TMDs and the Role of Nonperturbative Inputs in Determining the TMDs

Jianwei Qiu

Brookhaven National Laboratory

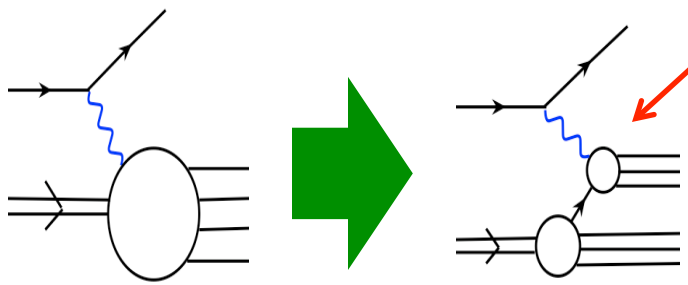
Stony Brook University

QCD Evolution 2015 Workshop

Jefferson Lab, Newport News, VA, May 26-30, 2015

Connecting hadrons to partons

- Experiments measure hadrons and leptons, not partons
- Large momentum transfer – sensitive to partons:



Sensitive to partonic dynamics

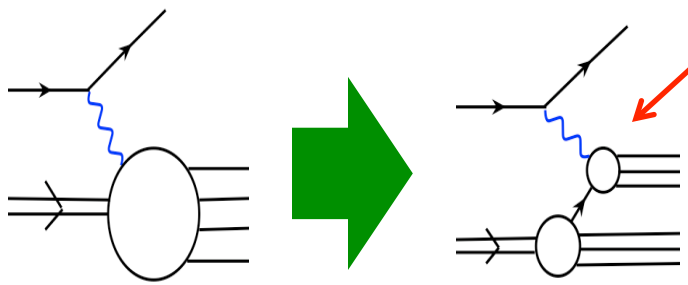
+...

(Diagrams with more active partons from each hadron!)

Connection between hadron and parton

Connecting hadrons to partons

- Experiments measure hadrons and leptons, not partons
- Large momentum transfer – sensitive to partons:



$+\ldots$

(Diagrams with more active partons from each hadron!)

Sensitive to partonic dynamics

Connection between hadron and parton

- QCD factorization – connecting partons to hadrons:

Hadronic matrix elements of parton fields:

$$\langle p,s|\mathcal{O}(\psi,A^\alpha)|p,s\rangle: \quad \langle p,s|\overline{\psi}(0)\gamma^+\psi(y)|p,s\rangle,\ \langle p,s|F^{+\alpha}(0)F^{+\beta}(y)|p,s\rangle(-g_{\alpha\beta})$$

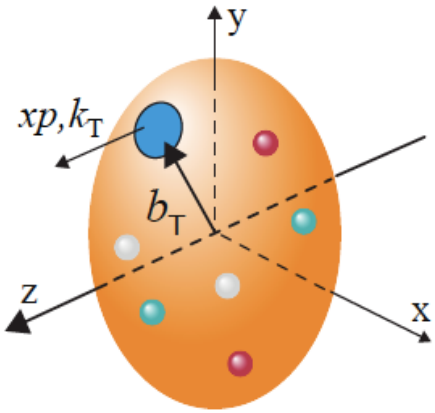
Isolate pQCD calculable short-distance partonic dynamics

Confined parton motion in a hadron

- High energy scattering with a large momentum transfer:
  - Momentum scale of the hard probe:

$$Q \gg 1/R \sim \Lambda_{\rm QCD} \sim 1/{\rm fm}$$

  - Combined motion ~ 1/R
    is too week to be sensitive to the hard probe
  - Collinear factorization – integrated into PDFs, …

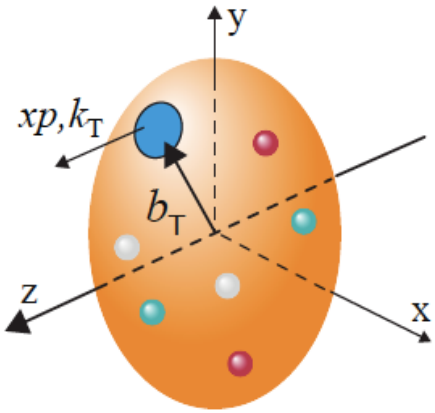


Confined parton motion in a hadron

- **High energy scattering with a large momentum transfer:**
  - Momentum scale of the hard probe:

$$Q \gg 1/R \sim \Lambda_{\rm QCD} \sim 1/{\rm fm}$$

  - Combined motion ~ 1/R is too week to be sensitive to the hard probe
  - Collinear factorization – integrated into PDFs, …



- **Scattering with multiple momentum scales observed:**
  - Two-scale observables, such as SIDIS, low p_T Drell-Yan, …

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

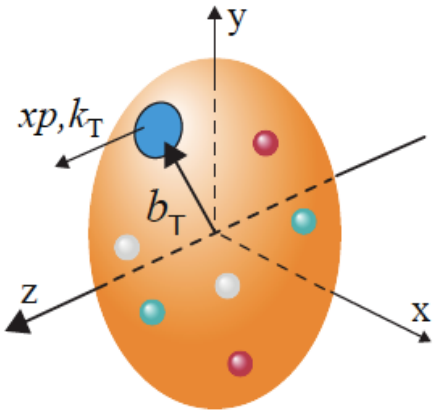
  - Hard scale Q_1 localizes the probe to see the quark or gluon d.o.f.
  - "Soft" scale Q_2 could be sensitive to the confined motion
  - TMD factorization: the confined motion is encoded into TMDs

Confined parton motion in a hadron

- **High energy scattering with a large momentum transfer:**
  - Momentum scale of the hard probe:

$$Q \gg 1/R \sim \Lambda_{\rm QCD} \sim 1/{\rm fm}$$

  - Combined motion ~ 1/R is too week to be sensitive to the hard probe
  - Collinear factorization – integrated into PDFs, …



- **Scattering with multiple momentum scales observed:**
  - Two-scale observables, such as SIDIS, low p_T Drell-Yan, …

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

  - Hard scale Q_1 localizes the probe to see the quark or gluon d.o.f.
  - "Soft" scale Q_2 could be sensitive to the confined motion
  - TMD factorization: the confined motion is encoded into TMDs

Confined motion is "unique" – the consequence of QCD, but, TMDs that represent it are not unique!

Outline

- ❑ Definitions of TMDs
- ❑ TMD factorization and TMD evolution
- ❑ The role of nonperturbative inputs in determining TMDs
- ❑ Summary and outlook

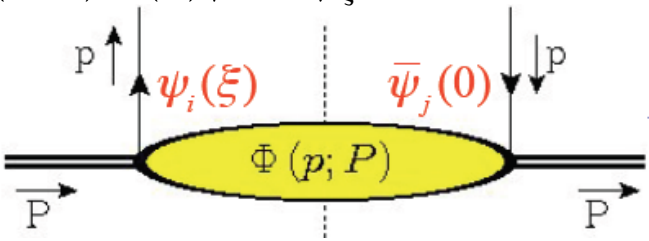
Definitions of TMDs

❑ Non-perturbative definition:

✧ **In terms of matrix elements of parton correlators:**

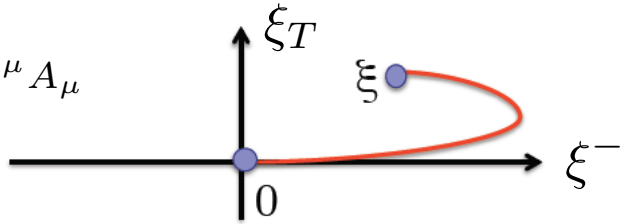
See Mulders' talk

$$\Phi^{[U]}(x, p_T; n) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{i\,p\cdot\xi} \langle P, S|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P, S\rangle_{\xi^+=0}$$



✧ **Depends on the choice of the gauge link:**

$$U(0,\xi) = e^{-ig\int_0^\xi ds^\mu A_\mu}$$



✧ **Decomposes into a list of TMDs:**

$$\Phi^{[U]}(x, p_T; n) = \Big\{ f_1^{[U]}(x, p_T^2) - f_{1T}^{\perp[U]}(x, p_T^2)\,\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x, p_T)\gamma_5$$
$$+ h_{1T}^{[U]}(x, p_T^2)\,\gamma_5\,\not{S}_T + h_{1s}^{\perp[U]}(x, p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + i h_1^{\perp[U]}(x, p_T^2)\,\frac{\not{p}_T}{M}\Big\}\frac{\not{P}}{2},$$

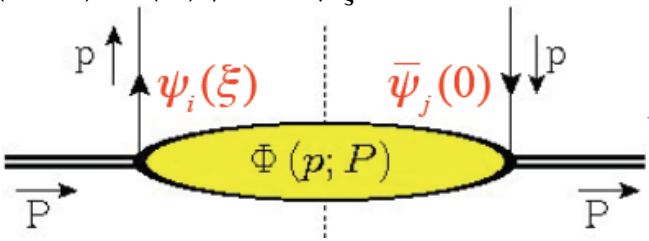
Definitions of TMDs

❑ Non-perturbative definition:

✧ In terms of matrix elements of parton correlators:

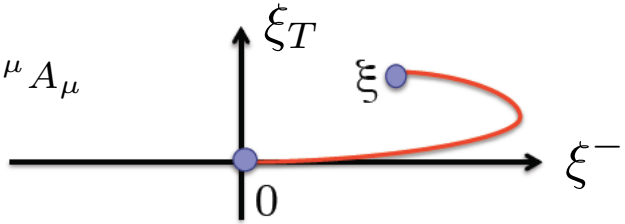
See Mulders' talk

$$\Phi^{[U]}(x, p_T; n) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{i\,p\cdot\xi} \langle P, S|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P, S\rangle_{\xi^+=0}$$



✧ Depends on the choice of the gauge link:

$$U(0,\xi) = e^{-ig\int_0^\xi ds^\mu A_\mu}$$



✧ Decomposes into a list of TMDs:

$$\Phi^{[U]}(x, p_T; n) = \Big\{ f_1^{[U]}(x, p_T^2) - f_{1T}^{\perp[U]}(x, p_T^2)\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x, p_T)\gamma_5$$
$$+ h_{1T}^{[U]}(x, p_T^2)\gamma_5 \not{S}_T + h_{1s}^{\perp[U]}(x, p_T)\frac{\gamma_5 \not{p}_T}{M} + i h_1^{\perp[U]}(x, p_T^2)\frac{\not{p}_T}{M}\Big\}\frac{\not{P}}{2},$$

✧ IF we knew proton wave function, this definition gives "unique" TMDs!

