QCD evolution for Collins asymmetries in $e^+e^-$ annihilation and SIDIS

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Outlines

- QCD evolution in TMD factorization
- Collins asymmetry in $e^+e^-$ annihilation and SIDIS
- Soft gluon resummation in dijet azimuthal angular correlation at hadron collider
Why do we need QCD evolution?

- Consider the production process $h_1 h_2 \rightarrow Z + X$

\[
\frac{d\sigma}{dQ_T^2} \sim \frac{1}{Q_T^2} \left\{ \alpha_s(L + 1) + \alpha_s^2(L^3 + L^2) + \alpha_s^3(L^5 + L^4) + \alpha_s^4(L^7 + L^6) + \ldots 
\right. \\
+ \alpha_s^2(L + 1) + \alpha_s^3(L^3 + L^2) + \alpha_s^4(L^5 + L^4) + \ldots \\
+ \alpha_s^3(L + 1) + \alpha_s^4(L^3 + L^2) + \ldots \right\}
\]

Where $Q_T$ is the transverse momentum, and $Q$ the mass of $Z$, and $L = \log[Q^2 / Q_T^2]$.

- We have to resum these large logs to make reliable predictions.
TMD factorization

At the small transverse momentum limitation

\[ W_{\text{(Drell-Yan)}} = H(M, \mu) f_1(x_1, b, M, \mu) f_2(x_2, b, M, \mu) S(b, \mu) \]

Fourier transformation

\[ Q_t \]

Every scale in each term can be evolved by RGE
TMD evolution in TMD parton distributions

With CSS evolution equation, evolution starts from \( \mu_b = c/b, \quad c = 2e^{-\gamma_E} \)

\[
\tilde{f}(x, b; Q) = \tilde{f}(x, b; \mu_b) e^{-S_{pert}(b)}
\]

where

\[
\tilde{f}(x, b; Q) = \int d^2k_\perp e^{-ik_\perp b} f(x, k_\perp; Q)
\]

\[
S_{PT}(b) = \int_{\mu_b}^{Q} \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right)
\]

Perturbative Sudakov factor

\[
A = \sum_{n=1} \left( \frac{\alpha_s}{\pi} \right)^n A^{(n)} \quad B = \sum_{n=1} \left( \frac{\alpha_s}{\pi} \right)^n B^{(n)}
\]
Calculation is perturbative, valid only in region $b \ll 1/\Lambda_{QCD}$

Fourier transform in momentum space involves non-perturbative region

$$f(x, k_{\perp}; Q) = \int_0^\infty \frac{b db}{2\pi} J_0(k_{\perp} b) \tilde{f}(x, b; Q)$$

Non-perturbative region needs to be treated.
Common method $b^*$ prescription

$$b^* = \frac{b}{\sqrt{1 + b^2/b_{max}^2}}$$

$$\tilde{f}(x, b; Q) = \tilde{f}(x, b^*; c/b^*) e^{-S_{pert}(b^*)} e^{-S_{NP}(b)}$$

Non-perturbative Sudakov factor
A new non-perturbative Sudakov factor is used. Where $Q_0^2 = 2.4\text{GeV}^2$

g_1$ and $g_q$ are free parameters, from the fitting of Drell-Yan processes, and $g_h$ is from SIDIS

In our fit, we choose $b_{\text{max}} = 1.5\text{GeV}^{-1}$

$$S^{\text{SIDIS}}_{\text{NP}}(Q, b) = g_2 \ln \left( \frac{b}{b_*} \right) \ln \left( \frac{Q}{Q_0} \right) + \left( g_q + \frac{g_h}{z_h^2} \right) b^2$$
In this work CSS scheme is applied

\[ \tilde{f}^j(x, b_\star; c/b_\star) = \sum_{j' = q, g} \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{j/j'} \left( \frac{x}{\hat{x}}, b_\star; c/b_\star \right) f^{j'}(x; c/b_\star) \]

\[ C = \sum_{n=1} \left( \frac{\alpha_s}{\pi} \right)^n C^{(n)} \]

Wilson coefficient  
Collinear PDF

transversity is related to a twist-2 collinear PDF

Collins function is related to twist-3 collinear FF
Collins asymmetries in $e^+e^- \rightarrow hh+X$

The Collins asymmetries is proportional to $\cos(\phi_1 + \phi_2)$ or $\cos(2\phi_0)$
Collins asymmetries in SIDIS

The Collins asymmetries is proportional to \( \sin(\phi + \phi_S) \)
Transversity and Collins FF

Parameterizations:

\[ h_1^q(x, Q_0) \propto N_q^h x^{a_q} (1 - x)^{b_q} \frac{1}{2} (f_1(x, Q_0) + g_1(x, Q_0)) \]

