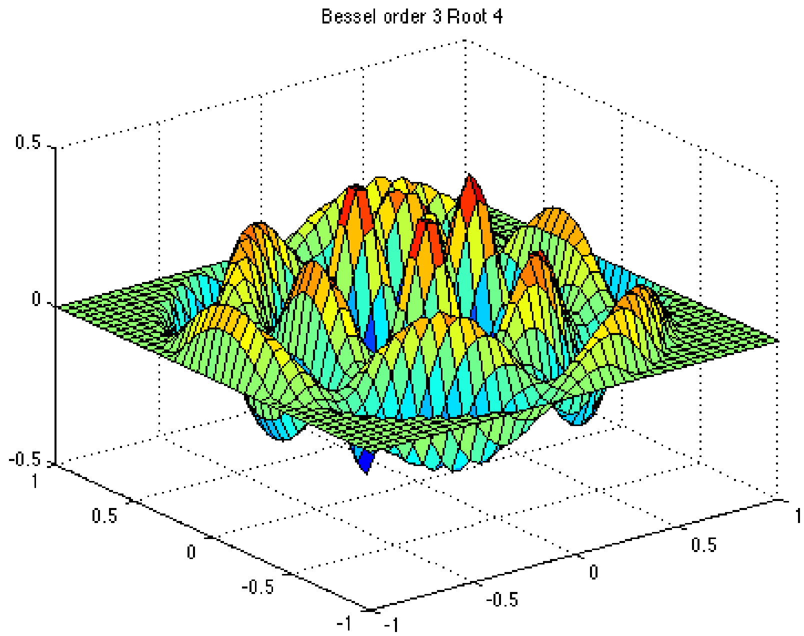
Bessel Weighted Asymmetries

Alexei Prokudin



May 29, 2015

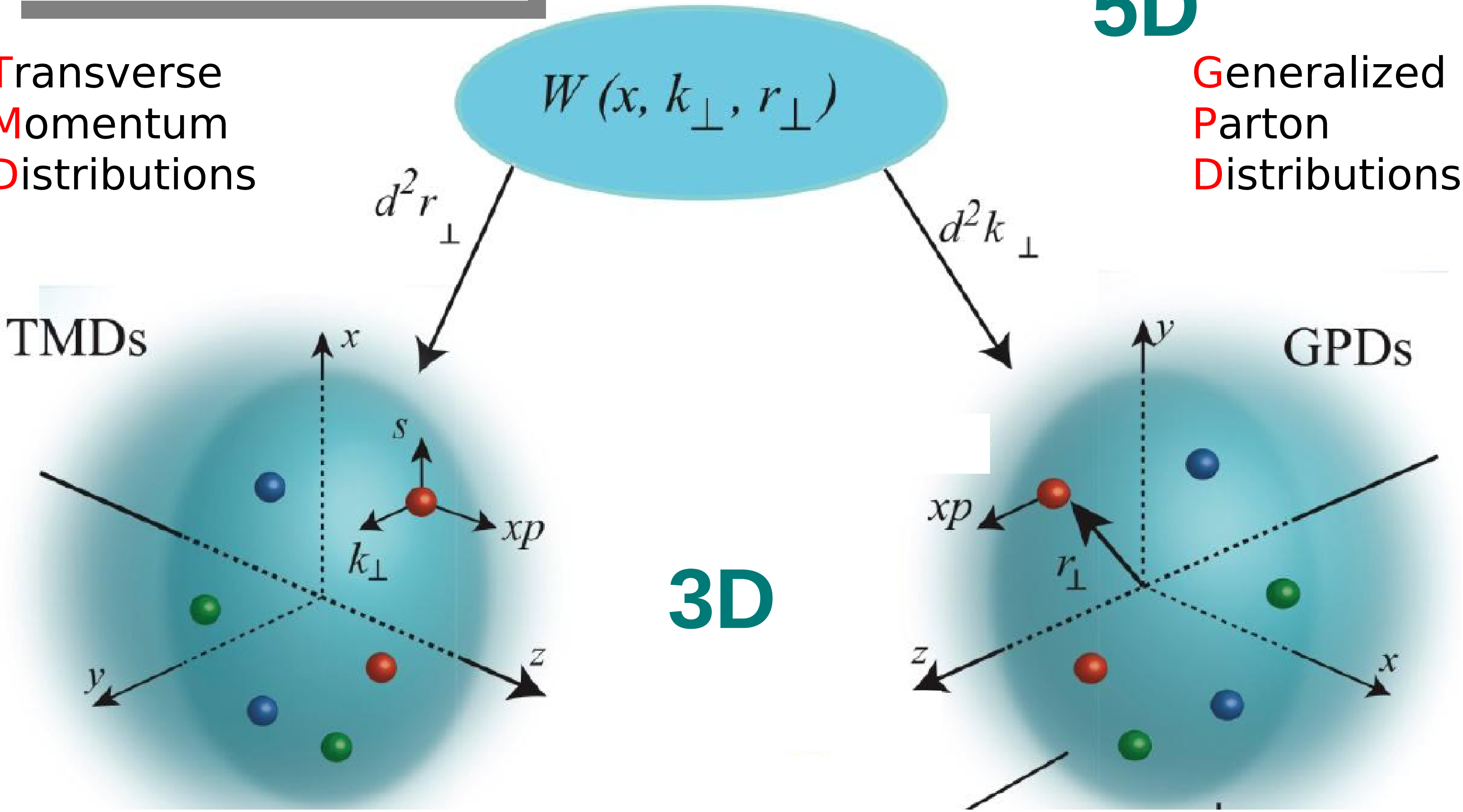
Unified View of Nucleon Structure

Wigner Distribution

5D

Transverse
Momentum
Distributions

Generalized
Parton
Distributions

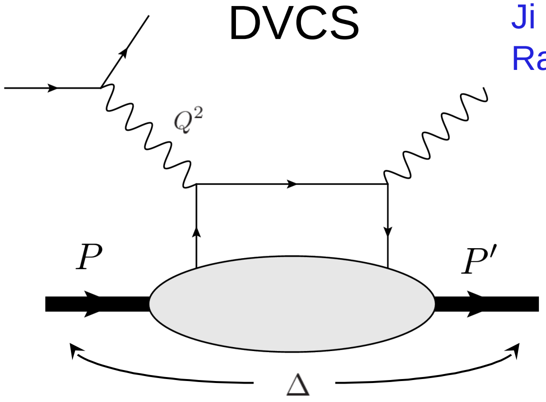


3D

GPDs

DVCS

Ji (1997)
Radyushkin (1997)



Q^2 ensures hard scale, pointlike interaction

$\Delta = P' - P$ momentum transfer can be varied independently

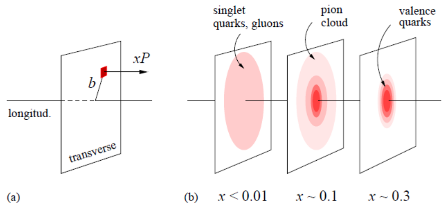
Connection to 3D structure

Burkardt (2000)
Burkardt (2003)

