Gluon Production in Heavy Ion Collisions: Beyond the Leading Order

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Based on:

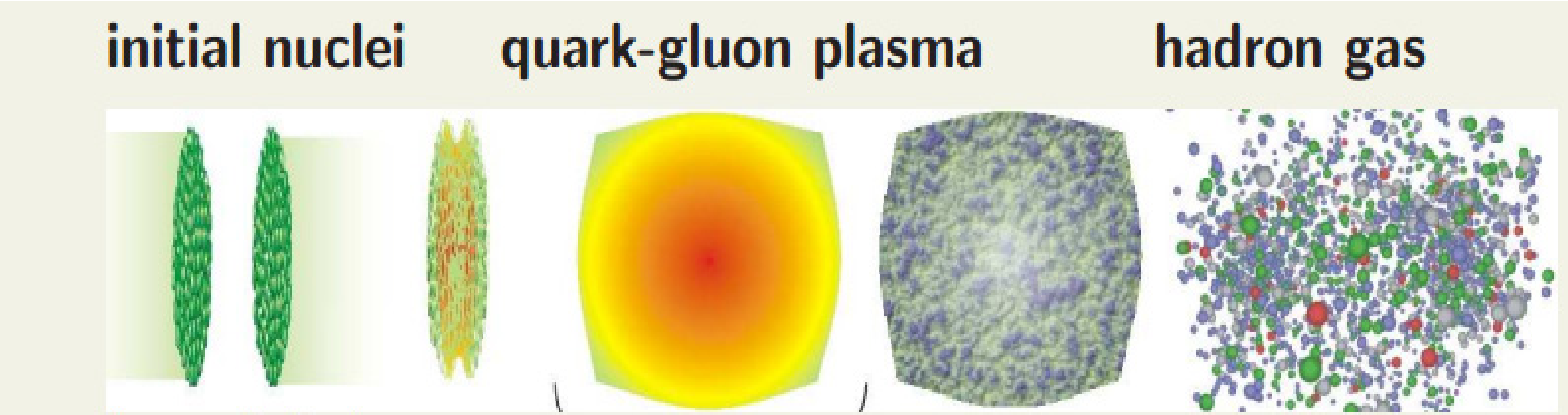
GA Chirilli, YV Kovchegov, DE Wertepny, Classical Gluon Production Amplitude for Nucleus-Nucleus Collisions: First Saturation Correction in the Projectile, JHEP (2015), arXiv:1501.03106

Outline

- Overall Goal: Calculate the single inclusive gluon production cross section in Heavy-Light ion collisions.

- Brief description of heavy ion collisions, why we need the single inclusive gluon production cross-section.
- Review of the p-A case.
- Current state of the Heavy-Light ion case calculation.
- Conclusions and Outlook

Heavy Ion Collisions



[image: S. Bass]

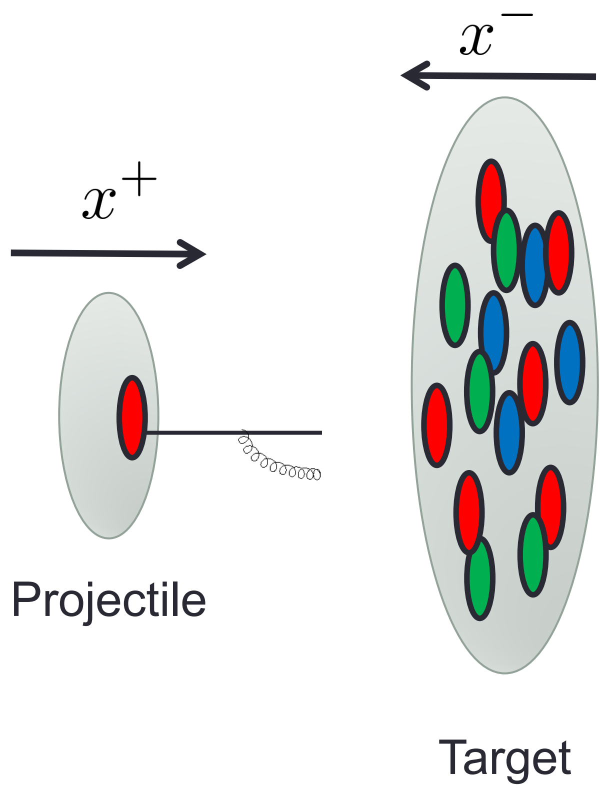
- Complicated system where different dynamics dominate at different time scales.
- Focus on the initial interactions, which are perturbative.
- Necessary to understand the rest of the system.
- Need the initial gluon distribution.

Heavy-Light Paradigm

- Need to know the single inclusive gluon production cross-section for Heavy Nucleus-Nucleus collisions at the classical level using perturbative QCD.
- Problem is too hard at the moment.
- Instead calculate a simpler problem, Heavy-Light Ion collisions.
- Take into account all nucleons in one of the ions (heavy) and only two in the other (light).

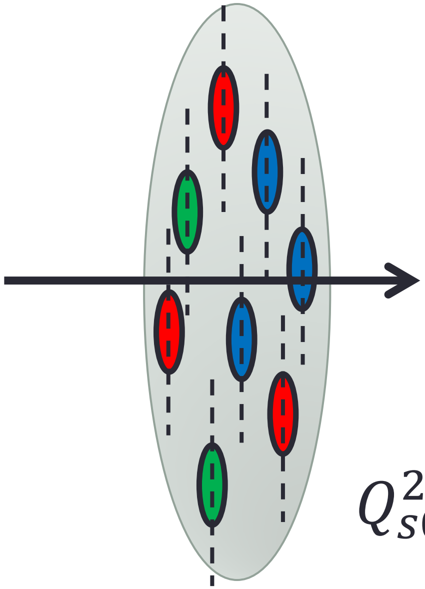
Proton-Nucleus Collison Case

- Due to the energy of the collision the projectile and target are Lorentz contracted.
- The interaction happens instantaneously compared to gluon emission time.
- View the projectile as emitting gluons which interact with the target instantaneously.
- Model using saturation physics.
- Using the Light-cone gauge, $A^+ = 0$.



The Interaction as a Wilson Line

- A quark/gluon propagating through a nucleus at high energy can be represented as a Wilson line. Recoilless in transverse spatial coordinate it interacts with many different “color patches”.



