QCD evolution for Collins asymmetries in e^+e^- annihilation and SIDIS

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in collaboration with Kang, Prokudin and Yuan

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Outlines

- QCD evolution in TMD factorization
- Collins asymmetry in e^+e^- annihilation and SIDIS
- soft gluon resummation in dijet azimuthal angular correlation at hadron collider

Why do we need QCD evolution?

- Consider the production process $h_1 h_2 \rightarrow Z+X$

$$\frac{d\sigma}{dQ_T^2} \sim \frac{1}{Q_T^2} \{ \alpha_S(L+1) \quad +\alpha_S^2(L^3+L^2) \quad +\alpha_S^3(L^5+L^4)+\alpha_S^4(L^7+L^6)+\ldots$$
$$+\alpha_S^2(\ L+\ \ 1) \quad +\alpha_S^3(L^3+L^2)+\alpha_S^4(L^5+L^4)+\ldots$$
$$+\alpha_S^3(\ L+\ \ 1)+\alpha_S^4(L^3+L^2)+\ldots \}$$

Where Q_T is the transverse momentum, and Q the mass of Z, and $L = \mathrm{Log}[Q^2 / Q_T^2]$.

- We have to resum these large logs to make reliable predictions

TMD factorization

At the small transverse momentum limitation

$$W_{(\text{Drell-Yan})} = H(M, \mu)\, f_1(x_1, \boxed{b}, M, \mu)\, f_2(x_2, b, M, \mu)\, S(b, \mu)$$

Fourier transformation $\Updownarrow$

$\boxed{Q_t}$

Every scale in each term can be evolved by RGE

TMD evolution in TMD parton distributions

With CSS evolution equation, evolution starts from $\mu_b = c/b,\ c = 2e^{-\gamma_E}$

$$\tilde{f}(x,b;Q) = \tilde{f}(x,b;\mu_b)e^{-S_{pert}(b)}$$

where

$$\tilde{f}(x,b;Q) = \int d^2k_\perp e^{-ik_\perp b} f(x,k_\perp;Q)$$

$$S^{PT}(b) = \int_{\mu_b}^{Q} \frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2} + B\right)$$ Perturbative Sudakov factor

$$A = \sum_{n=1}\left(\frac{\alpha_s}{\pi}\right)^n A^{(n)} \qquad B = \sum_{n=1}\left(\frac{\alpha_s}{\pi}\right)^n B^{(n)}$$

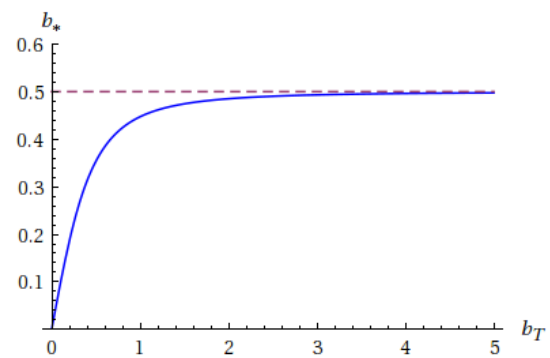
Calculation is perturbative, valid only in region $b \ll 1/\Lambda_{QCD}$

Fourier transform in momentum space involves non-perturbative region

$$f(x, k_\perp; Q) = \int_0^\infty \frac{bdb}{2\pi} J_0(k_\perp b) \tilde{f}(x, b; Q)$$

Non perturbative region needs to be treated.
Common method b* prescription

$$b_* = \frac{b}{\sqrt{1 + b^2/b_{max}^2}}$$



$$\tilde{f}(x, b; Q) = \tilde{f}(x, b_*; c/b_*) e^{-S_{pert}(b_*)} e^{-S_{NP}(b)}$$

Non perturbative Sudakov factor

$$S_{\rm NP}^{\rm SIDIS}(Q,b) = g_2 \ln\left(\frac{b}{b_*}\right)\ln\left(\frac{Q}{Q_0}\right) + \left(g_q + \frac{g_h}{z_h^2}\right) b^2$$

A new non-perturabtive Sudakov factor is used. Where Q_0^2=2.4GeV^2

g_1 and g_q are free parameters, from the fitting of Drell-Yan processes, and g_h is from SIDIS