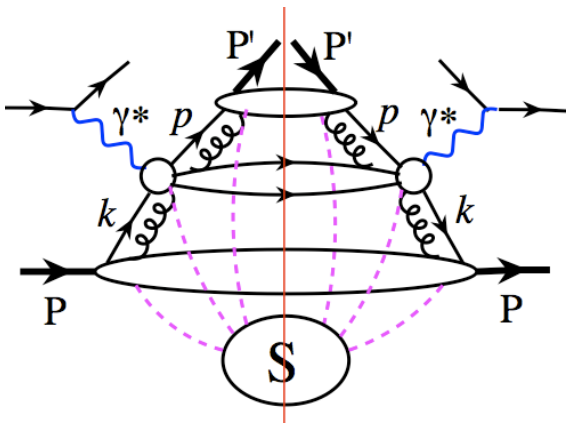
But, we do NOT know proton wave function (may calculate it using BSE?)

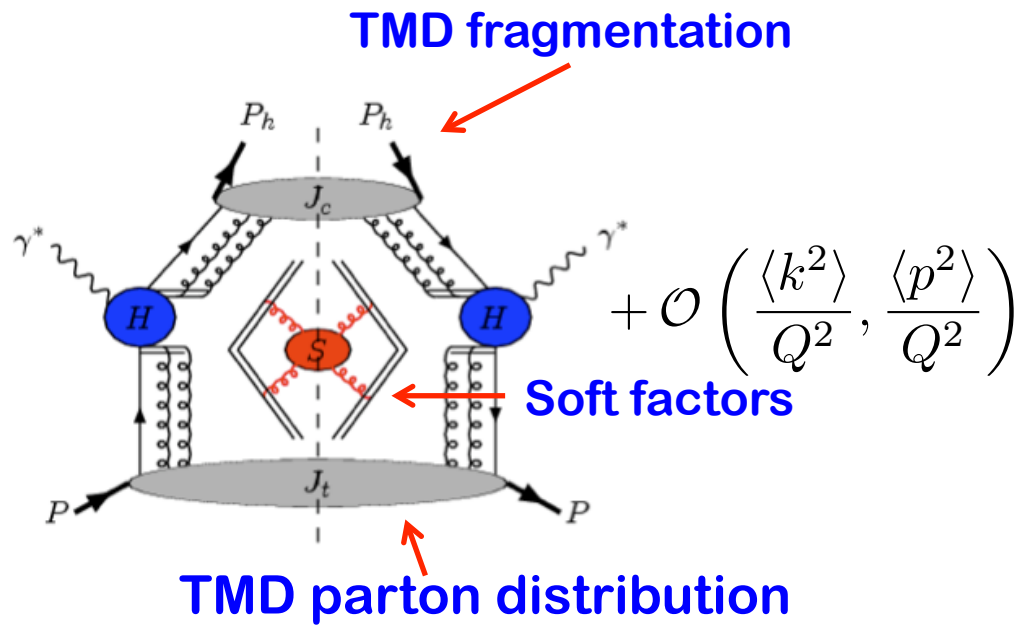
TMDs defined in this way are NOT direct physical observables!

Definitions of TMDs

- **Perturbative definition – in terms of TMD factorization:**

SIDIS as an example:



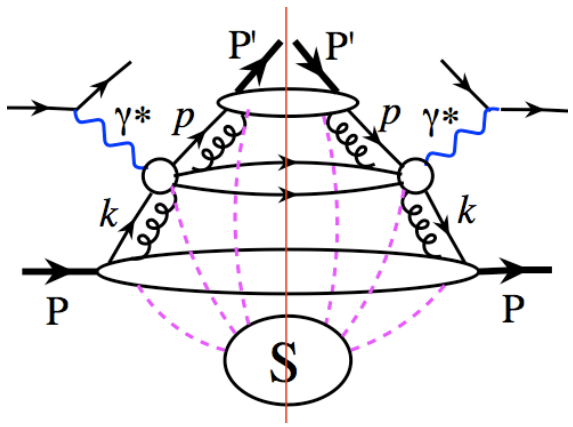


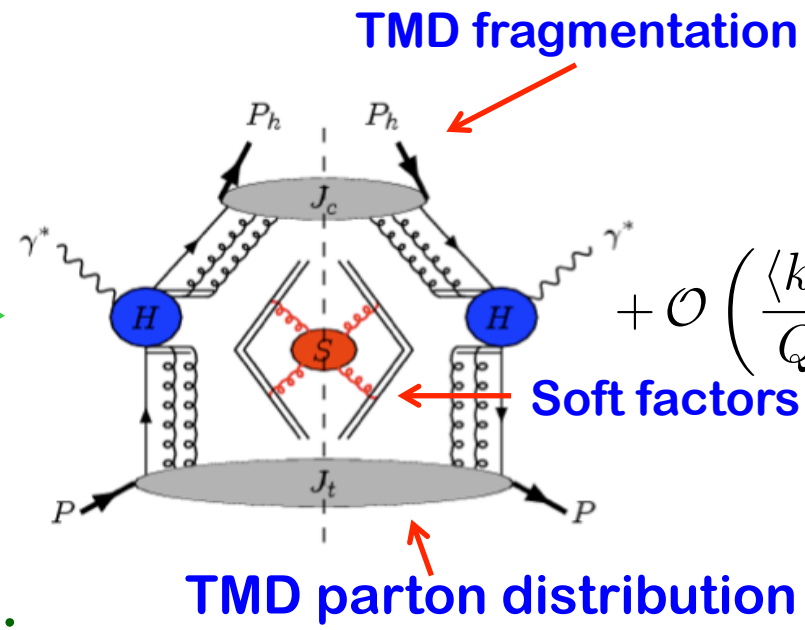
$$+\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

Definitions of TMDs

❑ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:





$$+\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

❑ Low P_{hT} – TMD factorization:

$$\sigma_{\rm SIDIS}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

❑ High P_{hT} – Collinear factorization:

$$\sigma_{\rm SIDIS}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q,P_{h\perp},\alpha_s)\otimes\phi_f\otimes D_{f\to h}+\mathcal{O}\left(\frac{1}{P_{h\perp}},\frac{1}{Q}\right)$$

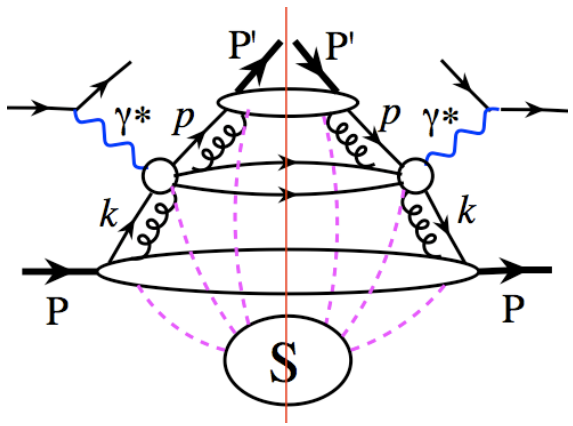
❑ P_{hT} Integrated - Collinear factorization:

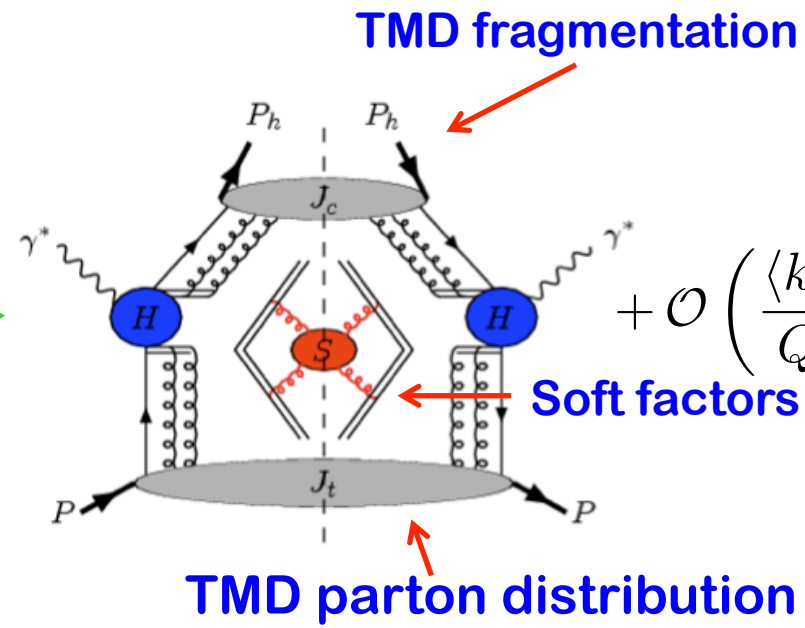
$$\sigma_{\rm SIDIS}(Q,x_B,z_h)=\tilde{H}(Q,\alpha_s)\otimes\phi_f\otimes D_{f\to h}+\mathcal{O}\left(\frac{1}{Q}\right)$$

Definitions of TMDs

❑ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:





$$+\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

❑ Operator definition:

$$\Phi^{[U]}(x,p_T;n,\mu)=\int\frac{d\xi^- d^2\xi_T}{(2\pi)^3}\,e^{i\,p\cdot\xi}\,\langle P,S|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P,S\rangle_{\xi^+=0}+\mathrm{UVCT}(\mu$$

