



Disentangling Spin Dependent GPDs through Deeply Virtual Processes with Polarizations and Azimuthal Correlations

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Presentation for QCD Evolution 2015 Workshop
Jefferson Lab, May.26-30 , 2015

These ideas were developed in Jlab, Trento ECT*, INT, DIS2011, SPIN, Frascati INF, Transversity 2011-2013, PANIC, POETIC, QCD2015 & in consultation with many of you



Collaborators

GPDs, Extension to Chiral Odd Sector

S. Liuti, O. Gonzalez Hernandez

- GRG, O. Gonzalez-Hernandez, S.Liuti, PRD(2015) arXiv: 1311.0483
- GRG, O. Gonzalez-Hernandez, S.Liuti, arXiv:1401.0438
- Ahmad, GRG, Liuti, PRD79, 054014, (2009)
- Gonzalez, GRG, Liuti PRD84, 034007 (2011)
- GRG, Gonzalez Hernandez, Liuti, J. Phys. G: Nucl. Part. Phys. 39 115001 (2012)
- Gonzalez Hernandez, Liuti, GRG, Kathuria, PRC88, 065206 (2013)



Outline

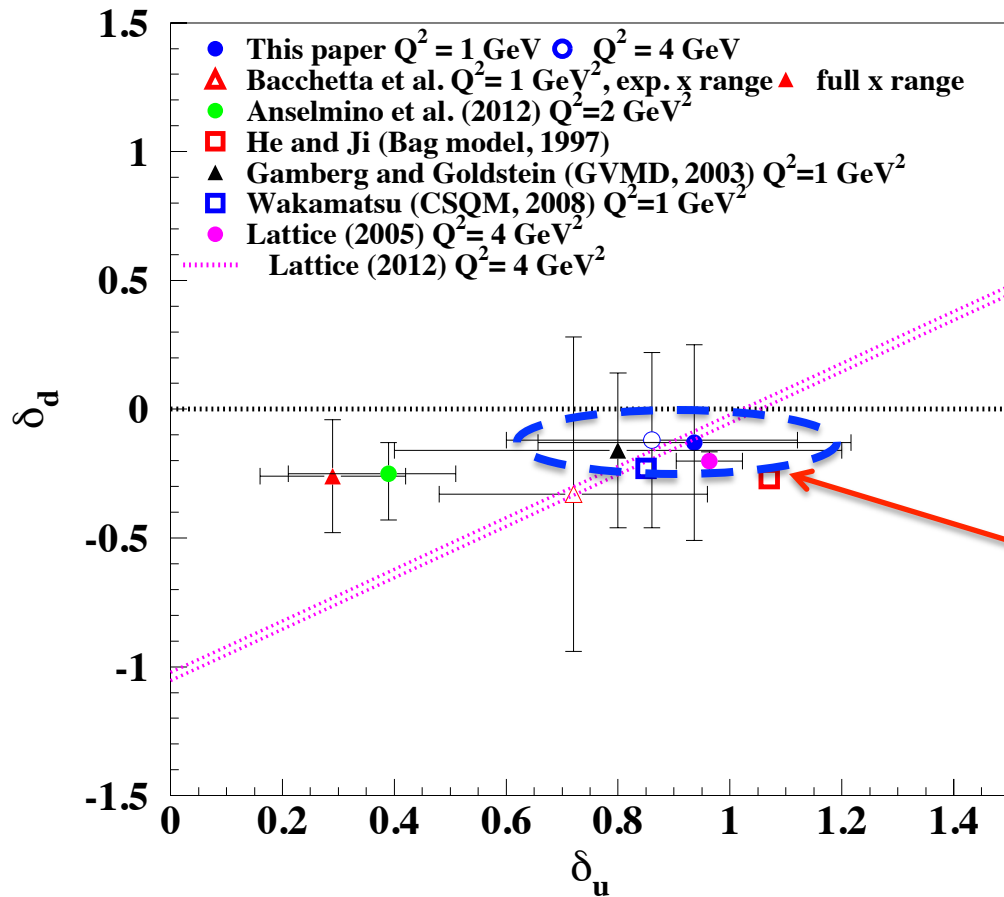
How to **model** & **measure** transversity, tensor charge & Chiral Odd GPDs

- Hadron Spatial Spin Structure from GPDs →
- Our “Flexible” parameterization for Chiral Even GPDs
 - Regge ✕ diquark spectator model: **R✕Dq**
 - Some results for DVCS (transverse γ^* → transverse γ)
 - EM Form factors
- Extend to Chiral Odd GPDs via diquark spin relations
- → Transversity
 - Model relations between Chiral even & odd helicity amps
 - π^0 & η production & flavor separation
 - Tensor charge δ_q
- Observables: Cross sections & Asymmetries
 - Which processes? Exclusive π^0 & η best candidates



Why look for Chiral odd GPDs? →

Transversity → tensor charges δ_q



GG, Gonzalez, Liuti,
arXiv:1401.0438



GPD definitions – 8 quark + 8 gluon

Momentum space nucleon matrix elements of quark field correlators

see, e.g. M. Diehl, Eur. Phys. J. C 19, 485 (2001).

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) \gamma^+ \psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, z_T=0}$$

$$= \frac{1}{2P^+} \bar{u}(p', \lambda') \left[\boxed{H^q} \gamma^+ + \boxed{E^q} \frac{i\sigma^{+\alpha} \Delta_\alpha}{2m} \right] u(p, \lambda),$$

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) \gamma^+ \gamma_5 \psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, z_T=0}$$

$$= \frac{1}{2P^+} \bar{u}(p', \lambda') \left[\boxed{\tilde{H}^q} \gamma^+ \gamma_5 + \boxed{\tilde{E}^q} \frac{\gamma_5 \Delta^+}{2m} \right] u(p, \lambda),$$

Chiral even GPDs
-> Ji sum rule

$$\langle J_q^x \rangle = \frac{1}{2} \int dx [H(x, 0, 0) + E(x, 0, 0)] x$$

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) i\sigma^{+i} \psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, z_T=0}$$

$$= \frac{1}{2P^+} \bar{u}(p', \lambda') \left[\boxed{H_T^q} i\sigma^{+i} + \boxed{\tilde{H}_T^q} \frac{P^+ \Delta^i - \Delta^+ P^i}{m^2} \right.$$

$$\left. + \boxed{E_T^q} \frac{\gamma^+ \Delta^i - \Delta^+ \gamma^i}{2m} + \boxed{\tilde{E}_T^q} \frac{\gamma^+ P^i - P^+ \gamma^i}{m} \right] u(p, \lambda)$$

Chiral odd GPDs
-> transversity
How to measure
and/or
parameterize them?



quark GPDs

- 8 quark GPDs per flavor (leading twist)
- $N \rightarrow q :: q' \rightarrow N'$ 8 independent helicity amps
 - 2 **questions**: how to **model** them?
 - how to **measure** them?
 - DVCS accesses 4 Chiral Even – $d\sigma/d\Omega$
 - **linearly** via BH X DVCS interference
- DV π^0 S accesses 2 Chiral Even + **4 Chiral Odd** See F. Sabatié Jlab/ HallA CIPANP talk
 - because $d\sigma_T > d\sigma_L$ ←
 - **bilinearly** via $d\sigma/d\Omega$ & polarization asymmetries
- See also - Goloskokov & Kroll EPJA47(2011)112 – Different approach

- **Other methods for extracting Chiral Odd**

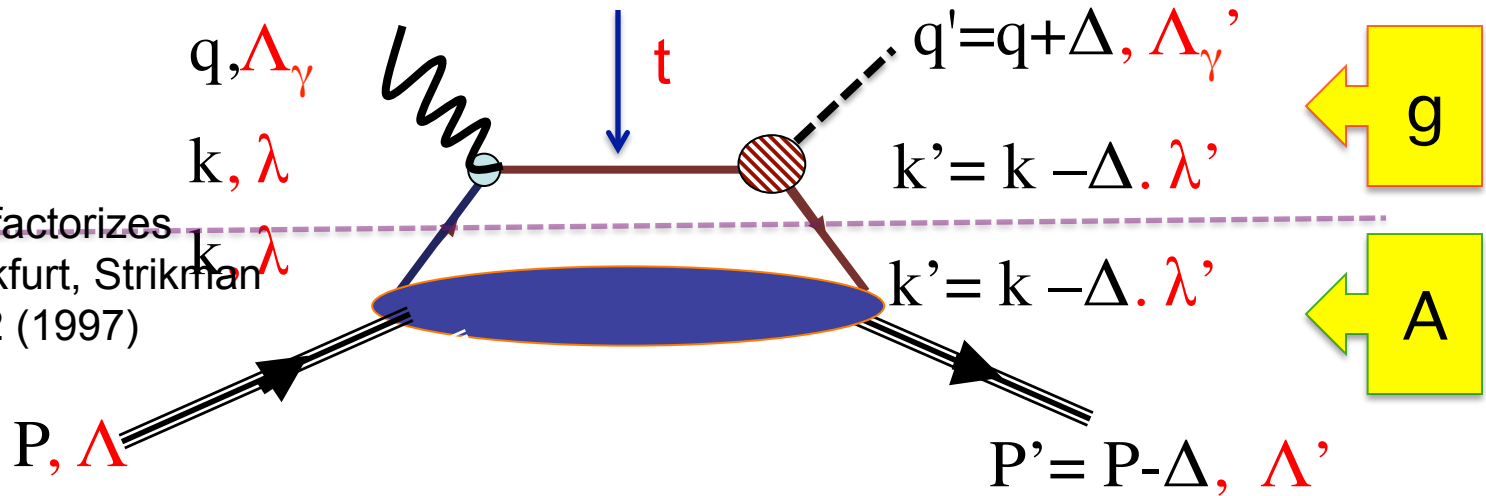
DV dihadron production - El Beiyad, et al. Phys.Lett. B688 (2010) 154; M.Radici CIPANP talk

- DV longitudinal Vector meson production - Goloskokov & Kroll EPJC74(2014)2725



Factorization in exclusive processes (DVCS, DVMP...)

$$\gamma^* p \rightarrow \gamma(M) p'$$



Longitudinal factorizes
Collins, Frankfurt, Strikman
PRD56, 2982 (1997)

Convolution of "hard part" with quark-proton **Helicity** amplitudes

Regge X diquark model: Chiral Even: Ahmad, Honkanen, Liuti, Taneja PRD75, 094003 (2007); EPJC63, 407 (2009).

$$f_{\Lambda_\gamma, \Lambda; \Lambda'_\gamma, \Lambda'} = \sum_{\lambda, \lambda'} g_{\lambda, \lambda'}^{\Lambda_\gamma, \Lambda'_\gamma(M)}(x, k_T, \zeta, t; Q^2) \otimes A_{\Lambda', \lambda'; \Lambda, \lambda}(x, k_T, \zeta, t),$$

$\lambda = +(-)$ λ' chiral even (odd)

see Ahmad, GG, Liuti, PRD79, 054014, (2009)

for first chiral odd GPD parameterization

Gonzalez, GG, Liuti PRD84, 034007 (2011) chiral even GPD



Normalizing GPDs - Chiral even

Form factor,

Forward limit

$$\int_0^1 H_q(x, \xi, t) dx = F_1^q(t), \quad H_q(x, 0, 0) = q(x) \quad \text{Integrates to charge}$$

$$\int_0^1 E_q(x, \xi, t) dx = F_2^q(t)$$

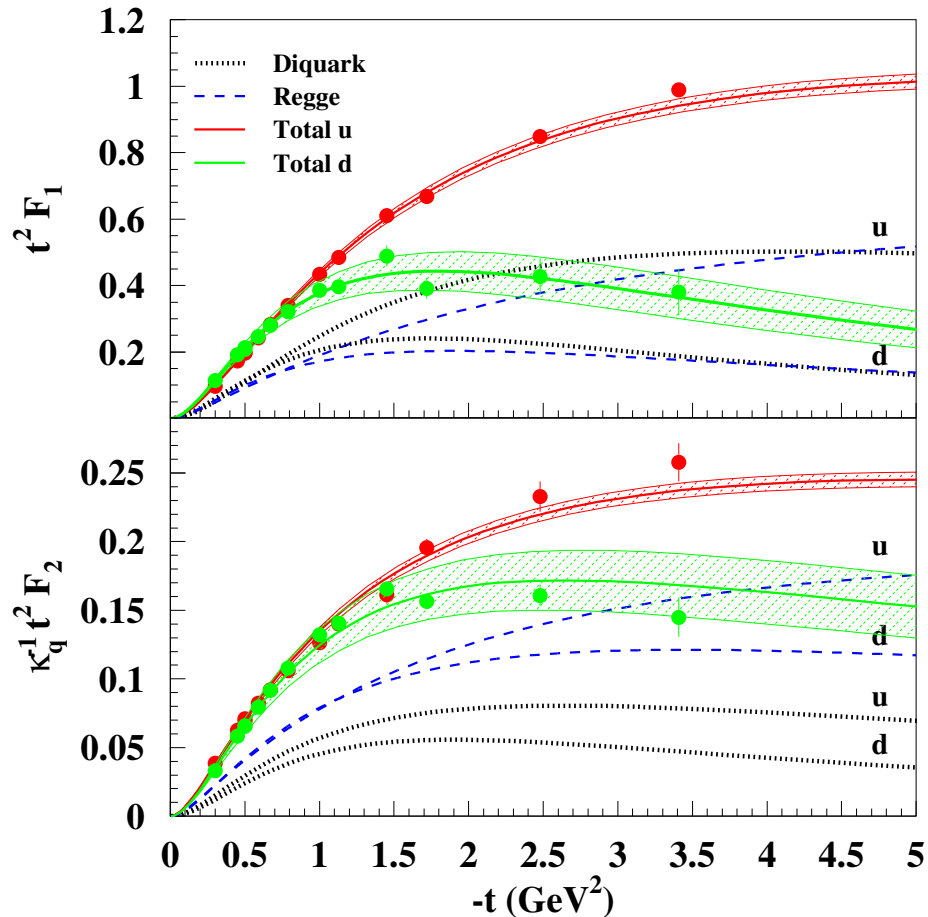
$$\int_0^1 \tilde{H}_q(x, \xi, t) dx = g_A^q(t), \quad \tilde{H}_q(x, 0, 0) = \Delta q(x) = q_{\Rightarrow}^{\rightarrow}(x) - q_{\Rightarrow}^{\leftarrow}(x)$$

Integrates to axial charge

$$\int_0^1 \tilde{E}_q(x, \xi, t) dx = g_P^q(t)$$



EM Form Factors (t dependence)



O.Gonzalez, GRG, S.Liuti, K.Kathuria PRC88, 065206(2013)
data: G.D. Cates, et al. PRL106,252003 (2011).



The question is: how do we normalize chiral-odd GPDs?

Only Physical constraints on the various chiral-odd GPDs are

Forward limit

$$H_T(x, 0, 0) = q_{\uparrow\uparrow}^{\uparrow}(x) - q_{\uparrow\uparrow}^{\downarrow}(x) = h_1(x) \quad \text{Transversity}$$

Form Factors

Integrates to tensor charge δ_q

$$\int H_T^q(x, \xi, t) dx = \delta q(t)$$

"Tensor form factor"

$$\int \bar{E}_T^q(x, \xi, t) dx = \int \left(2\tilde{H}_T^q + E_T^q \right) dx = \kappa_T^q(t)$$

Integrates to "transverse dipole moment" κ_T^q

$$\int \tilde{E}_T(x, \xi, t) dx = 0$$

No direct interpretation of E_T

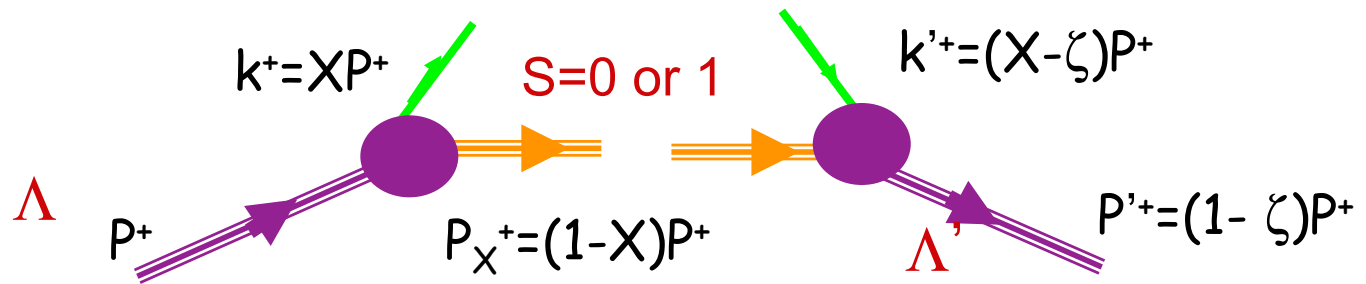
$$\lim_{t \rightarrow 0} \frac{t}{4M^2} \tilde{H}_T(x, 0, t) = h_1^\perp(x)$$



The Model – Reggeized Diquarks

The Model – first for Chiral Even – Reggeized Diquarks

Procedure to construct Chiral Odd GPDs & observables



Product of diquark l.c.w.f.'s $\rightarrow A_{\Lambda\Lambda; \Lambda'\lambda'=\lambda}$

$A_{\Lambda\Lambda; \Lambda'\lambda}$ \rightarrow chiral even GPDs

$g \otimes A \rightarrow$ exclusive process helicity amps

pdf's, FF's, $d\sigma/d\Omega$ & Asymmetries: parameters & predictions

vertex parity $\rightarrow A_{\Lambda\Lambda; -\Lambda'\lambda'}$ \rightarrow chiral odd GPDs \rightarrow pdf's, ...



