QCD Evolution 2015, Jefferson Lab, May 26–30, 2015

GPDs in the QCD multi-color limit

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work in collaboration with Christian Weiss

Overview:

- GPDs, brief overview, non-perturbative properties
- Limit of a large number of colors N_c in QCD
- Powerful first principle approach+guideline
- Applications to nucleon structure
- Chiral odd GPDs
- Conclusions

What are GPDs?

quark correlator: $\mathcal{M}(\Gamma)_{\lambda'\lambda} = \int \frac{\mathrm{d}P^+ z^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \overline{\psi}_q(-\frac{z}{2}) \, \Gamma \, [-\frac{z}{2}, \frac{z}{2}] \psi_q(\frac{z}{2}) \, | p, \lambda \rangle \Big|_{z^+ = z_T^j = 0}$

kinematic variables: $P = \frac{1}{2}(p' + p), \;\; \Delta = (p' - p), \;\; \xi = -\frac{\Delta^+}{P^+}, \;\; t = \Delta^2$

quark GPDs

$$\mathcal{M}(\gamma^+) \;=\; \overline{u}(p', \lambda') \left[\;\gamma^+ \;\; H^q + \frac{i\sigma^{+\mu}\Delta_\mu}{2M_N} \, E^q \right] u(p, \lambda)$$

$$\mathcal{M}(\gamma^+\gamma_5) \;=\; \overline{u}(p', \lambda') \left[\gamma^+\gamma_5 \; \tilde{H}^q \;+\; \frac{i\gamma_5\Delta^+}{2M_N} \; \tilde{E}^q \right] u(p, \lambda)$$

$$\begin{aligned} \mathcal{M}(i\sigma^{+j}) \;=\; & \overline{u}(p', \lambda') \left[\;\sigma^{+j} \;\; H_T^q + \frac{\gamma^+\Delta^j - \Delta^+\gamma^j}{M_N^2} \; E_T^q \right. \\ & \left. + \frac{P^+\Delta^j - \Delta^+P^j}{M_N^2} \; \tilde{H}_T^q + \frac{\gamma^+P^j - P^+\gamma^j}{M_N^2} \; \tilde{E}_T^q \right] u(p, \lambda) \end{aligned}$$

with $\mathrm{GPD}^q = \mathrm{GPD}^q(x, \xi, t; \mu^2)$

gluon GPDs analog (discussed elsewhere)

non-perturbative properties of GPDs

general constraints

- time-reversal & hermiticity: $\mathrm{GPD}(x,-\xi,t) = \pm \mathrm{GPD}(x,\xi,t) = \begin{cases} H, E, \tilde{H}, \tilde{E}, H_T, E_T, \tilde{H}_T \\ \tilde{E}_T \end{cases}$
- Lorentz-invariance: polynomiality $\int \mathrm{d}x\, x^N \mathrm{GPD}(x,\xi,t) = a_0(t) + a_2(t)\,\xi^2 + \ldots + a_N\,\xi^N$
- inequalities (. . . , Pobylitsa, Pobylitsa & Kirch)

first principles

- lattice QCD (valuable information on Mellin moments, form factors)
- chiral perturbation theory (Diehl, Manashov, Schäfer; Kivel, Polyakov, Vladimirov)
- **large N_c** $\Longleftarrow$ focus of this talk
- . . .

effective methods, models

- bag (Ji et al), chiral quark soliton (Petrov et al; . . .), light-cone models (Pasquini et al; Lorcé), diquark (Simonetta, Gary), AdS QCD (Stan Brodsky, . . .), Schwinger-Dyson (Moutarde, Roberts. . .)

What is large N_c limit?

experiment

- $N_c = 3$ pretty well established. Period.

theory

- limit $N_c \to \infty$ powerful tool, insightful guideline, first principle method

statement

- $\frac{1}{N_c}$ is the only small parameter of QCD at *all* energies (S. Coleman, "Aspects of Symmetry")

technical remarks

- $g_s^2 N_c =$ fixed as limit $N_c \to \infty$ is taken 't Hooft limit
- planar diagrams dominate; in 1+1 possible to resum 't Hooft NPB 72 & 75 (1974) 461
- in 3+1 QCD not solved (neither for finite nor for infinite N_c). But ...

conclusions in large N_c

- meson & glueballs are free, stable, non-interacting particles,
  decay widths $\sim 1/\sqrt{N_c}$, coupling constants $\sim 1/N_c$ ('t Hooft; Veneziano; Callan et al; ... 1970s)
- mesons $= \bar{q}q$ states
  (and not $\bar{q}q\bar{q}q$ or $\bar{q}gq$, which are not abundant in nature; maybe GluEx?)
- Zweig rule = exact, if $N_c \to \infty$ (only explanation for Zweig rule)
- $|\mathbf{baryons}\rangle = |\mathbf{solitons\ of\ meson\ fields}\rangle$ Witten, NPB 160 (1979) 57
  solitonic field not computed from first principles so far (QCD not solved ...)
  but **symmetry properties** of large-N_c mean field **known** $\Longleftarrow$ **explore!**