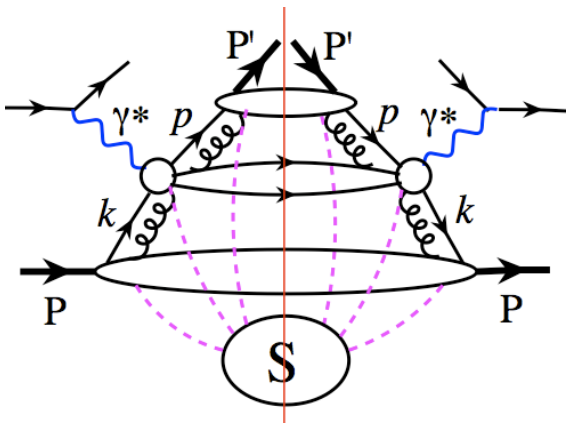
$$\equiv\int^{\mathcal{O}(\mu^2)}dp^2\,\widetilde{\Phi}^{[U]}(p;n)=\int^{\infty}dp^2\,\widetilde{\Phi}^{[U]}(p;n)-\int_{\mathcal{O}(\mu^2)}^{\infty}dp^2\,\widetilde{\Phi}^{[U]}(p;n)$$

This operator definition is scheme dependent, & needed for calculating the short-distance hard coefficients, order-by-order, in perturbation theory

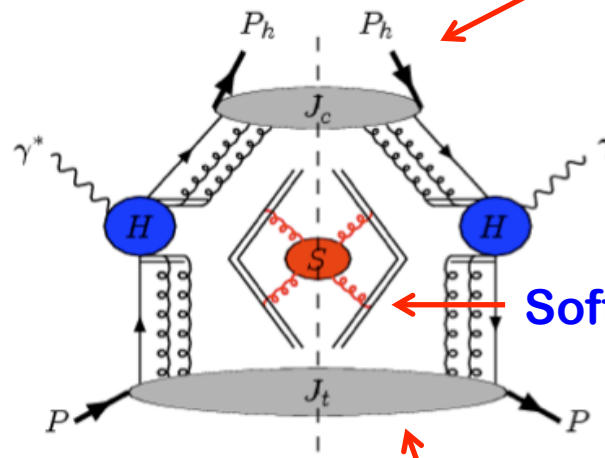
Definitions of TMDs

❑ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:



TMD fragmentation



$$+\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

Soft factors

TMD parton distribution

❑ Extraction of TMDs:

$$\sigma_{\rm SIDIS}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

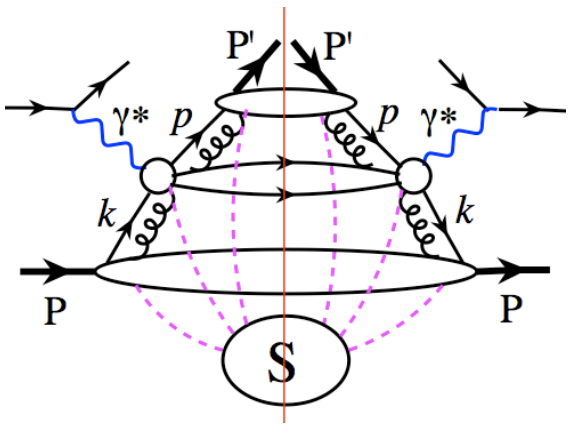
TMDs are extracted by fitting DATA using the factorization formula (approximation) and the perturbatively calculated $\hat{H}(Q;\mu)$.

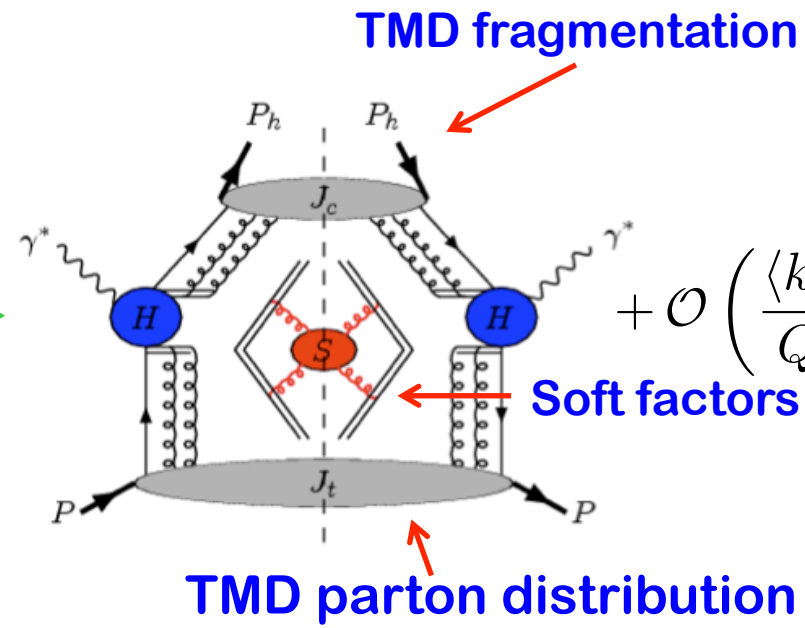
Extracted TMDs depend on the scheme (UV) and perturbative order (LO, NLO, NNLL, …) at which the hard part was calculated!

Definitions of TMDs

❑ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:





$$+\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

❑ Extraction of TMDs:

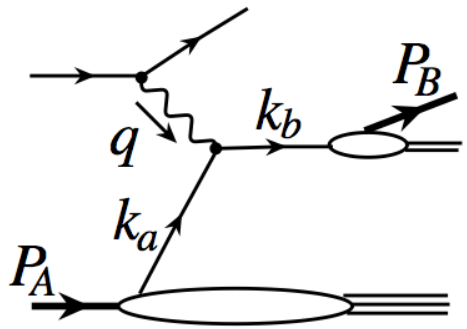
$$\sigma_{\rm SIDIS}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

TMDs are extracted by fitting DATA using the factorization formula (approximation) and the perturbatively calculated $\hat{H}(Q;\mu)$.

➡ **Extracted TMDs are valid only when the $\langle p^2\rangle \ll Q^2$**

TMD factorization in QCD

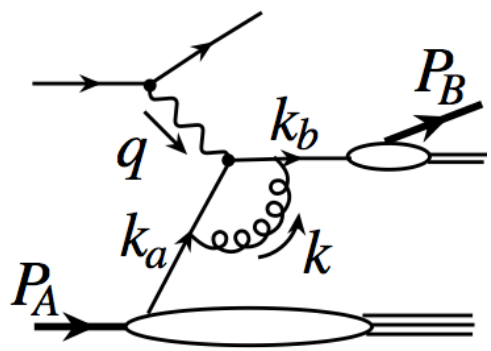
❑ SIDIS as an example – Parton model – LO QCD:



✧ Parton distribution function in a hadron

✧ parton-to-hadron fragmentation function

❑ QCD interaction and leading power regions:



✧ Collinear regions:

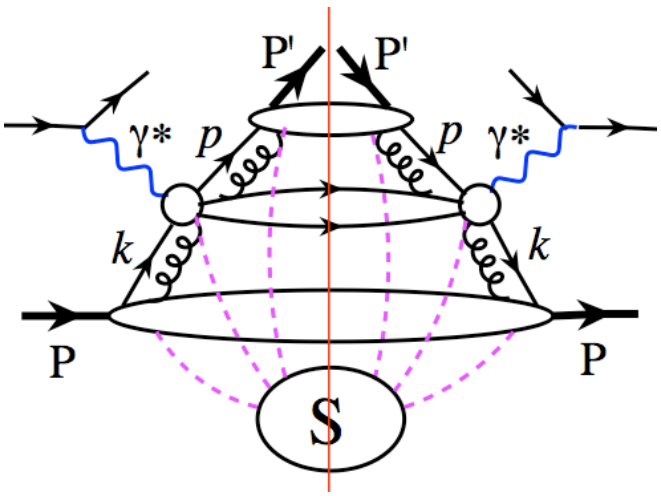
$$k \parallel P_A, \quad k \parallel P_B$$

✧ soft regions

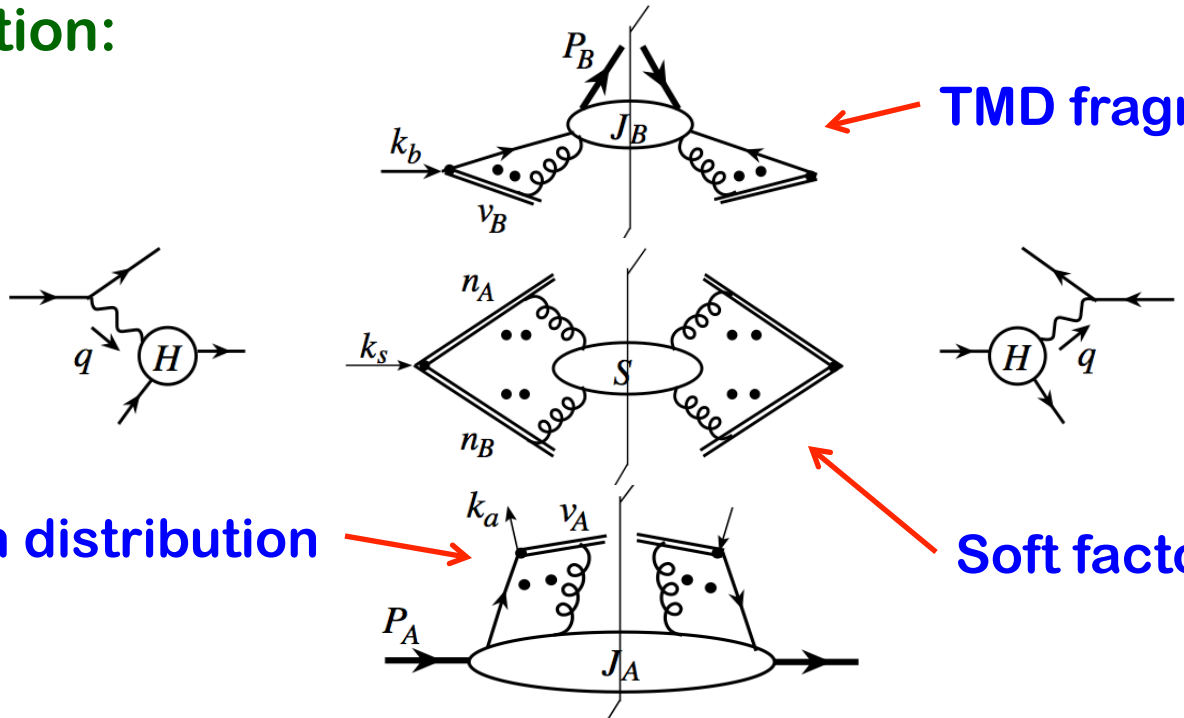
$$k^\mu \to 0$$

TMD factorization in QCD

- Leading pinch surface:
  - hard region
  - collinear to P region
  - collinear to P' region
  - soft region



