Bessel Weighted Asymmetries
Alexei Prokudin

May 29, 2015
Unified View of Nucleon Structure

Wigner Distribution

5D
Generalized Parton Distributions

3D
Transverse Momentum Distributions

\[ W(x, k_\perp, r_\perp) \]
GPDs

\[ Q^2 \] ensures hard scale, pointlike interaction
\[ \Delta = P' - P \] momentum transfer can be varied independently
Connection to 3D structure
Burkardt (2000)
Burkardt (2003)
Drell-Yan frame \[ \Delta^+ = 0 \]
Weiss (2009)

TMDs

\[ Q^2 \] ensures hard scale, pointlike interaction
\[ P_{hT} \] final hadron transverse momentum can be varied independently
Connection to 3D structure
Ji, Ma, Yuan (2004)
Collins (2011)
AP (2012)
### TMD distributions

<table>
<thead>
<tr>
<th></th>
<th>q</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td></td>
<td>f₁</td>
<td>g₁</td>
<td>h₁</td>
</tr>
<tr>
<td>U</td>
<td></td>
<td></td>
<td>h₁</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td></td>
<td>g₁</td>
<td>h₁</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>f₁</td>
<td>g₁</td>
<td>h₁</td>
<td>h₁</td>
</tr>
</tbody>
</table>

- **8 functions in total (at leading twist)**
- Each represents different aspects of partonic structure
- Each function is to be studied

Three types of modulations

\[ f(x, k^2) \]

\[ \frac{k_i S_{T i}}{M} f(x, k^2) \]

\[ \frac{k^i k^j - \frac{1}{2} k^2 g_{ij}}{M^2} g_T f(x, k^2) \]

Monopole
Dipole
Quadrupole

Extraction of the nucleon structure from the data is one of the main goals of modern facilities such as Jefferson Lab.

\[ f(x, k_{\perp}^2) \]

\[
\frac{k_{\perp i} S_{Ti}}{M} f(x, k_{\perp}^2)
\]

\[
\frac{k_{\perp i} k_{\perp j} - \frac{1}{2} k_{\perp}^2 g_{iT}}{M^2} f(x, k_{\perp}^2)
\]

How can TMDs be studied?
One can rewrite the cross-section in terms of 18 structure functions.

Each structure function encodes parton dynamics via convolutions of TMDs when factorization is applicable.

Mulders, Tangerman (1995),
Boer, Mulders (1998),
Bacchetta et al (2007)

\[
\frac{d\sigma}{dx \, dy \, d\psi \, dz \, d\phi_h \, dP_{h\perp}^2} = \\
\frac{\alpha^2}{xy Q^2} \frac{y^2}{2(1 - \varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1 + \varepsilon)} \cos \phi_h F_{UU}^{\cos \phi_h} \right. \\
+ \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda \varepsilon \sqrt{2\varepsilon(1 - \varepsilon)} \sin \phi_h F_{LU}^{\sin \phi_h} + \ldots \right\}
\]
Structure functions

Structure functions are convolutions of unobserved partonic momenta:

\[ F \sim \int d^2 \vec{k}_\perp d^2 \vec{p}_\perp \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp}) f(x, \vec{k}_\perp) D(z, \vec{p}_\perp) \]
Structure functions are convolutions of unobserved partonic momenta:

\[ F \sim \int d^2 \vec{k}_\perp d^2 \vec{p}_\perp \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp}) f(x, \vec{k}_\perp) D(z, \vec{p}_\perp) \]

Observed in experiment

The convolution is a complication!
Can we find a more direct measure?
Weighted structure functions

An attempt to unravel the convolution:

\[
\int d^2 \vec{P}_{h\perp} F \sim \int d^2 \vec{k}_\perp d^2 \vec{p}_\perp d^2 \vec{P}_{h\perp} \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp}) f(x, \vec{k}_\perp) D(z, \vec{p}_\perp)
\]

\[
= \left( \int d^2 \vec{k}_\perp f(x, \vec{k}_\perp) \right) \left( \int d^2 \vec{p}_\perp D(z, \vec{p}_\perp) \right) \equiv f^{(0)}(x) \cdot D^{(0)}(z)
\]

A product instead of the convolution!