Recursive fit

GRG, Gonzalez Hernandez, Liuti,
PRD84, 034007 (2011)

Advantage: control over the number of parameters to be fitted at different stages so that it can be optimized

Functional form:

From DIS

$$q(x, Q_o^2) = A_q x^{-\alpha_q} (1-x)^{\beta_q} F(x, c_q, d_q, \dots)$$

to DVCS, DVMP



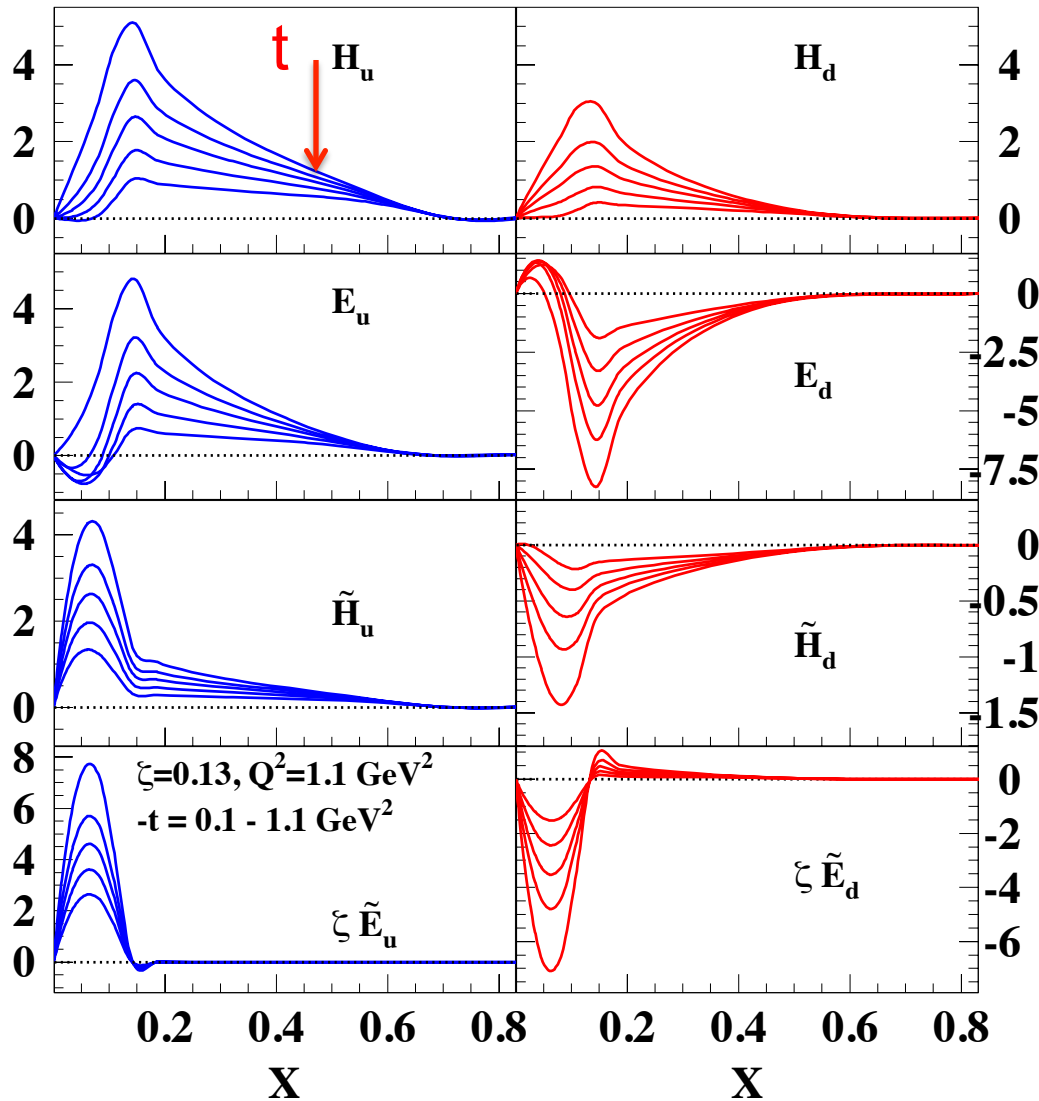
$$H_q(x, \xi, t; Q_o^2) = N_q x^{-[\alpha_q + \alpha'_q (1-x)^p t]} G^{a_1 a_2 a_3 \dots}(x, \xi, t)$$
$$a_1 = m_q, a_2 = M_X^q, a_3 = M_\Lambda^q, \dots$$

pdf's
Form Factors
 $d\sigma / d\Omega$
Asymmetries

“Flexible” parameterization based on the Reggeized quark-diquark model.



Chiral even GPDs



From GPDs
with evolution
to Compton
Form Factors

CFFs to helicity
amps

helicity amps to
observables

\leftrightarrow parameters



Observables

Chiral Even

Chiral Odd

$$A_{\Lambda'\pm, \Lambda\pm} \Leftrightarrow H, E, \tilde{H}, \tilde{E}$$

$$A_{\Lambda'\pm, \Lambda\mp} \Leftrightarrow H_T, E_T, \tilde{H}_T, \tilde{E}_T$$

Compton Form Factors

$$\mathcal{H}(\xi, t; Q^2) = \int dx \left[\frac{1}{x - \xi - i\epsilon} \mp \frac{1}{x + \xi - i\epsilon} \right] H(x, \xi, t; Q^2)$$

$$\rightarrow \left(P.V. \int dx \frac{H(x, \xi, t; Q^2)}{x - \xi} + i\pi H(\xi, \xi, t; Q^2) \right) \mp (\text{symm. term})$$

Re \mathcal{H}

Im \mathcal{H}



Having fit other data we predicted Hermes data

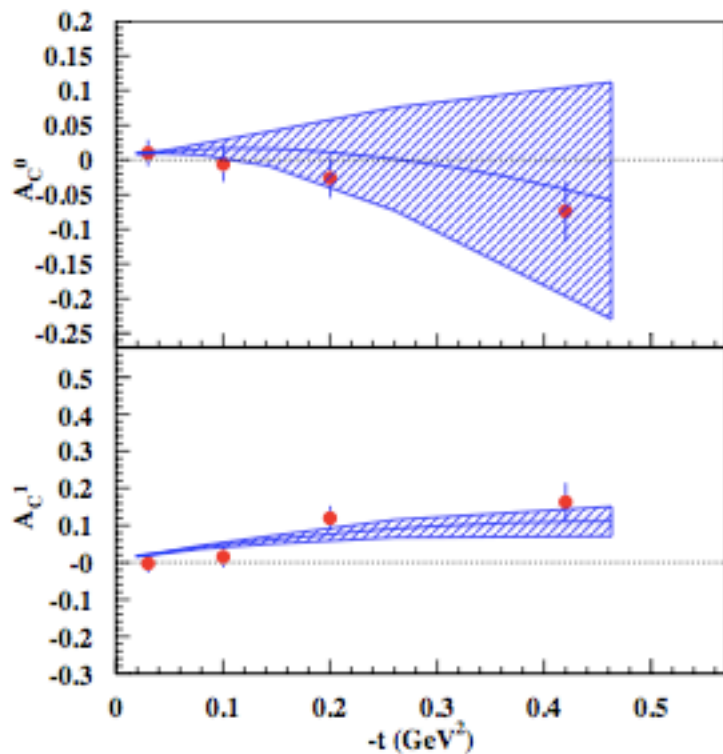


FIG. 20: Coefficients of the beam charge asymmetry, A_C , extracted from experiment [52, 53]. The lower panel is the coefficient for the $\cos \phi$ dependent term in Eq.(82), while the upper panel is the $\cos \phi$ independent term.

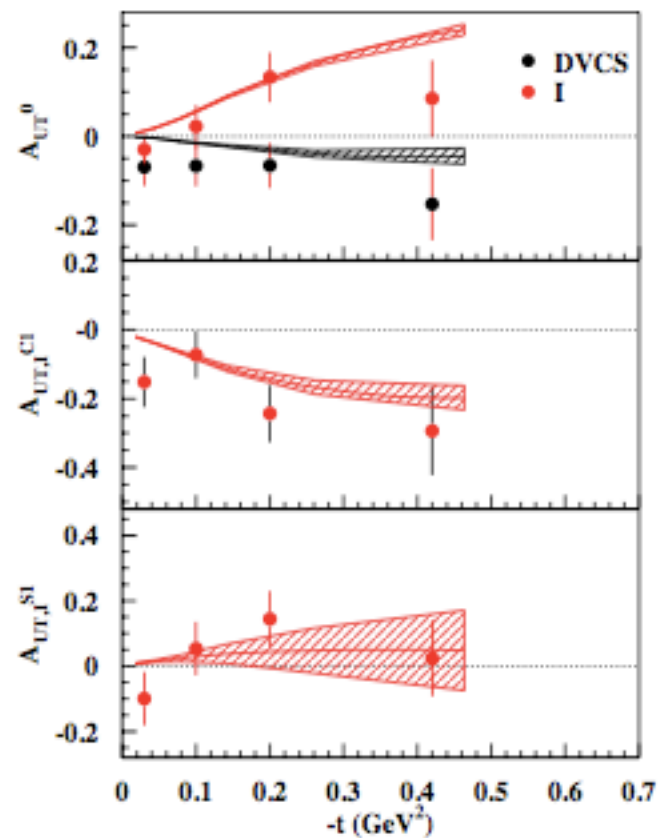


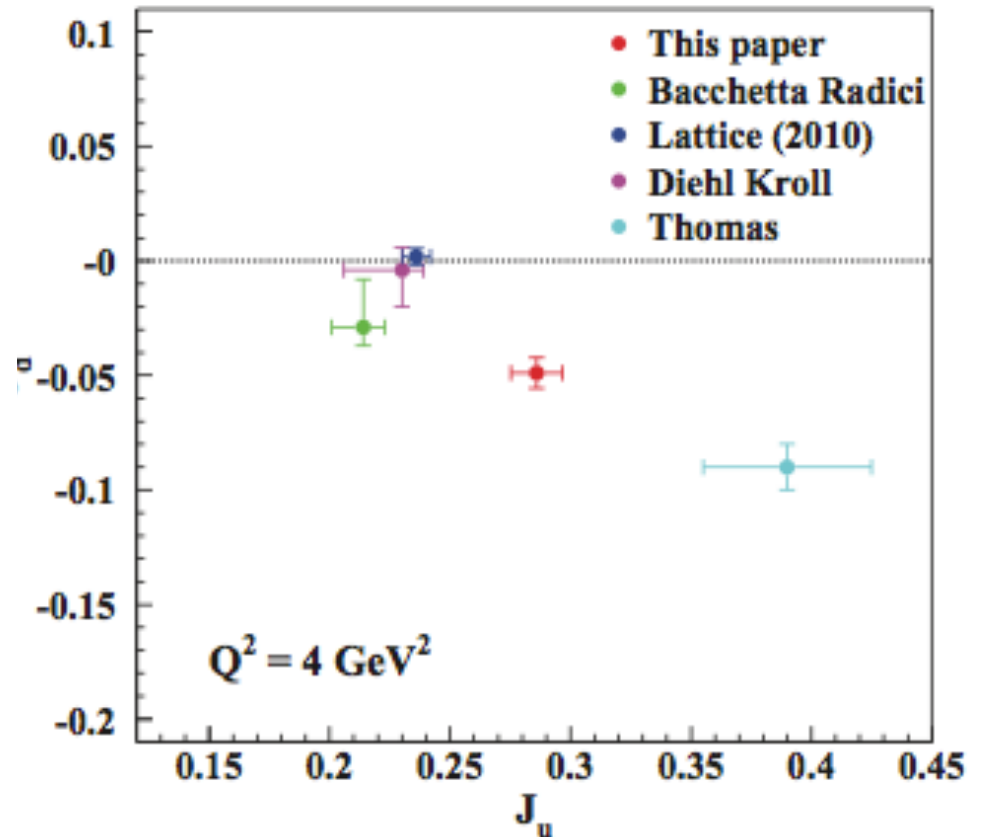
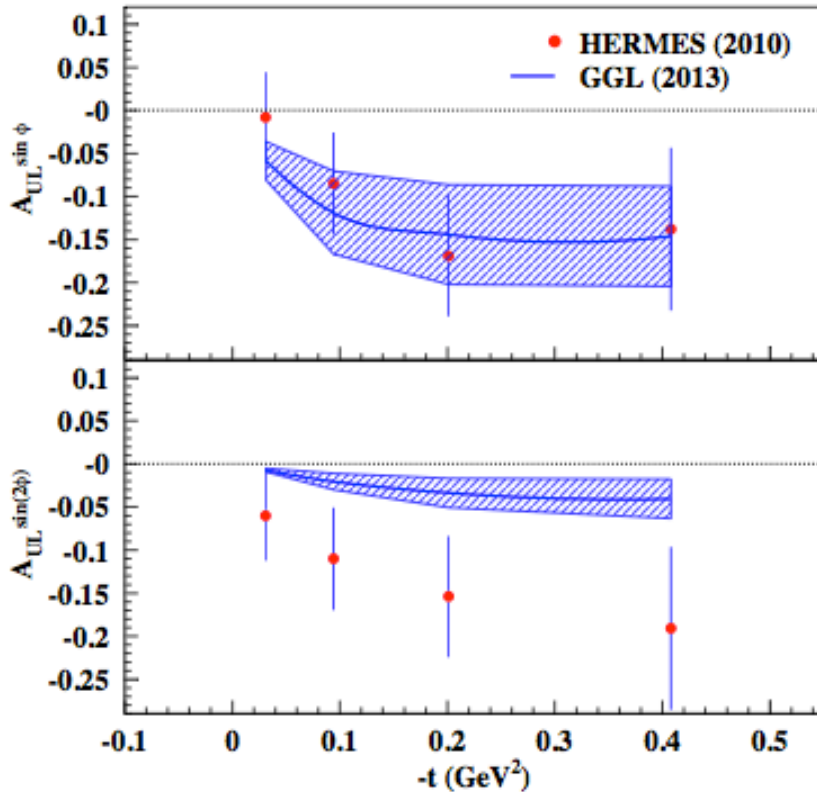
FIG. 21: Coefficients of the beam charge asymmetry, A_{UT} , extracted from experiment [51]. The upper panel shows the terms E and F from Eqs.(83) and (84), respectively; the middle panel shows G , and the lower panel H , both in Eq.(84). The curves are predictions obtained extending our quantitative fit of Jefferson lab data to the Hermes set of observables.



Other chiral even predictions

Gonzalez Hernandez, Liuti, GG, Kathuria

PHYSICAL REVIEW C 88, 065206 (2013)

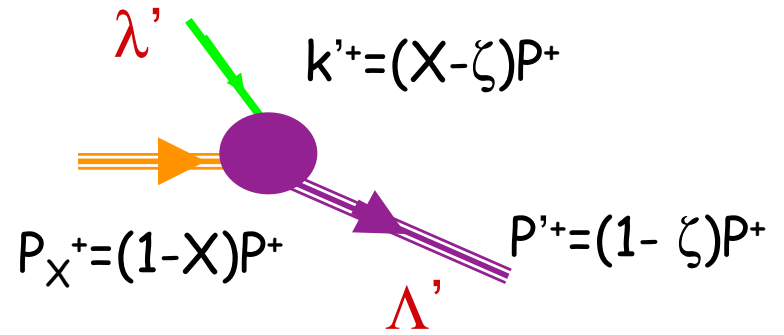
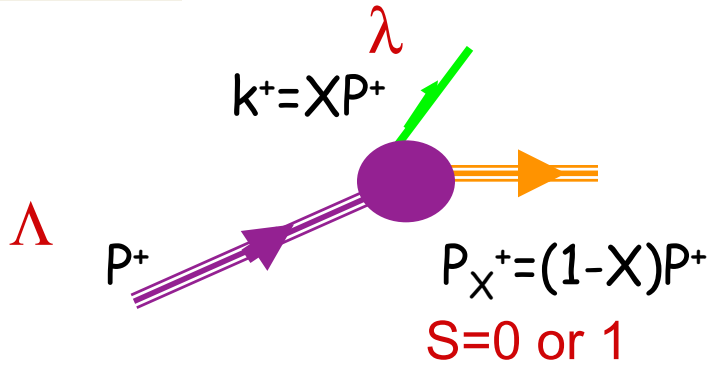




The Model for Chiral Odd – Reggeized Diquarks



Vertex Structures with Diquark Spectator



First focus on $S=0$ pure spectator - beginning

$$H \Rightarrow \varphi_{++}^*(k', P') \varphi_{++}(k, P) + \varphi_{-+}^*(k', P') \varphi_{-+}(k, P)$$

$$E \Rightarrow \varphi_{++}^*(k', P') \varphi_{+-}(k, P) + \varphi_{+-}^*(k', P') \varphi_{++}(k, P)$$

$$\tilde{H} \Rightarrow \varphi_{++}^*(k', P') \varphi_{++}(k, P) - \varphi_{-+}^*(k', P') \varphi_{-+}(k, P)$$

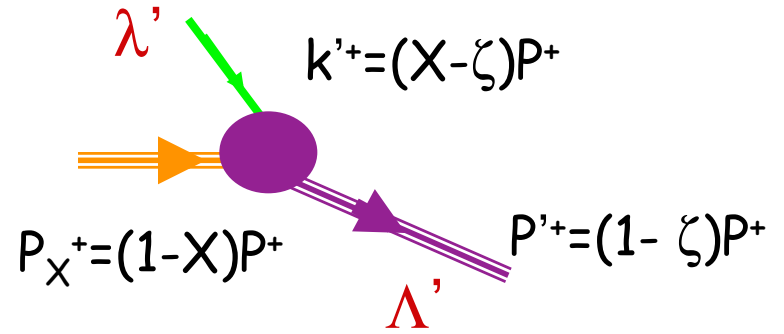
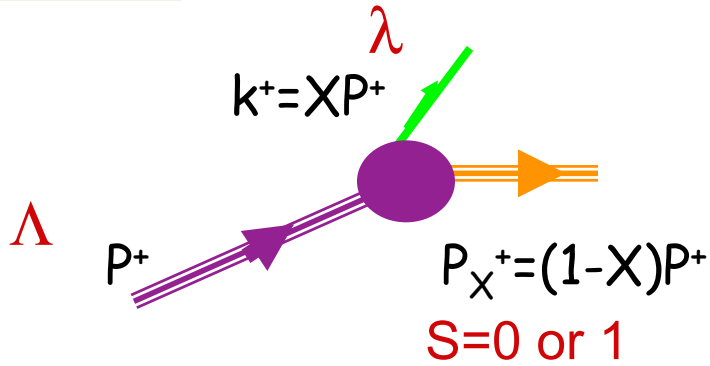
$$\tilde{E} \Rightarrow \varphi_{++}^*(k', P') \varphi_{+-}(k, P) - \varphi_{+-}^*(k', P') \varphi_{++}(k, P)$$