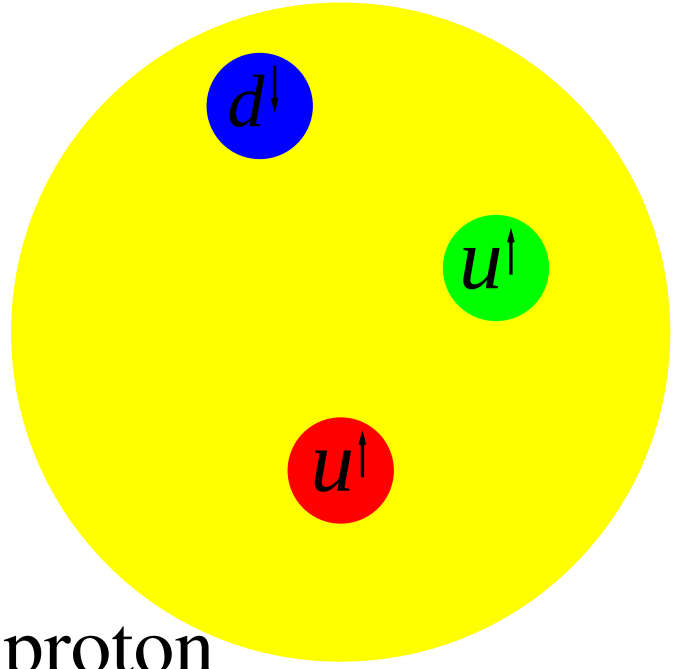
How to explore?

- "Witten-approach:" mean field + symmetries of mean field
- "Dashen-Jenkins-Manohar-approach:" abstract group theoretical exploration of symmetries
- equivalent methods

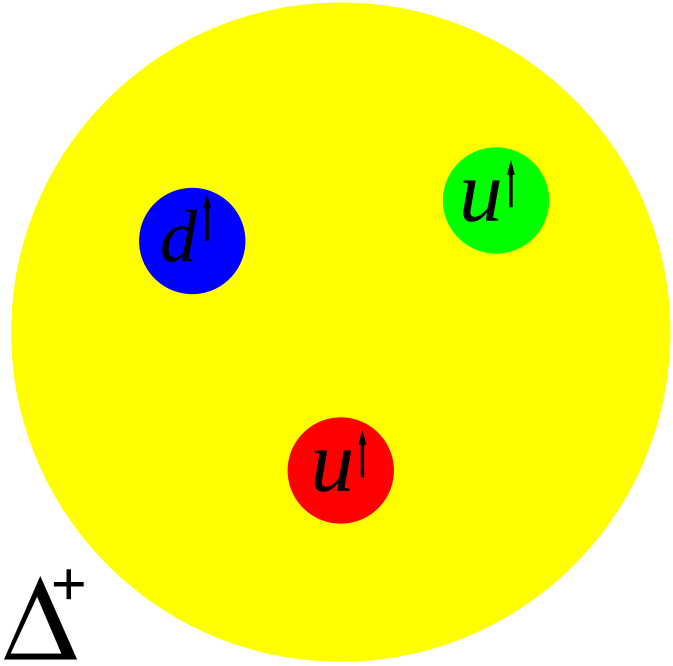
Remark: "color counting" is independent of dynamics!
same results from large-N_c QCD, Skyrme model, quark models.

In following: technical details of rigorous derivations skipped
when possible: quark-model-intuition used as illustration

for $N_c = 3$
baryon $\equiv$ 3q in antisymmetric color state $|B\rangle = \varepsilon^{abc}|q_a, q_b, q_c\rangle$, cartoon:



proton

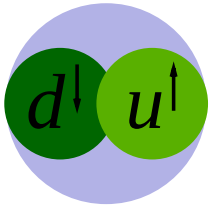


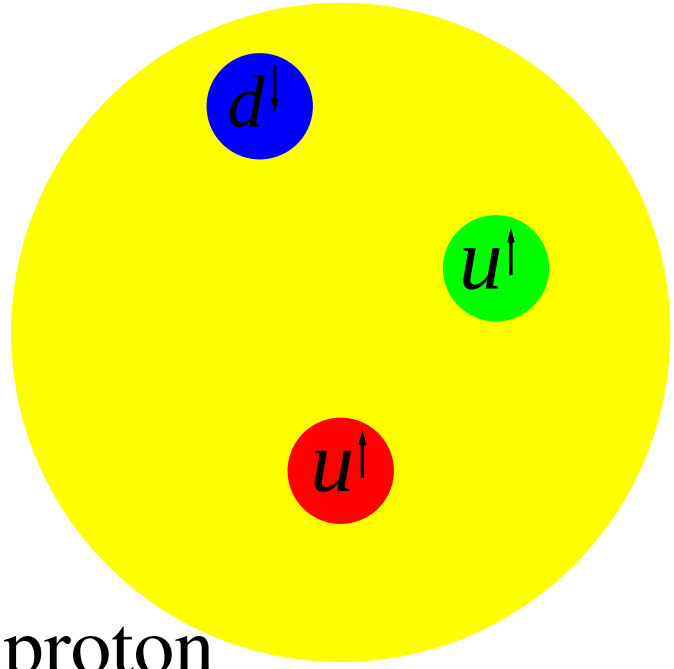
Δ^+

How would nucleon & Δ-resonance look for $N_c = 3, 5, 7, \ldots$?