Transversity

Favoured and unfavoured Collins FF

\[ \hat{H}^{(3)}_{fav}(z, Q_0) = N_u^c z^{\alpha_u} (1 - z)^{\beta_u} D_{\pi+u}(z, Q_0) \]
\[ \hat{H}^{(3)}_{unf}(z, Q_0) = N_d^c z^{\alpha_d} (1 - z)^{\beta_d} D_{\pi+d}(z, Q_0) \]

Total 13 parameters:

\[ N_u^h, N_d^h, a_u, a_d, b_u, b_d, N_u^c, N_d^c, \alpha_u, \alpha_d, \beta_d, \beta_u, g_c \]

SIDIS data used: HERMES, COMPASS, JLAB – 140 points

e+e- data used: BELLE, BABAR including P_T dependence – 122 points

\[ \chi^2_{min}/n_{d.o.f} = 0.89 \]
Transversity and Collins FF

$\ell P \rightarrow \pi^{\pm} X$

HERMES

$1 \lesssim \langle Q^2 \rangle \lesssim 6 \text{ GeV}^2$

COMPASS

$1 \lesssim \langle Q^2 \rangle \lesssim 21 \text{ GeV}^2$
$e^+ e^- \rightarrow \pi \pi X$

**BELLE**

$Q^2 = 110 \text{ GeV}^2$

**BABAR**

$Q^2 = 110 \text{ GeV}^2$
Transversity and Collins FF

Transversity

Positive $u$ and negative $d$ transversity

Collins

Positive favoured and negative unfavoured Collins FF

Compatible with LO extraction Anselmino et al 2009
What are evolution effects?

\[ e^+ e^- \rightarrow \pi \pi X \]

No evolution:

\[ Q^2 = 2.4 \text{ GeV}^2 \]

No \( A_2 \)
Scan $\delta q$ for transversity

\[ \delta u^{[0.0065, 0.35]} = +0.30^{+0.12}_{-0.11} \]
\[ \delta d^{[0.0065, 0.35]} = -0.20^{+0.35}_{-0.13} \]

\[ \delta q^{[x_{\text{min}}, x_{\text{max}}]} (Q^2) \equiv \int_{x_{\text{min}}}^{x_{\text{max}}} dx \, h_1^q(x, Q^2) \]

at $Q^2 = 10$ GeV$^2$
Summary

- TMD evolution is studied for the Collins effects in $e^+e^-$ annihilation and SIDIS with CSS resummation scheme

- The QCD evolution effect plays an important role in our fitting.
Soft Gluon Resummations in Dijet Azimuthal Angular Correlations at the Collider

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Abstract

We derive the all order soft gluon resummation in dijet azimuthal angular correlation in $pp$ collisions at the next-to-leading logarithmic level. The relevant coefficients for the resummation Sudakov factor, and the soft and hard factors are calculated. The theory predictions agree well with the experimental data from D0 collaboration at the Tevatron.

Ref: Peng Sun, C.-P. Yuan, Feng Yuan, PRL 113, 232001 (2014);

Motivations:
- reummation of large logs in dijet production
- factorization breaking effects
  Collins-Qiu, 2007; Vogelsang-Yuan, 2007;
  Rogers-Mulders 2010
Dijet production at the hadron colliders

- Most abundant events
- Almost back-to-back
- De-correlation comes
  - Hard gluon jet
  - Soft gluon radiation
QCD calculations

- Fixed order calculations divergent around $\pi$, where soft gluon radiation dominate.
- All order resummation is needed to understand the physics around here.
  - Two separate scales $P_T >> q_T$
- Leading $P_T$
- Total $q_T \approx P_T \sin(\pi - \Delta \phi)$

D0, PRL05
Beautiful data from Tevatron/LHC

DØ
- $p_T^{\text{max}} > 180$ GeV ($\times 8000$)
- $130 < p_T^{\text{max}} < 180$ GeV ($\times 400$)
- $100 < p_T^{\text{max}} < 130$ GeV ($\times 20$)
- $75 < p_T^{\text{max}} < 100$ GeV

CMS
- $p_T^{\text{max}} > 300$ GeV ($\times 10^4$)
- $200 < p_T^{\text{max}} < 300$ GeV ($\times 10^3$)
- $140 < p_T^{\text{max}} < 200$ GeV ($\times 10^2$)
- $110 < p_T^{\text{max}} < 140$ GeV ($\times 10$)
- $80 < p_T^{\text{max}} < 110$ GeV