- Factorization:



TMD fragmentation

TMD parton distribution

Soft factors

TMD factorization in QCD

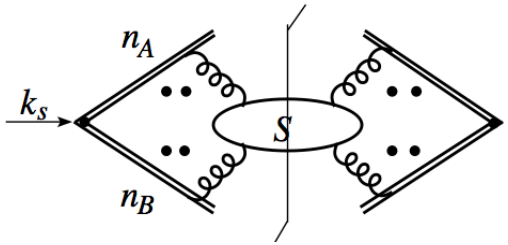
- **Factorization formalism in momentum space:**

$$W^{\mu\nu} \propto \sum_f \int d^2k_{aT} d^2k_{bT} \mathcal{S}(q_{hT} - k_{aT} - k_{bT}) \mathrm{Tr}\left[\mathcal{P}_A J_{f/A}(x, k_{aT}) \overline{\mathcal{P}}_A H_f^\nu(Q) \mathcal{P}_B J_{f\to B}(z, k_{bT}) \overline{\mathcal{P}}_B \overline{H}_f^\mu(Q)\right]$$

- **The soft factors:**

$$\mathcal{S} = Z_s \mathcal{S}^{(0)}$$

$$\mathcal{S}^{(0)}(k_{sT}) = \frac{1}{N_c} \int \frac{dk_s^+ dk_s^-}{(2\pi)^{4-2\epsilon}}$$



- **The soft factors in collinear factorization:**

$$\mathcal{S}^{(0)}(k_{sT}) \to \mathcal{S}^{(0)}(k_{sT} = 0) \to 1$$

Going to the b-space

Collins et al, 1995

- Factorization in b-space:

$$\tilde{D}_{1,h/f}(z,b_{\rm T}) = \int {\rm d}^{2-2\epsilon}\boldsymbol{k}_{\rm T}\, e^{i\boldsymbol{k}_{\rm T}\cdot\boldsymbol{b}_{\rm T}} D_{1,h/f}(z,z\boldsymbol{k}_{\rm T}),$$

$$W^{\mu\nu} \propto \sum_f W^{\mu\nu}_{(0)f} \int \frac{d^{2-2\epsilon}b_T}{(2\pi)^{2-2\epsilon}}\, e^{-i\vec{q}_{hT}\cdot\vec{b}_T}\, \widetilde{\mathcal{S}}(b_T)\, \widetilde{\phi}_{f/A}(x,b_T)\, \widetilde{D}_{f\to D}(z,b_T)$$

- Universality of the soft factor:

$$\tilde{S}_{(0)}(b_{\rm T};y_A,y_B) = \frac{1}{N_c}\langle 0|\; W(\boldsymbol{b}_{\rm T}/2;\infty,n_B)^\dagger_{ca}\, W(\boldsymbol{b}_{\rm T}/2;\infty,n_A)_{ad}\, W(-\boldsymbol{b}_{\rm T}/2;\infty,n_B)_{bc}\, W(-\boldsymbol{b}_{\rm T}/2;\infty,n_A)^\dagger_{db}\; |0\rangle$$

Wilson line: $W(x;\infty;n)_{ab} = P\left\{e^{-ig_0\int_0^\infty {\rm d}\lambda\, n\cdot A_{(0)\alpha}(x+\lambda n)t_\alpha}\right\}_{ab}$

- Evolution of TMDs:

from the wave function renormalization and the renormalization of the soft factors

Evolution of TMDs

- **Evolution of TMDs:**

$$\frac{\partial \ln \tilde{D}_{1,H_A/f}(z_A, b_T; \zeta_A, \ldots)}{\partial \ln \sqrt{\zeta_A}} = \tilde{K}(b_T; \mu)$$

$$\tilde{K}(b_T; \ldots) = \frac{\partial}{\partial y_n}\left[\frac{1}{2}\ln \tilde{S}(b_T; y_n, -\infty) - \frac{1}{2}\ln \tilde{S}(b_T; +\infty, y_n)\right]$$

$$= \frac{1}{2\tilde{S}(b_T; y_n, -\infty)}\frac{\partial \tilde{S}(b_T; y_n, -\infty)}{\partial y_n} - \frac{1}{2\tilde{S}(b_T; +\infty, y_n)}\frac{\partial \tilde{S}(b_T; +\infty, y_n)}{\partial y_n}$$

$$\frac{\mathrm{d} \ln \tilde{D}_{1,H_A/f}(z_A, b_T; \zeta_A, \ldots)}{\mathrm{d} \ln \mu} = \gamma_D(g; \zeta_A/\mu^2)$$

- **Evolved TMDs:**

$$\tilde{D}_{1,H_A/f}(z_A, b_T; \zeta_A; \mu) = \tilde{D}_{1,H_A/f}\left(z_A, b_T; m_A^2/z_A^2; \mu_0\right) \exp\left\{\ln \frac{\sqrt{\zeta_A} z_A}{m_A}\tilde{K}(b_T; \mu_0) + \right.$$

$$\left. + \int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'}\left[\gamma_D(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_A}}{\mu'}\gamma_K(g(\mu'))\right]\right\}.$$

Evolution of TMDs

- **Evolution of TMDs:**

$$\frac{\partial \ln \tilde{D}_{1,H_A/f}(z_A, b_T; \zeta_A, \ldots)}{\partial \ln \sqrt{\zeta_A}} = \tilde{K}(b_T; \mu)$$

$$\tilde{K}(b_T; \ldots) = \frac{\partial}{\partial y_n}\left[\frac{1}{2}\ln \tilde{S}(b_T; y_n, -\infty) - \frac{1}{2}\ln \tilde{S}(b_T; +\infty, y_n)\right]$$

$$= \frac{1}{2\tilde{S}(b_T; y_n, -\infty)}\frac{\partial \tilde{S}(b_T; y_n, -\infty)}{\partial y_n} - \frac{1}{2\tilde{S}(b_T; +\infty, y_n)}\frac{\partial \tilde{S}(b_T; +\infty, y_n)}{\partial y_n}$$

$$\frac{\mathrm{d} \ln \tilde{D}_{1,H_A/f}(z_A, b_T; \zeta_A, \ldots)}{\mathrm{d} \ln \mu} = \gamma_D(g; \zeta_A/\mu^2)$$

- **Evolved TMDs:**

$$\tilde{D}_{1,H_A/f}(z_A, b_T; \zeta_A; \mu) = \tilde{D}_{1,H_A/f}\left(z_A, b_T; m_A^2/z_A^2; \mu_0\right) \exp\left\{\ln\frac{\sqrt{\zeta_A} z_A}{m_A}\tilde{K}(b_T; \mu_0) + \right.$$

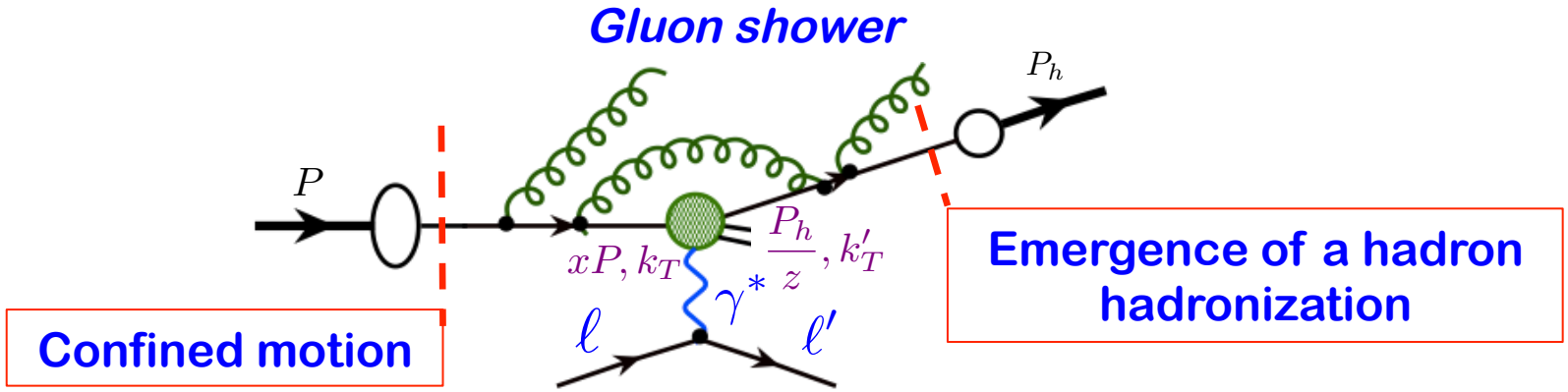
$$\left. + \int_{\mu_0}^{\mu}\frac{\mathrm{d}\mu'}{\mu'}\left[\gamma_D(g(\mu'); 1) - \ln\frac{\sqrt{\zeta_A}}{\mu'}\gamma_K(g(\mu'))\right]\right\}.$$