$$\rho(x,\vec{r}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp\cdot\vec{r}_\perp} H_q(x,\xi=0,t=-\vec{\Delta}_\perp^2)$$

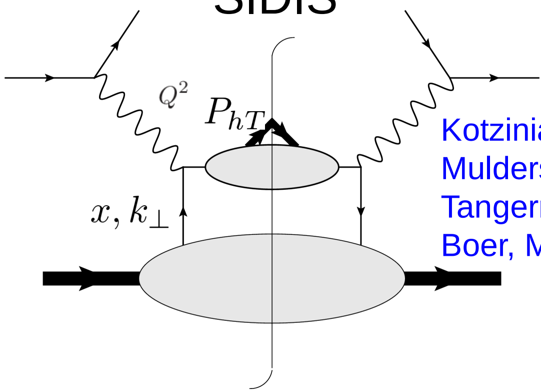
Drell-Yan frame $\Delta^+ = 0$

Weiss (2009)



TMDs

SIDIS



Kotzinian (1995),
Mulders,
Tangerman (1995),
Boer, Mulders (1998)

Q^2 ensures hard scale, pointlike interaction

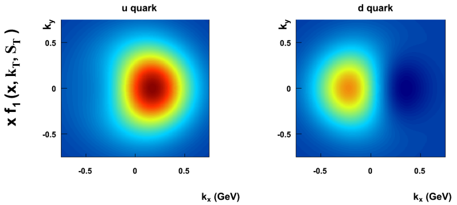
P_{hT} final hadron transverse momentum can be varied independently

Connection to 3D structure

Ji, Ma, Yuan (2004)
Collins (2011)

$$\tilde{f}(x,\vec{b}_T) = \int d^2k_\perp e^{i\vec{b}_T\cdot\vec{k}_\perp} f(x,\vec{k}_\perp)$$