L = 2.9 pb$^{-1}$
$\sqrt{s} = 7$ TeV
$|y| < 1.1$

D0, PRL05
CMS, PRL11
There are three kinds of large logarithms in the processes:

\[(\log(q_\perp/P_J))^2, \log(q_\perp/P_J)\text{ and } \log(R)\log(q_\perp/P_J)\]

\[
\frac{d^4\sigma}{dy_1dy_2dP_J^2d^2q_\perp} = \sum_{ab} \sigma_0 \left[ \int \frac{d^2b_\perp}{(2\pi)^2} e^{-iq_\perp\cdot b_\perp} W_{ab\rightarrow cd}(x_1, x_2, b_\perp) + Y_{ab\rightarrow cd} \right]
\]

where

\[
W_{ab\rightarrow cd}(x_i, b) = x_1 f_a(x_1, b, \zeta^2, \mu^2, \rho)x_2 f_b(x_2, b, \bar{\zeta}^2, \mu^2, \rho)\text{Tr} [H_{ab\rightarrow cd}(Q^2, \mu^2, \rho)S_{ab\rightarrow cd}(b, \mu^2, \rho)]
\]

It will contribute to \(\Delta \phi\) distribution.

It will not
Soft and collinear gluon at one-loop

Virtual
Ellis-Sexton 86

Jet (Narrow Jet Approx.)
Jager-Stratmann-Vogelsang 2004

Soft
Initial state

Soft
Final state (out of jet cone)

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\[ S_{IJ} = \int_{0}^{\pi} \frac{d\phi_0}{\pi} C_{I \mu}^{bb'} C_{J \nu}^{aa'} \langle 0| \mathcal{L}_{vcb'}^{\dagger}(b_\perp) \mathcal{L}_{vbc}(b_\perp) \mathcal{L}_{\overline{v}ca'}^{\dagger}(0) \mathcal{L}_{\overline{v}ac}(0) \mathcal{L}_{\overline{n}ji}^{\dagger}(b_\perp) \mathcal{L}_{\overline{n}i'k}(b_\perp) \mathcal{L}_{\overline{n}kl}^{\dagger}(0) \mathcal{L}_{n\nu'j}(0)|0 \rangle \]

The soft factor satisfies

\[ \frac{d}{d \ln \mu} S_{IJ}(\mu) = -\Gamma_{I,J,J'}^{s^\dagger} S_{J,J'}(\mu) - S_{IJ,J}(\mu) \Gamma_{J,J'}^{s} \]

\[ c_1 = f^{a_1a_2c} f_{a_3a_4}, \quad c_2 = f^{a_1a_2c} f_{a_3a_4} + f^{a_1a_4c} f_{a_3a_2}, \quad c_3 = d^{a_1a_2c} f_{a_3a_4}, \]

\[ c_4 = f^{a_1a_2c} d_{a_3a_4}, \quad c_5 = d^{a_1a_2c} f_{a_3a_4}, \quad c_6 = \delta^{a_1a_2}\delta^{a_3a_4}, \quad c_7 = \delta^{a_1a_2}\delta^{a_3a_4}, \quad c_8 = \delta^{a_1a_2}\delta^{a_3a_4} \]

\[ c_1 = \delta^{a_1a_2}\delta_{a_3a_4}, \quad c_2 = i f^{a_1a_2c} t_{a_3a_4}^{c}, \quad c_3 = d^{a_1a_2c} t_{a_3a_4}^{c} \]
Cross checks

- Divergences cancelled out between virtual, jet, sot contributions (dimension regulation applied)
- Final results: double logs, single logs, ..

\[
W^{(1)}(b_\perp)|_{\text{logs.}} = \frac{\alpha_s}{2\pi} \left\{ \mathcal{K}^{(0)}_{q_i q_j \rightarrow q_i q_j} \right\} \left[ -\ln \left( \frac{\mu^2 b_\perp^2}{b_0^2} \right) \left( \mathcal{P}_{qq}(\xi) \delta(1 - \xi') + \mathcal{P}_{qq}(\xi') \delta(1 - \xi) \right) - \delta(1 - \xi) \right.
\]
\[
\times \delta(1 - \xi') \left( C_F \ln^2 \left( \frac{Q^2 b_\perp^2}{b_0^2} \right) + \ln \left( \frac{Q^2 b_\perp^2}{b_0^2} \right) \left( -3C_F + C_F \ln \frac{1}{R_1} + C_F \ln \frac{1}{R_2} \right) \right) \right. 
\]
\[
- \delta(1 - \xi) \delta(1 - \xi') \ln \left( \frac{Q^2 b_\perp^2}{b_0^2} \right) \Gamma^{(qq')}_{\text{ssn}},
\]