Only valid when $b_T \ll 1/\mathrm{fm}$ and $\mu \gg \mu_0$

F.T. to momentum space → **Requires the nonperturbative inputs**

Parton k_T at the hard collision

- Sources of parton k_T at the hard collision:



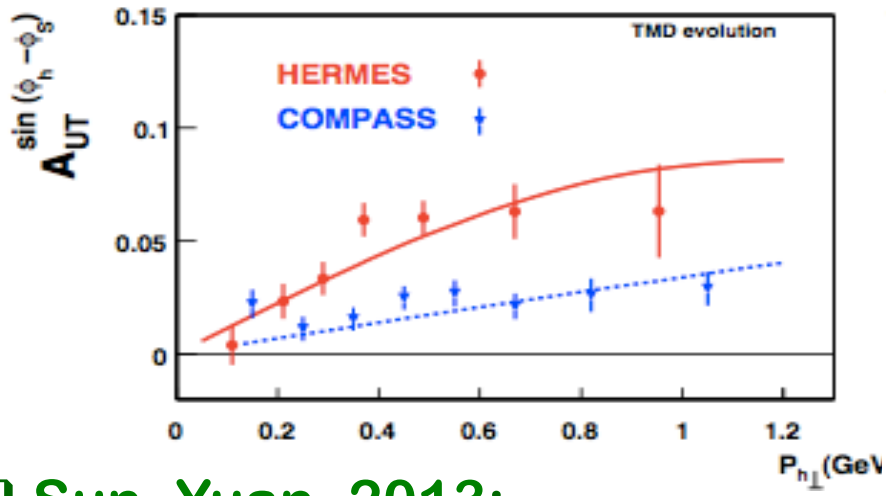
- Large k_T generated by the shower (caused by the collision):
  - ✧ Q^2-dependence – linear evolution equation of TMDs in b-space
  - ✧ The evolution kernels are perturbative at small b, but, not large b

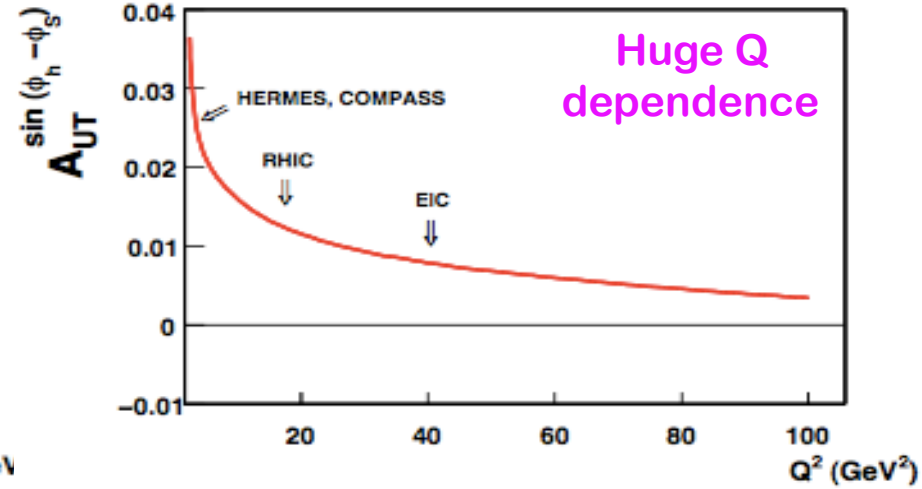
  ➡ The nonperturbative inputs at large b could impact TMDs at all Q^2

- Challenge: to extract the "true" parton's confined motion:
  - ✧ Separation of perturbative shower contribution from nonperturbative hadron structure – not as simple as PDFs

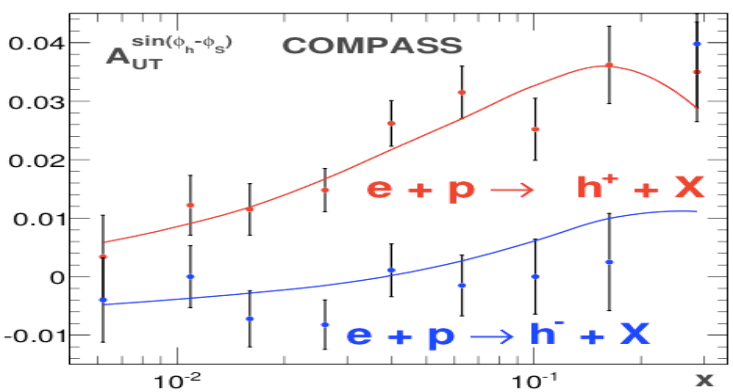
Importance of the nonperturbative inputs

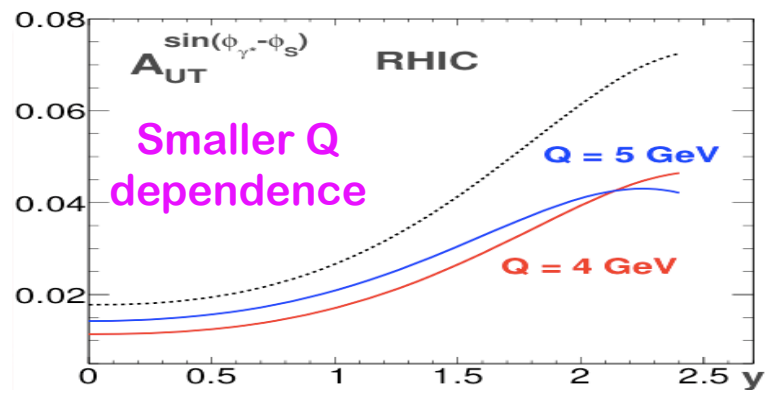
- Aybat, Prokudin, Rogers, 2012:





- Sun, Yuan, 2013:





No disagreement on evolution equations!

Issue: extrapolation to non-perturbative large b-region
choice of the Q-dependent "form factor"

Extrapolation to large b_T

❑ CSS b*-prescription:

Aybat and Rogers, arXiv:1101.5057
Collins and Rogers, arXiv:1412.3820

$$\tilde{F}_{f/P}(x, \mathbf{b}_T; Q, Q^2) = \overbrace{\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b)}^{\text{AA}}$$

$$\times \overbrace{\times \exp\left\{ \ln \frac{Q}{\mu_b} K(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\}}^{\text{BB}}$$

$$\times \boxed{\overbrace{\exp\left\{ g_{f/P}(x, b_T) + g_K(b_T) \ln \frac{Q}{Q_0} \right\}}^{\text{CC}}}$$

Nonperturbative "form factor"

$$b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}} \qquad \text{with } b_{\max} \sim 1/2 \text{ GeV}^{-1}$$

❑ Nonperturbative fitting functions

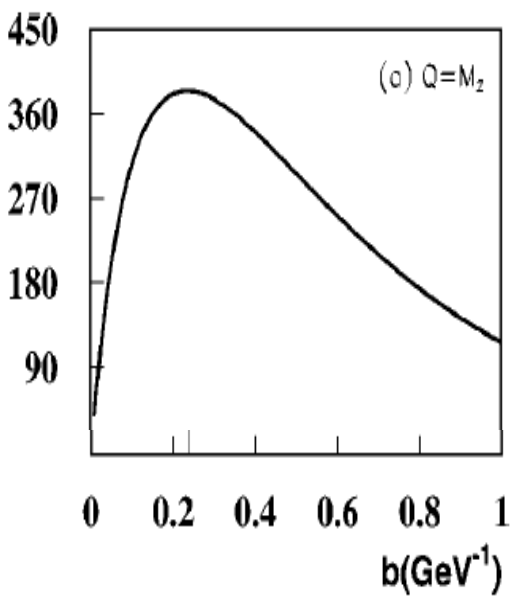
Various fits correspond to different choices for $g_{f/P}(x, b_T)$ **and** $g_K(b_T)$
e.g.

$$g_{f/P}(x, b_T) + g_K(b_T) \ln \frac{Q}{Q_0} \equiv -\left[g_1 + g_2 \ln \frac{Q}{2Q_0} + g_1 g_3 \ln(10x) \right] b_T^2$$

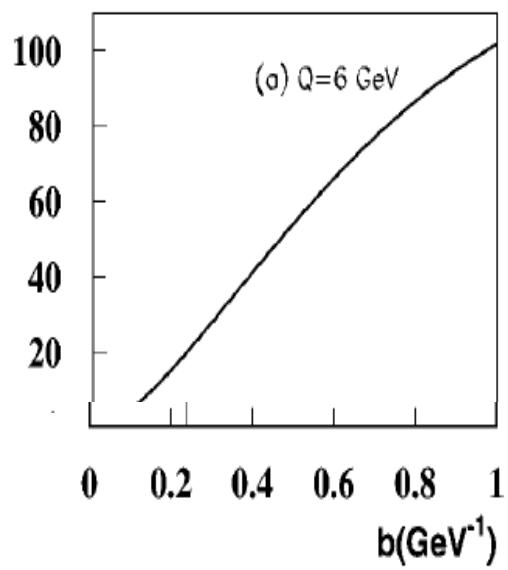
Some parameterizations could significantly change the small-b calculation!