AP (2012)



TMD distributions

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

8 functions in total (at leading twist)

Each represents different aspects of partonic structure

Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

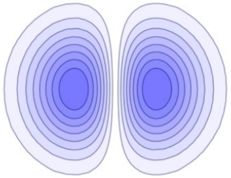
TMD distributions

Three types of modulations

$$f(x, \mathbf{k}_\perp^2)$$

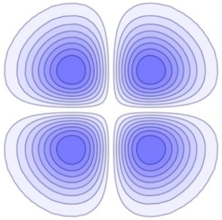
Monopole

$$\frac{\mathbf{k}_{\perp i} S_{Ti}}{M} f(x, \mathbf{k}_\perp^2)$$



Dipole

$$\frac{\mathbf{k}_\perp^i \mathbf{k}_\perp^j - \frac{1}{2}\mathbf{k}_\perp^2 g_T^{ij}}{M^2} f(x, \mathbf{k}_\perp^2)$$



Quadrupole

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

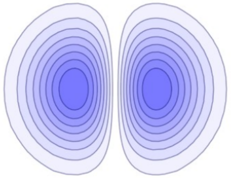
TMD distributions

Extraction of the nucleon structure from the data is one of the main goals of modern facilities such as Jefferson Lab 12

$$f(x, \mathbf{k}_\perp^2)$$

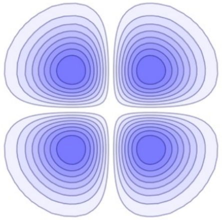
Monopole

$$\frac{\mathbf{k}_{\perp i} S_{Ti}}{M} f(x, \mathbf{k}_\perp^2)$$



Dipole

$$\frac{\mathbf{k}_\perp^i \mathbf{k}_\perp^j - \frac{1}{2}\mathbf{k}_\perp^2 g_T^{ij}}{M^2} f(x, \mathbf{k}_\perp^2)$$

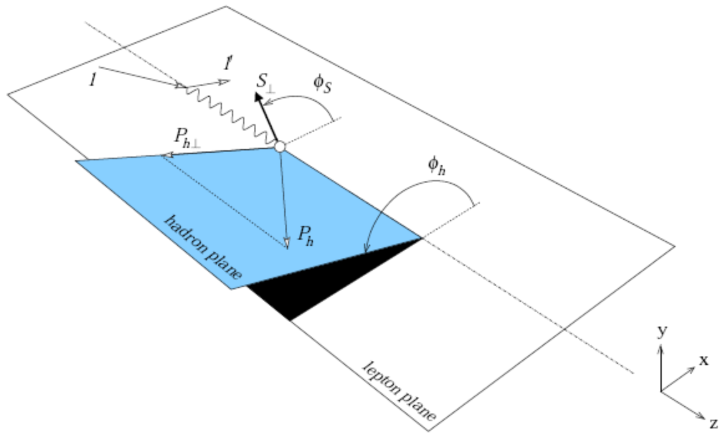


Quadrupole

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

How can TMDs be studied?

Semi Inclusive Deep Inelastic Scattering



One can rewrite the cross-section in terms of **18** structure functions

Each structure function encodes parton dynamics via convolutions of TMDs when factorization is applicable

Mulders, Tangerman (1995),
Boer, Mulders (1998)
Bacchetta et al (2007)

$$\frac{d\sigma}{dx\, dy\, d\psi\, dz\, d\phi_h\, dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\, F_{UU}^{\cos\phi_h}$$

$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \;+\ldots$$

Structure functions

Structure functions are convolutions of unobserved partonic momenta:

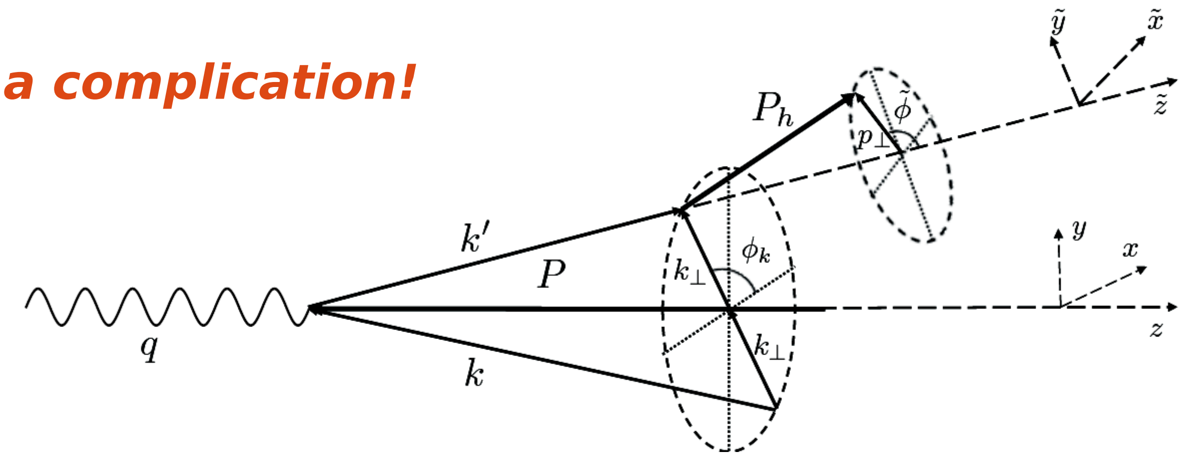
$$F \sim \int d^2\vec{k}_\perp d^2\vec{p}_\perp \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp}) f(x, \vec{k}_\perp) D(z, \vec{p}_\perp)$$

Structure functions are convolutions of unobserved partonic momenta:

$$F \sim \int d^2\vec{k}_\perp d^2\vec{p}_\perp \delta^{(2)}(\overbrace{z\vec{k}_\perp + \vec{p}_\perp}-\vec{P}_{h\perp}) f(x,\vec{k}_\perp) D(z,\vec{p}_\perp)$$

Observed in experiment

The convolution is a complication!



Can we find a more direct measure?

Weighted structure functions

An attempt to unravel the convolution:

$$\int d^2\vec{P}_{h\perp} F \sim \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{P}_{h\perp}\delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_{h\perp}) f(x,\vec{k}_\perp) D(z,\vec{p}_\perp)$$

$$= \left(\int d^2\vec{k}_\perp f(x,\vec{k}_\perp)\right)\left(\int d^2\vec{p}_\perp D(z,\vec{p}_\perp)\right) \equiv f^{(0)}(x)\cdot D^{(0)}(z)$$

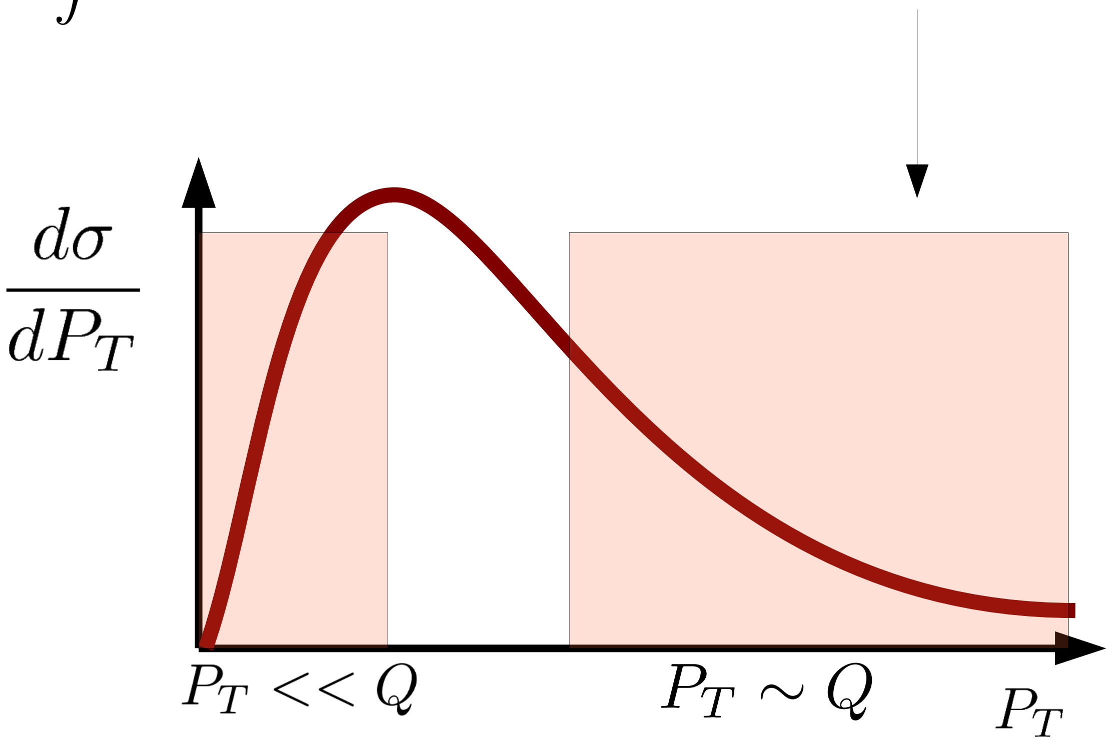
A product instead of the convolution!

