TMDs and the Role of Nonperturbative Inputs in Determining the TMDs

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QCD Evolution 2015 Workshop
Connecting hadrons to partons

- Experiments measure hadrons and leptons, not partons

- Large momentum transfer – sensitive to partons:

  Sensitive to partonic dynamics

  (Diagrams with more active partons from each hadron!)

  Connection between hadron and parton
Connecting hadrons to partons

- Experiments measure hadrons and leptons, not partons
- Large momentum transfer – sensitive to partons:

\[ \langle p, s | \mathcal{O}(\psi, A^\alpha) | p, s \rangle : \langle p, s | \overline{\psi}(0) \gamma^+ \psi(y) | p, s \rangle, \langle p, s | F^{+\alpha}(0) F^{+\beta}(y) | p, s \rangle (-g_{\alpha\beta}) \]

Isolate pQCD calculable short-distance partonic dynamics

- QCD factorization – connecting partons to hadrons:

Sensitive to partonic dynamics

(Diagrams with more active partons from each hadron!)

Connection between hadron and parton
Confined parton motion in a hadron

- High energy scattering with a large momentum transfer:
  - Momentum scale of the hard probe:
    \[ Q \gg 1/R \sim \Lambda_{\text{QCD}} \sim 1/\text{fm} \]
  - Combined motion \( \sim 1/R \) is too week to be sensitive to the hard probe
  - Collinear factorization – integrated into PDFs, …
Confined parton motion in a hadron

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  - Combined motion \( \sim 1/R \) is too weak to be sensitive to the hard probe
  - Collinear factorization – integrated into PDFs, ...

- Scattering with multiple momentum scales observed:
  - Two-scale observables, such as SIDIS, low \( p_T \) Drell-Yan, ...
    \[ Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\text{QCD}} \]
  - Hard scale \( Q_1 \) localizes the probe to see the quark or gluon d.o.f.
  - “Soft” scale \( Q_2 \) could be sensitive to the confined motion
  - TMD factorization: the confined motion is encoded into TMDs
Confined parton motion in a hadron

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  - TMD factorization: the confined motion is encoded into TMDs

Confined motion is “unique” – the consequence of QCD, but, TMDs that represent it are not unique!
Outline

- Definitions of TMDs
- TMD factorization and TMD evolution
- The role of nonperturbative inputs in determining TMDs
- Summary and outlook
Definitions of TMDs

- **Non-perturbative definition:**
  - In terms of matrix elements of parton correlators:
    \[ \Phi[U](x, p_T; n) = \int \frac{d\xi - d^2\xi_T}{(2\pi)^3} \, e^{i p \cdot \xi} \langle P, S | \bar{\psi}(0) U(0, \xi) \psi(\xi) | P, S \rangle_{\xi^+=0} \]
  - Depends on the choice of the gauge link:
    \[ U(0, \xi) = e^{-ig \int_0^\xi ds^\mu A_\mu} \]
  - Decomposes into a list of TMDs:
    \[
    \Phi[U](x, p_T; n) = \left\{ f_1^U(x, p_T^2) - f_{1T}^U(x, p_T^2) \frac{\epsilon_T}{M} + g_{1s}^U(x, p_T) \gamma_5 \\
    + h_1^U(x, p_T^2) \gamma_5 \not{s}_T + h_{1T}^U(x, p_T) \frac{\gamma_5 \not{p}_T}{M} + i h_{1s}^U(x, p_T^2) \frac{\not{p}_T}{M} \right\} \frac{\not{P}}{2},
    \]

See Mulders’ talk
Definitions of TMDs

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  - Decomposes into a list of TMDs:
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    \Phi[U](x, p_T; n) = \left\{ f_1[U](x, p_T^2) - f_{1T}[U](x, p_T^2) \frac{e_{ST}^p}{M} + g_{1s}[U](x, p_T) \gamma_5 \\
    + h_{1T}[U](x, p_T^2) \gamma_5 S_T + h_{1s}[U](x, p_T) \frac{\gamma_5 \Phi_T}{M} + ih_{1T}[U](x, p_T^2) \frac{\psi_T}{M} \right\} \frac{P^T}{2},
    \]
  - IF we knew proton wave function, this definition gives “unique” TMDs!
    But, we do NOT know proton wave function (may calculate it using BSE?)
    **TMDs defined in this way are NOT direct physical observables!**
Definitions of TMDs