$|\text{nucleon}\rangle = |S_N = \frac{1}{2},\, T_N = \frac{1}{2}\rangle,\ |\Delta\rangle = |S_\Delta = \frac{3}{2},\, T_\Delta = \frac{3}{2}\rangle$

constructing baryons in large N_c
(illustration in quark model, but result generic)
must preserve spin and isospin(!) $\rightarrow$ add light quarks in scalar-isoscalar pairs
$\rightarrow$ only in this way $|S, S_3\rangle$, $|T, T_3\rangle$ unchanged!





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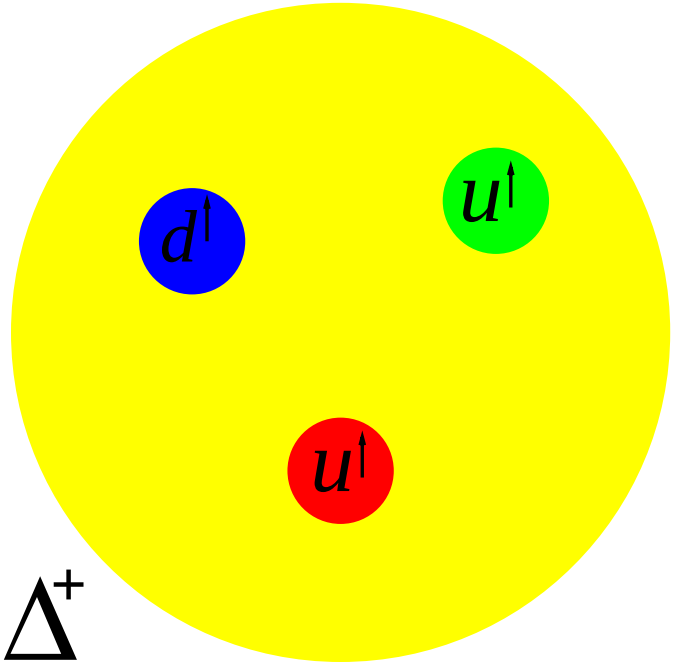

Δ^+

important:
all u with $S^\uparrow$
all d with $S^\downarrow$
w.f. symm. under
exchange of u (or d)
anti-symmetry due to
$\varepsilon^{12\ldots N_c}|q_1, q_2, \ldots, q_{N_c}\rangle$

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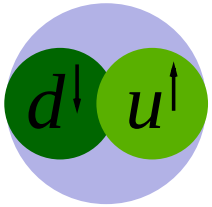
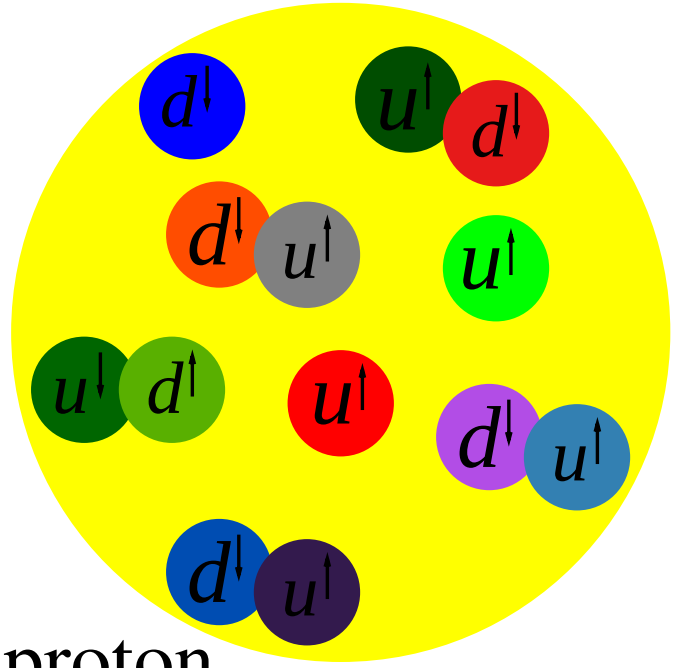
$\rightarrow$ **not "diquarks"** (quantum #'s correlated)