Extrapolation to large b_T

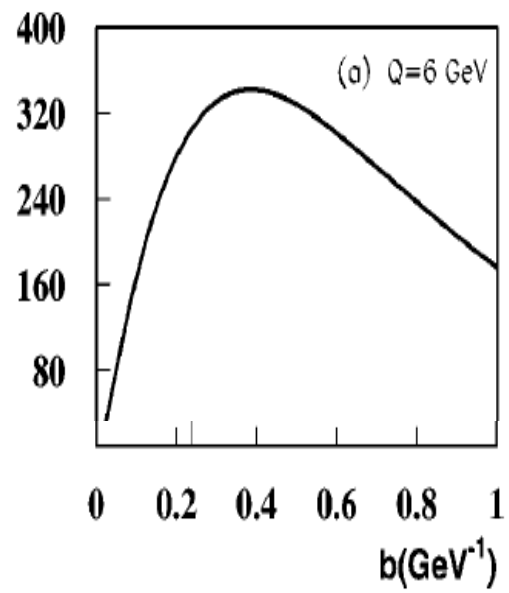
- **Features of perturbative calculation at small-b:** Qiu, Zhang, 2001
  - Sudakov form factor $\rightarrow$ $b_{sp} \propto (\frac{\Lambda_{QCD}}{Q})^{\lambda}, \lambda \sim 0.5$
  - evolution of $f_{a/A}$ and $D_{c\rightarrow h}$ also moves b_{sp}
    smaller $\xi \Rightarrow \mu\frac{\partial}{\partial\mu} f_{a/A}(\xi) > 0 \Rightarrow$ lower b_{sp}
- **Example for Q and √s dependence:** $W^{\rm DY}(x, b, Q)$



$\sqrt{s} = 1.8$ TeV



$\sqrt{s} = 27.4$GeV



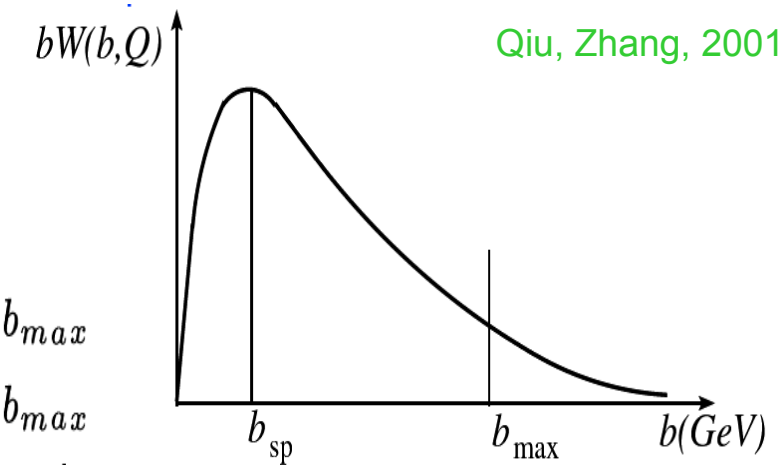
$\sqrt{s} = 1.8$ TeV

Extrapolation to large b_T

- **Another approach:**

$$\frac{d\sigma^{\rm resum}_{AB\to Z}}{dq_T^2} \propto \int_0^\infty db\, J_0(q_T b)\, b\, W(b,Q)$$

$$W = \begin{cases} W^{\rm pert}(b,x,z,Q) & b \le b_{max} \\ W^{\rm pert}(b_{max},x,z,Q)\, F^{NP}(b,Q;b_{max}) & b > b_{max} \end{cases}$$



$$W^{\rm pert}(b,x,z,Q) = \sum_j e_j^2 \left[f_{a/A} \otimes C^{in}_{a\to j}\right]\left[C^{out}_{j\to c} \otimes D_{b\to h}\right] \times {\rm e}^{-S(b,Q)}$$

$$F^{NP}_{QZ}(b,Q;b_{max}) = \exp\left\{ -\ln\left(\frac{Q^2 b_{max}^2}{c^2}\right)\left[g_1\left((b^2)^\alpha - (b_{max}^2)^\alpha\right) + g_2\left(b^2 - b_{max}^2\right)\right] - \bar{g}_2\left(b^2 - b_{max}^2\right)\right\}$$

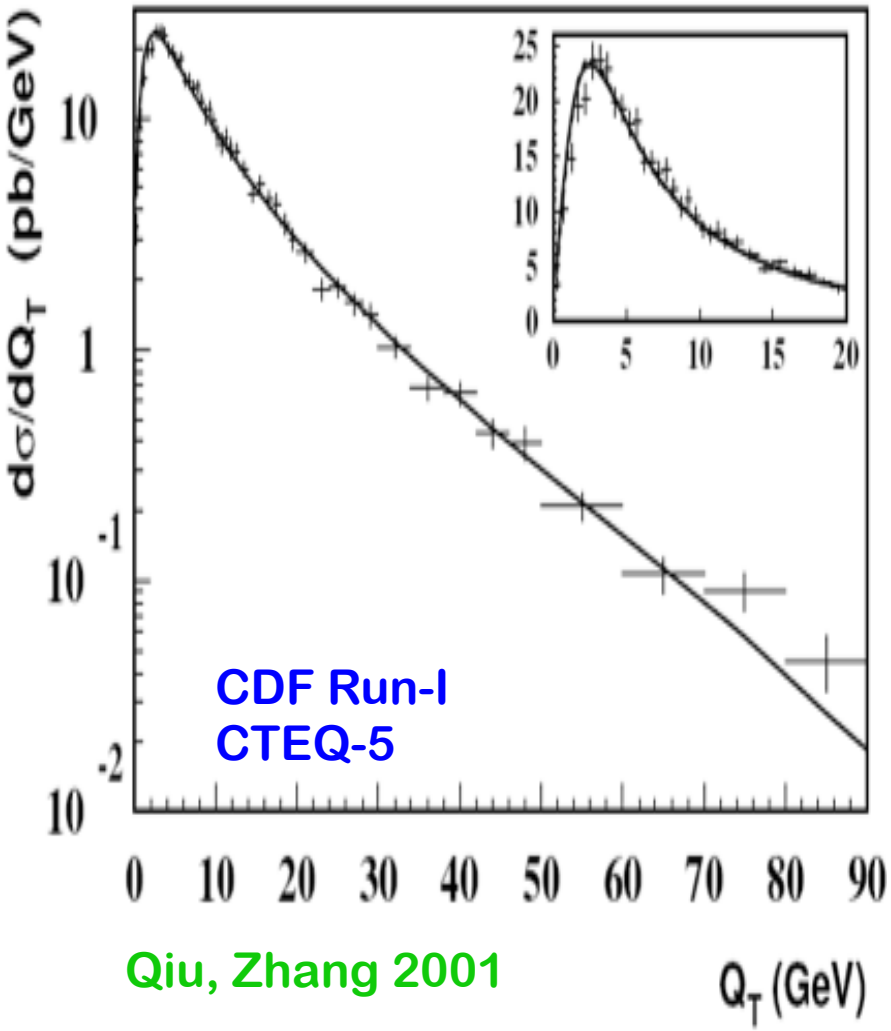
Leading twist

Intrinsic power corrections

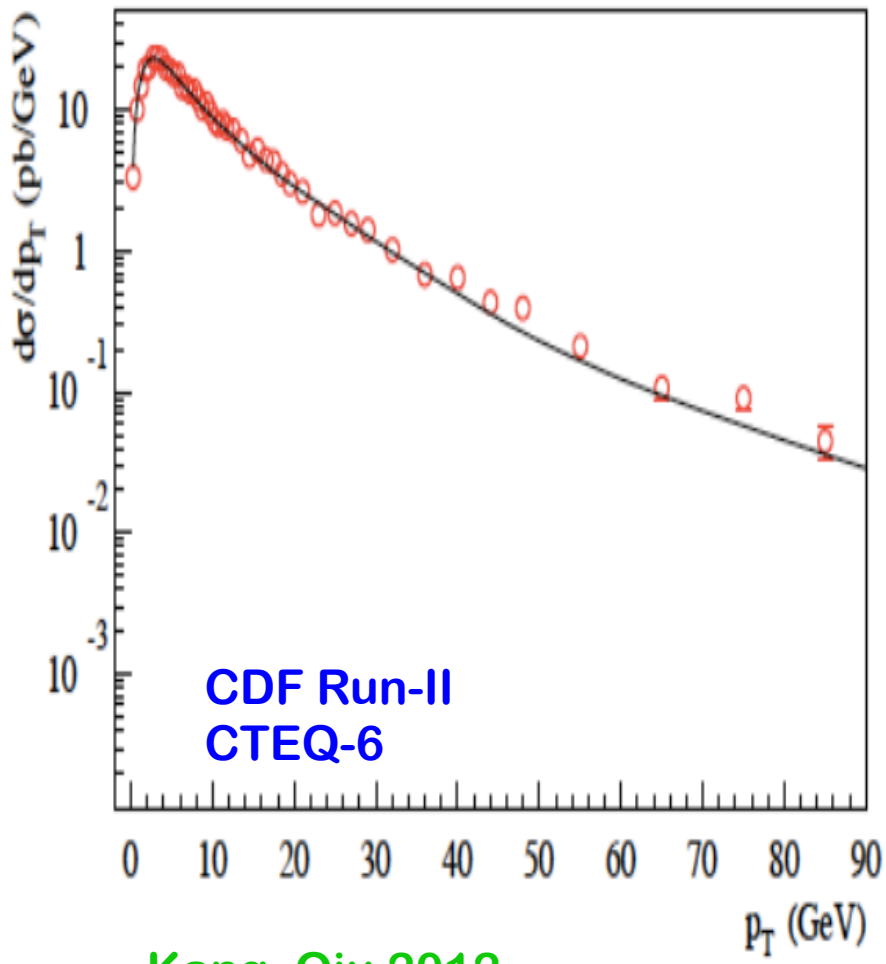
Dynamical power corrections

All parameters, α, g_1, g_2, are fixed by the continuity of the "W" and its derivatives at b_{max} – excellent predictive power for observables with the saddle point at small enough b_{sp}