- **Perturbative definition** – in terms of TMD factorization:

SIDIS as an example:

\[ TMD \text{ fragmentation} \]

\[ \text{TMD parton distribution} \]

\[ \text{Soft factors} \]

\[ + O \left( \frac{\langle k^2 \rangle}{Q^2}, \frac{\langle p^2 \rangle}{Q^2} \right) \]
Definitions of TMDs

- **Perturbative definition** – in terms of TMD factorization:

  SIDIS as an example:

  \[ \sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_{\perp}) \otimes D_{f \rightarrow h}(z, p_{\perp}) \otimes S(k_{s\perp}) + \mathcal{O}\left(\frac{P_{h\perp}}{Q^2}\right) \]

  - **Low** \( P_{hT} \) – TMD factorization:

    \[ \sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_{\perp}) \otimes D_{f \rightarrow h}(z, p_{\perp}) \otimes S(k_{s\perp}) + \mathcal{O}\left(\frac{P_{h\perp}}{Q^2}\right) \]

  - **High** \( P_{hT} \) – Collinear factorization:

    \[ \sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f \rightarrow h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q} \right) \]

  - **\( P_{hT} \) Integrated - Collinear factorization:**

    \[ \sigma_{\text{SIDIS}}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f \rightarrow h} + \mathcal{O}\left(\frac{1}{Q} \right) \]
Definitions of TMDs

- **Perturbative definition** – in terms of TMD factorization:

  SIDIS as an example:

  \[ \Phi^{[U]}(x, p_T; n, \mu) = \int \frac{d^2 \xi}{(2\pi)^3} \exp(i p \cdot \xi) \langle P, S | \bar{\psi}(0) U(0, \xi) \psi(\xi) | P, S \rangle_{\xi+ = 0} + \mathcal{O}(\mu^2) \]

  This operator definition is scheme dependent, & needed for calculating the short-distance hard coefficients, order-by-order, in perturbation theory.
Definitions of TMDs

- **Perturbative definition** – in terms of TMD factorization:

  SIDIS as an example:

  \[ q \theta P \]

  \[ \text{TMD fragmentation} \]

  \[ \text{Soft factors} \]

  \[ TMD + O \left( \frac{k^2}{Q^2}, \frac{p^2}{Q^2} \right) \]

- **Extraction of TMDs:**

  \[ \sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes D_{f \rightarrow h}(z, p_\perp) \otimes S(k_{s\perp}) + O \left( \frac{P_{h\perp}}{Q} \right) \]

  TMDs are extracted by fitting DATA using the factorization formula (approximation) and the perturbatively calculated \( \hat{H}(Q; \mu) \).

  Extracted TMDs depend on the scheme (UV) and perturbative order (LO, NLO, NNLL, …) at which the hard part was calculated!
Definitions of TMDs

- Perturbative definition – in terms of TMD factorization:
  
  SIDIS as an example:

- Extraction of TMDs:
  
  \[ \sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes D_{f\rightarrow h}(z, p_\perp) \otimes S(k_{s\perp}) + \mathcal{O}\left(\frac{P_{h\perp}}{Q^2}\right) \]

  TMDs are extracted by fitting DATA using the factorization formula (approximation) and the perturbatively calculated \( \hat{H}(Q; \mu) \).

  Extracted TMDs are valid only when the \( \langle p^2 \rangle \ll Q^2 \)
TMD factorization in QCD

- **SIDIS as an example – Parton model – LO QCD:**
  - Parton distribution function in a hadron
  - Parton-to-hadron fragmentation function

- **QCD interaction and leading power regions:**
  - Collinear regions:
    \[ k \parallel P_A, \quad k \parallel P_B \]
  - Soft regions
    \[ k^{\mu} \to 0 \]
TMD factorization in QCD

- **Leading pinch surface:**
  - ♦ hard region
  - ♦ collinear to $P$ region
  - ♦ collinear to $P'$ region
  - ♦ soft region

- **Factorization:**
  - TMD fragmentation
  - TMD parton distribution
  - Soft factors
TMD factorization in QCD

Factorization formalism in momentum space:

\[ W^{\mu\nu} \propto \sum_f \int d^2k_a d^2k_b S(q_\perp - k_a - k_b) \text{Tr} \left[ \mathcal{P}_{A} J_f(A, k_a T) \mathcal{P}_{A} H_f^\nu(Q) \mathcal{P}_{B} J_f(B, k_b T) \mathcal{P}_{B} H_f^\mu(Q) \right] \]