Vertex form factor function

$$\phi(k^2, \lambda) = \frac{k^2 - m^2}{|k^2 - \lambda^2|^2}$$



Vertex Structures with Diquark Spectator



First focus on $S=0$ pure spectator - beginning

$$H \Rightarrow \varphi_{++}^*(k', P') \varphi_{++}(k, P) + \varphi_{-+}^*(k', P') \varphi_{-+}(k, P)$$

$$E \Rightarrow \varphi_{++}^*(k', P') \varphi_{+-}(k, P) + \varphi_{+-}^*(k', P') \varphi_{++}(k, P)$$

$$\tilde{H} \Rightarrow \varphi_{++}^*(k', P') \varphi_{++}(k, P) - \varphi_{-+}^*(k', P') \varphi_{-+}(k, P)$$

$$\tilde{E} \Rightarrow \varphi_{++}^*(k', P') \varphi_{+-}(k, P) - \varphi_{+-}^*(k', P') \varphi_{++}(k, P)$$

Vertex form factor

$$\phi(k^2, \lambda) = \frac{k^2 - m^2}{|k^2 - \lambda^2|^2}$$

Parity at vertices:

By switching $\lambda \rightarrow -\lambda$ & $\Lambda \rightarrow -\Lambda$ (Parity) will have chiral evens go to \pm chiral odds giving relations – before k integrations

$$A(\Lambda' \lambda'; \Lambda \lambda) \rightarrow \pm A(\Lambda', \lambda'; -\Lambda, -\lambda)^*$$

but then $(\Lambda' - \lambda') - (\Lambda - \lambda) \neq (\Lambda' - \lambda') + (\Lambda - \lambda)$ unless $\Lambda = \lambda$



S=0 Chiral even \leftrightarrow odd helicity amps (+ S=1)

odd \leftrightarrow even

$$\begin{aligned} A_{++,--}^{(0)} &= A_{++,++}^{(0)} \\ A_{++,+-}^{(0)} &= -A_{++,--}^{(0)} \\ A_{+-,++}^{(0)} &= -A_{-+,++}^{(0)}, \end{aligned}$$

t-channel flip·flip \leftrightarrow nonflip·nonflip
flip·nonflip \leftrightarrow nonflip·flip

Invert both sides to get GPDs – same helicity amp sets

$$\begin{aligned} \tilde{H}_T^0 &= -(1-\zeta)^2 \frac{M(1-x)}{m+Mx'} \left[E^0 - \frac{\zeta}{2} \tilde{E}^0 \right] \\ E_T^0 &= -\frac{(1-\zeta/2)^2}{1-\zeta} \left[2\tilde{H}_T^0 - E^0 + \left(\frac{\zeta/2}{1-\zeta/2} \right)^2 \tilde{E}^0 \right] \end{aligned}$$

$$\tilde{E}_T^0 = -\frac{\zeta/2(1-\zeta/2)}{1-\zeta} \left[2\tilde{H}_T^0 - E^0 + \tilde{E}^0 \right]$$

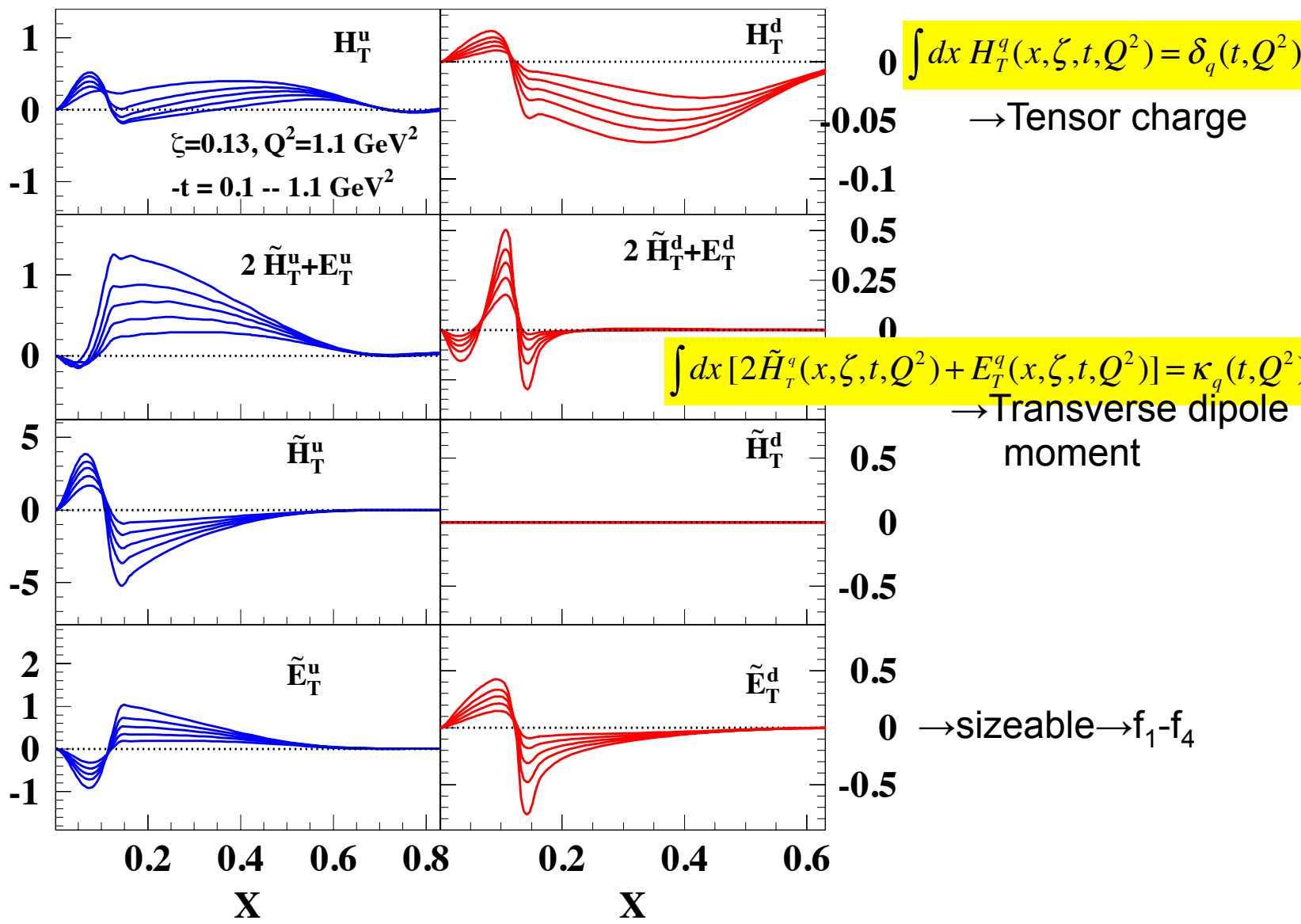
$$H_T^0 = \frac{H^0 + \tilde{H}^0}{2} - \frac{\zeta^2/4}{1-\zeta} \frac{E^0 + \tilde{E}^0}{2} - \frac{\zeta^2/4}{(1-\zeta/2)(1-\zeta)} E_T^0 + \frac{\zeta/4(1-\zeta/2)}{1-\zeta} \tilde{E}_T^0 + \tilde{H}_T^0,$$

S = 0 double helicity flip amplitude was calculated directly from Eq.(16),

$$A_{+-,-+}^{(0)} = \frac{t_0 - t}{4M} \frac{1}{\sqrt{1-\zeta}} \frac{1}{(1-\zeta/2)} \frac{\tilde{x}}{m+Mx'} \left[E^0 - \frac{\zeta}{2} \tilde{E}^0 \right].$$

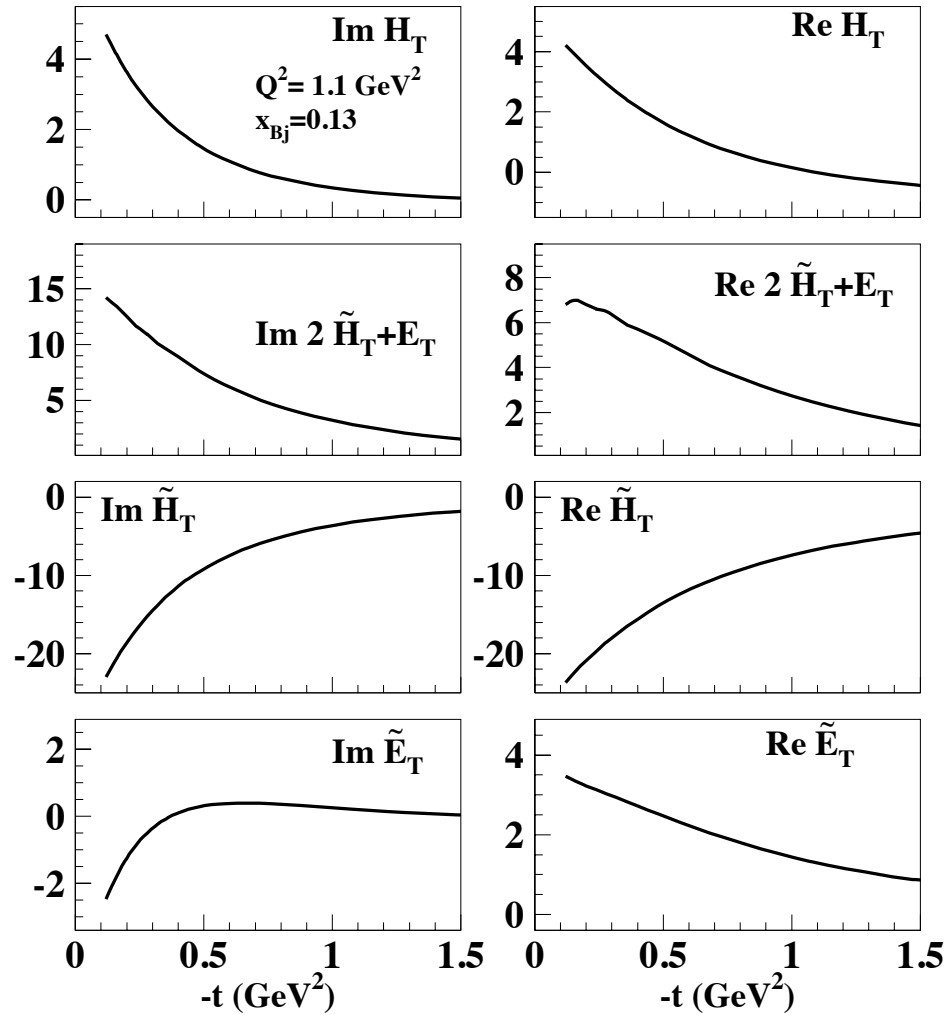


RESULT: Chiral odd GPDs





Compton form factors

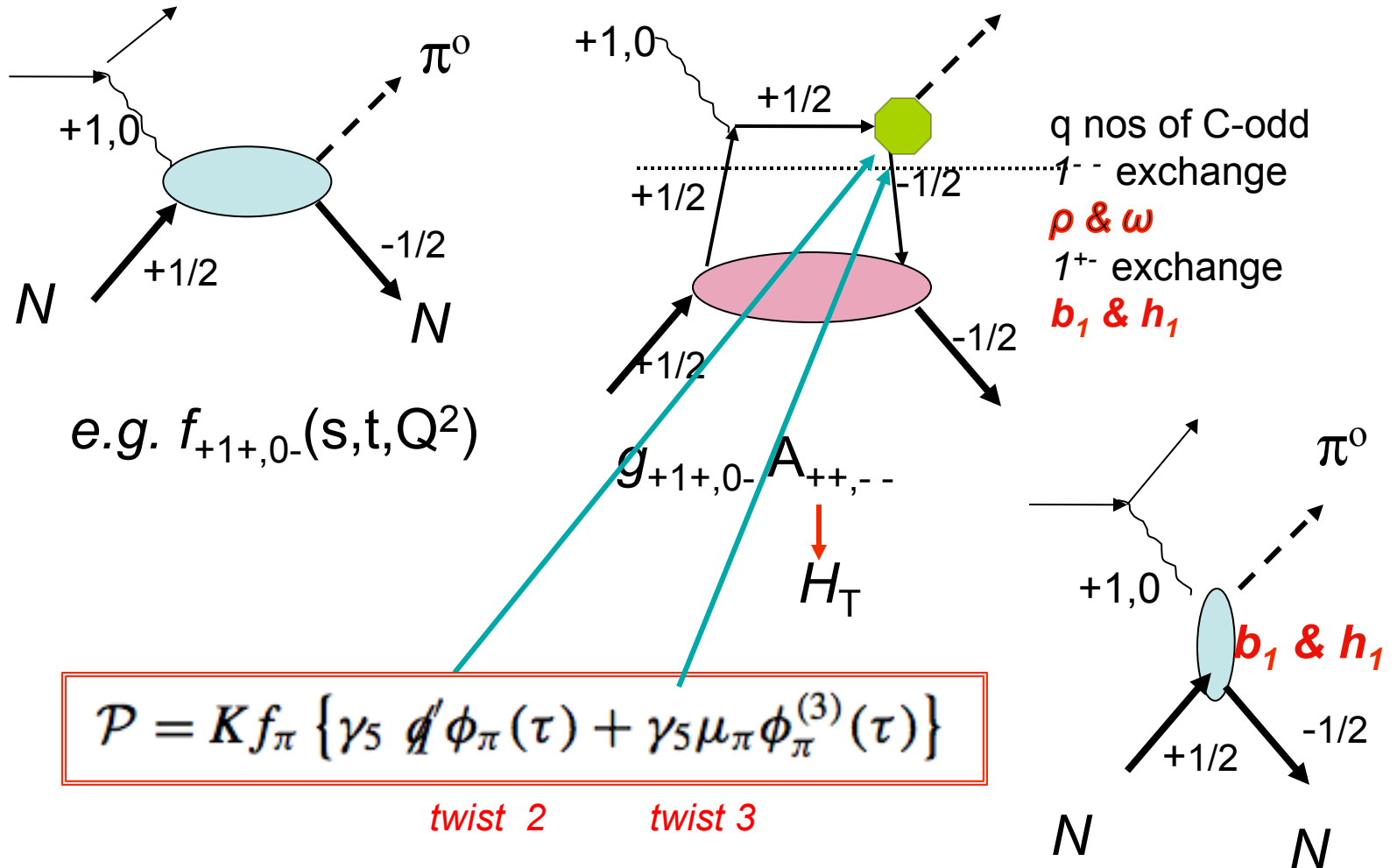




How to single out & measure **chiral odd GPDs**?

Exclusive Lepto-production of π^0 or η, η'

to measure **chiral odd GPDs & Transversity**

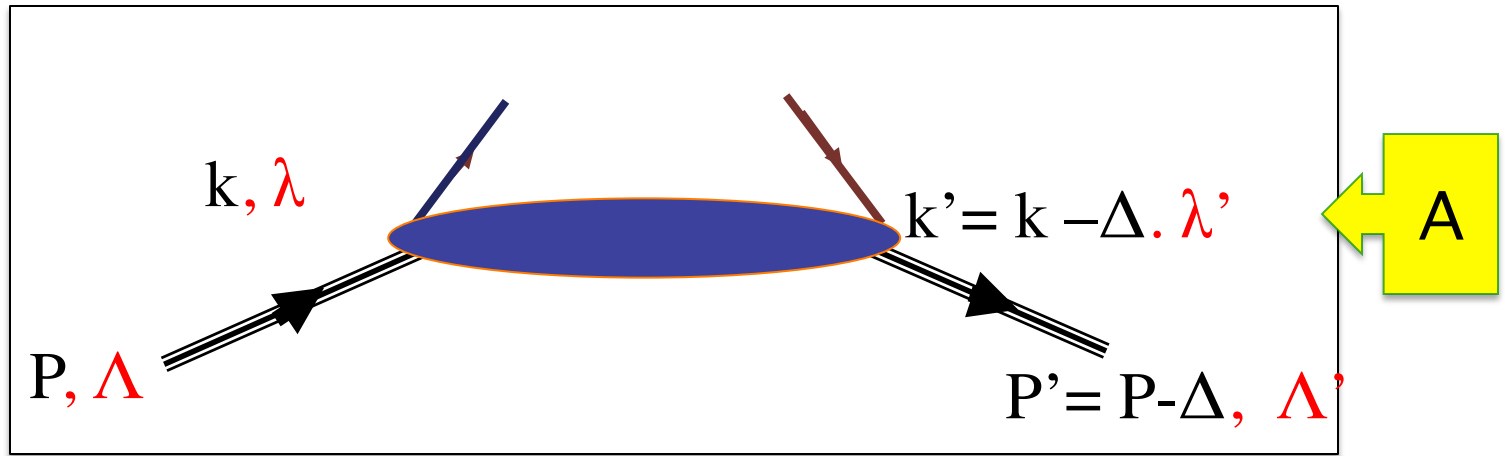


t-channel J^{PC} quantum numbers enhance chiral odd



6 independent helicity amps for π^0 or η, η' (& K, D_C)