Quark channel: $q_i q_j \rightarrow q_i q_j$

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After solving the evolution equations

\[ W_{ab\to cd}(x_1, x_2, b) = x_1 f_a(x_1, \mu = c_0/b_\perp) x_2 f_b(x_2, \mu = c_0/b_\perp) e^{-S_{Sud}(Q^2, b_\perp)} \]

\[ \times \text{Tr} [H_{ab\to cd} \exp[-\int_{c_0/b_\perp}^{Q} \frac{d\mu}{\mu} \gamma^s]] S_{ab\to cd} \exp[-\int_{c_0/b_\perp}^{Q} \frac{d\mu}{\mu} \gamma^s] \]

where

\[ S_{Sud}(Q^2, b_\perp, C_1, C_2) = \int_{C_1/2}^{C_2} \frac{d\mu^2}{\mu^2} \left[ \ln \left( \frac{Q^2}{\mu^2} \right) A + B + D_1 \ln \frac{2}{R_1} + D_2 \ln \frac{2}{R_2} \right] \]

For \( g g \to jj \) \( A_{gg} = C_A a_s/\pi \) \quad \( B_{gg} = -2C A \beta_0 a_s/\pi \)

For \( q q \to jj \) \( A_{qq} = C_F a_s/\pi \) \quad \( B_{qq} = -2C F/3 a_s/\pi \)

For \( q g \to jj \) \( A_{qg} = (A_{gg} + A_{qq})/2 \) \quad \( B_{qg} = -(B_{gg} + B_{qq})/2 \)

for quark jet \( D_i = C_F a_s/\pi \)

for gluon jet \( D_i = C_A a_s/\pi \)
Compared to full calculations

\[ \frac{\alpha_s}{2\pi^2} \frac{1}{q_1^2} \sum_{ab,a'b'} \sigma_0 \int \frac{dx'_1}{x'_1} \frac{dx'_2}{x'_2} f_a(x'_1, \mu) f_b(x'_2, \mu) \]
\[ \times \left\{ h_{a'b'\rightarrow cd}^{(0)} \left[ \xi_1 P^a_{a'/a} (\xi_1) \delta(1 - \xi_2) + \xi_2 P^b_{b'/b} (\xi_2) \delta(1 - \xi_1) + \delta(1 - \xi_1) \delta(1 - \xi_2) \delta_{aa'} \delta_{bb'} \left( (C_a + C_b) \ln \frac{Q^2}{q_1^2} + C_c \ln \frac{1}{R_1^2} + C_d \ln \frac{1}{R_2^2} \right) \right] \right\} , \]

(10)

Leading \( P_T \)

Total \( q_T \approx P_T \sin(\pi - \phi) \)

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Compared to the data

NLL Resummation:
Sun, C.P. Yuan, F. Yuan, PRL 2014

\[ x_1 f_a(x_1, \mu = b_0/b_\perp) x_2 f_b(x_2, \mu = b_0/b_\perp) e^{-S_{\text{Sud}}(Q^2, b_\perp)} \]

\[ \text{Tr} \left[ H_{ab\rightarrow cd} \exp \left[ - \int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu} \gamma^s \right] S_{ab\rightarrow cd} \exp \left[ - \int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu} \gamma^s \right] \right] \]

Full NLO: Nagy 2002, NLOJET++
At the LHC

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Higgs + jet production in pp collision

\[ \text{Higgs+Jet, Sun, C.-P. Yuan, F. Yuan, phys.Rev.Lett.(2015)} \]
Summary

- Soft gluon resummation for dijet correlation at the next-to-leading logarithmic order agrees well with the experimental data.

- Extending to EW boson plus jet production will be interesting to follow.
Thank you very much!
Approaches to TMD evolution

Collins-Soper-Sterman (CSS) resummation framework

Collins-Soper-Sterman 1985
ResBos: C.P. Yuan, P. Nadolsky
Qiu-Zhang 1999, Vogelsang etc...
Prokudin-Kang-Sun-Yuan 2014

“New” Collins approach

Collins 2011
Aybat-Rogers 2011,
Aybat-Collins-Rogers-Qiu, 2012
Aybat-Prokudin-Rogers 2012
Anselmino-Boglione-Melis 2012
Prokudin-Bacchetta 2013
Echevarria-Idilbi-Kang-Vitev 2014

Soft Collinear Effective Theory (SCET)

Echevarria-Idilbi-Schafer-Scimemi 2012
D'Alesio-Echevarria-Melis-Scimemi 2014
Why do we need QCD evolution?

TMD factorization is applicable in case two different scales are observed in processes such as SIDIS, Drell-Yan, W/Z production in hadron-hadron collisions. Kinematical regime: $Q_T \ll Q$

For SIDIS $Q_T$ is transverse momentum of final parton

And $Q$ is the invariant mass of virtual photon

*Double and Single* logarithms will appear order by order in perturbative calculations

\[
\left( \alpha_s \ln^2 \frac{Q^2}{Q_T^2} \right)^n
\]

\[
\left( \alpha_s \ln \frac{Q^2}{\mu^2} \right)^n
\]