The soft factors:

\[ S = Z_s S^{(0)} \]

\[ S^{(0)}(k_s T) = \frac{1}{N_c} \int \frac{dk_s^+ dk_s^-}{(2\pi)^{4-2\epsilon}} \]

The soft factors in collinear factorization:

\[ S^{(0)}(k_s T) \rightarrow S^{(0)}(k_s T = 0) \rightarrow 1 \]
Going to the b-space

- **Factorization in b-space:**
  
  \[ \tilde{D}_{1, h/f}(z, b_T) = \int d^{2-2\epsilon} k_T e^{ik_T \cdot b_T} D_{1, h/f}(z, z k_T). \]

  \[ W^{\mu\nu} \propto \sum_f W^{\mu\nu}_f(0) \int \frac{d^{2-2\epsilon} b_T}{(2\pi)^{2-2\epsilon}} e^{-i q h \cdot b_T} \tilde{S}(b_T) \tilde{\phi}_{f/A}(x, b_T) \tilde{D}_{f \rightarrow D}(z, b_T) \]

- **Universality of the soft factor:**
  
  \[ \tilde{S}_{(0)}(b_T; y_A, y_B) = \frac{1}{N_c} \langle 0 | W(b_T/2; \infty, n_B)^\dagger W(b_T/2; \infty, n_A)_{ad} : W(-b_T/2; \infty, n_B)_{bc} W(-b_T/2; \infty, n_A)^\dagger_{db} | 0 \rangle \]

  **Wilson line:**

  \[ W(x; \infty; n)_{ab} = P \left\{ e^{-i g_0 \int_0^\infty d\lambda n \cdot A(0)_{\alpha}(x+\lambda n)t_{\alpha}} \right\}_{ab} \]

- **Evolution of TMDs:**

  from the wave function renormalization and the renormalization of the soft factors
Evolution of TMDs

- Evolution of TMDs:

\[
\frac{\partial \ln \tilde{D}_{1, H_A/f}(z_A, b_T; \zeta_A, \ldots)}{\partial \ln \sqrt{\zeta_A}} = \tilde{K}(b_T; \mu)
\]

\[
\tilde{K}(b_T; \ldots) = \frac{\partial}{\partial y_n} \left[ \frac{1}{2} \ln \tilde{S}(b_T; y_n, -\infty) - \frac{1}{2} \ln \tilde{S}(b_T; +\infty, y_n) \right]
\]

\[
= \frac{1}{2 \tilde{S}(b_T; y_n, -\infty)} \frac{\partial \tilde{S}(b_T; y_n, -\infty)}{\partial y_n} - \frac{1}{2 \tilde{S}(b_T; +\infty, y_n)} \frac{\partial \tilde{S}(b_T; +\infty, y_n)}{\partial y_n}
\]

\[
\frac{\partial \ln \tilde{D}_{1, H_A/f}(z_A, b_T; \zeta_A, \ldots)}{\partial \ln \mu} = \gamma_D(g; \zeta_A/\mu^2)
\]

- Evolved TMDs:

\[
\tilde{D}_{1, H_A/f}(z_A, b_T; \zeta_A; \mu) = \tilde{D}_{1, H_A/f} \left( z_A, b_T; m_A^2/z_A^2; \mu_0 \right) \exp \left\{ \ln \frac{\sqrt{\zeta_A} z_A}{m_A} \tilde{K}(b_T; \mu_0) + \int_{\mu_0}^\mu \frac{d\mu'}{\mu'} \left[ \gamma_D(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_A}}{\mu'} \gamma_K(g(\mu')) \right] \right\}
\]
Evolution of TMDs

Evolution of TMDs:

\[
\frac{\partial \ln \tilde{D}_{1,H_A/f}(z_A, b_T; \zeta_A, \ldots)}{\partial \ln \sqrt{\zeta_A}} = \tilde{K}(b_T; \mu)
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\tilde{K}(b_T; \ldots) = \frac{\partial}{\partial y_n} \left[ \frac{1}{2} \ln \tilde{S}(b_T; y_n, -\infty) - \frac{1}{2} \ln \tilde{S}(b_T; +\infty, y_n) \right]
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\[
= \frac{1}{2\tilde{S}(b_T; y_n, -\infty)} \frac{\partial \tilde{S}(b_T; y_n, -\infty)}{\partial y_n} - \frac{1}{2\tilde{S}(b_T; +\infty, y_n)} \frac{\partial \tilde{S}(b_T; +\infty, y_n)}{\partial y_n}
\]

\[
\frac{d \ln \tilde{D}_{1,H_A/f}(z_A, b_T; \zeta_A, \ldots)}{d \ln \mu} = \gamma_D(g; \zeta_A/\mu^2)
\]

