wakamatsu