In our fit, we choose b_{max}=1.5GeV^{-1}

In this work CSS scheme is applied

$$\tilde{f}^j(x, b_*; c/b_*) = \sum_{j'=q,g} \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{j/j'}\left(\frac{x}{\hat{x}}, b_*; c/b_*\right) f^{j'}(x; c/b_*)$$

$$C = \sum_{n=1} \left(\frac{\alpha_s}{\pi}\right)^n C^{(n)}$$

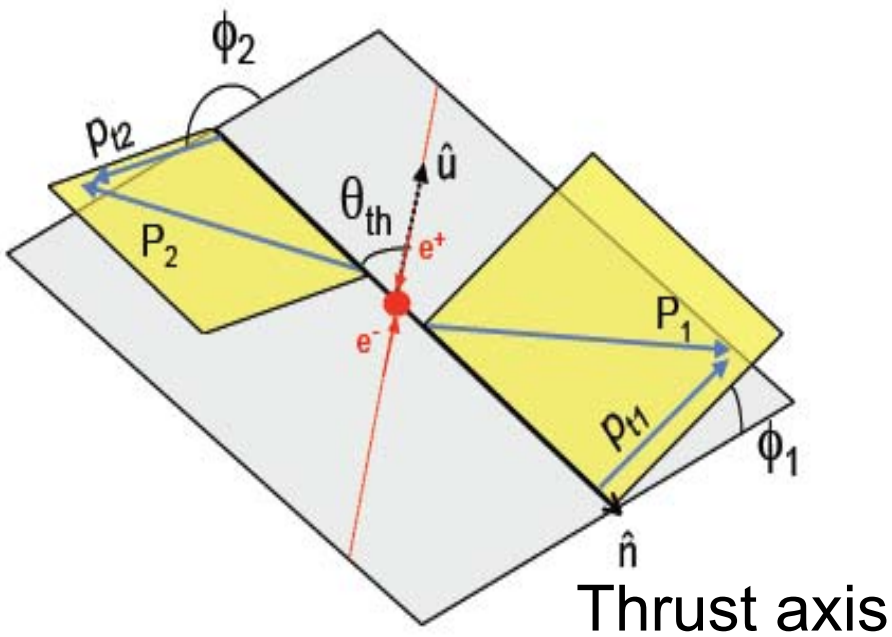
Wilson coefficient

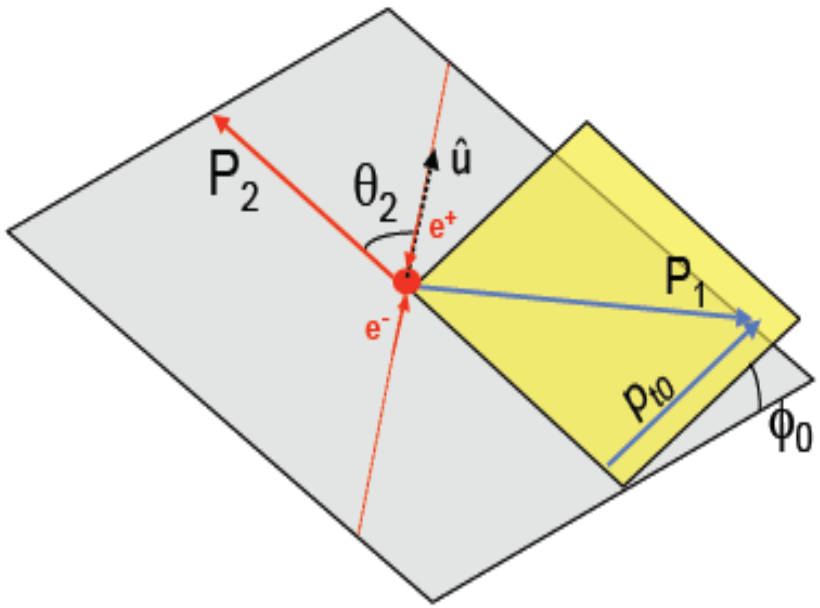
Collinear PDF

transversity is related to a twist-2 colinear PDF

Collins function is related to twist-3 collinear FF

Collins asymmetries in $e^+e^- \rightarrow hh+X$

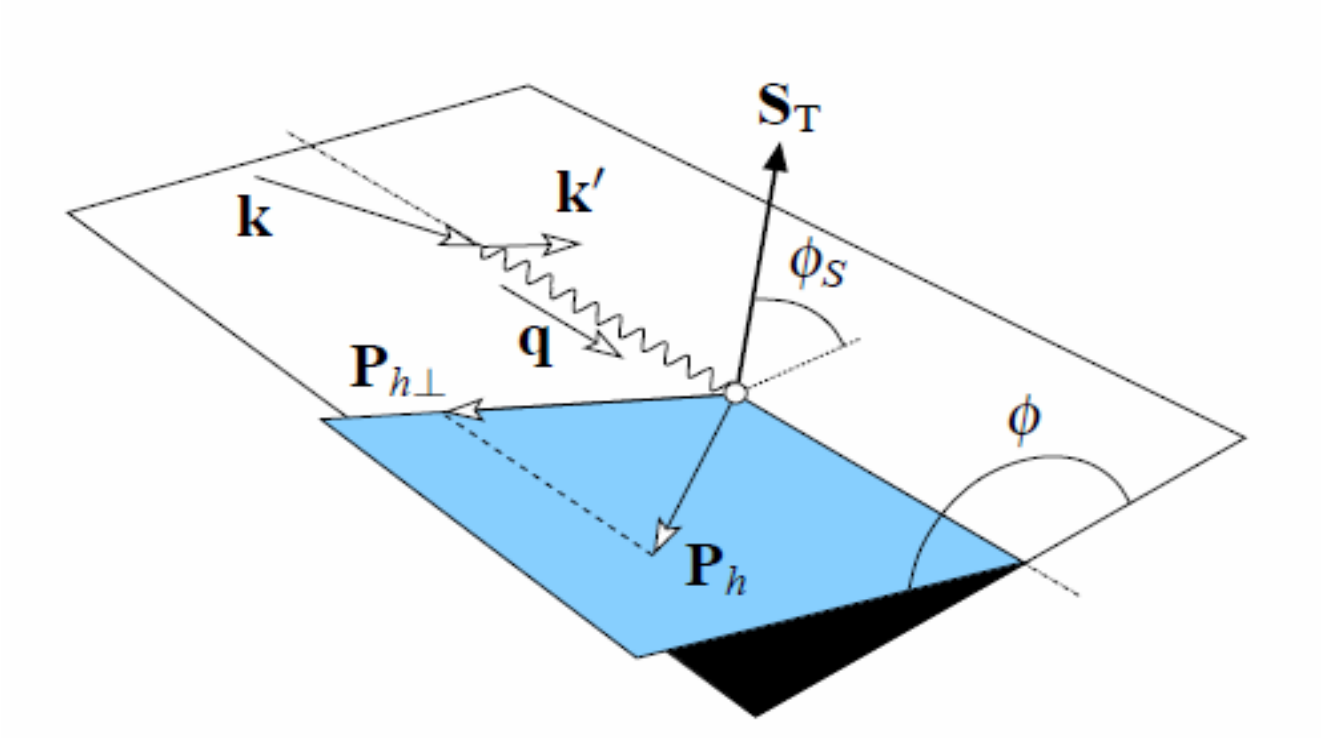




A_{12} A_0

The Collins asymmetries is proportional to $\cos(\phi_1+\phi_2)$ or $\cos(2\phi_0)$

Collins asymmetries in SIDIS



The Collins asymmetries is proportional to $\sin(\phi+\phi_S)$

Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

Parameterizations:

$$h_1^q(x,Q_0) \propto N_q^h x^{a_q}(1-x)^{b_q}\frac{1}{2}\left(f_1(x,Q_0)+g_1(x,Q_0)\right)$$

Transversity

Favoured and unfavoured Collins FF

$$\hat{H}_{fav}^{(3)}(z,Q_0) = N_u^c z^{\alpha_u}(1-z)^{\beta_u} D_{\pi^+/u}(z,Q_0)$$

$$\hat{H}_{unf}^{(3)}(z,Q_0) = N_d^c z^{\alpha_d}(1-z)^{\beta_d} D_{\pi^+/d}(z,Q_0)$$

Total 13 parameters:

$$N_u^h, N_d^h, a_u, a_d, b_u, b_d, N_u^c, N_d^c, \alpha_u, \alpha_d, \beta_d, \beta_u, g_c$$