Evolved TMDs:

\[
\tilde{D}_{1,H_A/f}(z_A, b_T; \zeta_A; \mu) = \tilde{D}_{1,H_A/f}(z_A, b_T; m_A^2/z_A^2; \mu_0) \exp \left\{ \ln \frac{\zeta_A z_A}{m_A} \tilde{K}(b_T; \mu_0) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_D(g(\mu'); 1) - \ln \frac{\zeta_A}{\mu'} \gamma_K(g(\mu')) \right] \right\}
\]

Only valid when \( b_T \ll 1/fm \) and \( \mu \gg \mu_0 \)

F.T. to momentum space \( \rightarrow \) Requires the nonperturbative inputs
Parton $k_T$ at the hard collision

- Sources of parton $k_T$ at the hard collision:

  - Gluon shower
  - Emergence of a hadron hadronization

- Large $k_T$ generated by the shower (caused by the collision):
  - $Q^2$-dependence – linear evolution equation of TMDs in $b$-space
  - The evolution kernels are perturbative at small $b$, but, not large $b$

- Challenge: to extract the “true” parton’s confined motion:
  - Separation of perturbative shower contribution from nonperturbative hadron structure – not as simple as PDFs
Importance of the nonperturbative inputs

- Aybat, Prokudin, Rogers, 2012:
  - Huge Q dependence

- Sun, Yuan, 2013:
  - Smaller Q dependence

No disagreement on evolution equations!

Issue: extrapolation to non-perturbative large b-region choice of the Q-dependent “form factor”
Extrapolation to large $b_T$

- **CSS $b^*$-prescription:**

\[
\tilde{F}_{f/P}(x, b_T; Q, Q^2) = \sum_j \int_x^1 \frac{d\tilde{x}}{\tilde{x}} \tilde{C}_{f/j}(x/\tilde{x}, b_\star; \mu_0^2, \mu_b, g(\mu_b)) f_{j/P}(\tilde{x}, \mu_b)
\]

\[
\times \exp \left\{ \ln \frac{Q}{\mu_b} K(b_\star; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[ \gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\}
\]

\[
b_\star = \frac{b_T}{\sqrt{1 + b_T^2/b_{\text{max}}^2}} \quad \text{with} \quad b_{\text{max}} \sim 1/2 \text{ GeV}^{-1}
\]

- **Nonperturbative fitting functions**

Various fits correspond to different choices for $g_{f/P}(x, b_T)$ and $g_K(b_T)$, e.g.

\[
g_{f/P}(x, b_T) + g_K(b_T) \ln \frac{Q}{Q_0} \equiv - \left[ g_1 + g_2 \ln \frac{Q}{2Q_0} + g_1 g_3 \ln(10x) \right] b_T^2
\]

Some parameterizations could significantly change the small-$b$ calculation!
Extrapolation to large $b_T$

- **Features of perturbative calculation at small-$b$:**
  - Sudakov form factor $\rightarrow b_{sp} \propto \left( \frac{\Lambda_{\text{QCD}}}{Q} \right)^\lambda, \lambda \sim 0.5$
  - Evolution of $f_{a/A}$ and $D_{c\rightarrow h}$ also moves $b_{sp}$
    - smaller $\xi \Rightarrow \mu \frac{\partial}{\partial \mu} f_{a/A}(\xi) > 0 \Rightarrow$ lower $b_{sp}$

- **Example for Q and $\sqrt{s}$ dependence:** $W^{DY}(x, b, Q)$
  - $\sqrt{s} = 1.8$ GeV
  - $\sqrt{s} = 27.4$ GeV
  - $\sqrt{s} = 1.8$ TeV