Phenomenology – Z^0 at Tevatron



Qiu, Zhang 2001

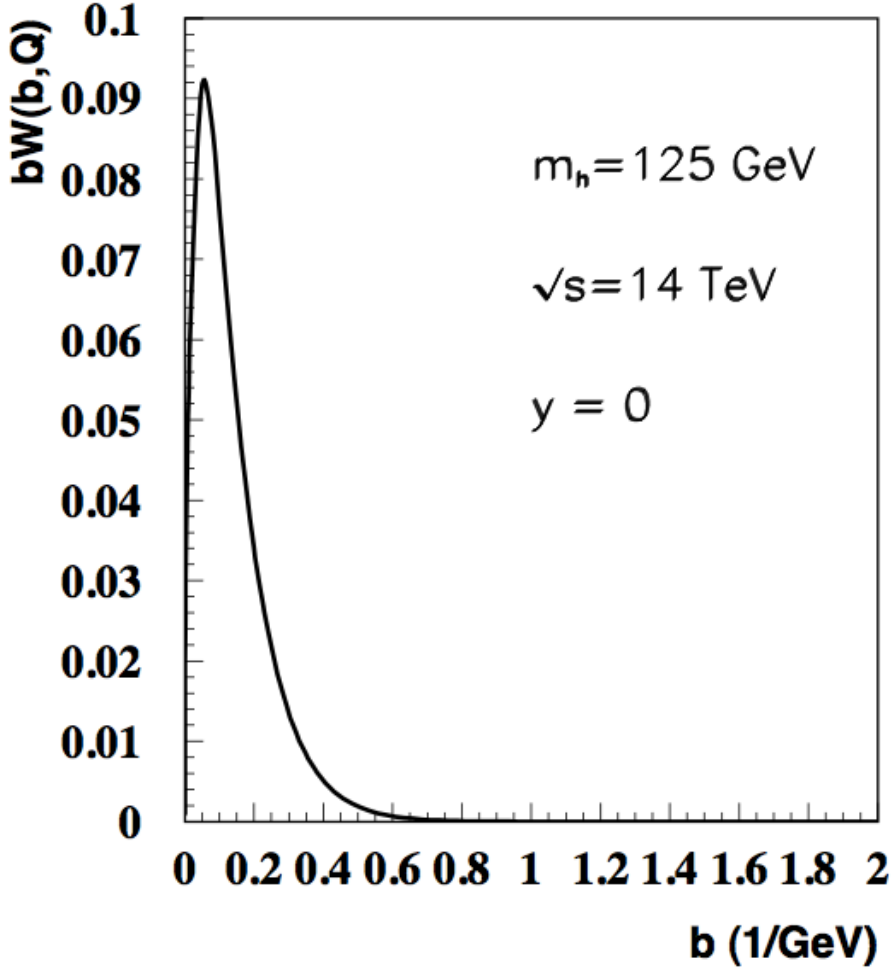


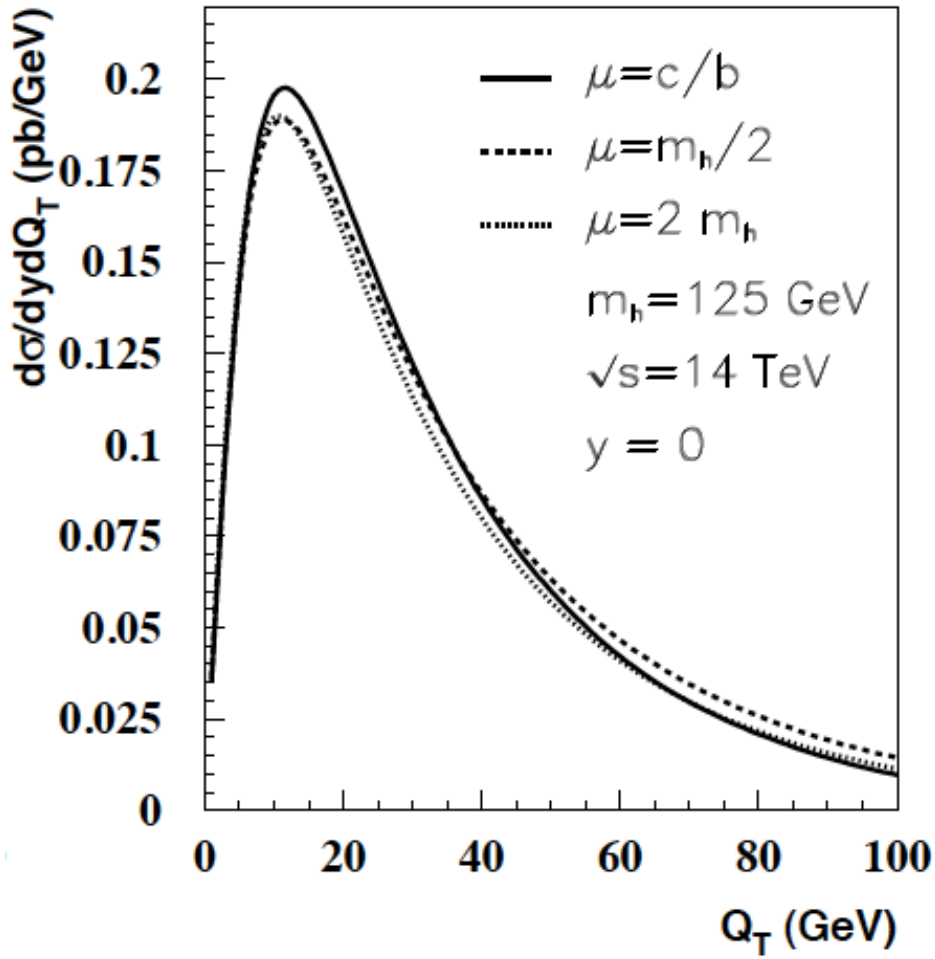
Kang, Qiu 2012

No free fitting parameter!

Phenomenology – Higgs

Berger, Qiu, 2003

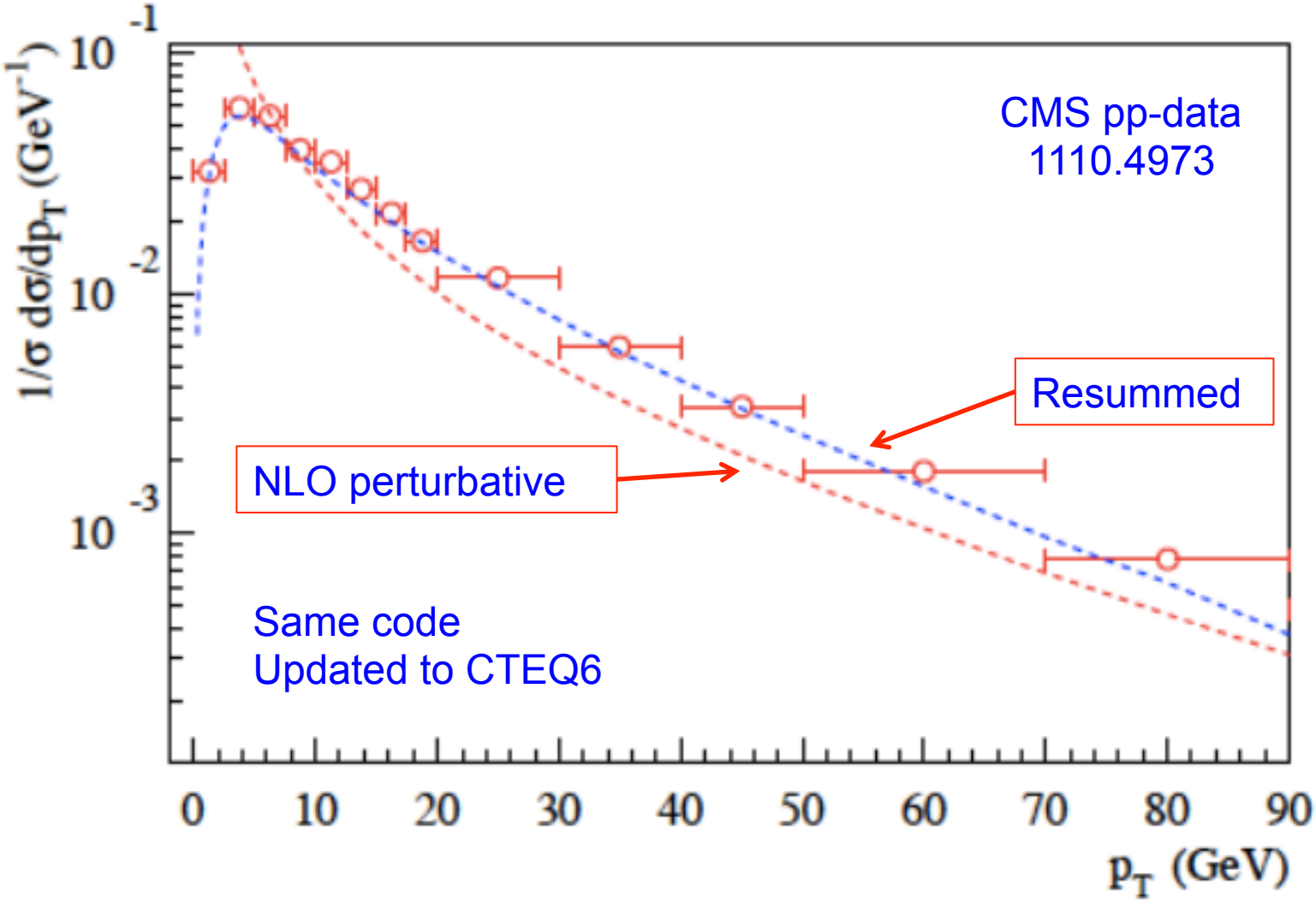




Effectively no non-perturbative uncertainty!