$Q_{s0}^2 \sim A^{1/3}$

$$U_{\boldsymbol{x}} = \mathrm{P}\exp\left\{ i\, g \int_{-\infty}^{\infty} dx^+ \, \mathcal{A}^-(x^+, x^- = 0, \boldsymbol{x}) \right\}$$

$$\mathcal{A}^- = \sum_i T^a A_i^{a-}$$

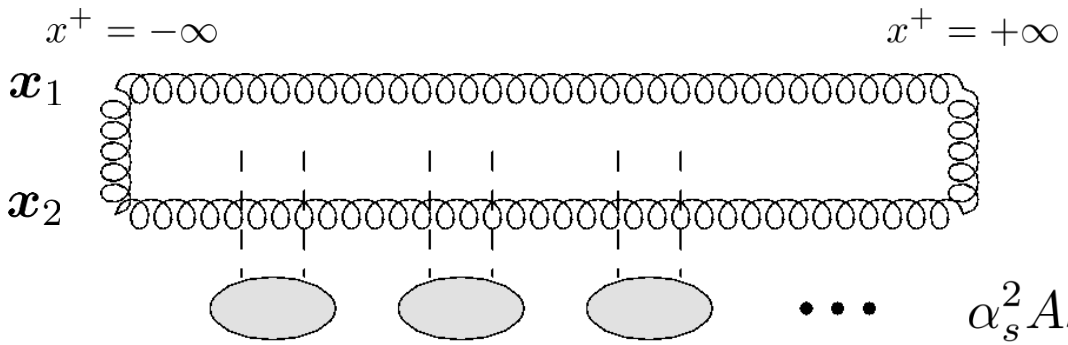
- In the heavy light collision case:

$$A_P \ll A_T \quad \rightarrow \quad Q_{s\,P} \ll Q_{s\,T}$$

Analysis of the Gluon Dipole – Saturation effects

- The gluon dipole resums a series of tree level scatterings off many nucleons. The dotted lines represent gluons.

$$S_G(\boldsymbol{x}_1, \boldsymbol{x}_2, y) \equiv \frac{1}{N_c^2 - 1} \left\langle Tr[U_{\boldsymbol{x}_1} U^\dagger_{\boldsymbol{x}_2}] \right\rangle$$



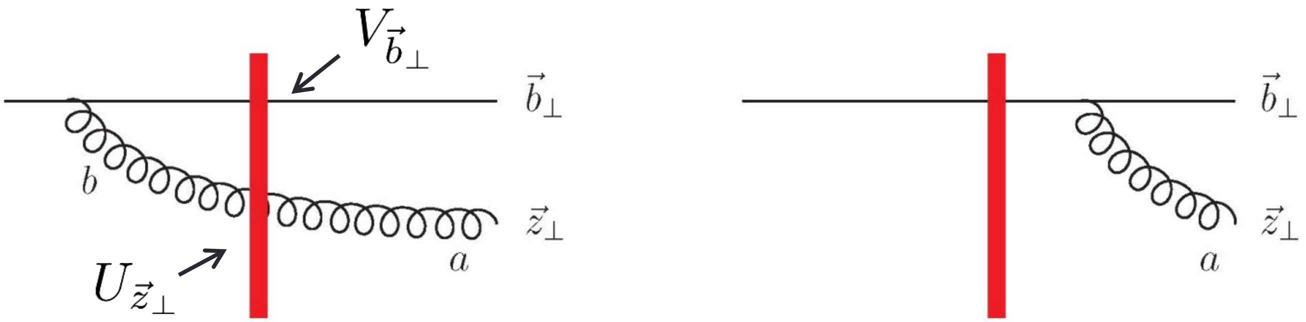
$$\alpha_s^2 A_T^{\frac{1}{3}} \sim 1$$

$$S_G(\boldsymbol{x}_1, \boldsymbol{x}_2, y = 0) = \exp\left[-\frac{1}{4} \left| \boldsymbol{x}_1 - \boldsymbol{x}_2 \right|^2 Q_{s0}^2 \left(\frac{\boldsymbol{x}_1 + \boldsymbol{x}_2}{2} \right) \ln\left(\frac{1}{\left| \boldsymbol{x}_1 - \boldsymbol{x}_2 \right| \Lambda} \right) \right]$$

- The forward scattering amplitude is given by.

$$N_G(\boldsymbol{x}_1, \boldsymbol{x}_2, y = 0) = 1 - S_G(\boldsymbol{x}_1, \boldsymbol{x}_2, y = 0)$$

Diagrams for pA Collisions



- High energy scattering between the projectile and the target is an instantaneous interaction (shockwave) at $x^+ = 0$.
- Gluon emission can happen before or after, not during.
- The projectile interacting with the target results in a power counting of

$$|M|^2 \sim \frac{1}{\alpha_s}(\alpha_s^2 A_P^{\frac{1}{3}})(\alpha_s^2 A_T^{\frac{1}{3}})^N \qquad A_P^{\frac{1}{3}} = 1 \quad \alpha_s^2 A_T^{\frac{1}{3}} \sim 1$$

- In total the amplitude is

$$M(\vec{z}_\perp, \vec{b}_\perp) = \frac{i\,g}{\pi} \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_\perp)}{|\vec{z}_\perp - \vec{b}_\perp|^2} \left[U_{\vec{z}_\perp}^{ab} - U_{\vec{b}_\perp}^{ab}\right] \left(V_{\vec{b}_\perp} t^b\right)$$

- Used the relation: $\left(t^a V_{\vec{b}_\perp}\right) = \left(V_{\vec{b}_\perp} t^b\right) U_{\vec{b}_\perp}^{ab}$

Result for p-A

- The resulting cross section is

$$\frac{d\sigma}{d^2k_T\,dy} = \underbrace{\frac{\alpha_s\,C_F}{4\,\pi^4}} \int d^2z\,d^2z'\,d^2b\,e^{-i\vec{k}_\perp\cdot(\vec{z}_\perp-\vec{z}\,'_\perp)}\,\frac{\vec{z}_\perp-\vec{b}_\perp}{|\vec{z}_\perp-\vec{b}_\perp|^2}\cdot\frac{\vec{z}\,'_\perp-\vec{b}_\perp}{|\vec{z}\,'_\perp-\vec{b}_\perp|^2}$$
$$\times\underbrace{\left[S_G(\vec{z}_\perp,\vec{z}\,'_\perp)-S_G(\vec{b}_\perp,\vec{z}\,'_\perp)-S_G(\vec{z}_\perp,\vec{b}_\perp)+1\right]}$$

- Overall power counting:
  - One emitted gluon (classical): $\frac{1}{\alpha_s}$
  - Single nucleon in the projectile: $\alpha_s^2 A_P^{\frac{1}{3}}$
  - Interactions in the target: $(\alpha_s^2 A_T^{\frac{1}{3}})^N \sim 1$
- In total: $\frac{1}{\alpha_s}(\alpha_s^2 A_P^{\frac{1}{3}})$