SIDIS data used: HERMES, COMPASS, JLAB – 140 points

e+e- data used: BELLE, BABAR including P_T dependence – 122 points

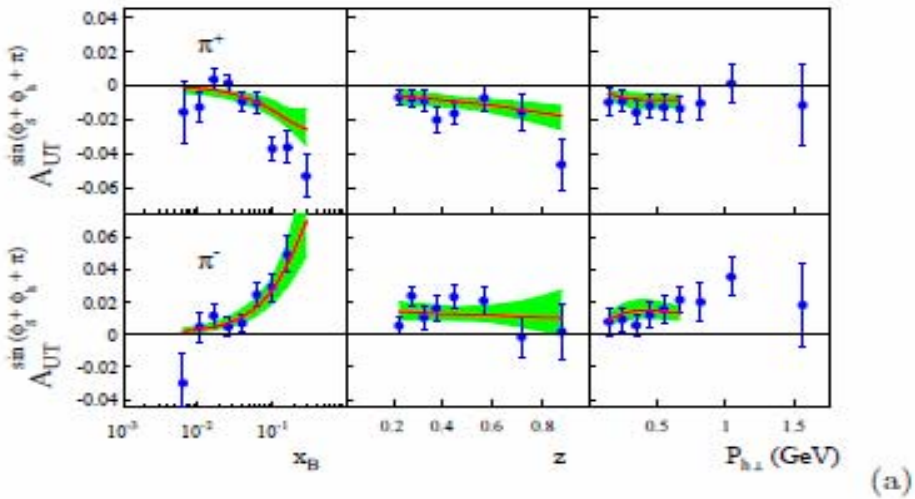
$$\chi^2_{min}/n_{d.o.f} = 0.89$$

Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

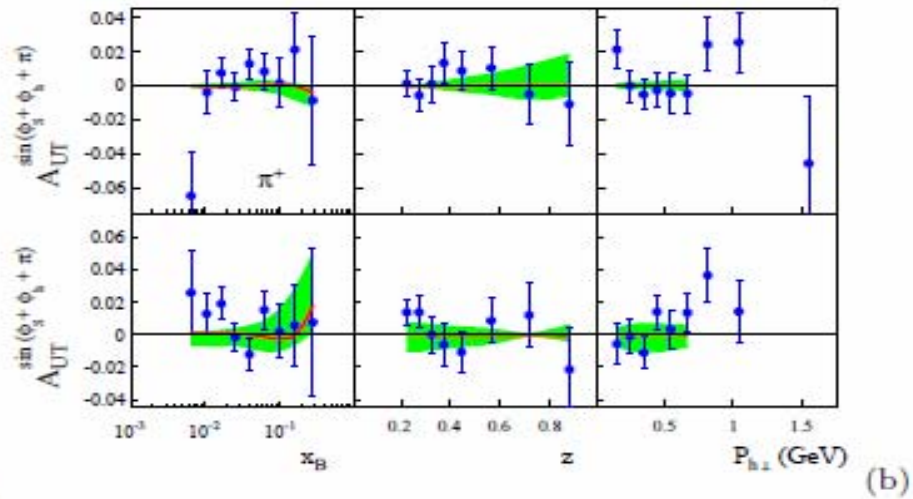
$$\ell P \to \pi^{\pm} X$$

HERMES



$$1 \lesssim \langle Q^2 \rangle \lesssim 6 \text{ GeV}^2$$

COMPASS



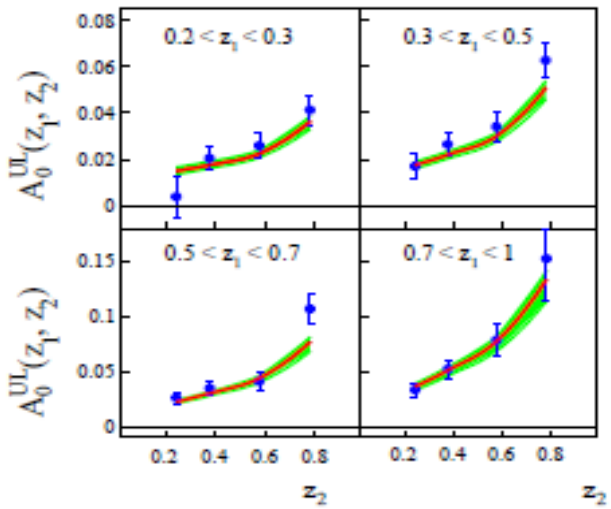
$$1 \lesssim \langle Q^2 \rangle \lesssim 21 \text{ GeV}^2$$

Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

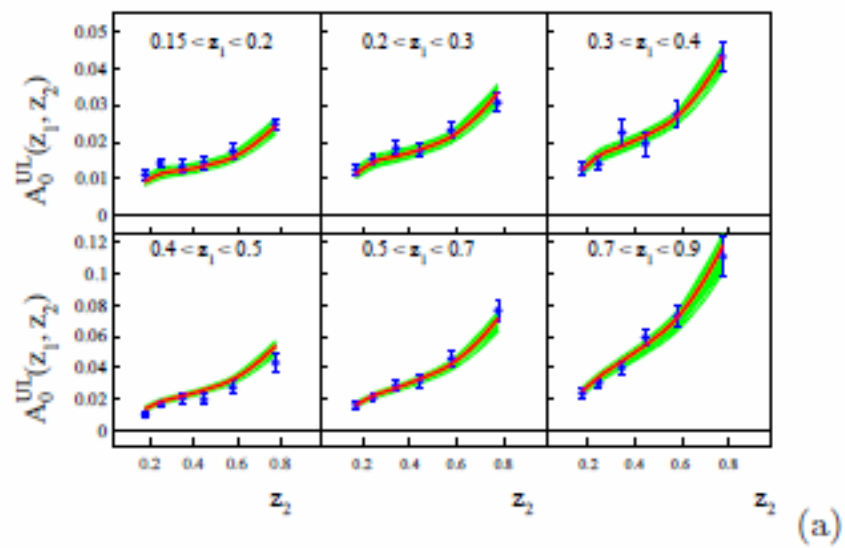
$$e^+e^- \to \pi\pi X$$

BELLE



$Q^2 = 110\ \mathrm{GeV}^2$

BABAR

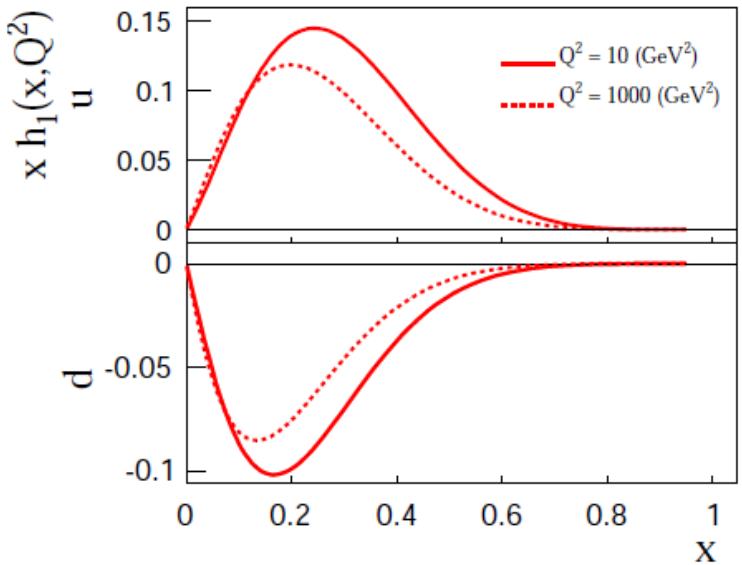


$Q^2 = 110\ \mathrm{GeV}^2$

Transversity and Collins FF

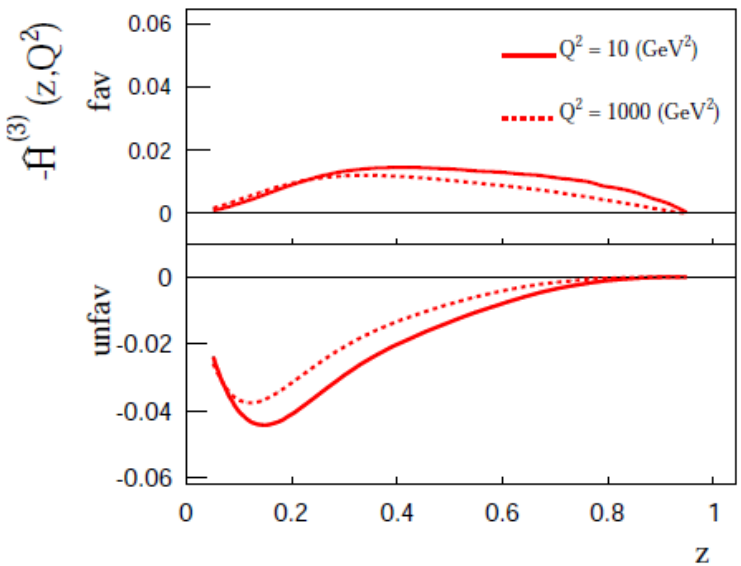