Phenomenology – Z^0 at the LHC

Kang, Qiu, 2012



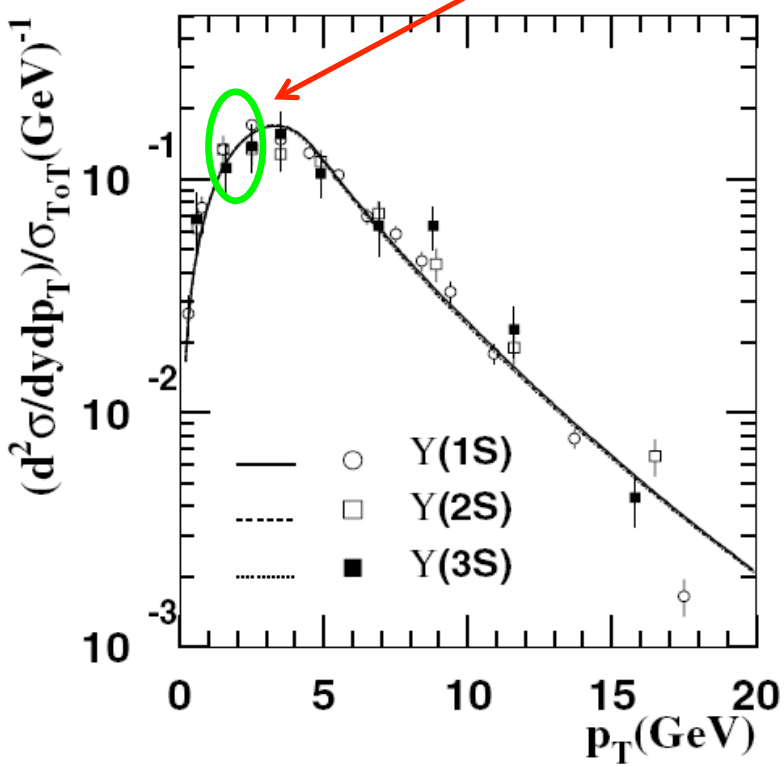
Effectively no non-perturbative uncertainty!

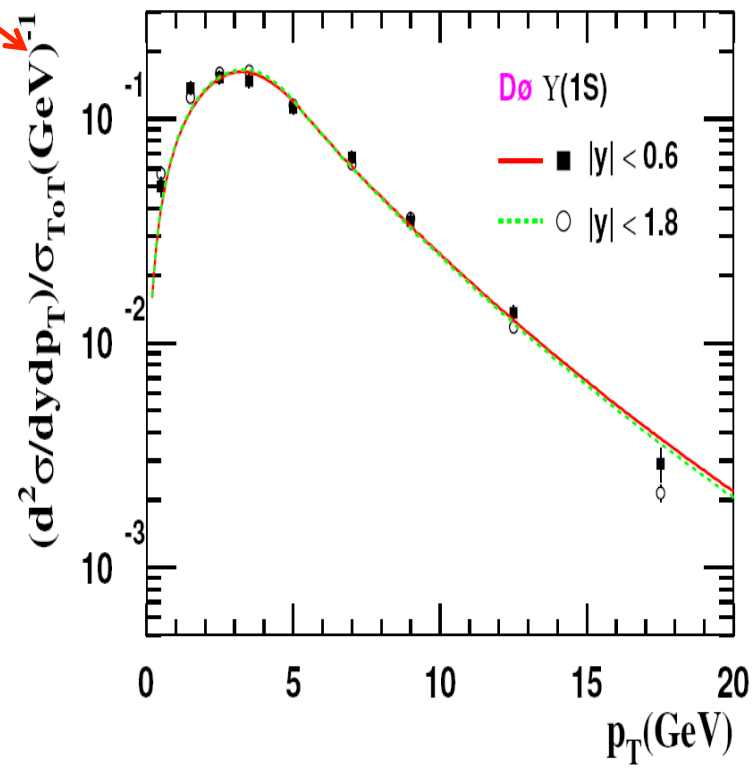
Phenomenology – Upsilon

Berger, Qiu, Wang, 2005

❑ Upsilon at Tevatron:

Dominated by perturbative small-b contribution in its Fourier conjugate space



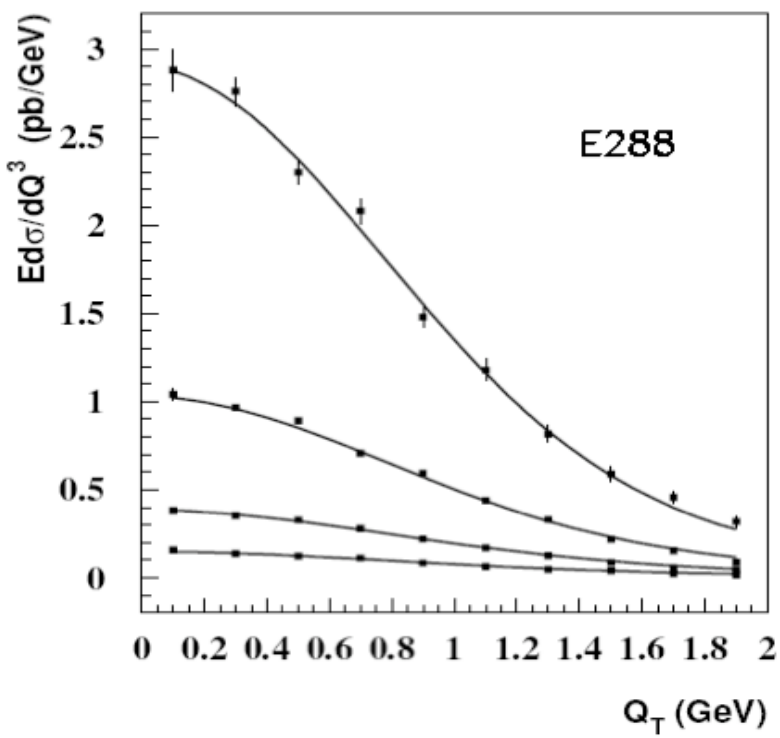


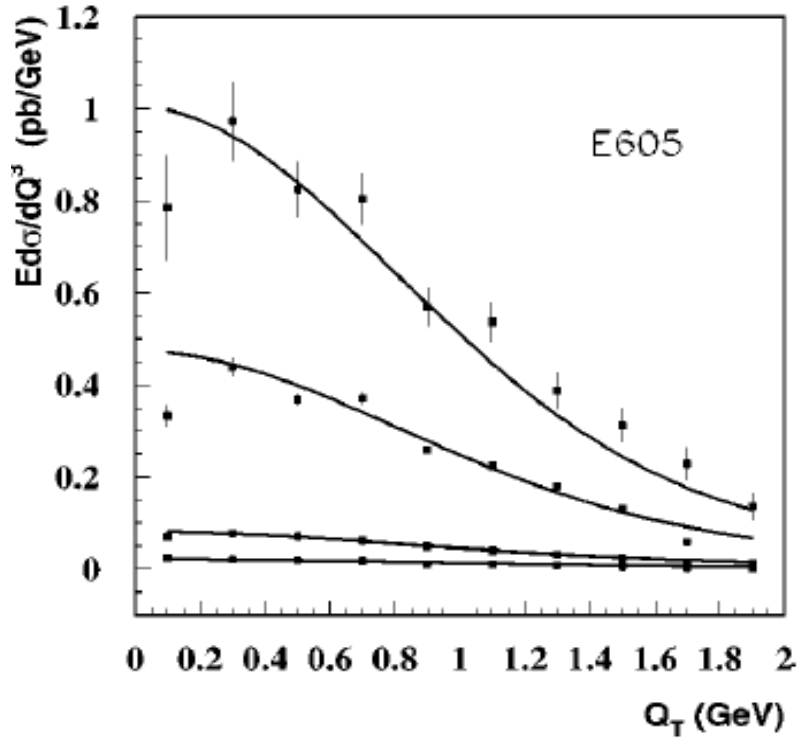
Predictions

Phenomenology – Drell-Yan

- **Saddle point is in nonperturbative regime:**

Qiu, Zhang, 2001





$$F_{QZ}^{NP}(b,Q;b_{max}) = \exp\left\{ -\ln\left(\frac{Q^2 b_{max}^2}{c^2}\right)\left[g_1\left((b^2)^\alpha - (b_{max}^2)^\alpha\right) + g_2\left(b^2 - b_{max}^2\right)\right] - \bar{g}_2\left(b^2 - b_{max}^2\right)\right\}$$

Leading twist

Intrinsic power corrections

Dynamical power corrections

Summary

- TMDs are NOT direct physical observables – could be defined differently

  Relevant definition arises from the approximation used in deriving the factorization!

- The evolution equation of the TMDs is the consequence of the factorization

- Knowledge of nonperturbative inputs at large b is crucial in determining the TMDs from fitting the data

- The TMD Collaboration – a topical theory collaboration was formed to pull together expertise from theory, lattice and phenomenology to address issues concerning TMDs

Thank you!

Modified TMD distributions

- Modified TMD distribution – "remove" the soft factor:

$$\tilde{D}_{1,H_A/f}(z_A, b_T; \zeta_A; \mu)$$

$$\stackrel{\text{def}}{=} \lim_{\substack{y_A \to +\infty \\ y_B \to -\infty}} \tilde{D}^{\text{unsub}}_{1,H_A/f}(z_A, b_T; y_{p_A} - y_B) \sqrt{\frac{\tilde{S}_{(0)}(b_T; y_A, y_n)}{\tilde{S}_{(0)}(b_T; y_A, y_B)\ \tilde{S}_{(0)}(b_T; y_n, y_B)}}$$

$$\times \text{ UV renormalization factor}$$

- The TMD distribution:

$$\tilde{f}_{f/H_A}(x, \boldsymbol{b}_T; \zeta_A; \mu) \stackrel{\text{def}}{=}$$

$$= \lim_{\substack{y_A \to +\infty \\ y_B \to -\infty}} \tilde{f}^{\text{unsub}}_{f/H_A}(x, \boldsymbol{b}_T; y_{P_A} - y_B) \sqrt{\frac{\tilde{S}_{(0)}(b_T, y_A, y_n)}{\tilde{S}_{(0)}(b_T, y_A, y_B)\ \tilde{S}_{(0)}(b_T, y_n, y_B)}}$$

$$\times \text{ UV renormalization factor}$$

Collins book