Qiu, Zhang, 2001
Extrapolation to large $b_T$

Another approach:

\[
\frac{d\sigma_{AB\rightarrow Z}^{\text{resum}}}{dq_T^2} \propto \int_0^\infty db \, J_0(q_T b) \, b \, W(b, Q)
\]

\[
W = \begin{cases} 
W^{\text{pert}}(b, x, z, Q) & b \leq b_{\text{max}} \\
W^{\text{pert}}(b_{\text{max}}, x, z, Q) \, F^{NP}(b, Q; b_{\text{max}}) & b > b_{\text{max}}
\end{cases}
\]

\[
W^{\text{pert}}(b, x, z, Q) = \sum_i e_j^2 \left[ f_A / A \otimes C_{a \rightarrow j}^{\text{in}} \right] \left[ C_{j \rightarrow c}^{\text{out}} \otimes D_{b \rightarrow h} \right] \times e^{-S(b, Q)}
\]

\[
F^{NP}(b, Q; b_{\text{max}}) = \exp \left\{ -\ln\left( \frac{Q^2 b_{\text{max}}^2}{c^2} \right) \left[ g_1 \left( (b^2)^\alpha - (b_{\text{max}}^2)^\alpha \right) + g_2 \left( b^2 - b_{\text{max}}^2 \right) \right] - g_2 \left( b^2 - b_{\text{max}}^2 \right) \right\}
\]

All parameters, $\alpha, g_1, g_2$, are fixed by the continuity of the “W” and its derivatives at $b_{\text{max}}$ – excellent predictive power for observables with the saddle point at small enough $b_{\text{sp}}$
Phenomenology – $Z^0$ at Tevatron

CDF Run-I
CTEQ-5

Qiu, Zhang 2001

CDF Run-II
CTEQ-6

Kang, Qiu 2012

No free fitting parameter!
Effectively no non-perturbative uncertainty!
Effectively no non-perturbative uncertainty!

Phenomenology – $Z^0$ at the LHC

Kang, Qiu, 2012

CMS pp-data 1110.4973

Same code Updated to CTEQ6

NLO perturbative

Resummed

$1/\sigma \, d\sigma/dp_T \, (\text{GeV}^{-1})$

$p_T \, (\text{GeV})$
Phenomenology – Upsilon

- Upsilon at Tevatron:

Dominated by perturbative small-b contribution in its Fourier conjugate space

![Graph showing predictions](Image)

Predictions
Saddle point is in nonperturbative regime:

\[ F_{QZ}^{NP}(b, Q; b_{max}) = \exp \left\{ - \ln\left( \frac{Q^2 b^2}{c^2} \right) \left[ g_1 \left( (b^2)^\alpha - (b_{max}^2)^\alpha \right) 
+ g_2 \left( b^2 - b_{max}^2 \right) \right] 
- \bar{g}_2 \left( b^2 - b_{max}^2 \right) \right\} \]
TMDs are NOT direct physical observables
– could be defined differently

Relevant definition arises from the approximation used in deriving the factorization!

The evolution equation of the TMDs is the consequence of the factorization

Knowledge of nonperturbative inputs at large $b$ is crucial in determining the TMDs from fitting the data

The TMD Collaboration – a topical theory collaboration was formed to pull together expertise from theory, lattice and phenomenology to address issues concerning TMDs

Thank you!
Modified TMD distributions

- Modified TMD distribution – “remove” the soft factor:

\[
\tilde{D}_{1, H_A/f}(z_A, b_T; \zeta_A; \mu) \\
\overset{\text{def}}{=} \lim_{y_A \to +\infty} \lim_{y_B \to -\infty} \tilde{D}^{\text{unsub}}_{1, H_A/f}(z_A, b_T; y_{P_A} - y_B) \sqrt{\frac{\tilde{S}(0)(b_T; y_A, y_n)}{\tilde{S}(0)(b_T; y_A, y_B) \tilde{S}(0)(b_T; y_n, y_B)}} \times \text{UV renormalization factor}
\]

- The TMD distribution:

\[
\tilde{f}_{f/H_A}(x, b_T; \zeta_A; \mu) \overset{\text{def}}{=} \lim_{y_A \to +\infty} \lim_{y_B \to -\infty} \tilde{f}^{\text{unsub}}_{f/H_A}(x, b_T; y_{P_A} - y_B) \sqrt{\frac{\tilde{S}(0)(b_T; y_A, y_n)}{\tilde{S}(0)(b_T; y_A, y_B) \tilde{S}(0)(b_T; y_n, y_B)}} \times \text{UV renormalization factor}
\]