Kotzinian, Mulders (1997), Boer, Mulders (1998)

The problem:

TMD factorization has a validity region $P_T \ll Q$

Integration $\int dP_{h\perp} d\phi_h P_{h\perp}$ emphasizes high momentum region

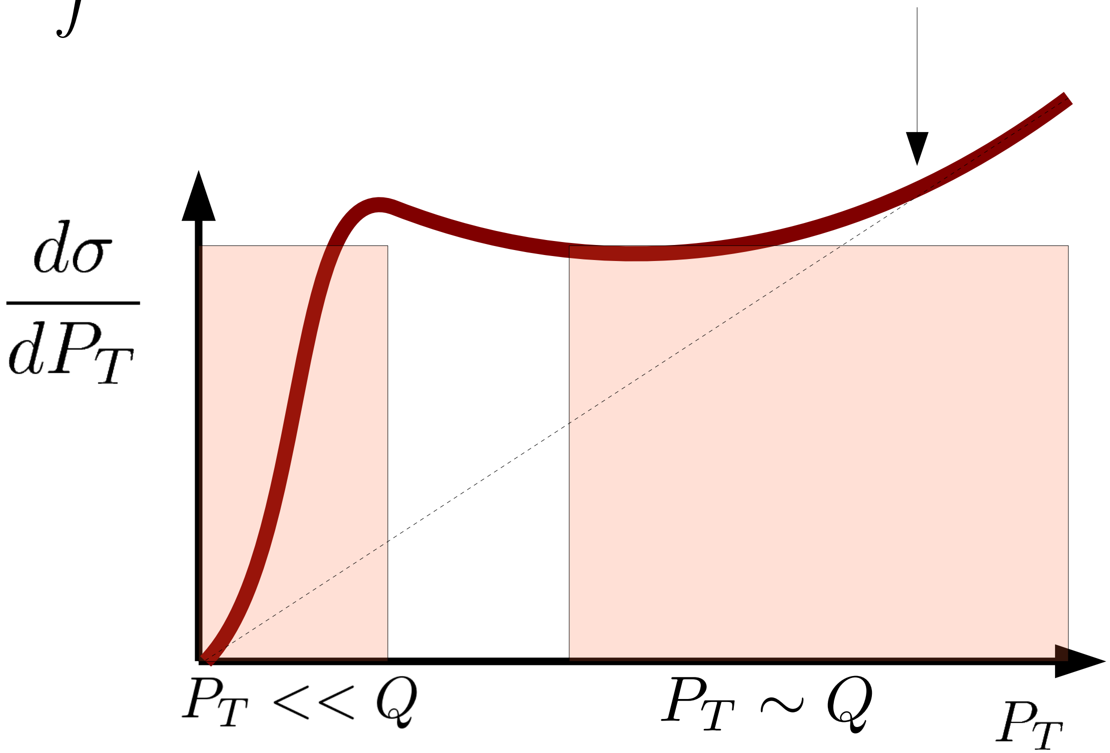


Collinear contributions will "spoil" our interpretation!

The problem:

TMD factorization has a validity region $P_T \ll Q$

Integration $\int dP_{h\perp} d\phi_h P_{h\perp}$ emphasizes high momentum region



Collinear contributions will "spoil" our interpretation!

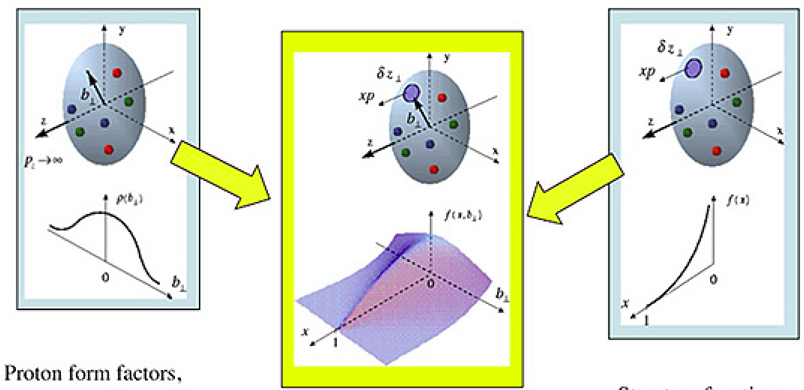
Can we circumvent the problem and emphasize the region of TMD?