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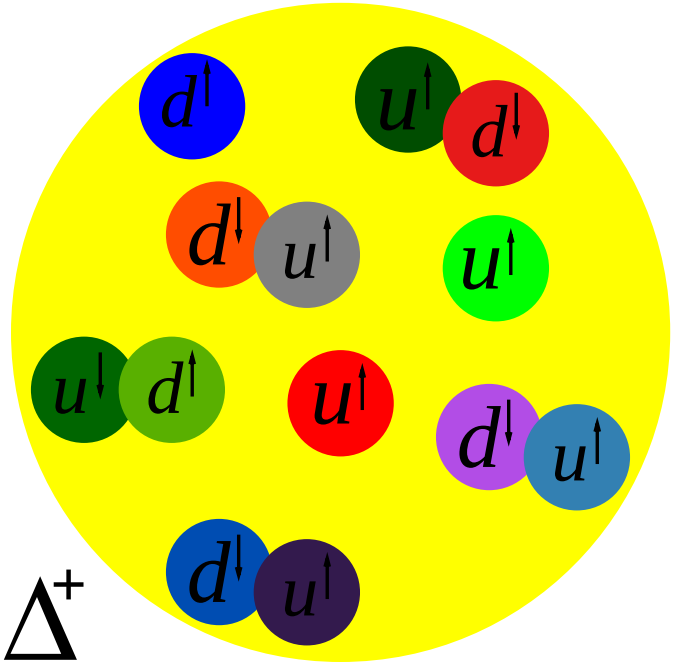

Δ^+

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anti-symmetry due to $\varepsilon^{12\ldots N_c}|q_1, q_2, \ldots, q_{N_c}\rangle$

Observations:

- not surprising we have a static mean field ("soliton")!
- total nucleon spin = result of enormous cancellations
- if for some reason $\exists$ spin of one flavor tends to be (anti-)aligned with nucleon spin
  $\Rightarrow$ the other other flavor automatically exhibits opposite polarization

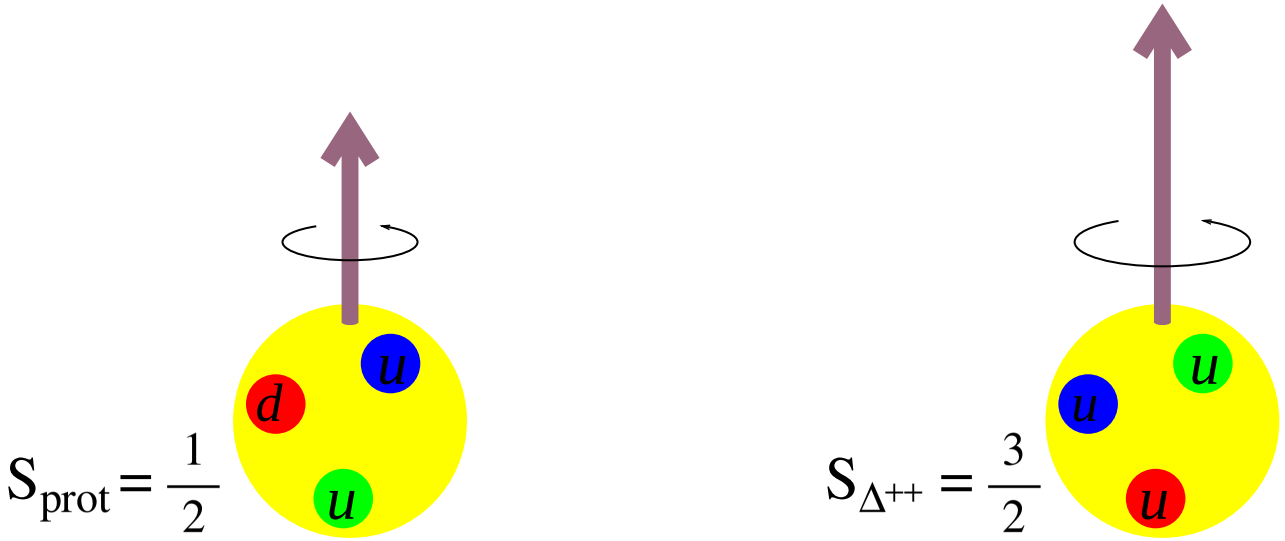
  (does not tell the magnitude of a correlation between flavor- and nucleon-polarization,
  $\rightarrow$ need dynamics e.g. chiral dynamics)

technical description

- $\boldsymbol{\langle B', p'| \hat{O}_{\rm QCD} |B, p\rangle = 2M_N \int {\rm d}^3 X\, e^{i(\vec{p}-\vec{p}')\vec{X}} \int {\rm d}R\, \phi^*_{B'}(R)\, O(R)_{\rm eff}\, \phi_B(R) + \ldots}$
  $R \in$ SU(2) describes collective coordinates in 3-space & flavor-space
  $\phi_B(R) =$ rotational wave-function (defined in terms of Wigner matrices)
  $O(R)_{\rm eff}$ is the effective mean-field function associated with the operator $\hat{O}_{\rm QCD}$
- $O(R)_{\rm eff}$ is unknown (because 3+1 QCD in large N_c not solved)
  but it's symmetry properties are known!
  $\Rightarrow$ sufficient!

large-N_c mean field picture:

- baryons are massive objects $M_B \sim N_c$
- nucleon and Δ different rotational states of the *same* soliton solution



- this has consequences.
  E.g. baryon masses $M_B = N_c\, a_1 + N_c^0\, a_0 + \ldots$
  but . . . (see next page)

results (static properties):