8 Quark-nucleon helicity amps for u,d (& s,c)



odd

$$f_{10}^{++} = g_{10}^{+-} \otimes A_{+-,++}$$

$$f_{10}^{+-} = g_{10}^{+-} \otimes A_{--,++}$$

$$f_{10}^{-+} = g_{10}^{+-} \otimes A_{+-,-+}$$

$$f_{10}^{--} = g_{10}^{+-} \otimes A_{++,+-}$$

$$f_{00}^{+-} = g_{00}^{+-} \otimes (A_{--,++} - A_{+-,-+})$$

$$f_{00}^{++} = g_{00}^{+-} \otimes (A_{++,+-} - A_{+-,++}),$$

$$f_{00}^{+-, even} = \frac{\zeta}{\sqrt{1-\zeta}} \frac{1}{1-\zeta/2} \frac{\sqrt{t_o-t}}{2M} \tilde{\mathcal{E}},$$

$$f_{00}^{++, even} = \frac{\sqrt{1-\zeta}}{1-\zeta/2} \tilde{\mathcal{H}} + \frac{-\zeta^2/4}{(1-\zeta/2)\sqrt{1-\zeta}} \tilde{\mathcal{E}},$$

even



6 helicity amps for π^0

Compton Form Factors

$$\begin{aligned}
 f_1 \quad f_{10}^{++} &= g_{\pi}^{V,odd}(Q) \frac{\sqrt{t_0-t}}{4M} \left[2\tilde{\mathcal{H}}_T + (1+\xi) (\mathcal{E}_T + \tilde{\mathcal{E}}_T) \right] \\
 &= g_{\pi}^{V,odd}(Q) \frac{\sqrt{t_0-t}}{2M} \left[\tilde{\mathcal{H}}_T + \frac{1}{2-\zeta} \mathcal{E}_T + \frac{1}{2-\zeta} \tilde{\mathcal{E}}_T \right], \\
 f_2 \quad f_{10}^{+-} &= \frac{g_{\pi}^{V,odd}(Q) + g_{\pi}^{A,odd}(Q)}{2} \sqrt{1-\xi^2} \left[\mathcal{H}_T + \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T + \frac{\xi^2}{1-\xi^2} \mathcal{E}_T + \frac{\xi}{1-\xi^2} \tilde{\mathcal{E}}_T \right] \\
 &= \frac{g_{\pi}^{V,odd}(Q) + g_{\pi}^{A,odd}(Q)}{2} \frac{\sqrt{1-\zeta}}{1-\zeta/2} \left[\mathcal{H}_T + \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T + \frac{\zeta^2/4}{1-\zeta} \mathcal{E}_T + \frac{\zeta/2}{1-\zeta} \tilde{\mathcal{E}}_T \right] \\
 f_3 \quad f_{10}^{-+} &= -\frac{g_{\pi}^{A,odd}(Q) - g_{\pi}^{V,odd}(Q)}{2} \sqrt{1-\xi^2} \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T \\
 &= -\frac{g_{\pi}^{A,odd}(Q) - g_{\pi}^{V,odd}(Q)}{2} \frac{\sqrt{1-\zeta}}{1-\zeta/2} \frac{t_0-t}{4M^2} \tilde{\mathcal{H}}_T \\
 f_4 \quad f_{10}^{--} &= g_{\pi}^{V,odd}(Q) \frac{\sqrt{t_0-t}}{4M} \left[2\tilde{\mathcal{H}}_T + (1-\xi) (\mathcal{E}_T - \tilde{\mathcal{E}}_T) \right] \\
 &= g_{\pi}^{V,odd}(Q) \frac{\sqrt{t_0-t}}{2M} \left[\tilde{\mathcal{H}}_T + \frac{1-\zeta}{2-\zeta} \mathcal{E}_T + \frac{1-\zeta}{2-\zeta} \tilde{\mathcal{E}}_T \right] \\
 f_5 \quad f_{00}^{+-} &= g_{\pi}^{A,odd}(Q) \sqrt{1-\xi^2} \left[\mathcal{H}_T + \frac{\xi^2}{1-\xi^2} \mathcal{E}_T + \frac{\xi}{1-\xi^2} \tilde{\mathcal{E}}_T \right] \sqrt{t_0-t} \\
 f_6 \quad f_{00}^{++} &= -g_{\pi}^{A,odd}(Q) \frac{\sqrt{t_0-t}}{2M} \left[\xi \mathcal{E}_T + \tilde{\mathcal{E}}_T \right] \sqrt{t_0-t}
 \end{aligned}$$

Also Chiral Even CFFs



Observables

Cross sections
Asymmetries



Cross section with ϕ modulations & beam/target polarized

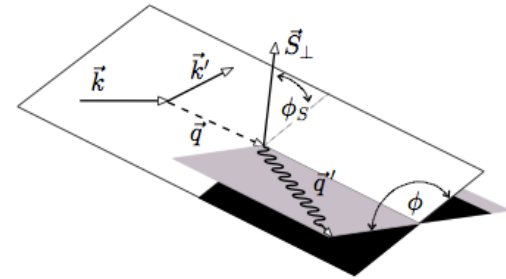
$$\frac{d^4\sigma}{dx_{Bj}dyd\phi dt} = \Gamma \left\{ \begin{aligned} & \left[F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{2\epsilon(\epsilon+1)} \cos \phi F_{UU}^{\cos \phi} + h \sqrt{2\epsilon(1-\epsilon)} \sin \phi F_{LU}^{\sin \phi} \right. \\ & + S_{\parallel} \left[\sqrt{2\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} + h \left(\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi} \right) \right] \\ & + S_{\perp} \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) + \epsilon \left(\sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right) \right. \\ & + \left. \left. \sqrt{2\epsilon(1+\epsilon)} \left(\sin \phi_S F_{UT}^{\sin \phi_S} + \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right) \right] \right. \\ & + \left. S_{\perp} h \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \left(\cos \phi_S F_{LT}^{\cos \phi_S} + \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \right\} \end{aligned} \right.$$

$$A_{LU} = \sqrt{\epsilon(1-\epsilon)} \frac{F_{LU}^{\sin \phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$

$$A_{UL} = \frac{N_{s_z=+} - N_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\epsilon \sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$

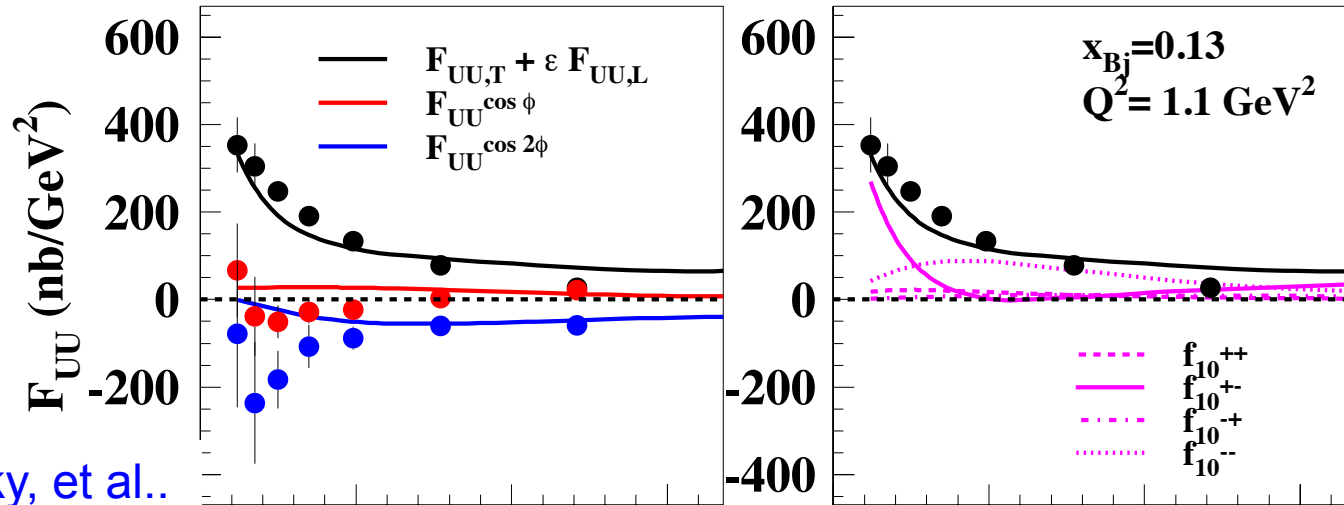
$$\begin{aligned} F_{UU,T} &= \frac{d\sigma_T}{dt}, & F_{UU,L} &= \frac{d\sigma_L}{dt}, & F_{UU}^{\cos \phi} &= \frac{d\sigma_{LT}}{dt}, \\ F_{UU}^{\cos 2\phi} &= \frac{d\sigma_{TT}}{dt}, & F_{LU}^{\sin \phi} &= \frac{d\sigma_{LT'}}{dt} \end{aligned}$$

$$A_{LL} = \frac{N_{s_z=+}^{\rightarrow} - N_{s_z=-}^{\rightarrow} + N_{s_z=+}^{\leftarrow} - N_{s_z=-}^{\leftarrow}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{1-\epsilon^2} F_{LL}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$

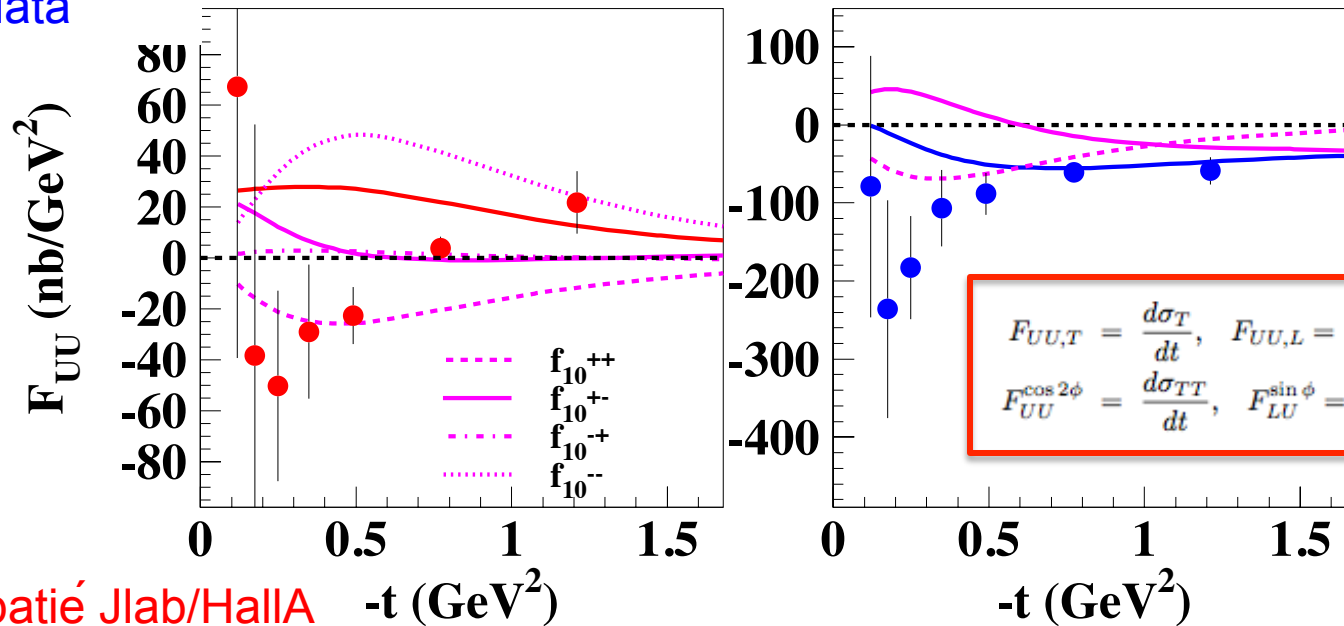




Unpolarized cross sections



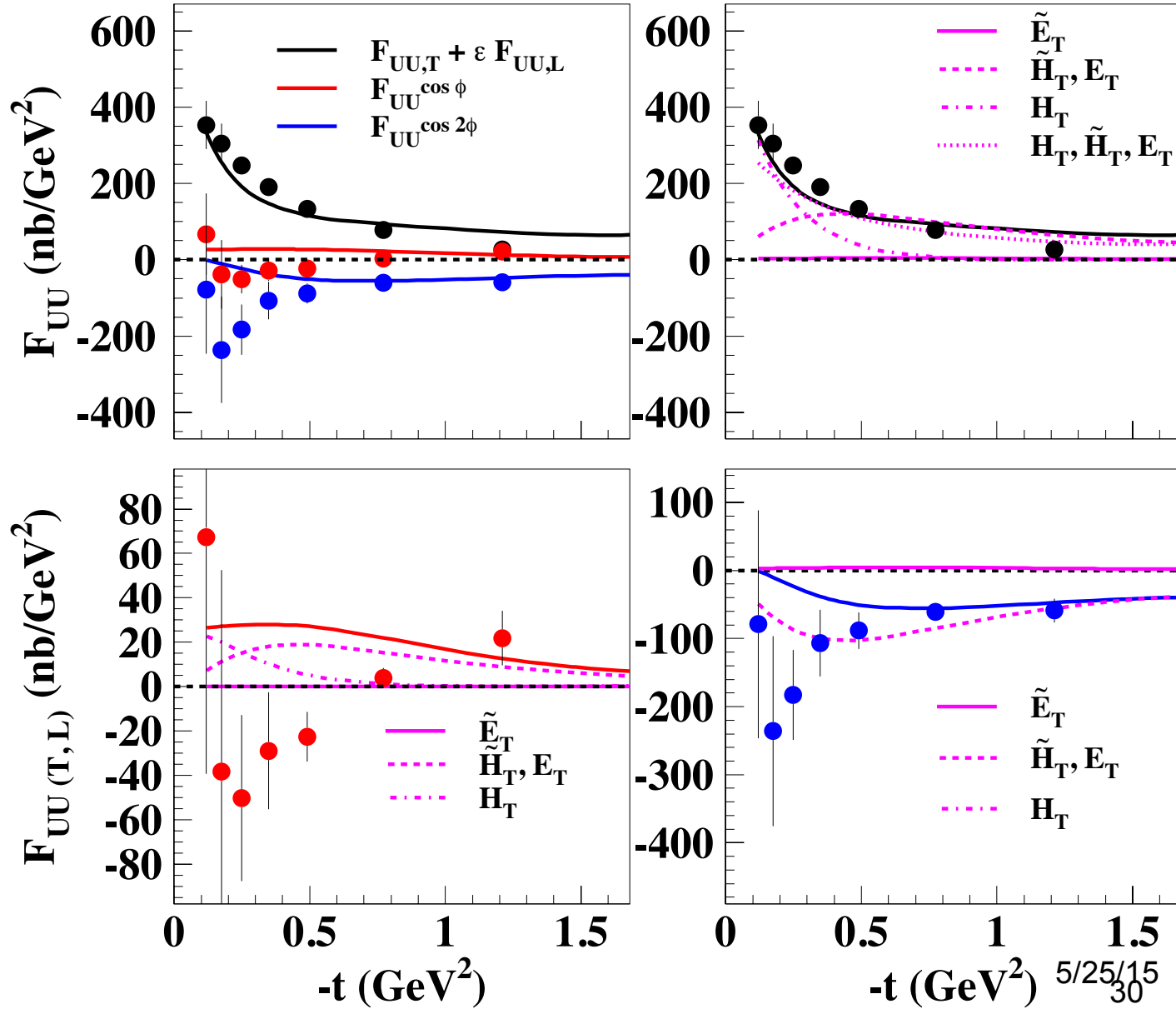
V.Kubarovsky, et al.
Jlab/CLAS data



See F. Sabatié Jlab/HallA
CIPANP talk for L & T
separation



Same, separating the GPDs contribution





Unpolarized cross section components

$$F_{UU,T} = \frac{d\sigma_T}{dt}, \quad F_{UU,L} = \frac{d\sigma_L}{dt}, \quad F_{UU}^{\cos\phi} = \frac{d\sigma_{LT}}{dt},$$

$$F_{UU}^{\cos 2\phi} = \frac{d\sigma_{TT}}{dt}, \quad F_{LU}^{\sin\phi} = \frac{d\sigma_{LT'}}{dt}$$

$$F_{UU,T} = \mathcal{N} [|f_{10}^{++}|^2 + |f_{10}^{+-}|^2 + |f_{10}^{-+}|^2 + |f_{10}^{--}|^2]$$

$|f_{10}^{+-}|$ dominates small t & $> |f_{10}^{\pm\pm}|$

$$F_{UU,L} = \mathcal{N} [|f_{00}^{++}|^2 + |f_{00}^{+-}|^2]$$

Jlab/HallA
L & T separation

$$F_{UU}^{\cos 2\phi} = -\mathcal{N} 2\Re [(f_{10}^{++})^* (f_{10}^{--}) - (f_{10}^{+-})^* (f_{10}^{-+})]$$

Or form of $-|f_{1+}|^2 + |f_{1-}|^2$

$$F_{UU}^{\cos\phi} = -\mathcal{N} \Re [(f_{00}^{+-})^* (f_{10}^{-+} + f_{10}^{+-}) + (f_{00}^{++})^* (f_{10}^{++} - f_{10}^{--})]$$

Polarized beam

$$F_{LU}^{\sin\phi} = \mathcal{N} \Im [(f_{00}^{+-})^* (f_{10}^{-+} + f_{10}^{+-}) + (f_{00}^{++})^* (f_{10}^{++} - f_{10}^{--})]$$



Comparing to other models

- The $t \rightarrow 0$ feature for us is that f_{10}^{+-} dominates & it is driven by H_T . But f_{10}^{++} & f_{10}^{--} also contribute as $\sim \sqrt{(t_0 - t)}$, however weaker.
- f_{10}^{++} & f_{10}^{--} are not equal in magnitude, especially vs. ζ or ξ .
- In $A_{LL} \sim |f_{10}^{++}|^2 + |f_{10}^{+-}|^2 - |f_{10}^{-+}|^2 - |f_{10}^{--}|^2$ sensitive to differences



Comparison between models

$$\frac{d\sigma_T}{dt} \propto |\mathcal{H}_T|^2 + \tau |\bar{\mathcal{E}}_T|^2$$

Goloskokov and Kroll

$$\tau = \frac{t_o - t}{8M^2}$$

$$\frac{d\sigma_T}{dt} \propto g_\pi^{(1)}(Q^2) \tau |\bar{\mathcal{E}}_T + (1 + \xi) \tilde{\mathcal{E}}_T|^2 + g_\pi^{(2)}(Q^2) \tau |\bar{\mathcal{E}}_T + (1 - \xi) \tilde{\mathcal{E}}_T|^2 + g_\pi^{(3)}(Q^2) |\mathcal{H}_T|^2$$

Goldstein, Gonzalez, Liuti

$$\frac{d\sigma_{TT}}{dt} \propto |\mathcal{E}_T|^2$$

Goloskokov and Kroll

$$\frac{d\sigma_{TT}}{dt} \propto (\bar{\mathcal{E}}_T + (1 + \xi) \tilde{\mathcal{E}}_T)^* (\bar{\mathcal{E}}_T + (1 - \xi) \tilde{\mathcal{E}}_T)$$

Goldstein, Gonzalez, Liuti



CLAS π^0 : Bedlinskiy, et al. PRL109, 112001 (2012).