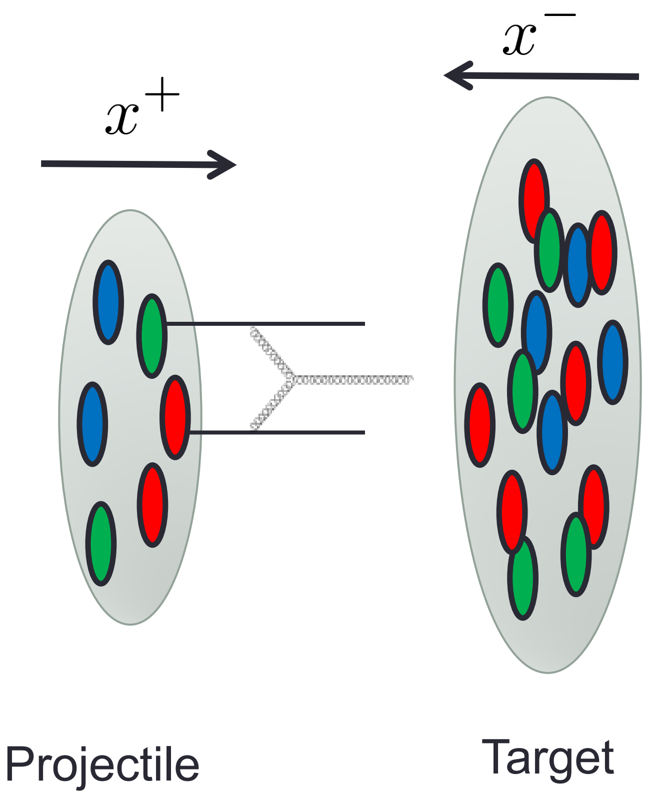
Heavy-Light Collision Case

- Target nucleus has same power counting as before.
- Projectile has many nucleons, but not too many such that

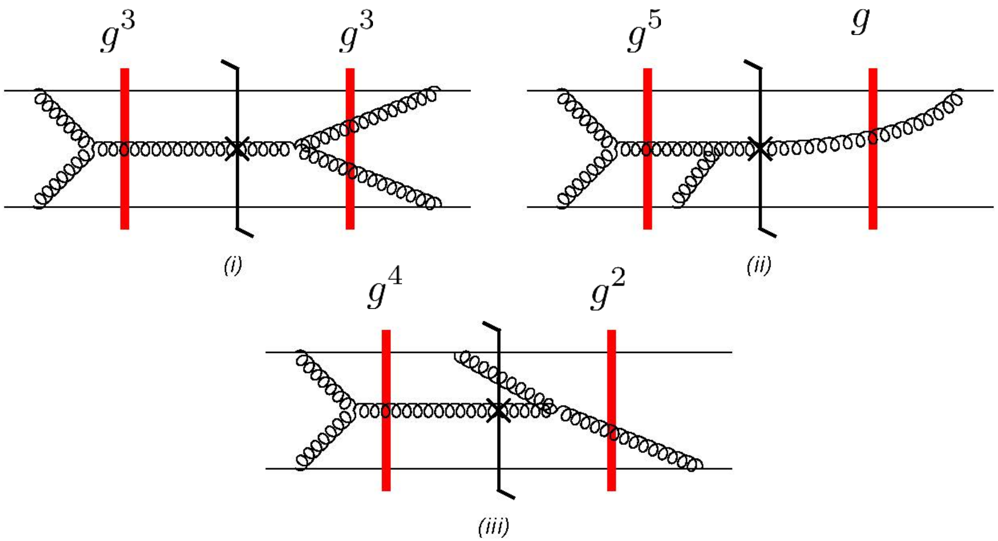
$$\alpha_s \ll \alpha_s^2 A_P^{\frac{1}{3}} \lesssim 1 \qquad \alpha_s^2 A_T^{\frac{1}{3}} \sim 1$$

- No Quantum corrections.
- Two nucleons from projectile.
- Overall power counting for the cross section,

$$\sim \frac{1}{\alpha_s}(\alpha_s^2 A_P^{\frac{1}{3}})^2(\alpha_s^2 A_T^{\frac{1}{3}})^N$$