INT 2010 Think Tank

Gluons and the quark sea at high energies: distributions, polarization, tomography

September 13 to November 19, 2010

Report from the INT program "Gluons and the quark sea at high energies: distributions, polarization, tomography"



Proton form factors, transverse charge & current densities

Correlated quark momentum and helicity distributions in transverse space - GPDs

Structure functions, quark longitudinal momentum & helicity distributions

Daniel Boer
Leonard Gamberg
Berni Musch
AP

Came up with an idea!

Boer, Gamberg, Musch, Prokudin JHEP 1110 (2011)

The idea:
Use a function that suppresses large momenta

The cross-section is a Fourier transform of b-space expression:

$$\frac{d\sigma}{dx_B\, dy\, d\psi\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}| d|\boldsymbol{P}_{h\perp}|} = \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2}\, e^{-i\boldsymbol{b}_T\cdot \boldsymbol{P}_{h\perp}} \left\{ \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{(1-\varepsilon)} \left(1+\frac{\gamma^2}{2x_B}\right) L_{\mu\nu}\tilde{W}^{\mu\nu} \right\}$$

b-space is natural for TMD evolution and has clear physical interpretation of transverse separation of quark fields in the bi-local matrix element

$$\tilde{f}(x,\vec{b}_T) = \int d^2k_\perp e^{i\vec{b}_T\cdot\vec{k}_\perp} f(x,\vec{k}_\perp)$$

Boer, Gamberg, Musch, Prokudin JHEP 1110 (2011)

Bessel Weighting

The idea:
Use a function that suppresses large momenta

Unpolarized cross-section

$$\sigma = \sigma_0 \int \frac{db_T b_T d\varphi}{(2\pi)^2} e^{-i|b_T||P_{h\perp}|cos\varphi} \mathcal{F}_{UU}$$

$$= \sigma_0 \int \frac{db_T b_T}{(2\pi)} J_0(|b_T||P_{h\perp}|) \mathcal{F}_{UU}$$

Structure functions in b-space

$$\mathcal{F}_{UU} = x \sum_a e_a^2 \tilde{f}(x, z^2 b_T^2) \cdot \tilde{D}(z, b_T^2)$$

A product of FT TMDs instead of the convolution!

Boer, Gamberg, Musch, Prokudin JHEP 1110 (2011)

Bessel Weighting

Fourier transformed TMDs

$$\tilde{f}(x, b_T^2) = \int d^2k_\perp e^{ib_T k_\perp} f(x, k_\perp^2) = 2\pi \int dk_\perp k_\perp J_0(b_T k_\perp) f(x, k_\perp^2)$$

Fourier transformed derivatives TMDs

$$\tilde{f}^{(n)}(x, b_T^2) \equiv \left(-\frac{2}{M^2} \partial_{b_T^2} \right) \tilde{f}(x, b_T^2)$$

Related to “conventional” moments

$$\tilde{f}^{(n)}(x, 0) \equiv \int d^2k_\perp \left(\frac{k_\perp^2}{2M^2} \right)^n f(x, k_\perp^2) \equiv f^{(n)}(x)$$

Boer, Gamberg, Musch, Prokudin JHEP 1110 (2011)

Bessel Weighting

The idea:
Use a function that suppresses large momenta

Unpolarized cross-section

$$\sigma = \sigma_0 \int \frac{db_T b_T}{(2\pi)} J_0(|b_T||P_{h\perp}|)\mathcal{F}_{UU}$$

Bessel functions are orthogonal!

$$\int_0^\infty d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,J_n(|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{b}_T|)\,J_n(|\boldsymbol{P}_{h\perp}|\,\mathcal{B}_T) = \frac{1}{\mathcal{B}_T}\delta(|\boldsymbol{b}_T| - \mathcal{B}_T)$$

$$\int dP_{h\perp} P_{h\perp} d\Phi_h J_0(\mathcal{B}_T|P_{h\perp}|)\sigma = 2\pi\sigma_0 x \sum_a e_a^2 \tilde{f}(x, z^2\mathcal{B}_T^2)\cdot\tilde{D}(z, \mathcal{B}_T^2)$$

$\mathcal{B}_T\ (\mathrm{GeV}^{-1})$ is a parameter

Boer, Gamberg, Musch, Prokudin JHEP 1110 (2011)