Kotzinian, Mulders (1997), Boer, Mulders (1998)
The problem:

TMD factorization has a validity region

\[ P_T \ll Q \]

Integration \[ \int dP_{h\perp} d\phi_h P_{h\perp} \] emphasizes high momentum region

Collinear contributions will “spoil” our interpretation!
The problem:

TMD factorization has a validity region

Integrating \( \int dP_{h \perp} d\phi_h P_{h \perp} \) emphasizes the high momentum region.

\[ P_T \ll Q \]

Collinear contributions will "spoil" our interpretation!
Can we circumvent the problem and emphasize the region of TMD?
Gluons and the quark sea at high energies: distributions, polarization, tomography

September 13 to November 19, 2010

Report from the INT program "Gluons and the quark sea at high energies: distributions, polarization, tomography"

Daniel Boer
Leonard Gamberg
Berni Musch
AP

Came up with an idea!

Boer, Gamberg, Musch, Prokudin JHEP 1110 (2011)
The cross-section is a Fourier transform of $b$-space expression:

$$
\frac{d\sigma}{dx_B \, dy \, d\psi \, d\phi_h \, dP_{h\perp}} = \int \frac{d^2 b_T}{(2\pi)^2} \, e^{-ib_T \cdot P_{h\perp}} \left\{ \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{(1 - \varepsilon)} \left(1 + \frac{\gamma^2}{2x_B}\right) L_{\mu\nu} \tilde{W}^{\mu\nu} \right\}
$$

$b$-space is natural for TMD evolution and has clear physical interpretation of transverse separation of quark fields in the bi-local matrix element.

$$
\tilde{f}(x, \vec{b}_T) = \int d^2 k_{\perp} \, e^{i\vec{b}_T \cdot \vec{k}_{\perp}} \, f(x, \vec{k}_{\perp})
$$

Boer, Gamberg, Musch, Prokudin JHEP 1110 (2011)
Unpolarized cross-section

\[
\sigma = \sigma_0 \int \frac{db_T b_T d\varphi}{(2\pi)^2} e^{-i|b_T|P_{h\perp} \cos \varphi} F_{UU} \\
= \sigma_0 \int \frac{db_T b_T}{(2\pi)} J_0(|b_T|P_{h\perp}) F_{UU}
\]

Structure functions in b-space

\[
F_{UU} = \sum_a e_a^2 \tilde{f}(x, z^2 b_T^2) \cdot \tilde{D}(z, b_T^2)
\]

**A product of FT TMDs instead of the convolution!**

Boer, Gamberg, Musch, Prokudin JHEP 1110 (2011)
Bessel Weighting

Fourier transformed TMDs

$$\tilde{f}(x, b_T^2) = \int d^2 k_\perp e^{ib_T k_\perp} f(x, k_\perp^2) = 2\pi \int dk_\perp J_0(b_T k_\perp) f(x, k_\perp^2)$$

Fourier transformed derivatives TMDs

$$\tilde{f}^{(n)}(x, b_T^2) \equiv \left( -\frac{2}{M^2} \partial_{b_T^2} \right) \tilde{f}(x, b_T^2)$$

Related to “conventional” moments

$$\tilde{f}^{(n)}(x, 0) \equiv \int d^2 k_\perp \left( \frac{k_\perp^2}{2M^2} \right)^n f(x, k_\perp^2) \equiv f^{(n)}(x)$$

---

Boer, Gamberg, Musch, Prokudin JHEP 1110 (2011)
Bessel Weighting

Unpolarized cross-section

\[ \sigma = \sigma_0 \int \frac{d b_T b_T}{(2\pi)} J_0(|b_T||P_{h\perp}|)\mathcal{F}_{UU} \]