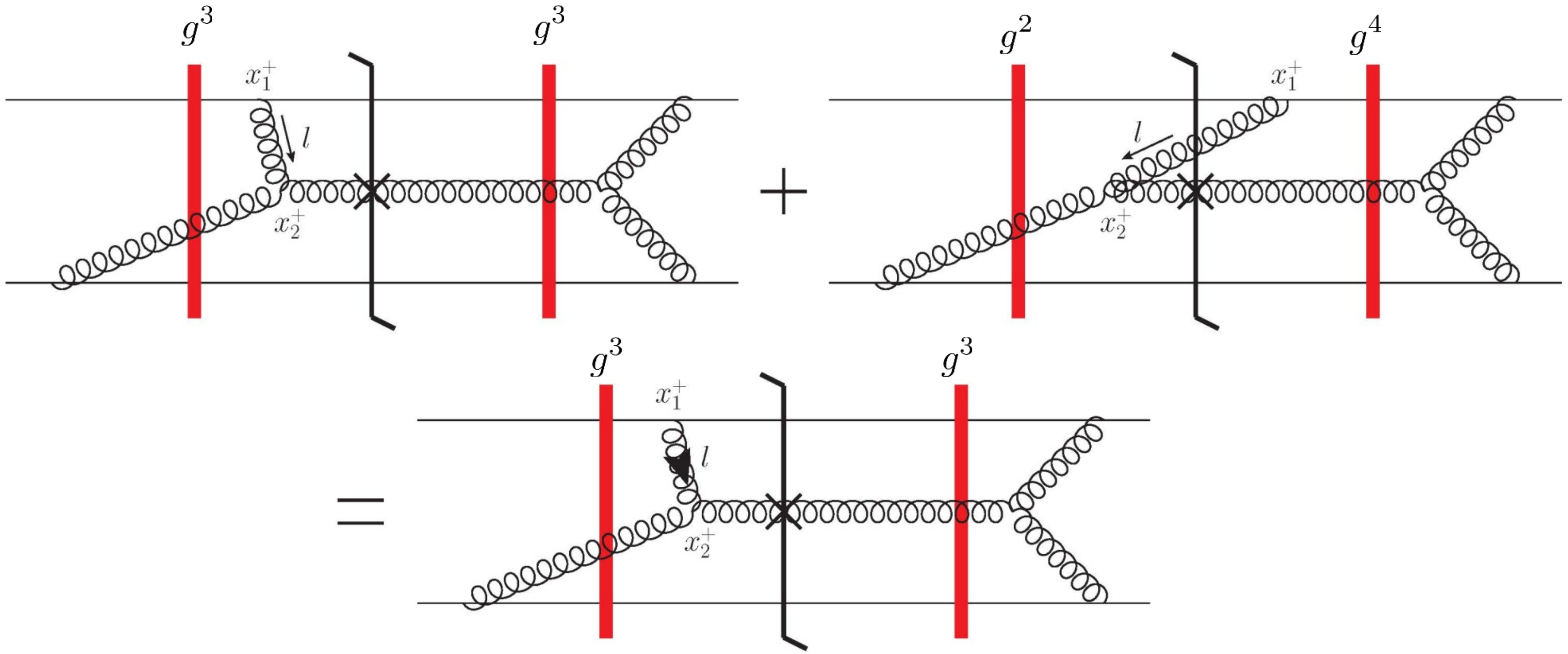
Types of Diagrams



- Diagrams have two quarks from the projectile and 6 vertices (order g^6).
- Have a huge number of diagrams.
- Diagrams can be separated into three classes:
  i) Square of order-g^3 amplitudes
  ii) Interference between order-g^5 and order-g amplitudes
  iii) Interference between order-g^4 and order-g^2 amplitudes
- These can be combined together in various ways to reduce the number of diagrams.
- Light-cone gauge, $A^+ = 0$

$$\frac{-i\,D_{\mu\nu}(l)}{l^2 + i\,\epsilon} \qquad \text{where} \qquad D_{\mu\nu}(l) = g_{\mu\nu} - \frac{1}{\eta \cdot l}(\eta_\mu\, l_\nu + \eta_\nu\, l_\mu)$$

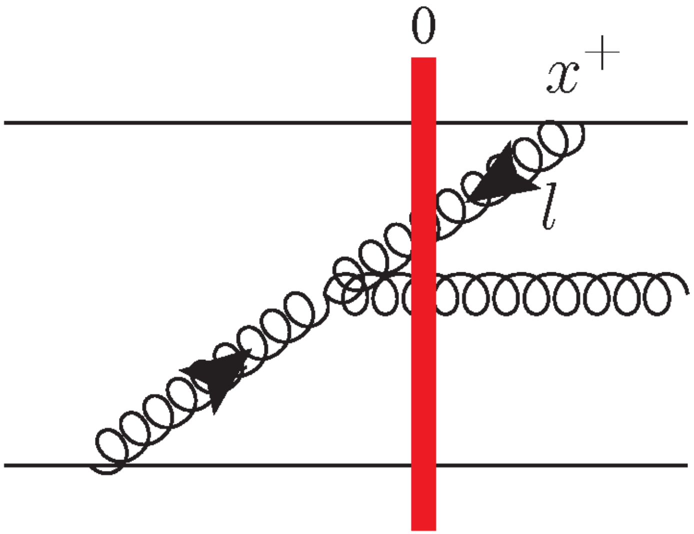
Retarded Green Function

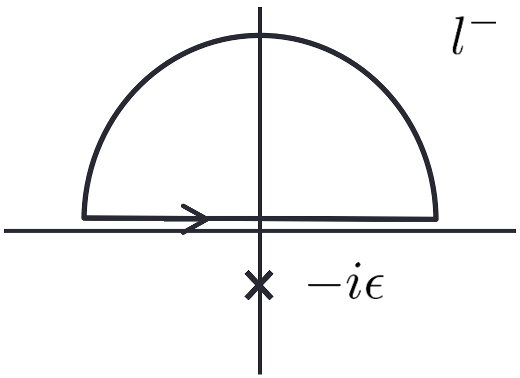


- Adding the top two diagrams turns the propagator into a retarded propagator, represented by the arrow.

$$\frac{-i\,D_{\mu\nu}(l)}{l^2 + i\,\epsilon} + 2\pi\,\theta(-l^+)\,\delta(l^2)\,D_{\mu\nu}(l) = \frac{-i\,D_{\mu\nu}(l)}{l^2 + i\,\epsilon\,l^+}$$

Backwards Propagators



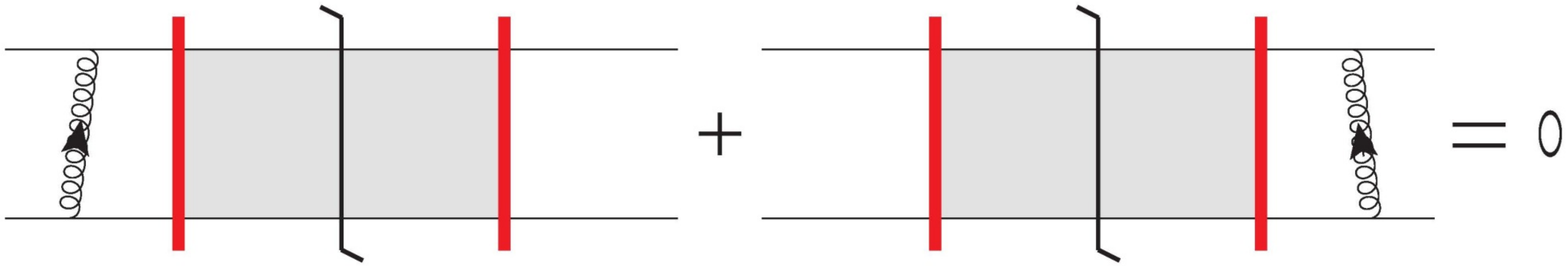


- "l-" momentum must flow forwards in "x+" time.

$$\propto \int dl^- \frac{\Theta(x^+)}{2l^+l^- - l_\perp^2 + i\,\epsilon\, l^+} e^{il^- x^+} = 0$$