GGL vs. G&K

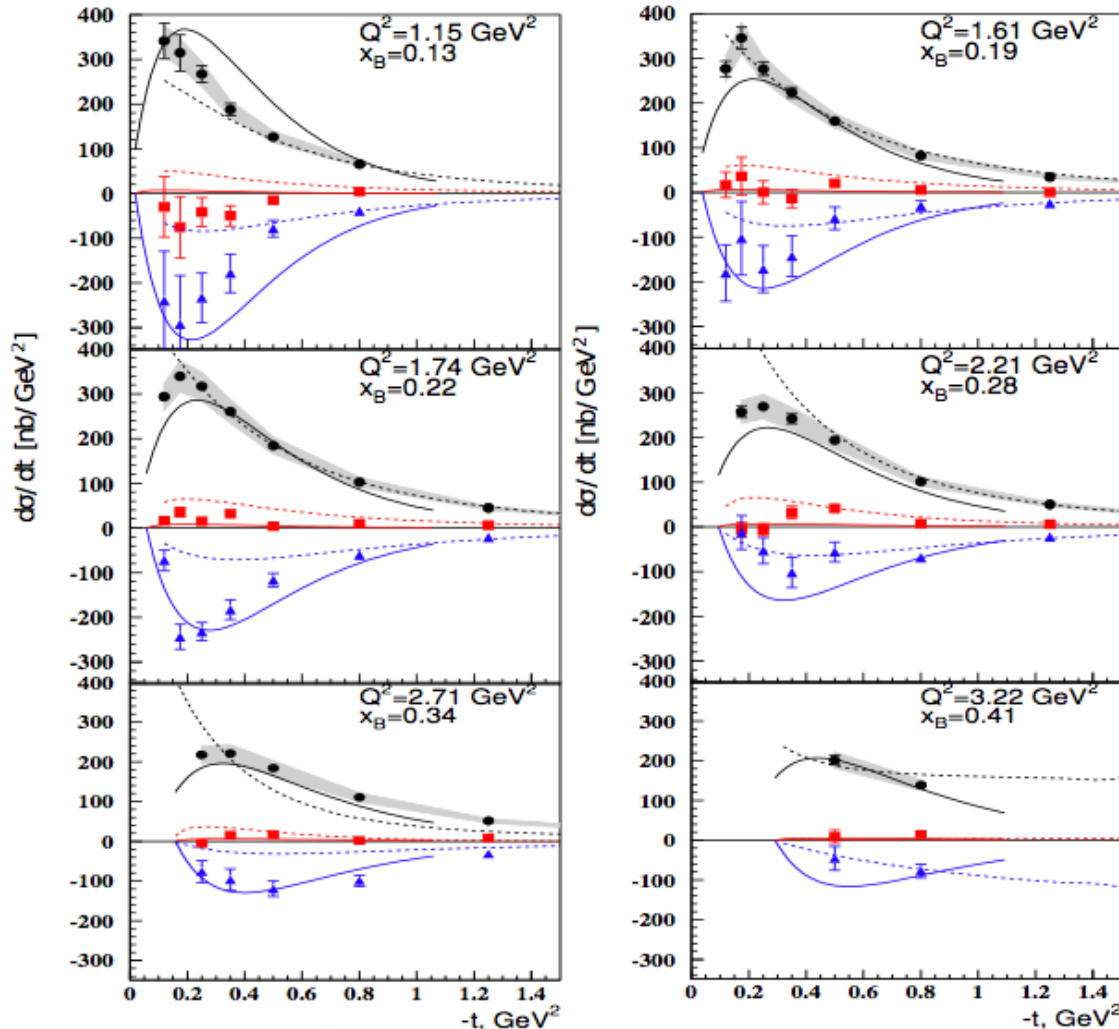


FIG. 2: The extracted structure functions vs. t for the bins with the best kinematic coverage and for which there are theoretical calculations. The data and curves are as follows: black- $\sigma_U (= \sigma_T + \epsilon\sigma_L)$, blue- σ_{TT} , and red- σ_{LT} . The shaded bands reflect the experimental systematic uncertainties. The curves are theoretical predictions produced with the models of Refs. [14] (solid) and [15] (dashed).



Important combinations of overall helicity amps

$$f_{10}^{++} + f_{10}^{--} \propto \frac{\sqrt{t_0 - t}}{4M} \left[2\tilde{\mathcal{H}}_T + (\mathcal{E}_T - \xi\tilde{\mathcal{E}}_T) \right] \quad \text{“natural”}$$
$$f_{10}^{++} - f_{10}^{--} \propto \frac{\sqrt{t_0 - t}}{4M} \left[\xi\mathcal{E}_T - \tilde{\mathcal{E}}_T \right] \quad \text{“unnatural”}$$

→ 0 for $\xi \rightarrow 0$

Quite sizable effect for $\xi > 0$

Goloskokov&Kroll take **only** H_T and $(2\tilde{H}_T + E_T)$ non-zero
EPJA47, (2011) 112 for π^0 and EPJC74 (2014)2725 for V_0 .

What observables reveal this? $A_{LU}^{\sin\varphi}, \dots$



Asymmetries:
 Longitudinal polarizations
 Sensitive to $f_{10}^{++} - f_{10}^{--}$

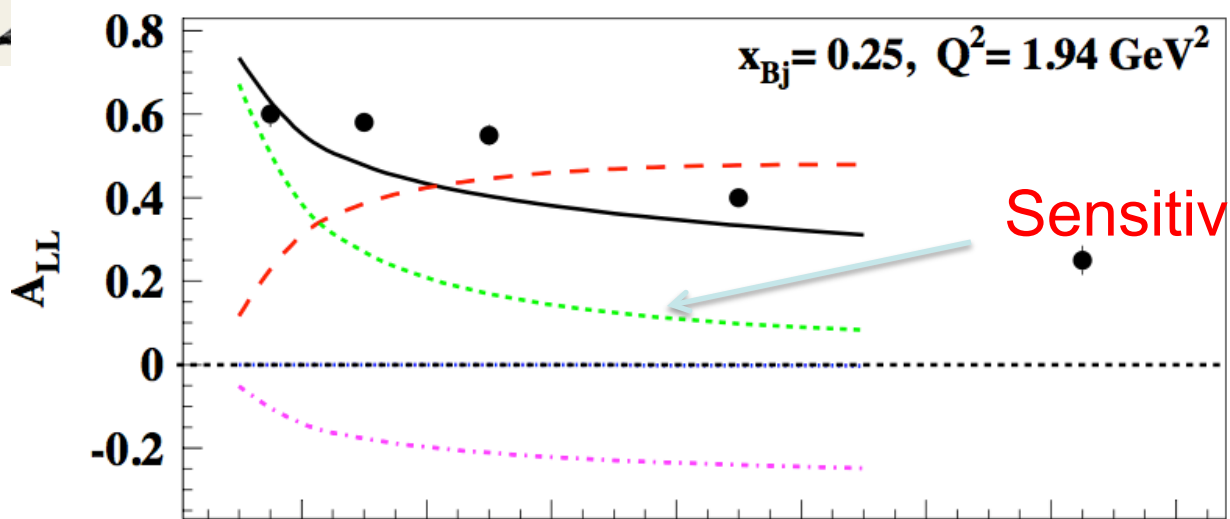
$$F_{UL}^{\sin \phi} = \frac{1}{\sqrt{2}} \mathcal{N} \Im m [(f_{00}^{+-})^* (f_{10}^{+-} - f_{10}^{-+}) + (f_{00}^{++})^* (f_{10}^{++} + f_{10}^{--})]$$

$$F_{UL}^{\sin 2\phi} = -\mathcal{N} \Im m [(f_{10}^{++})^* (f_{10}^{--}) - (f_{10}^{+-})^* (f_{10}^{-+})]$$

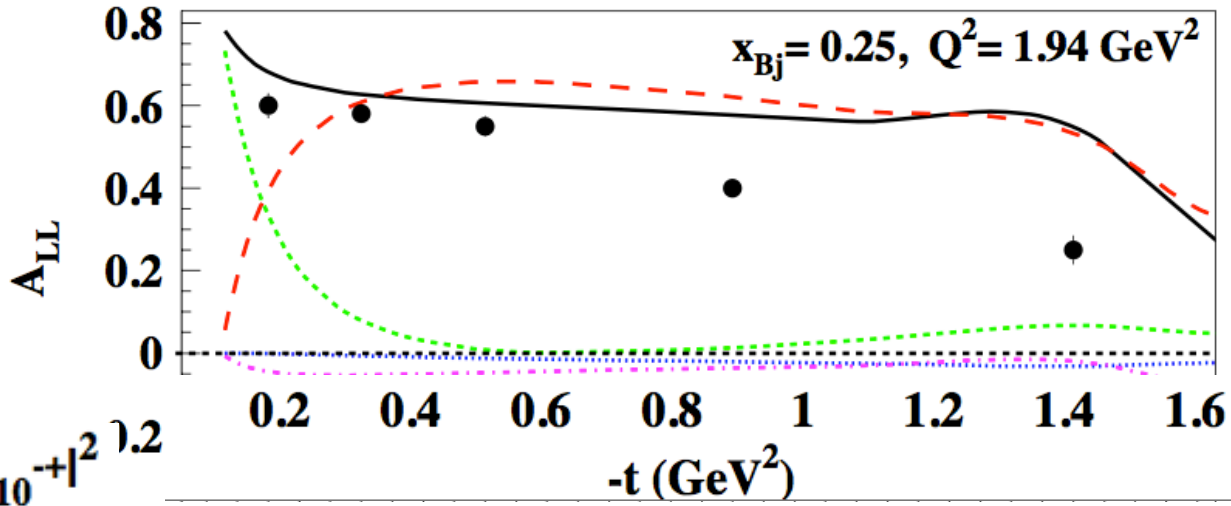
$$F_{LL}^{\cos \phi} = \frac{1}{\sqrt{2}} \mathcal{N} \Re e [(f_{00}^{+-})^* (f_{10}^{+-} - f_{10}^{-+}) + (f_{00}^{++})^* (f_{10}^{++} + f_{10}^{--})]$$

$$F_{LL} = \frac{1}{2} \mathcal{N} [|f_{10}^{++}|^2 + |f_{10}^{+-}|^2 - |f_{10}^{-+}|^2 - |f_{10}^{--}|^2]$$

$$A_{LL} = \frac{\sqrt{1 - \epsilon^2} F_{LL} + \sqrt{2\epsilon_L(\epsilon - 1)} \cos \phi F_{LL}^{\cos \phi}}{F_{UU,T} + \epsilon_L F_{UU,L}}$$



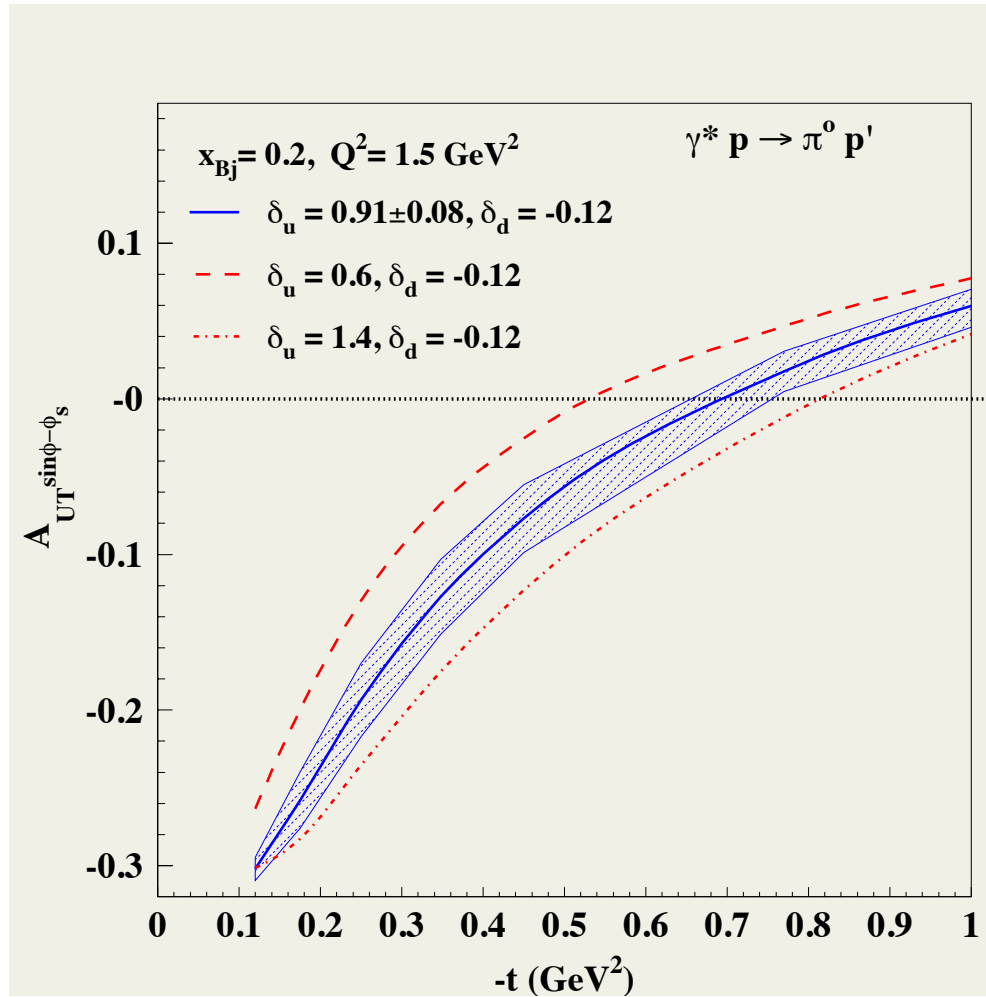
Role of parameters



- - $|f_{10}^{++}|^2$
- $|f_{10}^{-+}|^2$
- $|f_{10}^{+-}|^2$
- $|f_{10}^{--}|^2$



Asymmetry sensitive to tensor charge





Chiral odd GPD's & helicity/transversity interpretation?

$$\begin{aligned}
 \tau \left[2\tilde{H}_T(X, 0, t) + E_T(X, 0, t) \right] &= A_{++,+-} + A_{-+,-} \\
 &= A_{++,++}^{T_Y} - A_{+-,+}^{T_Y} + A_{-+,-}^{T_Y} - A_{--,--}^{T_Y} \\
 H_T(X, 0, t) &= A_{++,--} + A_{-+,+-} \\
 &= A_{++,++}^{T_X} - A_{+-,+}^{T_X} - A_{-+,-}^{T_X} + A_{--,--}^{T_X} \\
 \tau^2 \tilde{H}_T(X, 0, t) &= -A_{-+,+-} \\
 &= A_{++,++}^{T_Y} - A_{+-,+}^{T_Y} - A_{-+,-}^{T_X} + A_{--,--}^{T_X} \\
 \tilde{E}_T(X, 0, t) &= A_{++,+-} - A_{-+,-} = 0 \quad \text{From } T \text{ invariance} \\
 &\quad \text{for } \xi = 0
 \end{aligned}$$



Chiral odd GPDs & TMDs ?

Transversity transfer; Boer-Mulders; pretzelosity; worm gears

$$H_T(X, 0, 0) = h_1(X), \quad \text{Transversity pdf}$$

Model dependent relations

with the Boer-Mulders, $h_1^\perp(X)$, and $h_{1T}^\perp(X)$ functions, :

Pretzelosity

$$\lim_{t \rightarrow 0} \frac{t}{4M^2} \tilde{H}_T(X, 0, t) = h_{1T}^\perp(X) = \int d^2 k_T h_{1T}^\perp(X, k_T)$$

B-M

$$2\tilde{H}_T(X, 0, t) + E_T(X, 0, t) = h_1^\perp(X) = \int d^2 k_T h_1^\perp(X, k_T),$$

by the integration to form factors at $t = 0$, giving the tensor charge,

$$\delta_q = \int_0^1 dx H_T(X, 0, 0)$$

$$\tilde{E}_T(X, \xi, t) = A_{++,+-} - A_{-+,-}$$

h_{1L}^\perp worm-gear

****Tensor charge can be related to BSM couplings**

A.~Courtoy, S.~Baessler, M.~Gonzalez-Alonso and S.~Liuti, arXiv:1503.06814.



Connecting tensor charge to BSM:

Courtoy, Baessler, Gonzalez-Alonso and Liuti, arXiv:1503.06814.