- **mass-splittings $M_\Delta - M_N \sim \mathcal{O}(1/N_c)$**

$$\frac{M_\Delta - M_N}{M_\Delta + M_N} = \begin{cases} \mathcal{O}(\frac{1}{N_c^2}) & \text{vs} \\ 0.135 \sim \frac{1}{7} & \text{exp.} \end{cases} \quad \text{(reasonable)} \surd$$

- **anomalous magnetic moments:**

$$\kappa^u - \kappa^d \sim \mathcal{O}(N_c) \ \text{vs} \ \ 3.71|_{\text{exp}}$$
$$\kappa^u + \kappa^d \sim \mathcal{O}(N_c^0) \ \text{vs} \ -0.36|_{\text{exp}} \quad \text{(extremely reasonable)} \surd$$

- **axial properties:**

$$g_A^{(3)} = \Delta u - \Delta d \sim \mathcal{O}(N_c) \ \text{vs} \ 1.26|_{\text{exp}}$$
$$g_A^{(0)} = \Delta u + \Delta d \sim \mathcal{O}(N_c^0) \ \text{vs} \ 0.30|_{\text{exp, scheme, at few GeV}} \quad \text{(works just fine)} \surd$$

- **etc ...**

partonic (dynamical) properties:

- nucleon heavy: $P^+ \sim N_c \quad \rightarrow \quad x = \dfrac{p^+}{P^+} \sim \dfrac{1}{N_c}$

  xN_c fixed for $N_c \to \infty$ ("non-exceptional" values)

- normalization $\langle B, p'|\, B, p\rangle = 2P^0(2\pi)^3\delta^{(3)}(\vec{p} - \vec{p}\,')$ with $P^0 \sim \boldsymbol{N_c}$

  operator is color-singlet: $\bar{\psi}\ldots\psi \sim \boldsymbol{N_c}$

  generic expansion for correlator $\langle \boldsymbol{B, p'}|\overline{\psi}_{\boldsymbol{q}}\,\boldsymbol{\Gamma}\psi_{\boldsymbol{q}}\,|\boldsymbol{B, p}\rangle = \boldsymbol{N_c^2 a_\Gamma^q + N_c b_\Gamma^q + N_c^0 \ldots}$

- $\mathbf{PDF}_{\boldsymbol{i}}(\boldsymbol{x}) = \boldsymbol{N_c^{A_i} F_i(xN_c)}$ with $A_i = 1,\, 2$ (Diakonov et al, Pobylitsa+Polyakov 1996)

  $(f_1^u + f_1^d)(x) = N_c^2 F_{f_1}^{u+d}(xN_c) \quad \gg \quad (f_1^u - f_1^d)(x) = N_c F_{f_1}^{u-d}(xN_c)$ √

  $(g_1^u - g_1^d)(x) = N_c^2 F_{g_1}^{u-d}(xN_c) \quad \gg \quad (g_1^u + g_1^d)(x) = N_c F_{g_1}^{u+d}(xN_c)$ √

  $(h_1^u - h_1^d)(x) = N_c^2 F_{h_1}^{u-d}(xN_c) \quad \gg \quad (h_1^u + h_1^d)(x) = N_c F_{h_1}^{u+d}(xN_c)$ (prediction!) √

- $\boldsymbol{\bar{q}}$ **analog(!)**
  exciting: $W^\pm$ RHIC (Akio Ogawa), $\boldsymbol{\Delta\bar{u}(x) \approx -\Delta\bar{d}(x)}$ (de Florian, Sassot, Stratmann, Vogelsang)

TMDs (nothing known, helpful predictions!)

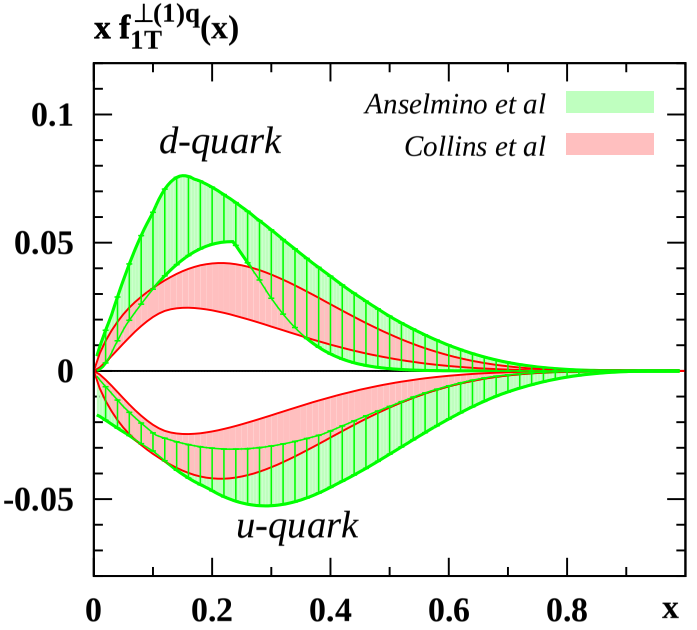
- here $\mathrm{TMD}_i(x, p_T) = N_c^{B_i} F_i(xN_c, p_T)$, $p_T \sim N_c^0$

  predictions (Pobylitsa hep-ph/0301236)