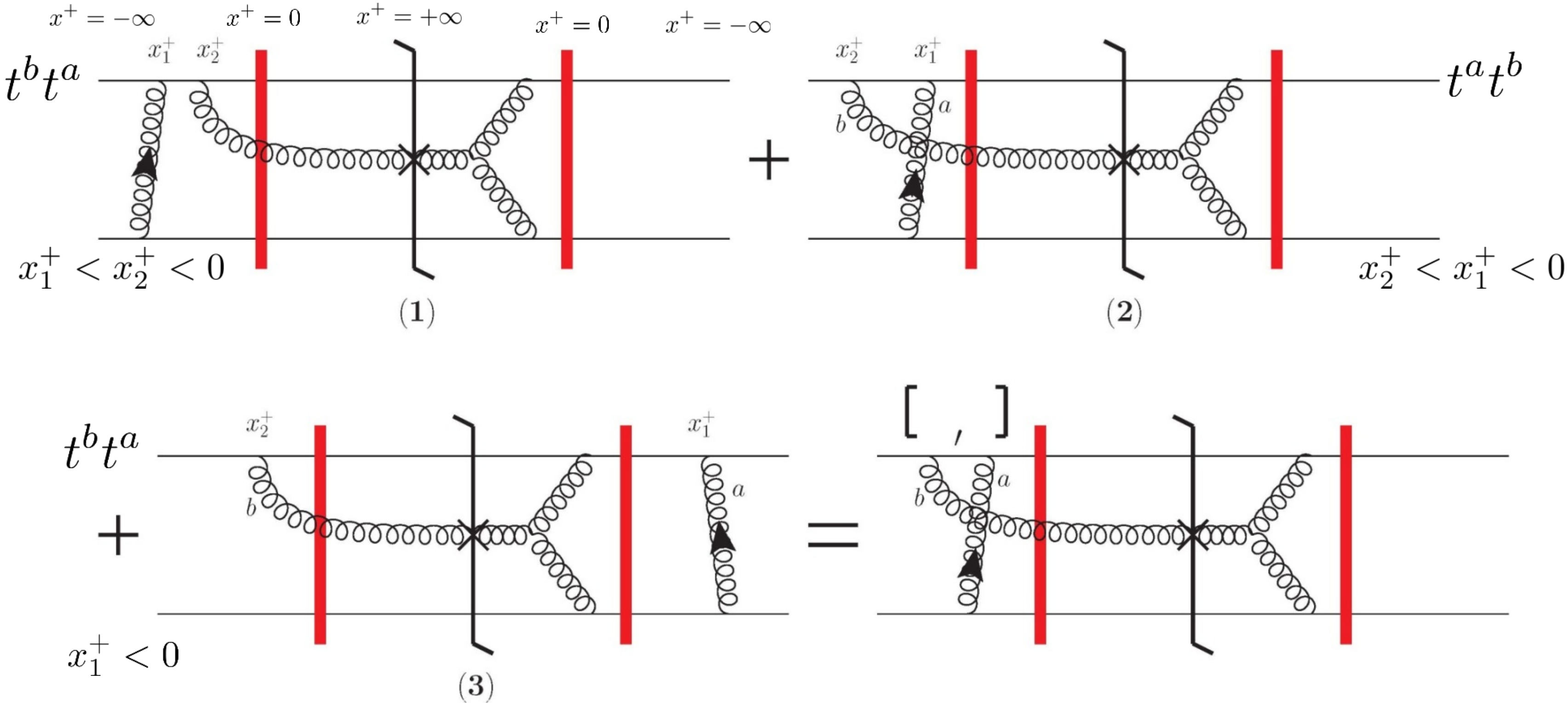
- Flowing backwards into the shock wave is not allowed.

Cancellations



- Shaded region represents any late-time interaction.
- Moving the retarded propagator across the cut gives rise to a minus sign.
- The sum of the diagrams is zero.

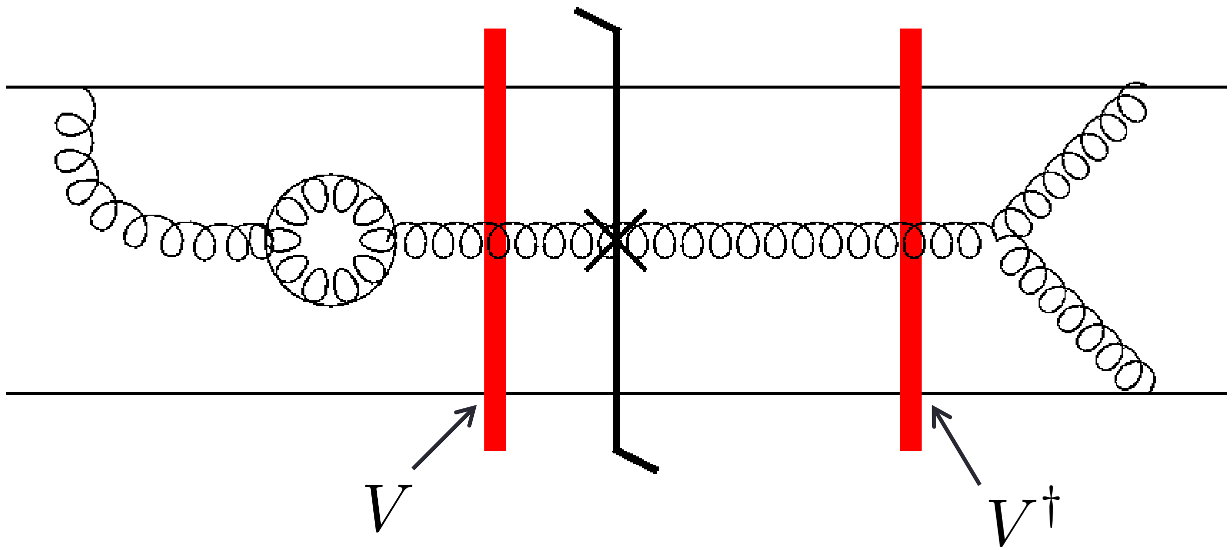
Commutators



- Using the cancellation shown previously diagrams (1), (2), and (3) can be combined into a single diagram. Diagram (2) with the color structure of the quark line replaced by a commutator, notated by the square brackets.