BSM couplings $1, \gamma_5, (\gamma_\mu + \gamma_\mu \gamma_5), i\sigma^{j+} \gamma_5$

$$C_T = \frac{G_F}{\sqrt{2}} V_{ud} g_T \epsilon_T$$

$$b = \frac{2}{1 + 3\lambda^2} [g_S \epsilon_S - 12g_T \epsilon_T \lambda]$$

$$b_\nu = \frac{2}{1 + 3\lambda^2} [g_S \epsilon_S \lambda - 4g_T \epsilon_T (1 + 2\lambda)],$$

The observable is always the product of the fundamental coupling times a hadronic matrix element!

g_T and g_S are the flavor- non-singlet/isovector hadronic matrix elements

$$\begin{aligned} \langle p'_p, S_p | \bar{u}u - \bar{d}d | p_p, S_p \rangle &= g_S(-t) \bar{U}(p'_p, S_p) U(p_p, S_p) \quad , \\ \langle p'_p, S_p | \bar{u}\sigma_{\mu\nu}u - \bar{d}\sigma_{\mu\nu}d | p_p, S_p \rangle &= g_T(-t) \bar{U}(p'_p, S_p) \sigma_{\mu\nu} U(p_p, S_p), \end{aligned}$$

... or by using isospin symmetry:

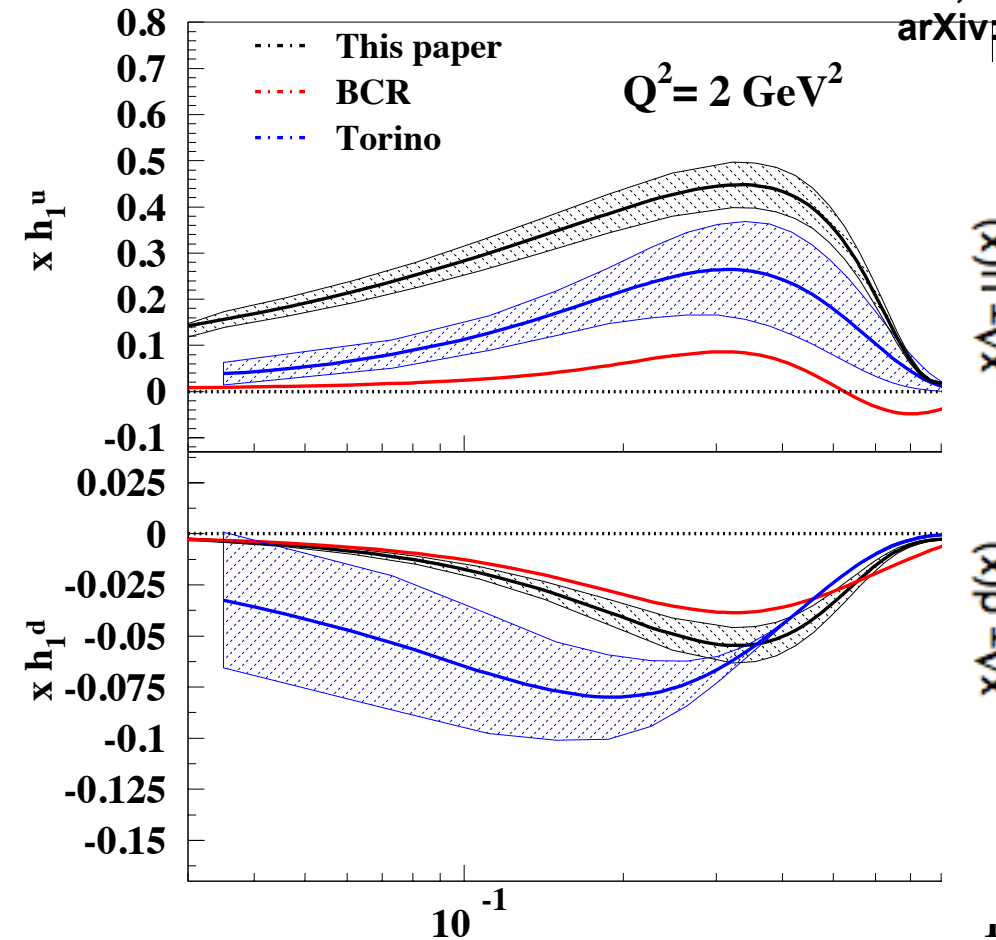
$$\begin{aligned} \langle p_p, S_p | \bar{u}d | p_n, S_n \rangle &= g_S(-t) \bar{U}(p_p, S_p) U(p_n, S_n) \quad , \\ \langle p_p, S_p | \bar{u}\sigma_{\mu\nu}d | p_n, S_n \rangle &= g_T(-t) \bar{U}(p_p, S_p) \sigma_{\mu\nu} U(p_n, S_n), \end{aligned}$$

The precision with which ϵ_T can be measured depends on the uncertainty on g_T

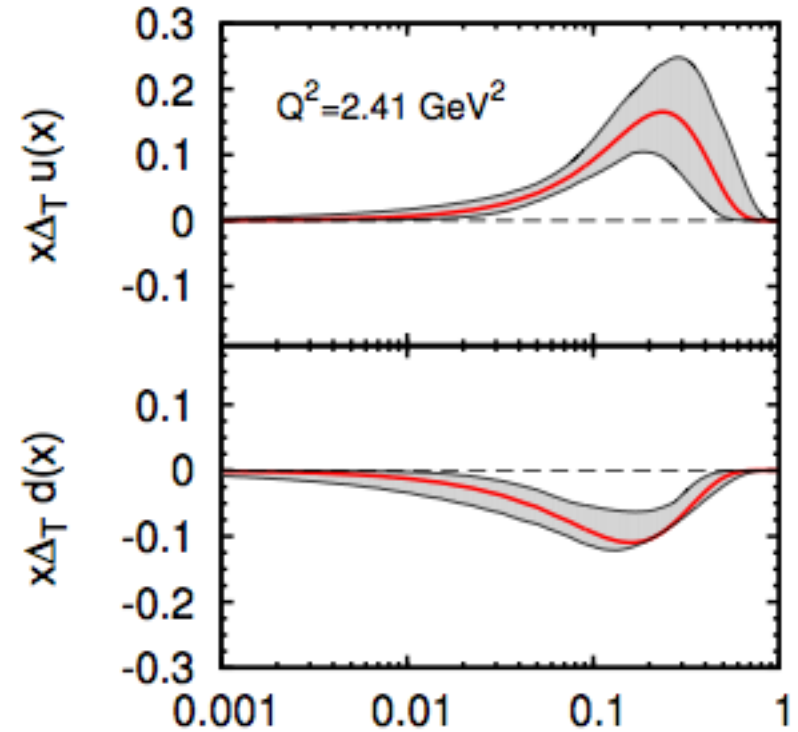


Extraction of transversity after using DVCS data via chiral even \leftrightarrow odd **Transversity** \rightarrow pdf's: $h_1^q(x, Q^2)$

GG, Gonzalez, Liuti,
arXiv:1311.0483 PRD



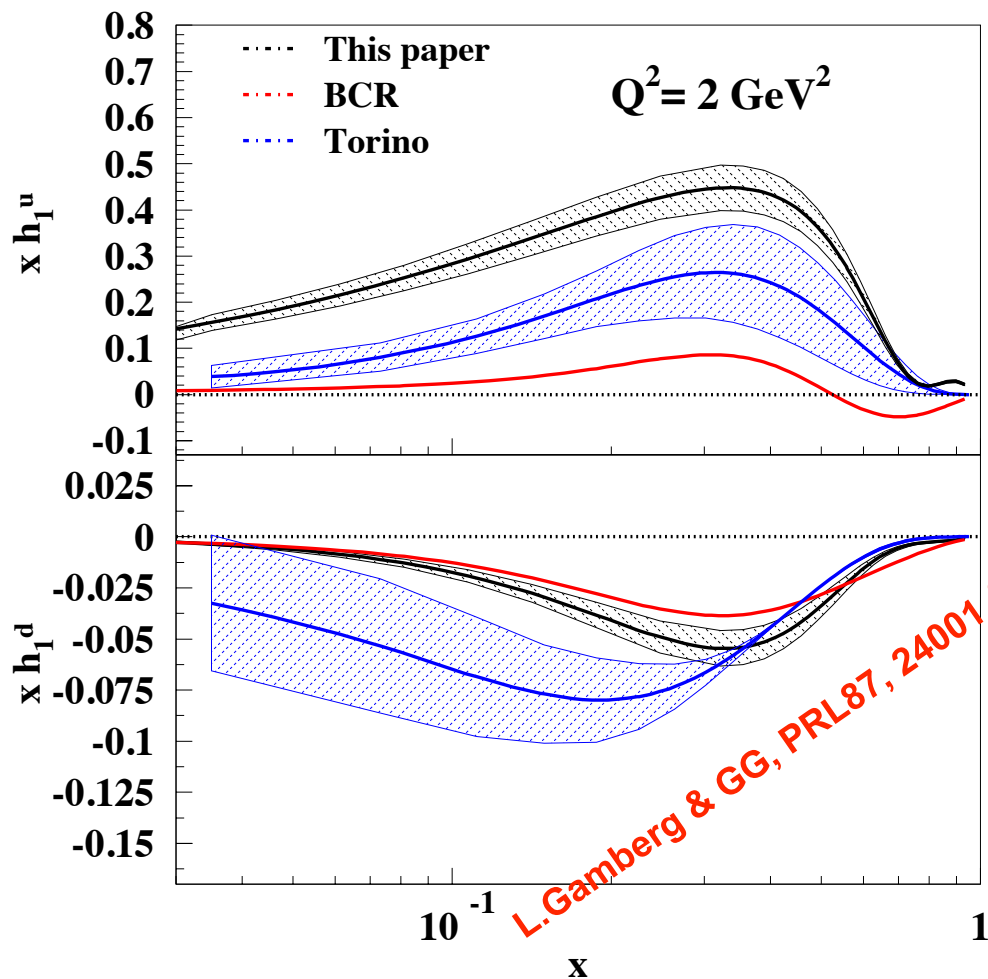
BCR=Bacchetta, Conte, Radici^x



Anselmino, Boglione, et al.,
Phys.Rev. D87 (2013) 094019
 $\delta u = 0.31_{-0.12}^{+0.16}$ $\delta d = -0.27_{-0.10}^{+0.10}$

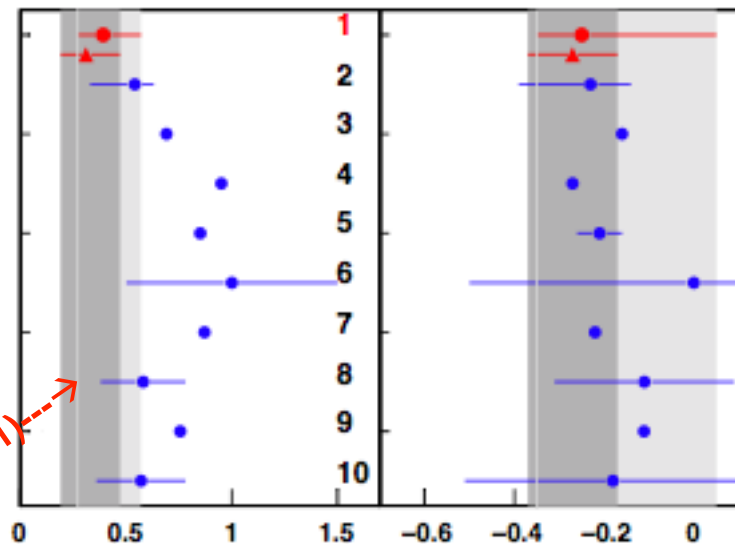


Extraction of tensor charge



L. Gamberg & GG, PRL87, 24001 (2001)

$\bullet \delta u = 0.39^{+0.18}_{-0.12}$ $\bullet \delta d = -0.25^{+0.30}_{-0.10}$
 $\blacktriangle \delta u = 0.31^{+0.16}_{-0.12}$ $\blacktriangle \delta d = -0.27^{+0.10}_{-0.10}$



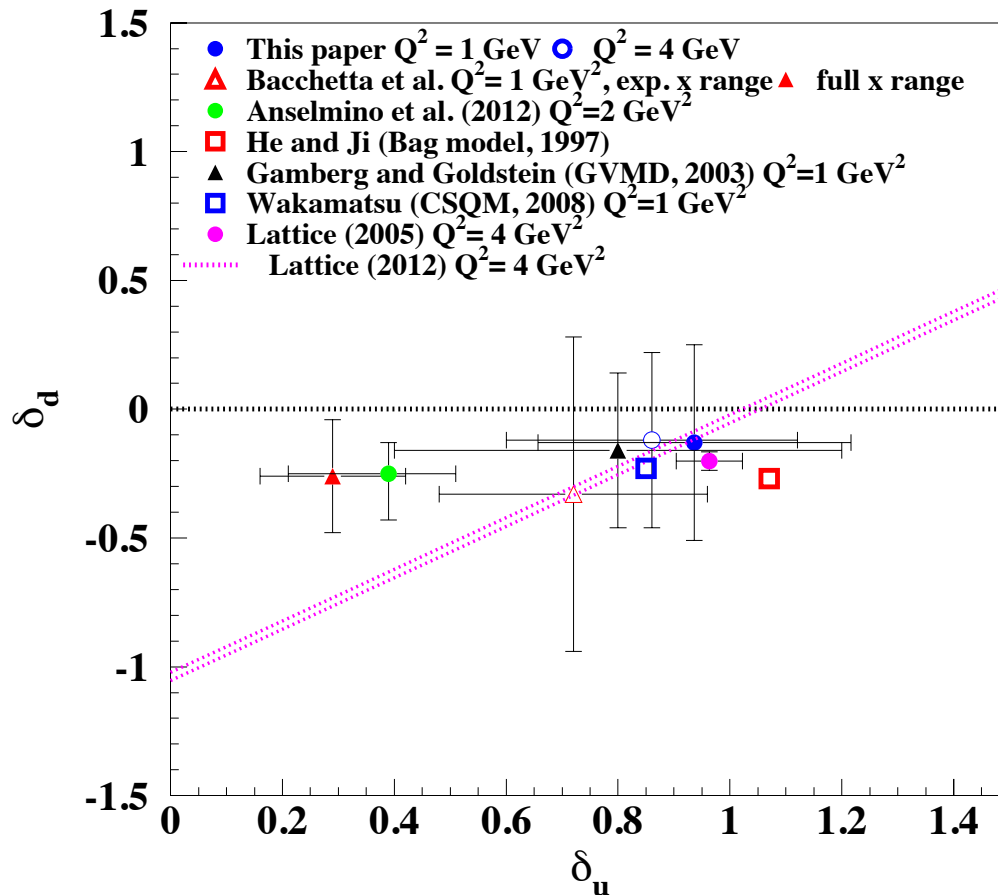
Anselmino, Boglione, et al.,
 Phys.Rev. D87 (2013) 094019
 $\delta u = 0.31^{+0.16}_{-0.12}$ $\delta d = -0.27^{+0.10}_{-0.10}$

From our Reggeized form
 $\delta u \approx 1.2$ $\delta d \approx -0.08$
 Closer to QCD sum rule values



Chiral odd GPDs \rightarrow

Transversity \rightarrow tensor charges δ_q



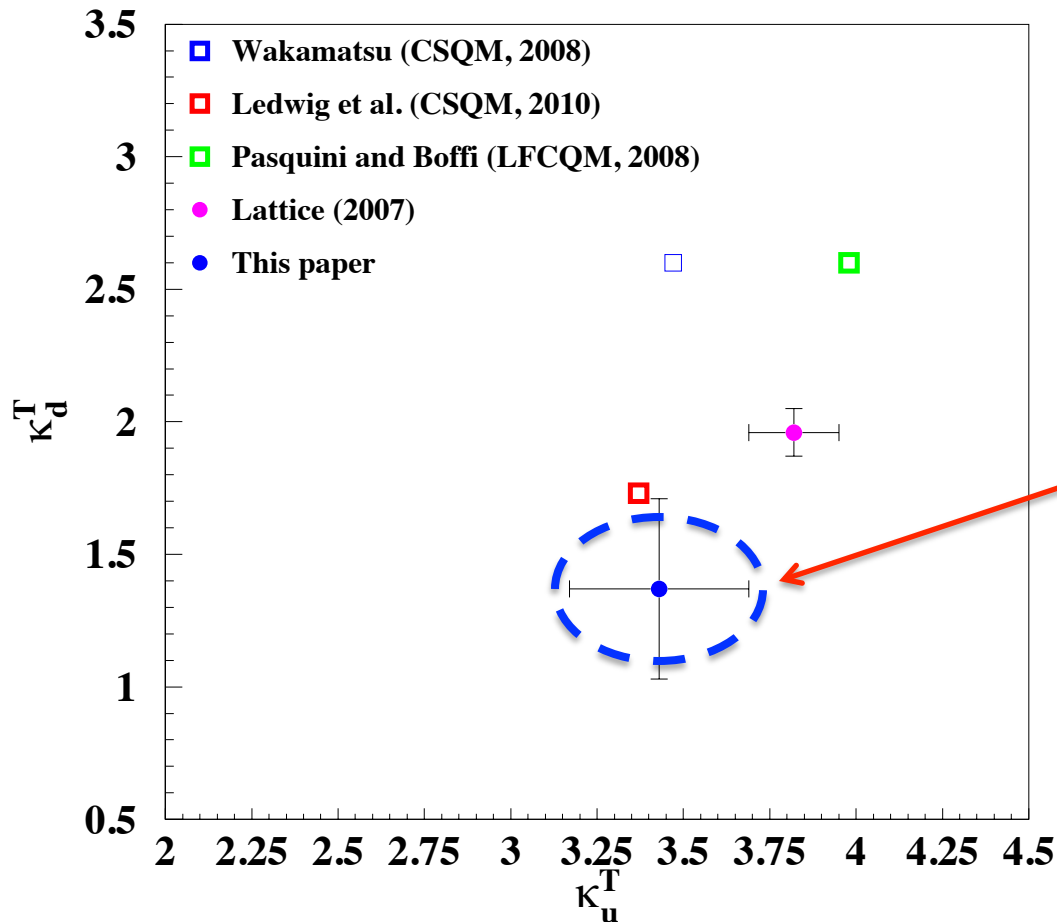
GG, Gonzalez, Liuti,
arXiv:1401.0438 PRD



Chiral odd GPDs \rightarrow

transverse spin-flavor "dipole moments" κ_T^q

defined by M. Burkardt, PRD72,094020(2005)