Bessel functions are orthogonal!

\[ \int_0^\infty d|P_{h\perp}| |P_{h\perp}| J_n(|P_{h\perp}| |b_T|) J_n(|P_{h\perp}| B_T) = \frac{1}{B_T} \delta(|b_T| - B_T) \]

\[ \int d|P_{h\perp}| |P_{h\perp}| d\Phi_h J_0(B_T |P_{h\perp}|)\sigma = 2\pi \sigma_0 x \sum_a e_a x f(x, z^2 B_T^2) \cdot \tilde{D}(z, B_T^2) \]

\[ B_T \text{ (GeV}^{-1}) \text{ is a parameter} \]

Boer, Gamberg, Musch, Prokudin JHEP 1110 (2011)
Bessel Weighting: asymmetries

SIDIS cross-section contains structure functions with

\[ \sigma \sim \int \frac{db_T b_T}{(2\pi)} \left( \ldots J_0(|b_T||P_{h\perp}|) + \ldots J_1(|b_T||P_{h\perp}|) + \ldots J_2(|b_T||P_{h\perp}| + \ldots J_3(|b_T||P_{h\perp}|) \right) \]

Introduce a weight:

\[ w_n \equiv J_n(|P_{h\perp}|B_T) \ n! \left( \frac{2}{B_T} \right)^n \]

n=0,1,2,3

Use the weight with the experimental data. Collins effect as an example:

\[ A_{UT}^{2J_1(|P_{h\perp}|B_T)} \sin(\phi_h+\phi_s) (B_T) = 2 \frac{\alpha^2}{yQ^2} \frac{y^2}{(1-\varepsilon)} \left( 1 + \frac{\gamma^2}{2x_B} \right) \varepsilon \]

\[ \times \sum_a e_a^2 H_{UT}^{\sin(\phi_h+\phi_s)}(Q^2, \mu^2, \rho) \tilde{h}_1^{(0)a}(x, z^2 B_T^2; \mu^2, \zeta, \rho) \tilde{H}_1^{(1)a}(z, B_T^2; \mu^2, \zeta, \rho) \]

Boer, Gamberg, Musch, Prokudin JHEP 1110 (2011)
Can we do it?

$|P_{h\perp}|_{\text{res}}$ resolution of binning. $\Lambda_{TMD} < Q$ region where TMD is still valid.

Boer, Gamberg, Musch, Prokudin JHEP 1110 (2011)
Feasibility study

Agasyan, Avakyan, De Sactis, Gamberg, Mirazita, Musch, Prokudin, Rossi JHEP 1110 (2015)

\[
\frac{d\sigma}{dxdydzd\phi_h dP_{h\perp}^2} = K(x, y) \int \frac{db_T b_T}{2\pi} J_0(b_T P_{h\perp}) \left( \mathcal{F}_{UU,T}(b_T) + S_\parallel \lambda e^{\sqrt{1 - \epsilon^2}} \mathcal{F}_{LL}(b_T) \right)
\]

Generated in JLab 12 kinematics. Acceptance effects and smearing are Studied.

\[\langle x \rangle = 0.22, \quad \langle z \rangle = 0.51\]

\[b_T < 6 \text{ (GeV}^{-1})\]

Systematical errors \(~\text{few }\%\)

\[A_{LL}^J(b_T P_{h\perp}) \text{ plotted versus } b_T\]
Conclusions

• Our new method is a generalization of naïve weighting and resolves the problems of the latter

• TMDs can be studied directly in b-space thus allowing for complimentary information to momentum space measurements

• Instead of complicated convolutions one will have to deal with simple products in b-space
Conclusions

• Our new method is a generalization of naïve weighting and resolves the problems of the latter

• TMDs can be studied directly in b-space thus allowing for complimentary information to momentum space measurements

• Instead of complicated convolutions one will have to deal with simple products in b-space

THANK YOU!