$$t^a t^b \to [t^a, t^b]$$

No Quantum Contributions

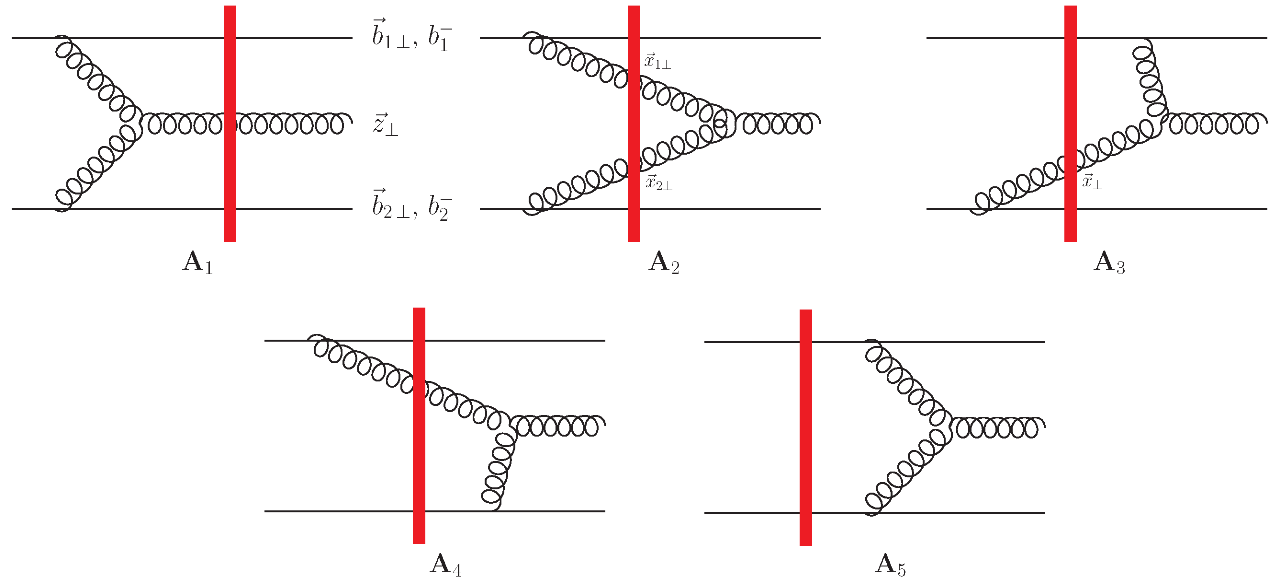


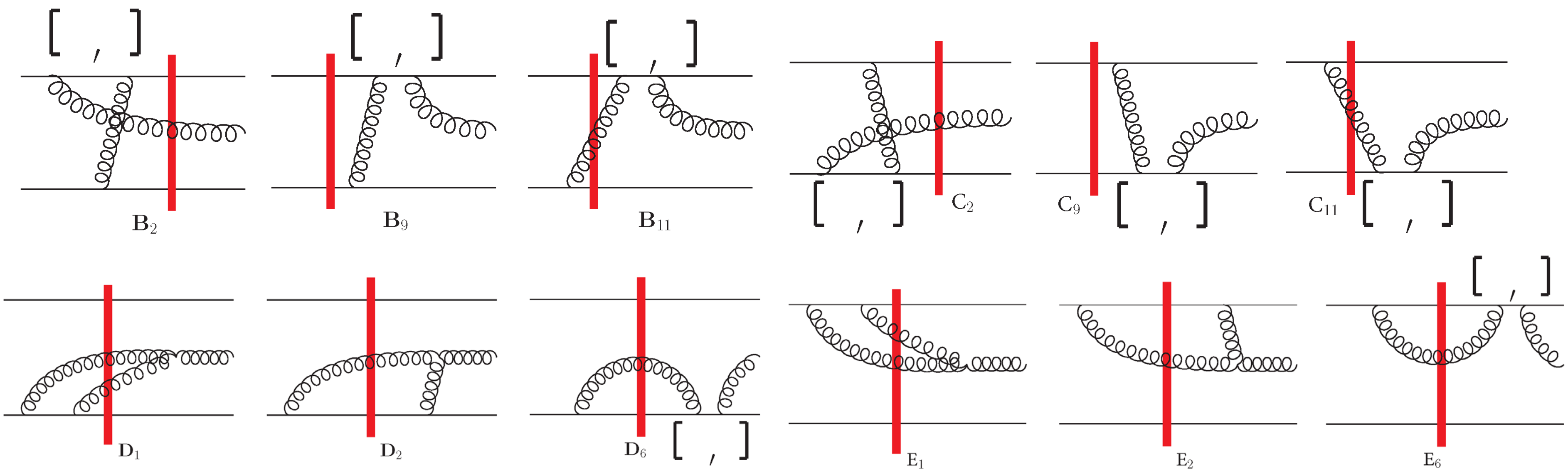
- Quantum corrections go away at this order.
- Left with classical fields
- Zero due to color averaging of quark two.

$$tr[t^a V^\dagger V] = tr[t^a] = 0$$

Final Diagrams

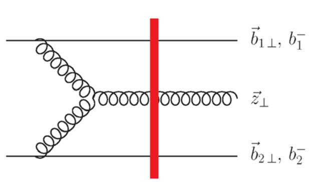
Using these tricks the number of diagrams are reduced to a manageable amount.
All diagrams are classical.
Not a complete set.