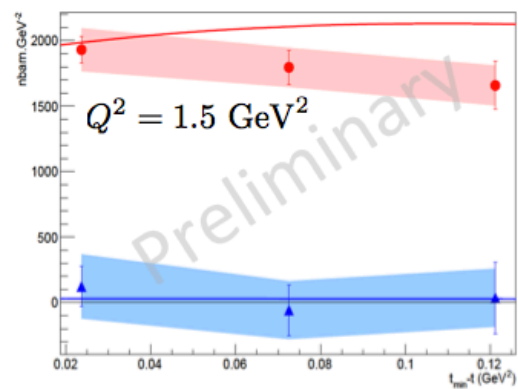
GG, Gonzalez, Liuti,
arXiv:1401.0438



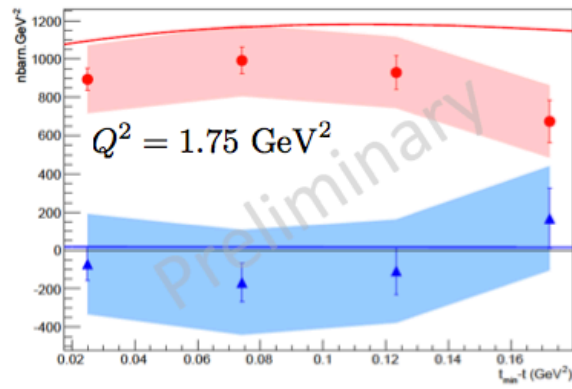
Comparing with new data from Hall A

courtesy F. Sabatie, CIPANP

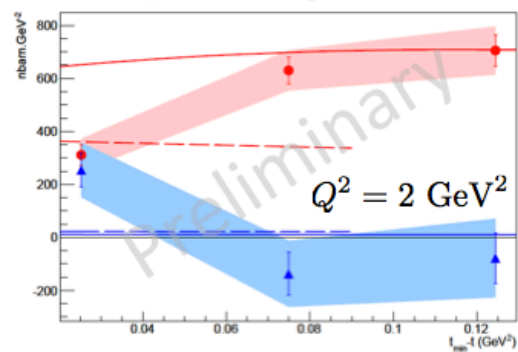
σ_T (red circle) and σ_L (blue triangle)



σ_T (red circle) and σ_L (blue triangle)



σ_T (red circle) and σ_L (blue triangle)

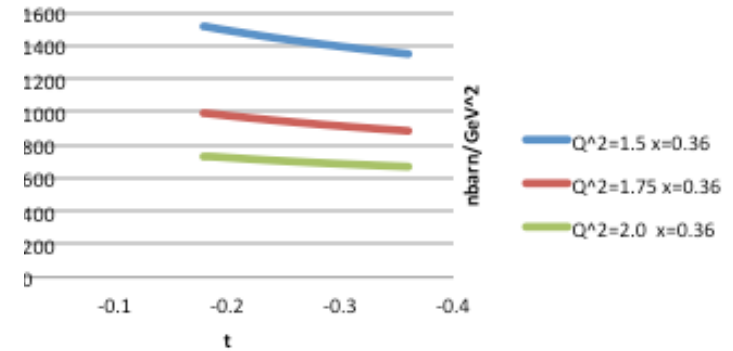


Shaded area: 2% normalizatio

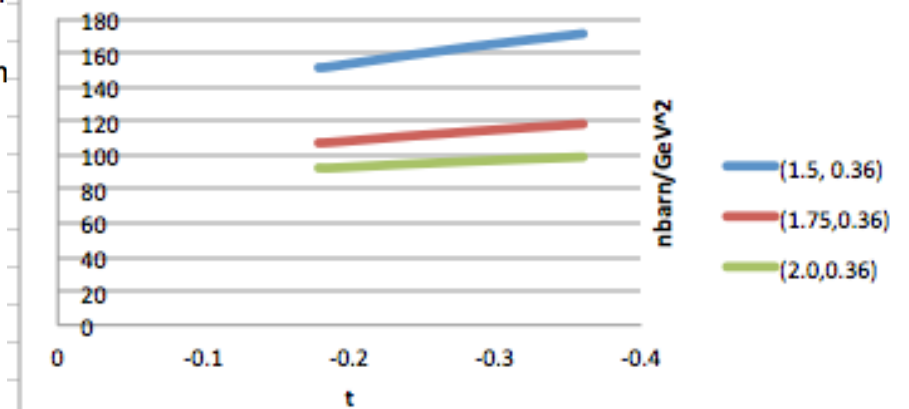
Solid line: GK11 model (descri

Dashed line: Goldstein-Liuti m
(waiting for updated values)

sigma_T

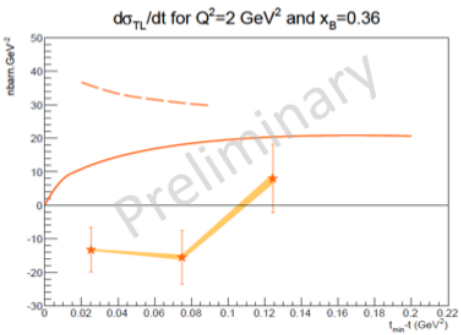
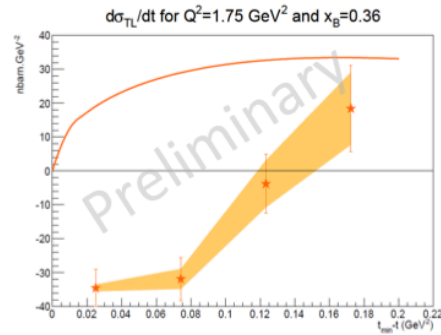
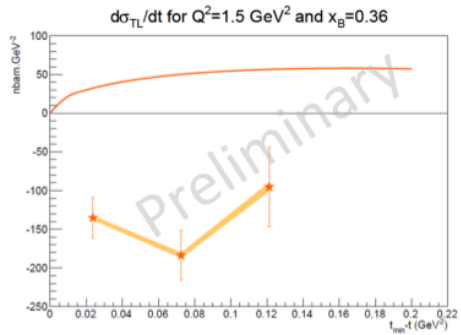


sigma_L





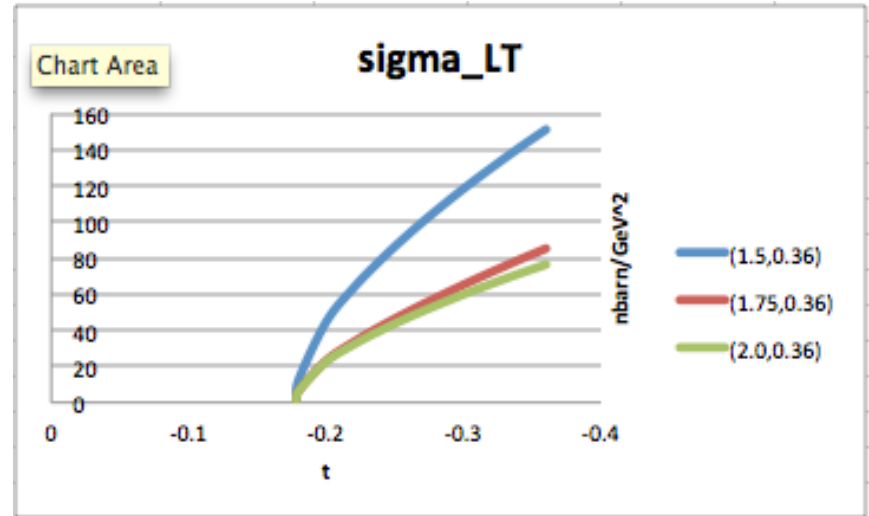
Comparing with new data from Hall A courtesy F. Sabatie, CIPANP



Shaded area: 2% normalization uncertainty

Solid line: GK11 model (described earlier)

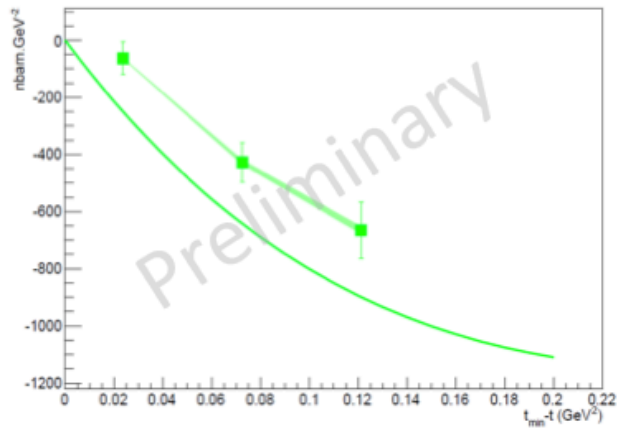
Dashed line: Goldstein-Liuti model
(waiting for updated values)



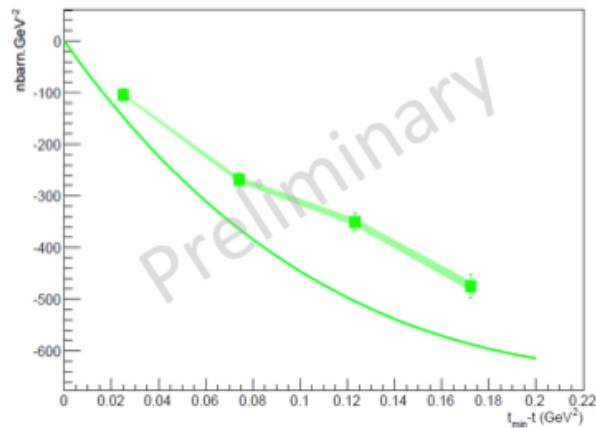


Comparing with new data from Hall A courtesy F. Sabatie, CIPANP

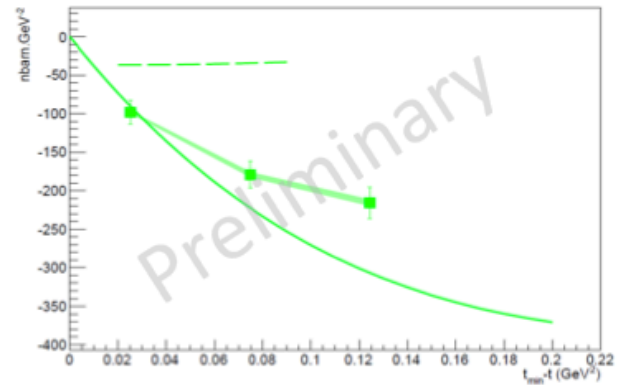
$d\sigma_{TT}/dt$ for $Q^2=1.5 \text{ GeV}^2$ and $x_B=0.36$



$d\sigma_{TT}/dt$ for $Q^2=1.75 \text{ GeV}^2$ and $x_B=0.36$



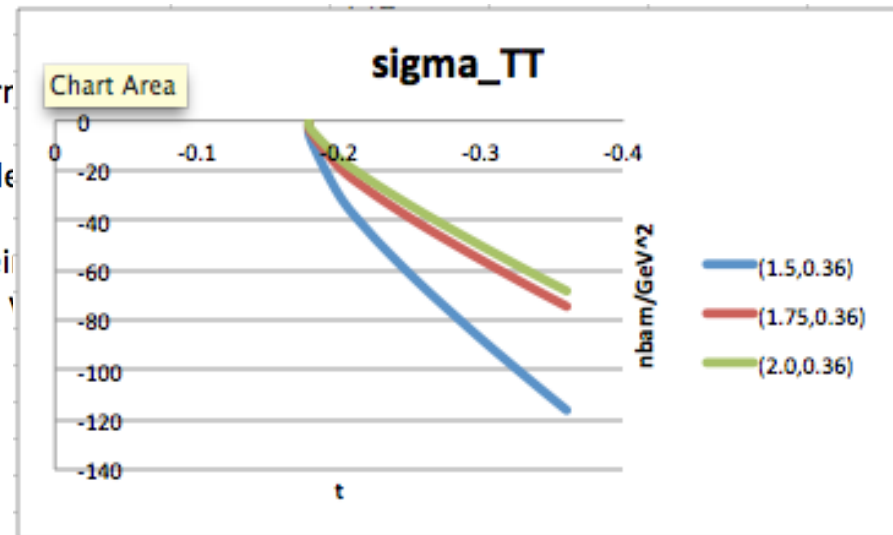
$d\sigma_{TT}/dt$ for $Q^2=2 \text{ GeV}^2$ and $x_B=0.36$



Shaded area: 2% norm

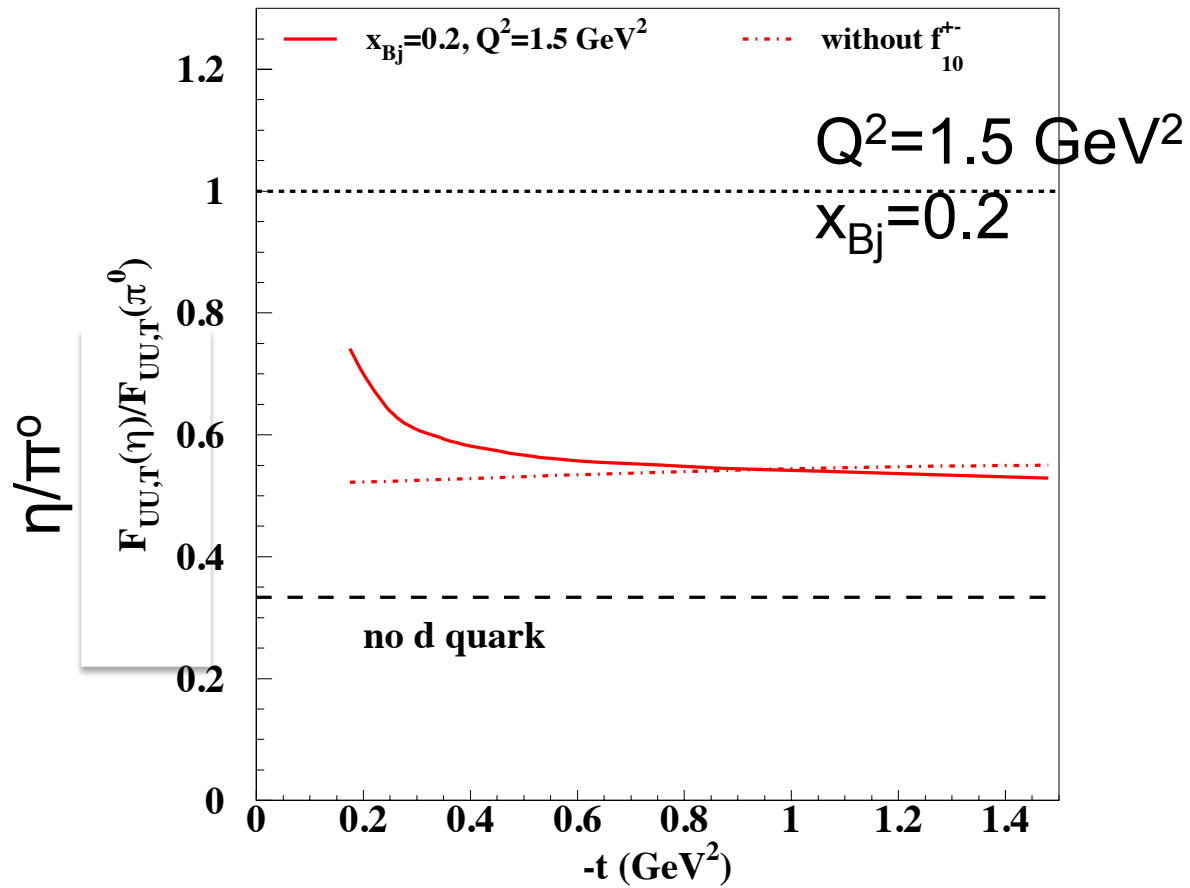
Solid line: GK11 model

Dashed line: Goldstein
(waiting for updated)





Ratio of unpolarized η / π^0





Summary

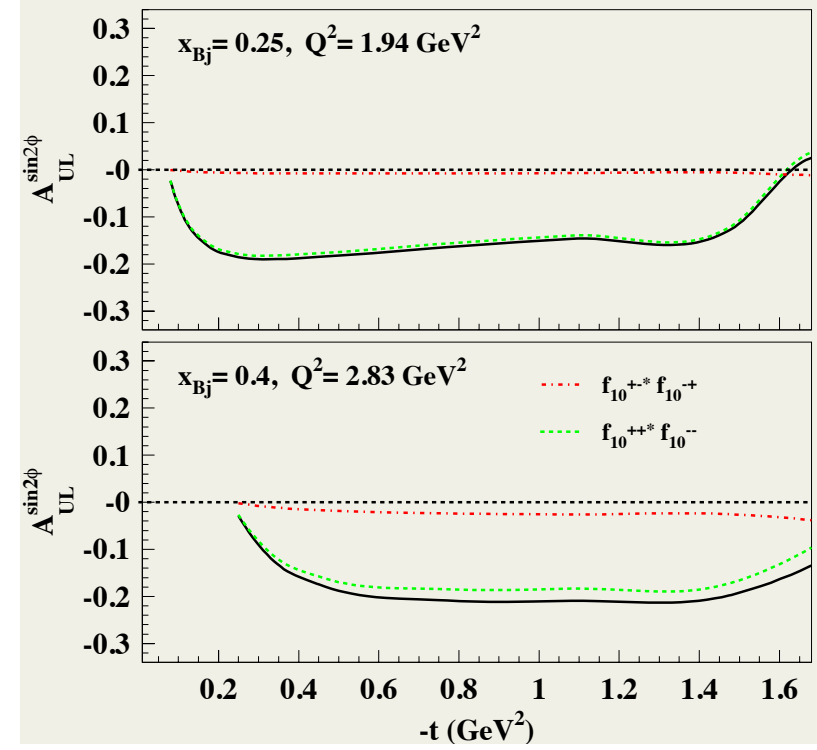
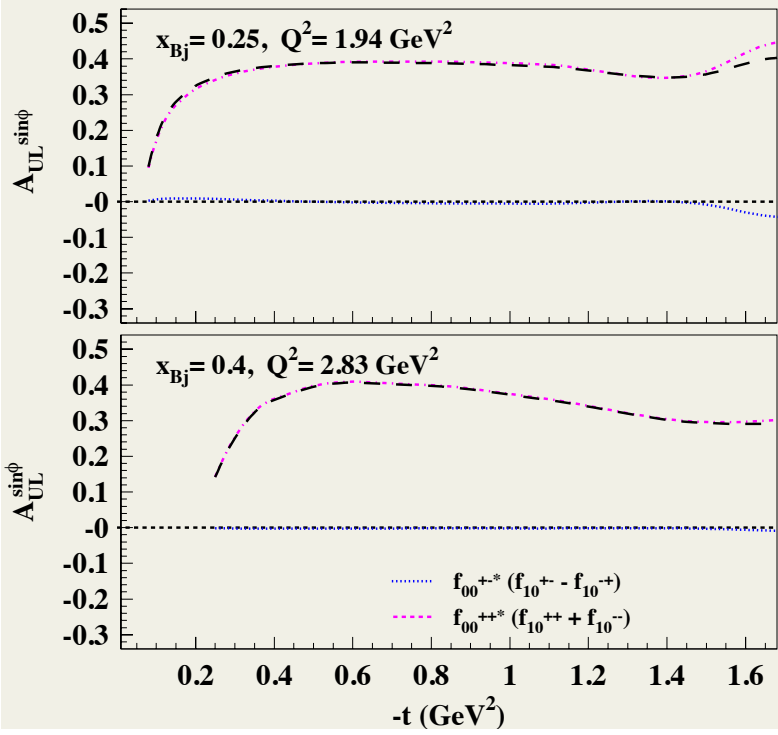
- Flexible parameterization for chiral even from form factors, pdfs & DVCS $R \times Dq$
- Extended $R \times Dq$ to chiral odd sector
- DVMP – π^0 many $d\sigma$'s & Asymmetries measure *Transversity*