$$
\begin{aligned}
(f_1^u + f_1^d) &\sim N_c^2 \gg |f_1^u - f_1^d| \sim N_c \quad \surd \\
|g_1^u - g_1^d| &\sim N_c^2 \gg |g_1^u + g_1^d| \sim N_c \quad \surd \\
|h_1^u - h_1^d| &\sim N_c^2 \gg |h_1^u + h_1^d| \sim N_c \quad \surd
\end{aligned}
$$

$$
\begin{aligned}
|f_{1T}^{\perp u} - f_{1T}^{\perp d}| &\sim N_c^3 \gg |f_{1T}^{\perp u} + f_{1T}^{\perp d}| \sim N_c^2 \quad \leftarrow \text{used}^* \surd \\
|g_{1T}^{\perp u} - g_{1T}^{\perp d}| &\sim N_c^3 \gg |g_{1T}^{\perp u} + g_{1T}^{\perp d}| \sim N_c^2 \\
|h_1^{\perp u} + h_1^{\perp d}| &\sim N_c^3 \gg |h_1^{\perp u} - h_1^{\perp d}| \sim N_c^2 \\
|h_{1L}^{\perp u} - h_{1L}^{\perp d}| &\sim N_c^3 \gg |h_{1L}^{\perp u} + h_{1L}^{\perp d}| \sim N_c^2 \\
|h_{1T}^{\perp u} - h_{1T}^{\perp d}| &\sim N_c^4 \gg |h_{1T}^{\perp u} + h_{1T}^{\perp d}| \sim N_c^3
\end{aligned}
$$

Sivers function $|u - d| \gg |u + d|$ $\surd$ fit-Ansatz with
$\boldsymbol{f_{1T}^{\perp u} = -f_{1T}^{\perp d}}$, all data fitted with 2 parameters (decade ago)
Efremov et al PLB 612 (2005) 233, Collins et al PRD 73 (2006) 014021
meanwhile updated, modern parametrizations (see talks)
indications for BM, other TMDs to be tested



proceedings "Transversity 2005" Como

Gluon PDFs/TMDs (if nothing known, helpful guideline!)

- also for gluon distributions large N_c counting possible
- $\Delta G(x)/G(x) = \mathcal{O}(1/N_c)$ Efremov, Goeke, Pobylitsa PLB 488 (2000) 182
- (gluon Sivers)/(quark Sivers) $= \mathcal{O}(1/N_c)$ Efremov et al, PLB 612 (2005) 233
- can be helpful guideline, if otherwise we do not know anything!

Remark: Sivers function $\sim N_c^{2+1}$

while $f_1 \sim N_c^2$ (?), does Sivers asymmetry $A_{UT}^{\text{Siv}} \propto \dfrac{f_{1T}^{\perp}}{f_1^q}$ grow with N_c ?

No!

"additional power" of N_c due to: $\quad \text{correlator}^i \sim \dfrac{p_T^i}{M_N} f_{1T}^{\perp}(x, p_T) \sim N_c^2$

correlators of PDFs, TMDs, GPDs $\sim N_c^2$ at most

(analog for GPDs below)

chiral-even GPDs (Petrov et al 1998; Pentinnen et al; Goeke, Polyakov, Vanderhaeghen)

- heavy nucleon moves slowly: $\vec{p},\ \vec{p}^{\,\prime} \sim N_c^0$ i.e. $|t| \sim N_c^0 \ll M_N^2 \sim N_c^2$

  xN_c and ξN_c = fixed for $N_c \to \infty$ ("non-exceptional" values)

  $\mathrm{GPD}_i(x,\xi,t) = N_c^{A_i}\, F(xN_c, \xi N_c, t)$

- $(H^u + H^d)(x,\xi,t) \sim N_c^2 \quad \gg \quad (u-d) =$ small

- $(E^u - E^d)(x,\xi,t) \sim N_c^3 \quad \gg \quad (u+d) =$ small

- $(\tilde{H}^u - \tilde{H}^d)(x,\xi,t) \sim N_c^2 \quad \gg \quad (u+d) =$ small

- $(\tilde{E}^u - \tilde{E}^d)(x,\xi,t) \sim N_c^4 \quad \gg \quad (u+d) =$ small

predictions compatible with forward limit, form factors
being tested in phenomenology: so far, so good ✓

chiral-odd GPDs: mentioned in literature (works & proceedings by Matthias Burkardt)
current study: first rigorous and systematic study

chiral-odd GPDs (new results)

- $(H_T^u - H_T^d)(x,\xi,t) = N_c^2\ D_{H_T}^{u-d}(xN_c,\xi N_c,t) \quad \gg \quad (u+d) = \text{smaller}$ (cf. transversity ✓)
- $(E_T^u + E_T^d)(x,\xi,t) = N_c^3\ D_{E_T}^{u+d}(xN_c,\xi N_c,t) \quad \gg \quad (u - d) = \text{smaller}$
- $(\tilde{H}_T^u + \tilde{H}_T^d)(x,\xi,t) = N_c^3\ D_{\tilde{H}_T}^{u-d}(xN_c,\xi N_c,t) \quad \gg \quad (u-d) = \text{smaller}$
- $(\tilde{E}_T^u - \tilde{E}_T^d)(x,\xi,t) = N_c^3\ D_{\tilde{E}_T}^{u-d}(xN_c,\xi N_c,t) \quad \gg \quad (u+d) = \text{smaller}$