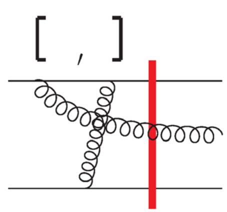


Results: Amplitude – A, B, and C graphs

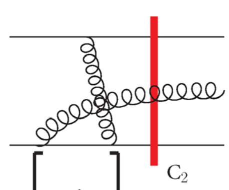
$$
\begin{aligned}
&\sum_i A_i + \sum_i B_i + \sum_i C_i \\
&= -\frac{g^3}{4\pi^4}\int d^2x_1\, d^2x_2\, \delta[(\vec{z}_\perp - \vec{x}_{1\perp})\times(\vec{z}_\perp - \vec{x}_{2\perp})] \Bigg[\frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{x}_{2\perp}-\vec{x}_{1\perp})}{|\vec{x}_{2\perp}-\vec{x}_{1\perp}|^2}\, \frac{\vec{x}_{1\perp}-\vec{b}_{1\perp}}{|\vec{x}_{1\perp}-\vec{b}_{1\perp}|^2}\cdot\frac{\vec{x}_{2\perp}-\vec{b}_{2\perp}}{|\vec{x}_{2\perp}-\vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{x}_{1\perp}-\vec{b}_{1\perp})}{|\vec{x}_{1\perp}-\vec{b}_{1\perp}|^2}\, \frac{\vec{z}_\perp-\vec{x}_{1\perp}}{|\vec{z}_\perp-\vec{x}_{1\perp}|^2}\cdot\frac{\vec{x}_{2\perp}-\vec{b}_{2\perp}}{|\vec{x}_{2\perp}-\vec{b}_{2\perp}|^2} \\
&\quad - \frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{x}_{2\perp}-\vec{b}_{2\perp})}{|\vec{x}_{2\perp}-\vec{b}_{2\perp}|^2}\, \frac{\vec{x}_{1\perp}-\vec{b}_{1\perp}}{|\vec{x}_{1\perp}-\vec{b}_{1\perp}|^2}\cdot\frac{\vec{z}_\perp-\vec{x}_{2\perp}}{|\vec{z}_\perp-\vec{x}_{2\perp}|^2}\Bigg]\, f^{abc}\left[U^{bd}_{\vec{x}_{1\perp}} - U^{bd}_{\vec{b}_{1\perp}}\right]\left[U^{ce}_{\vec{x}_{2\perp}} - U^{ce}_{\vec{b}_{2\perp}}\right]\left(V_{\vec{b}_{1\perp}}\, t^d\right)_1 \left(V_{\vec{b}_{2\perp}}\, t^e\right)_2 \\
&\quad + \frac{i\,g^3}{4\pi^3}\, f^{abc}\left(V_{\vec{b}_{1\perp}}\, t^d\right)_1 \left(V_{\vec{b}_{2\perp}}\, t^e\right)_2 \int d^2x\, \Bigg[U^{bd}_{\vec{b}_{1\perp}}\left(U^{ce}_{\vec{x}_\perp} - U^{ce}_{\vec{b}_{2\perp}}\right)\Bigg(\frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{z}_\perp-\vec{x}_\perp)}{|\vec{z}_\perp-\vec{x}_\perp|^2}\, \frac{\vec{x}_\perp-\vec{b}_{1\perp}}{|\vec{x}_\perp-\vec{b}_{1\perp}|^2}\cdot\frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2} \\
&\quad - \frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{z}_\perp-\vec{b}_{1\perp})}{|\vec{z}_\perp-\vec{b}_{1\perp}|^2}\, \frac{\vec{z}_\perp-\vec{x}_\perp}{|\vec{z}_\perp-\vec{x}_\perp|^2}\cdot\frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{z}_\perp-\vec{b}_{1\perp})}{|\vec{z}_\perp-\vec{b}_{1\perp}|^2}\, \frac{\vec{x}_\perp-\vec{b}_{1\perp}}{|\vec{x}_\perp-\vec{b}_{1\perp}|^2}\cdot\frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2}\Bigg) \\
&\quad - \left(U^{bd}_{\vec{x}_\perp} - U^{bd}_{\vec{b}_{1\perp}}\right)U^{ce}_{\vec{b}_{2\perp}}\Bigg(\frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{z}_\perp-\vec{x}_\perp)}{|\vec{z}_\perp-\vec{x}_\perp|^2}\, \frac{\vec{x}_\perp-\vec{b}_{1\perp}}{|\vec{x}_\perp-\vec{b}_{1\perp}|^2}\cdot\frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{z}_\perp-\vec{b}_{2\perp})}{|\vec{z}_\perp-\vec{b}_{2\perp}|^2}\, \frac{\vec{z}_\perp-\vec{x}_\perp}{|\vec{z}_\perp-\vec{x}_\perp|^2}\cdot\frac{\vec{x}_\perp-\vec{b}_{1\perp}}{|\vec{x}_\perp-\vec{b}_{1\perp}|^2} \\
&\quad - \frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{z}_\perp-\vec{b}_{2\perp})}{|\vec{z}_\perp-\vec{b}_{2\perp}|^2}\, \frac{\vec{x}_\perp-\vec{b}_{1\perp}}{|\vec{x}_\perp-\vec{b}_{1\perp}|^2}\cdot\frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2}\Bigg)\Bigg] \\
&\quad - \frac{i\,g^3}{4\pi^2}\, f^{abc}\left(V_{\vec{b}_{1\perp}}\, t^d\right)_1 \left(V_{\vec{b}_{2\perp}}\, t^e\right)_2 \Bigg[\left(U^{bd}_{\vec{z}_\perp} - U^{bd}_{\vec{b}_{1\perp}}\right)U^{ce}_{\vec{b}_{2\perp}}\, \frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{z}_\perp-\vec{b}_{1\perp})}{|\vec{z}_\perp-\vec{b}_{1\perp}|^2}\, \ln\frac{1}{|\vec{z}_\perp-\vec{b}_{2\perp}|\,\Lambda} - U^{bd}_{\vec{b}_{1\perp}}\left(U^{ce}_{\vec{z}_\perp} - U^{ce}_{\vec{b}_{2\perp}}\right)\frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{z}_\perp-\vec{b}_{2\perp})}{|\vec{z}_\perp-\vec{b}_{2\perp}|^2}\, \ln\frac{1}{|\vec{z}_\perp-\vec{b}_{1\perp}|\,\Lambda}\Bigg] \\
&\quad - \frac{i\,g^3}{4\pi^3}\int d^2x\, \left[U^{ab}_{\vec{x}_\perp} - U^{ab}_{\vec{z}_\perp}\right] f^{bde}\left(V_{\vec{b}_{1\perp}}\, t^d\right)_1 \left(V_{\vec{b}_{2\perp}}\, t^e\right)_2 \frac{\vec{\epsilon}_\perp^{\,\lambda *}\cdot(\vec{z}_\perp-\vec{x}_\perp)}{|\vec{z}_\perp-\vec{x}_\perp|^2}\, \frac{\vec{x}_\perp-\vec{b}_{1\perp}}{|\vec{x}_\perp-\vec{b}_{1\perp}|^2}\cdot\frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2}\, \mathrm{Sign}(b_2^- - b_1^-)
\end{aligned}
$$



A$_1$



B$_2$



C$_2$

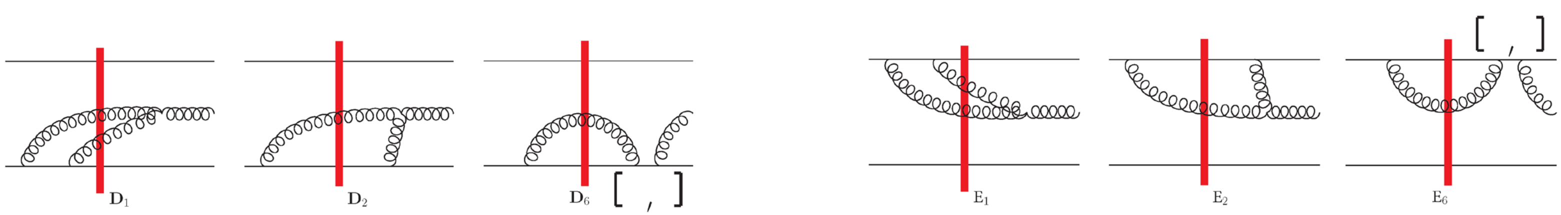
$$\vec{x}_\perp \times \vec{y}_\perp = x_1\, y_2 - x_2\, y_1$$