Bessel Weighting: asymmetries

SIDIS cross-section contains structure functions with

$$\sigma \sim \int \frac{db_T b_T}{(2\pi)} (...J_0(|b_T||P_{h\perp}|) + ...J_1(|b_T||P_{h\perp}|) + ...J_2(|b_T||P_{h\perp}| + ...J_3(|b_T||P_{h\perp}|))$$

Introduce a weight:

$$w_n \equiv J_n(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)\, n! \left(\frac{2}{\mathcal{B}_T}\right)^n$$

n=0,1,2,3

Use the weight with the experimental data. Collins effect as an example:

<u>Collins</u>

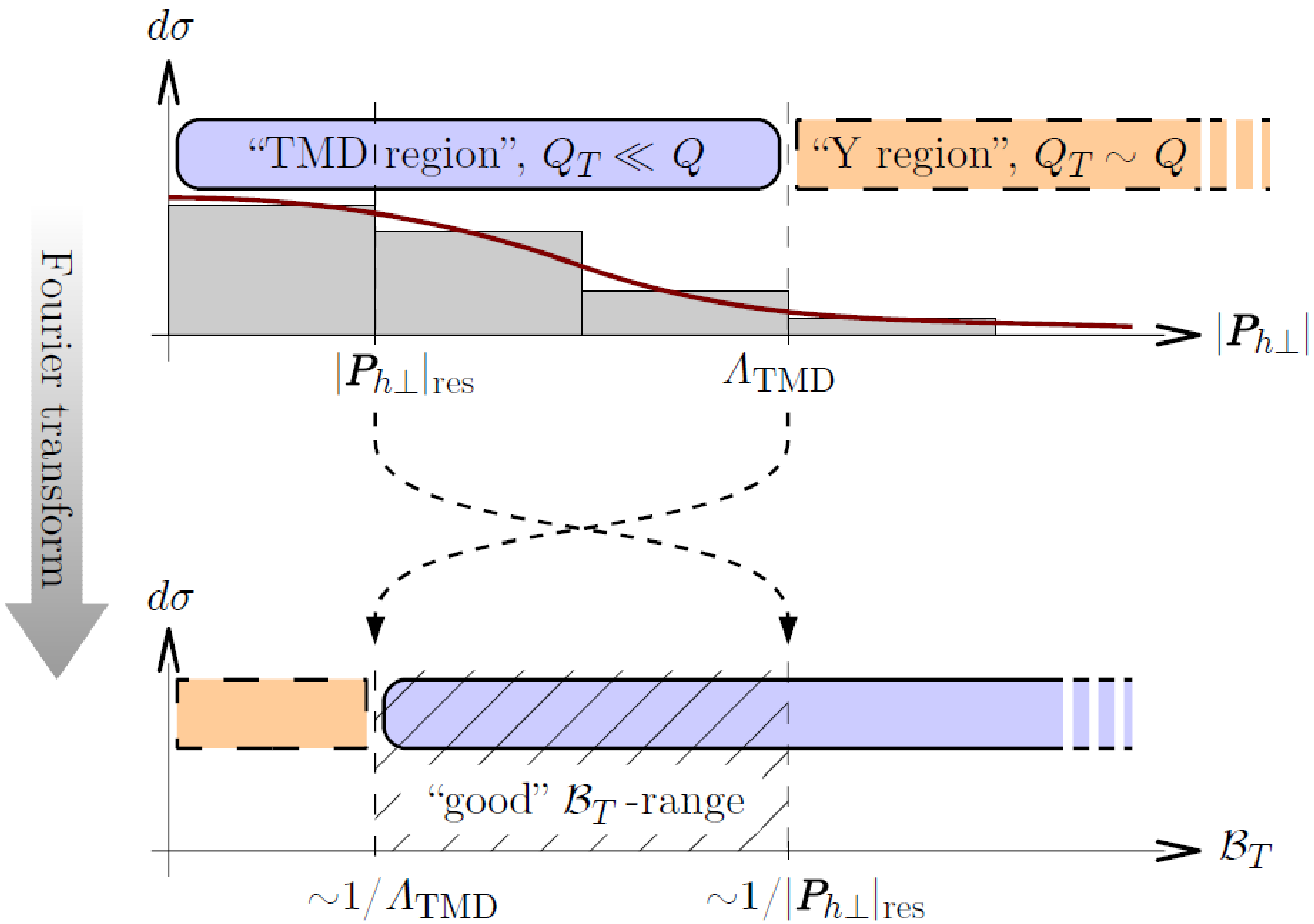
$$A_{UT}^{\frac{2\,J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)}{zM_h\mathcal{B}_T}\sin(\phi_h+\phi_s)}(\mathcal{B}_T) = 2\frac{\frac{\alpha^2}{yQ^2}\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)\varepsilon}{\frac{\alpha^2}{yQ^2}\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)}$$