Important: the same large-N_c scaling in the linear combination

(this linear combination naturally appears in cross sections)

$$\boldsymbol{2\tilde{H}_T + E_T}$$

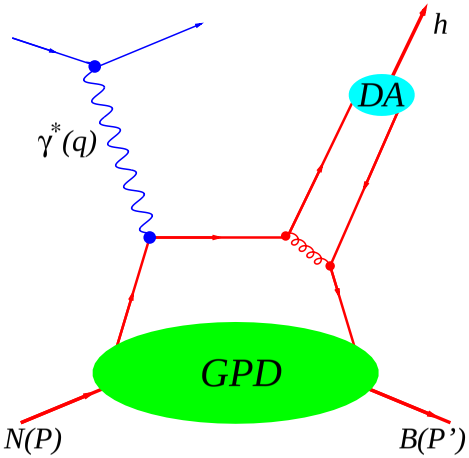
compatible with lattice, models & intuition (linear combination $2\tilde{H}_T + E_T$ in cross sections)

experiment (?)

JLab data PRC 90 (2014) 039901

prelim. data V. Kubarovsky, Excl. Meson Workshop, JLab, Jan.2015

hard meson production $ep \to e'p'h$ with $h = \pi^0,\ \eta$



- transversity GPDs at subleading twist (handbag)
  Gary Goldstein, Simonetta Liuty et al, Goloskokov+Kroll

- generalized form factors
  $\langle \mathrm{GPD} \rangle = \int \mathrm{d}x\ M(x,\xi,t;Q^2)\,\mathrm{GPD}(x,\xi,t;Q^2)$

  sensitive to H_T^q and $\overline{E}_T^q = 2\tilde{H}_T^q + E_T^q$

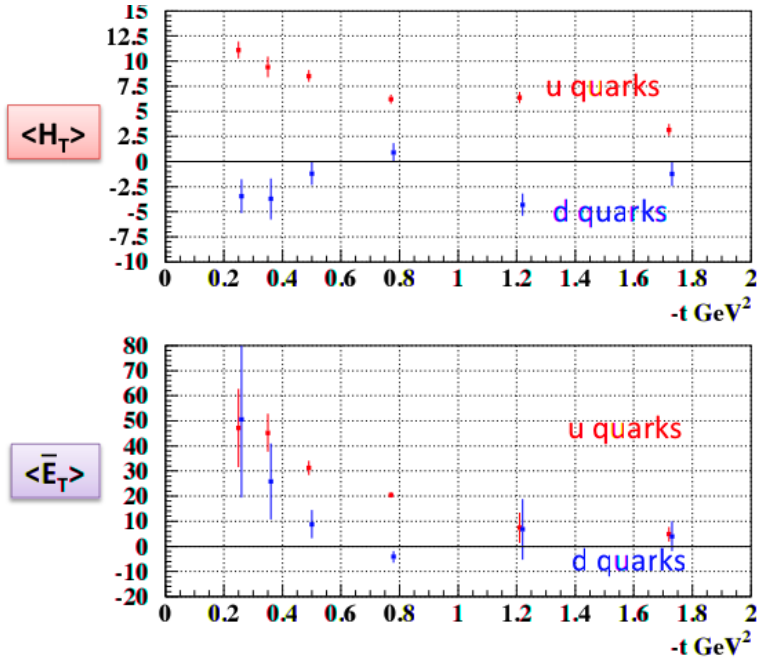
  $\left.\begin{array}{l}\pi^0 \to (2u+d)\\ \eta \to (2u-d)\end{array}\right\}$ + assumptions $\Rightarrow$ $\boldsymbol{u},\ \boldsymbol{d}$

- GPDs require $|t| \ll Q^2$, large-N_c needs $|t| \ll M_N^2$

  (i) $\langle \boldsymbol{H_T^u} \rangle - \langle \boldsymbol{H_T^d} \rangle > \langle \boldsymbol{H_T^u} \rangle + \langle \boldsymbol{H_T^d} \rangle$

  (ii) $\langle \boldsymbol{\overline{E}_T^u} \rangle + \langle \boldsymbol{\overline{E}_T^d} \rangle > \langle \boldsymbol{\overline{E}_T^u} \rangle - \langle \boldsymbol{\overline{E}_T^d} \rangle$

  as predicted from large N_c(!)