Backup slides



Longitudinally polarized target

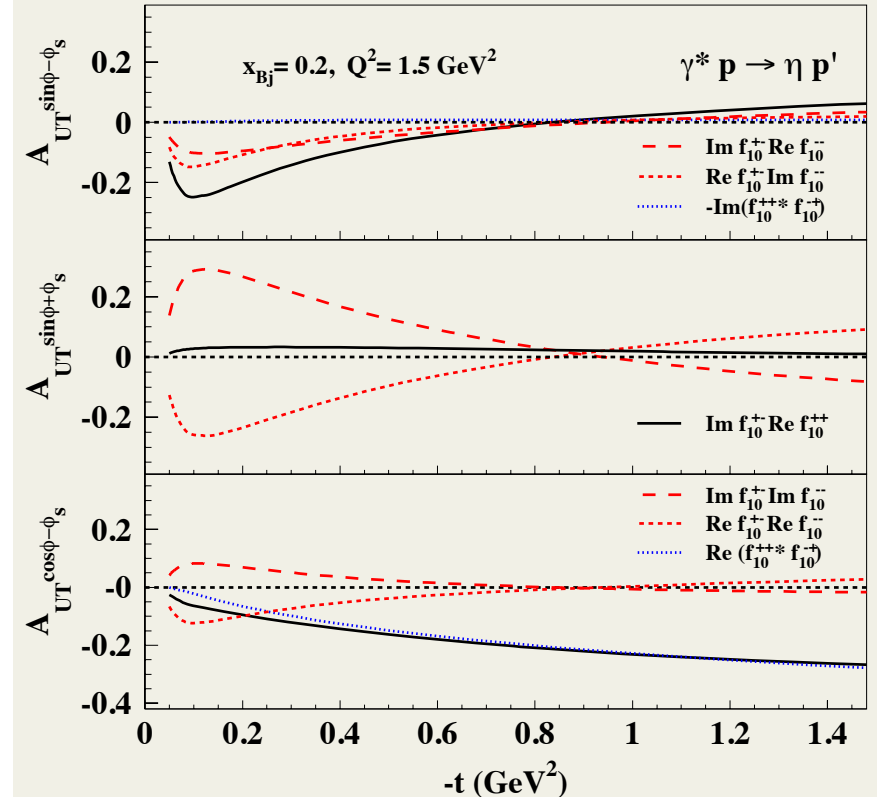
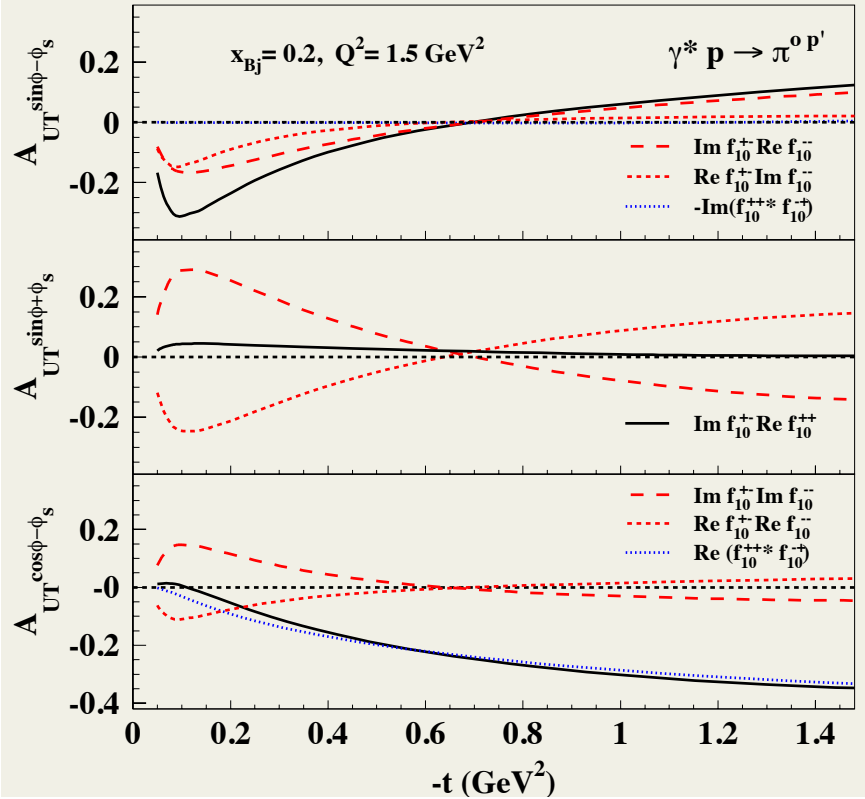


Look for tensor charge in f_{10}^{+-}

Transverse dipole moment in f_{10}^{++}, f_{10}^{--}



Transverse target



Look for tensor charge in f_{10}^{+-}

Transverse dipole moment in f_{10}^{++}, f_{10}^{--}



Reggeization via spectator diquark mass formulation

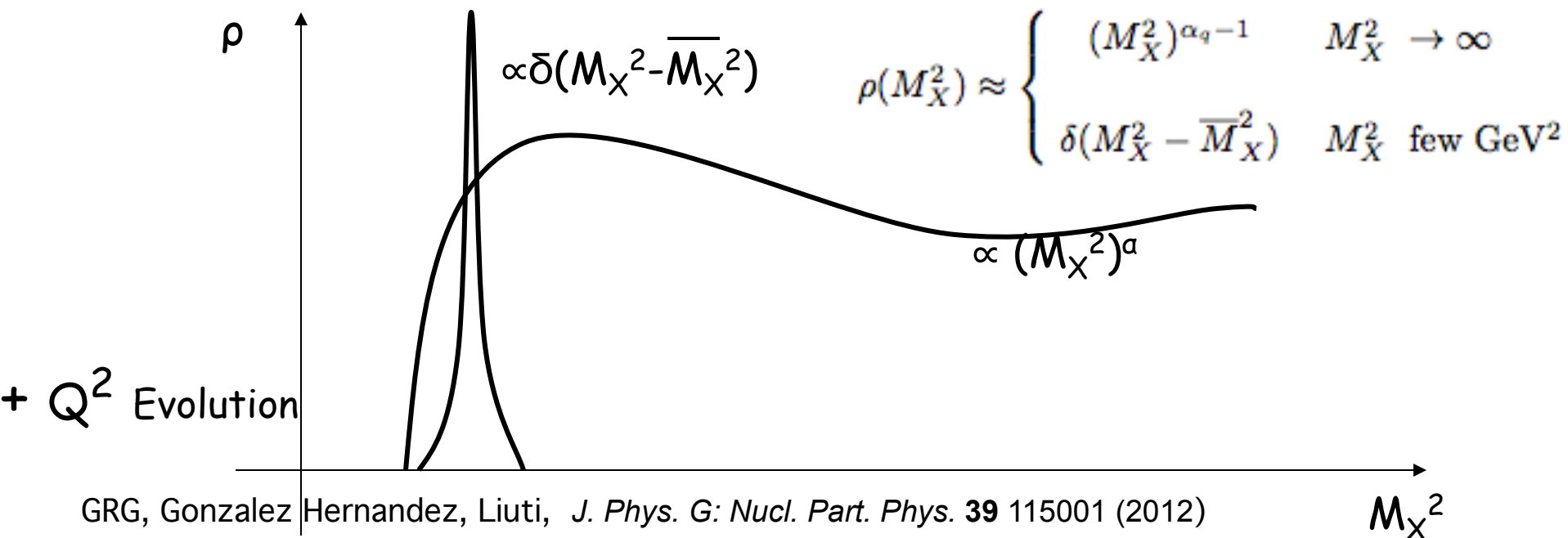
Where does the Regge behavior come from?

$$F_T^q(X, \zeta, t) = \mathcal{N}_q \int_0^\infty dM_X^2 \rho(M_X^2) F_T^{(m_q, M_\Lambda^q)}(X, \zeta, t; M_X),$$

Diquark spectral function

$$F(X, \zeta, t) \cong \mathcal{N} G_{M_X, m}^{M_\Lambda}(X, \zeta, t) R_p^{\alpha, \alpha'}(X, \zeta, t)$$

“Regge”

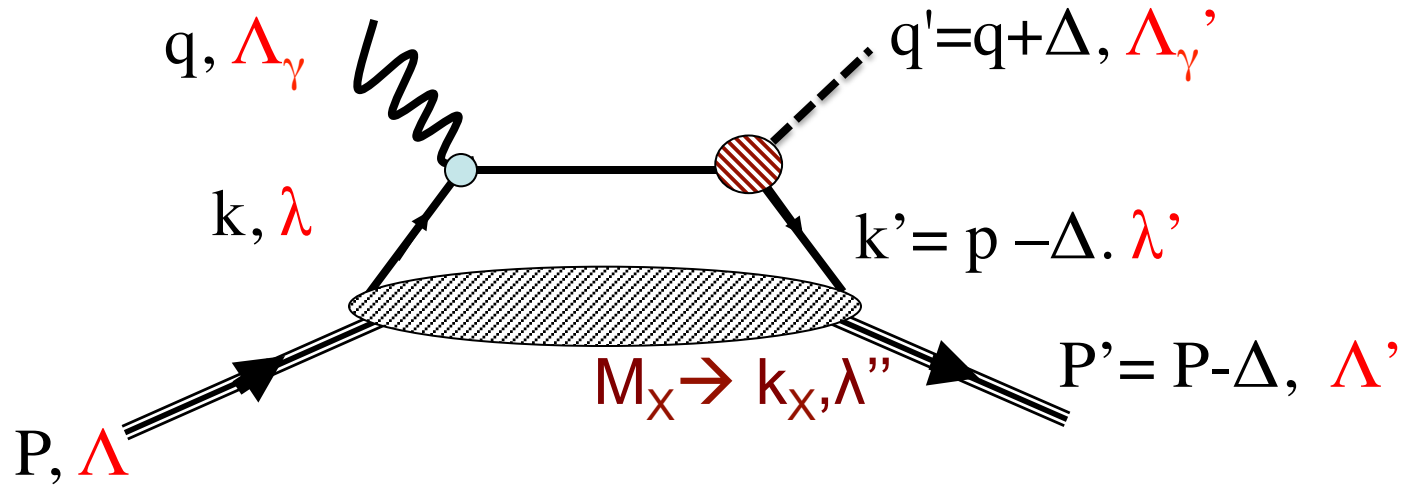


GRG, Gonzalez Hernandez, Liuti, *J. Phys. G: Nucl. Part. Phys.* **39** 115001 (2012)

Following DIS work by Brodsky, Close, Gunion (1973)



Reggeization



$$A = \mathcal{N} \int \frac{dk_X^2 dk^2}{(k^2 - m^2 - i\epsilon)(k'^2 - m^2 - i\epsilon)} \frac{\rho(k_X^2, k^2) \times (\text{spin structure})}{(k_X^2 - M_X^2 - i\epsilon)}$$

Landshoff, Polkinghorn, Short '71
 Brodsky, Close, Gunion '71 Regge
 behavior required for Compton
 Ahmad, Honkanen, Liuti, Taneja '07, '09
 Gorshteyn & Szczepaniak (PRD, 2010)
 Brodsky, Llanes-Estrada '07
 Brodsky, Llanes, Szczepaniak '08

GRG, Gonzalez Hernandez, Liuti, *J. Phys. G: Nucl. Part. Phys.* **39** 115001 (2012)

5/25/15



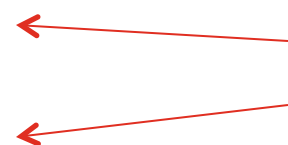
Helicity amps ($q'+N \rightarrow q+N'$) are linear combinations of GPDs

$$A_{+,+;+,+} = \sqrt{1-\xi^2} \left[\frac{H + \tilde{H}}{2} - \frac{\xi^2}{1-\xi^2} \frac{E + \tilde{E}}{2} \right]$$

$$A_{-,+;-,+} = \sqrt{1-\xi^2} \left[\frac{H - \tilde{H}}{2} - \frac{\xi^2}{1-\xi^2} \frac{E - \tilde{E}}{2} \right]$$

$$A_{+,+;-,+} = -\frac{\sqrt{t_0-t}}{4M} (E - \xi \tilde{E})$$

$$A_{-,+;+,+} = \frac{\sqrt{t_0-t}}{4M} (E + \xi \tilde{E})$$



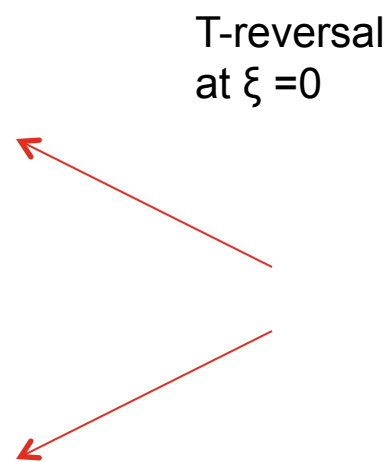
for chiral even GPDs and

$$A_{+,-;+,+} = -\frac{\sqrt{t_0-t}}{2M} \left[\tilde{H}_T + \frac{1+\xi}{2} E_T - \frac{1+\xi}{2} \tilde{E}_T \right]$$

$$A_{++,--} = \sqrt{1-\xi^2} \left[H_T + \frac{t_0-t}{4M^2} \tilde{H}_T - \frac{\xi^2}{1-\xi^2} E_T + \frac{\xi}{1-\xi^2} \tilde{E}_T \right]$$

$$A_{+,-,-+} = -\sqrt{1-\xi^2} \frac{t_0-t}{4M^2} \tilde{H}_T$$

$$A_{++,+-} = \frac{\sqrt{t_0-t}}{2M} \left[\tilde{H}_T + \frac{1-\xi}{2} E_T + \frac{1-\xi}{2} \tilde{E}_T \right],$$



T-reversal at $\xi=0$

for chiral odd GPDs, where for consistency with previous literature we have

In diquark spectator models $A_{++,++}$, etc. are calculated directly. Inverted \rightarrow GPDs



Invert to obtain model parameterization for GPDs

S=0 diquark

Spectator model

$$A_{++, -} = -A_{++, +}^*$$

$$A_{-, ++} = -A_{+, ++}^*$$

$$A_{++, ++} = A_{++, --}$$

$$H(x, \xi, t) = \frac{1}{\sqrt{1-\xi^2}}(A_{+, +; +, +} + A_{-, +; -, +}) - \frac{2M\xi^2}{\Delta(1-\xi^2)}(A_{+, +; -, +} - A_{-, +; +, +})$$

$$E(x, \xi, t) = -\frac{2M}{\Delta}(A_{+, +, -, +} - A_{-, +; +, +})$$

$$\tilde{H}(x, \xi, t) = \frac{1}{\sqrt{1-\xi^2}}(A_{+, +; +, +} - A_{-, +; -, +}) + \frac{2M\xi}{\Delta(1-\xi^2)}(A_{+, +, -, +} + A_{-, +; +, +})$$

$$\tilde{E}(x, \xi, t) = \frac{2M}{\Delta\xi}(A_{+, +, -, +} + A_{-, +; +, +})$$

for chiral even GPDs and

$$H_T(x, \xi, t) = \frac{1}{\sqrt{1-\xi^2}}(A_{+, +; -, -} + A_{-, +; +, -}) + \frac{2M\xi}{\Delta(1-\xi^2)}(A_{+, +; +, -} - A_{-, +; -, -})$$

$$\xi E_T(x, \xi, t) - \tilde{E}_T(x, \xi, t) = \frac{2M}{\Delta}(A_{+, +; +, -} - A_{-, +; -, -})$$

$$E_T(x, \xi, t) + \tilde{E}_T(x, \xi, t) = \frac{\Delta}{2M(1-\xi)}[2A_{+, +; +, -} + \frac{4M}{\Delta\sqrt{1-\xi^2}}A_{-, +; +, -}]$$

double flip

$$\tilde{H}_T(x, \xi, t) = \frac{4M^2}{\Delta^2\sqrt{1-\xi^2}}A_{-, +; +, -}$$



Unpolarized Helicity Amplitudes