$$\times\frac{\sum_a e_a^2\, H_{UT}^{\sin(\phi_h+\phi_S)}(Q^2,\mu^2,\rho)\,\tilde{h}_1^{(0)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{H}_1^{\perp(1)a}(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}{\sum_a e_a^2\, H_{UT}(Q^2,\mu^2,\rho)\,\tilde{f}_1^{(0)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^{(0)a}(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}$$

Boer, Gamberg, Musch, Prokudin JHEP 1110 (2011)

Can we do it?

$|P_{h\perp}|_{res}$ resolution of binning. $\Lambda_{TMD} < Q$ region where TMD is still valid

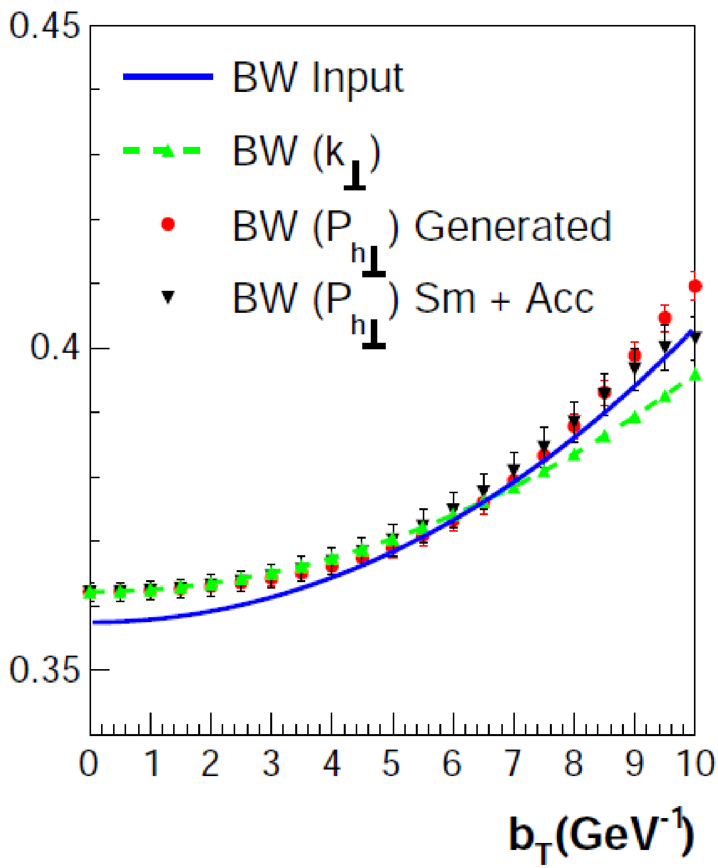


Boer, Gamberg, Musch, Prokudin JHEP 1110 (2011)

Feasibility study

Agasyan, Avakyan, De Sactis, Gamberg, Mirazita, Musch, Prokudin, Rossi JHEP 1110 (2015)

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = K(x,y)\int\frac{db_T\,b_T}{2\pi}J_0(b_T\,P_{h\perp})\left(\mathcal{F}_{UU,T}(b_T)+S_{||}\lambda_e\sqrt{1-\varepsilon^2}\mathcal{F}_{LL}(b_T)\right)$$



Generated in JLab 12 kinematics. Acceptance effects and smearing are Studied.

$$\langle x\rangle = 0.22,\ \langle z\rangle = 0.51$$

$$b_T < 6\ (\mathrm{GeV}^{-1})$$

Systematical errors ~ few %

$A_{LL}^{J_0(b_T P_{h\perp})}$ plotted versus b_T.

Conclusions

Conclusions

- Our new method is a generalization of naïve weighting and resolves the problems of the latter

- TMDs can be studied directly in b-space thus allowing for complimentary information to momentum space measurements

- Instead of complicated convolutions one will have to deal with simple products in b-space

Conclusions

- Our new method is a generalization of naïve weighting and resolves the problems of the latter

- TMDs can be studied directly in b-space thus allowing for complimentary information to momentum space measurements

- Instead of complicated convolutions one will have to deal with simple products in b-space

THANK YOU!