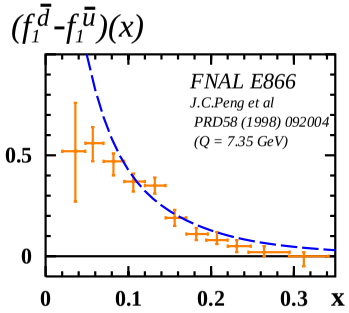


$Q^2 = 1.8\ \mathrm{GeV}^2$, $x_{\mathrm{Bj}} = 0.22$

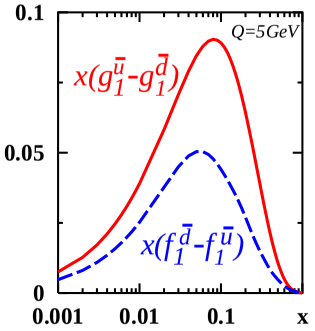
from V. Kubarovsky, Excl. Meson Workshop

Outlook look back at "history" to set expectations for future

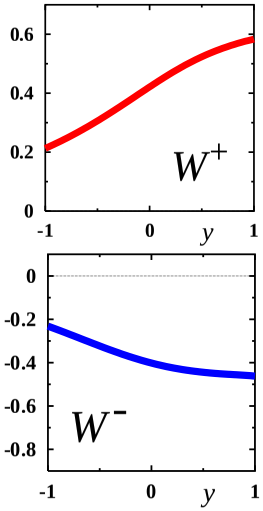
- large-N_c says $|(\Delta\bar{u} - \Delta\bar{d})(x)| > |(\Delta\bar{u} + \Delta\bar{d})(x)|$. So what? 15 years ago nothing known about $\Delta\bar{q}(x)$ ...
- large-N_c cannot tell anything about **different functions**. Models can.
  Need dynamics (chiral physics) → chiral quark soliton model Diakonov et al 1996; Dressler et al 2000



non-trivial result from
effective chiral theory
(large-N_c is tool to perform
non-perturbative calculation):
$(\Delta\bar{u} - \Delta\bar{d})(x) \sim N_c^2$
$> |(\bar{u} - \bar{d})(x)| \sim N_c$



- consequences:

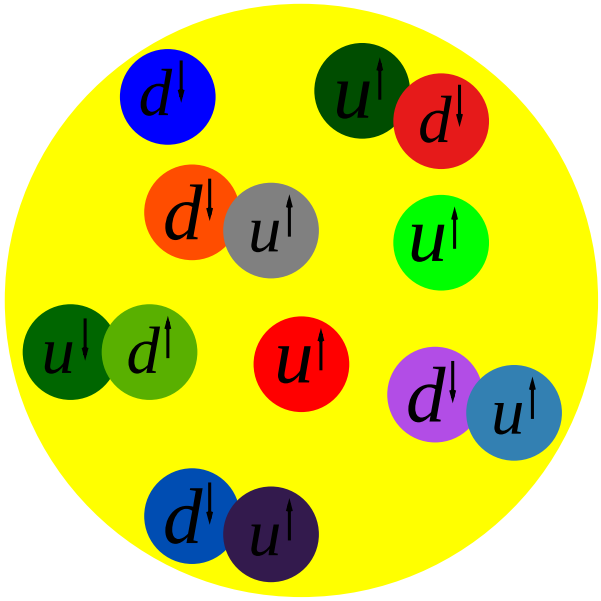


$W^{\pm}$ in DY at RHIC
exciting result!
Dressler et al 2000 (me $\in$ et al) **Eur. Phys. J. C18 (2001) 719**
PS: when will RHIC measure?
CW: will take a while ...
... 15 years later:
pattern seen at RHIC! (Akio Ogawa)
BUT was worth waiting!

- Now: dynamical calculation of chiral odd GPDs. What will we learn?
  Model-dependent but interesting, and needed!

Conclusions

- **large N_c**: first-principle model-independent theoretical tool, powerful method, worth it!
- gives insights in non-perturbative physics, provides guidelines, **works** phenomenologically
- we presented a study of **chiral odd GPDs** in **QCD multi-color limit**
- **JLab** data give first insights in flavor structure of chiral odd GPDs
- observed patterns **explained** in the large-N_c limit! $\surd$
- future: study dynamics, see what we learn



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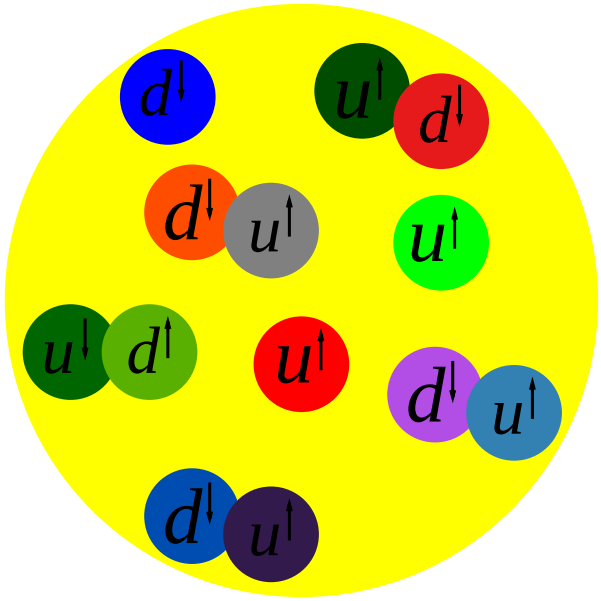

Thank you!