$$\Lambda = \text{IR cutoff}$$

Results: Amplitude – D graphs

$$
\begin{aligned}
\sum_i D_i = & -\frac{g^3}{8\,\pi^4} \int d^2x_1\, d^2x_2\, \delta[(\vec{z}_\perp - \vec{x}_{1\perp}) \times (\vec{z}_\perp - \vec{x}_{2\perp})] \left[\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{2\perp} - \vec{x}_{1\perp})}{|\vec{x}_{2\perp} - \vec{x}_{1\perp}|^2} \, \frac{\vec{x}_{1\perp} - \vec{b}_{2\perp}}{|\vec{x}_{1\perp} - \vec{b}_{2\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \right. \\
& \left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{1\perp} - \vec{b}_{2\perp})}{|\vec{x}_{1\perp} - \vec{b}_{2\perp}|^2} \, \frac{\vec{z}_\perp - \vec{x}_{1\perp}}{|\vec{z}_\perp - \vec{x}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} + \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{2\perp} - \vec{b}_{2\perp})}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \, \frac{\vec{x}_{1\perp} - \vec{b}_{2\perp}}{|\vec{x}_{1\perp} - \vec{b}_{2\perp}|^2} \cdot \frac{\vec{z}_\perp - \vec{x}_{2\perp}}{|\vec{z}_\perp - \vec{x}_{2\perp}|^2} \right] \\
& \times f^{abc} \left[U^{bd}_{\vec{x}_{1\perp}} - U^{bd}_{\vec{b}_{2\perp}} \right] \left[U^{ce}_{\vec{x}_{2\perp}} - U^{ce}_{\vec{b}_{2\perp}} \right] \left(V_{\vec{b}_{1\perp}} \right)_1 \left(V_{\vec{b}_{2\perp}}\, t^e\, t^d \right)_2 \\
& + \frac{i\, g^3}{4\,\pi^3} \int d^2x\, f^{abc}\, U^{bd}_{\vec{b}_{2\perp}} \left[U^{ce}_{\vec{x}_\perp} - U^{ce}_{\vec{b}_{2\perp}} \right] \left(V_{\vec{b}_{1\perp}} \right)_1 \left(V_{\vec{b}_{2\perp}}\, t^e\, t^d \right)_2 \left(\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \, \frac{1}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right. \\
& \left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \, \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \, \frac{1}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right) \\
& + \frac{i\, g^3}{4\,\pi^2} f^{abc}\, U^{bd}_{\vec{b}_{2\perp}} \left[U^{ce}_{\vec{z}_\perp} - U^{ce}_{\vec{b}_{2\perp}} \right] \left(V_{\vec{b}_{1\perp}} \right)_1 \left(V_{\vec{b}_{2\perp}}\, t^e\, t^d \right)_2 \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{2\perp}|\, \Lambda}
\end{aligned}
$$

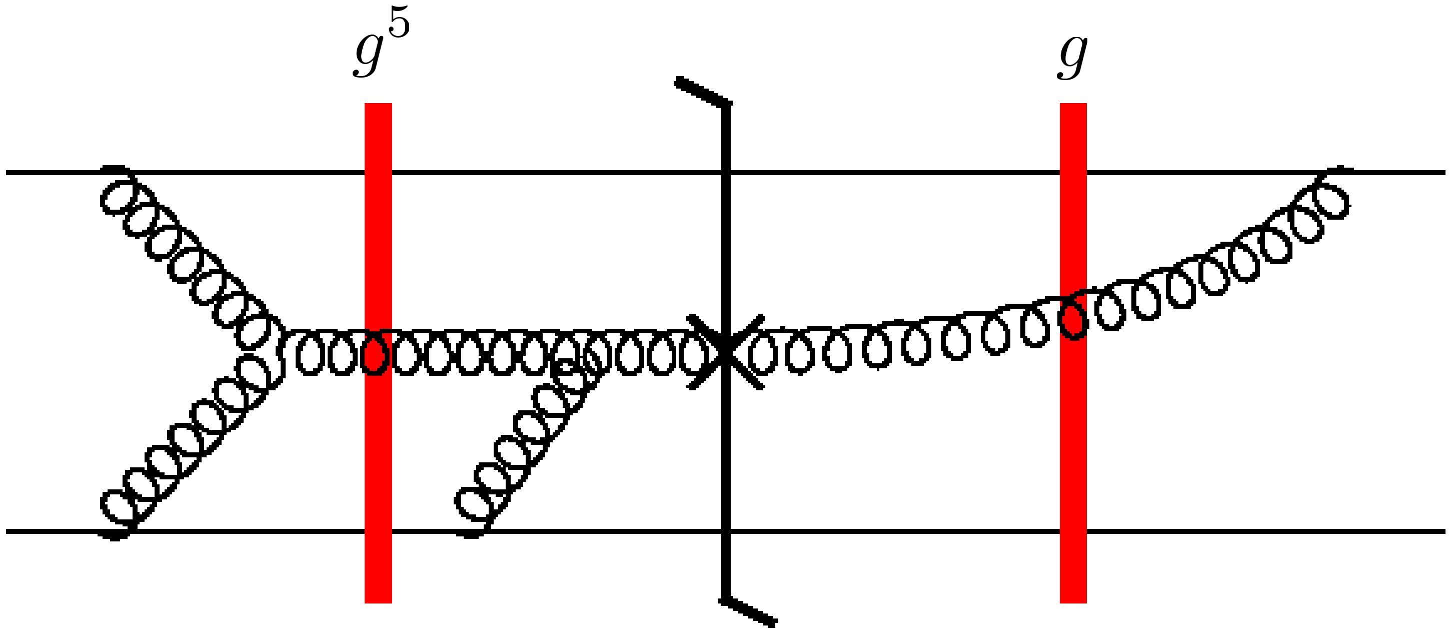
- To get the E graph results switch quark 1 with quark 2 (1 ↔2)



Conclusion

- A serious attempt at analytically calculating the single inclusive gluon production cross-section for Heavy-Light Ion collisions at the classical level.
- Going beyond the p-A result from 1997. [Kovchegov, Mueller]
- Reduced all of the possible diagrams into a few classical field diagrams with a single produced gluon in the amplitude.
- Ended up with a compact result in transverse coordinate space for the g^3 amplitude.
- Similar results in this direction have been obtained by Balitsky (2004).

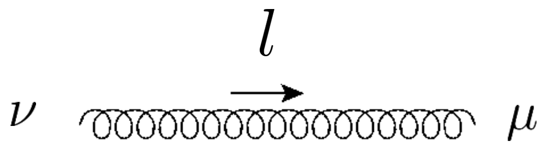
Outlook



- Need to calculate g^5 amplitude to get the single inclusive gluon production cross-section.
- Work in progress.

Back up

Light-Cone Propagator



- Different prescriptions for regulating the pole:
  - Principle Value (PV):

$$D^{(PV)}_{\mu\nu}(l) = g_{\mu\nu} - PV\left(\frac{1}{\eta \cdot l}\right)(\eta_\mu\, l_\nu + \eta_\nu\, l_\mu)$$

  - For boundary condition: $\vec{A}_\perp(x^- \to \pm\infty)$

$$D^{\pm}_{\mu\nu}(l) = g_{\mu\nu} - \frac{\eta_\mu\, l_\nu}{\eta \cdot l \pm i\epsilon} - \frac{\eta_\nu\, l_\mu}{\eta \cdot l \mp i\epsilon}$$

- All three lead to the same g^3 amplitude result.
- PV prescription ends up being more economical.