Kang-Prokudin-Sun-Yuan 2014

Transversity



Positive u and negative d transversity

Collins



Positive favoured and negative unfavoured Collins FF

Compatible with LO extraction Anselmino et al 2009

Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

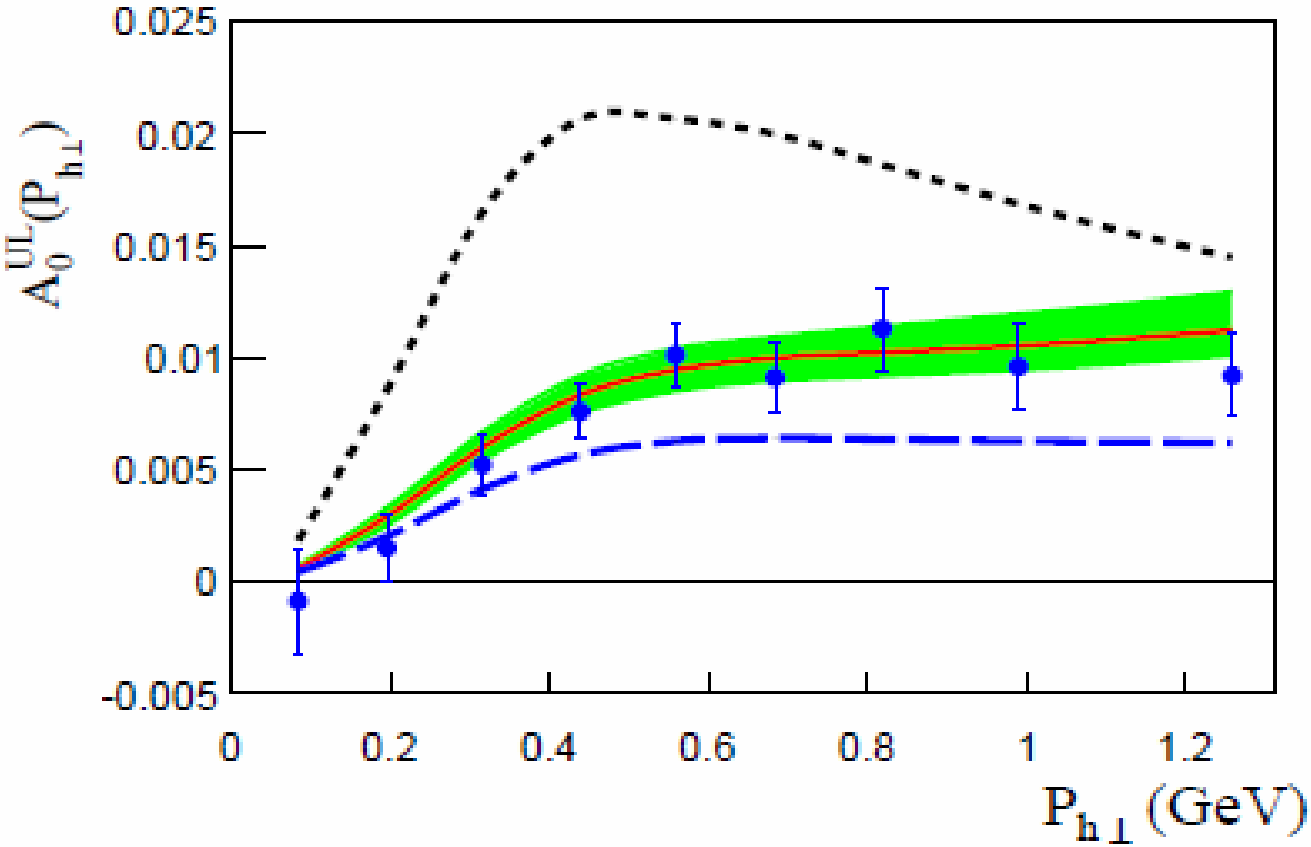
What are evolution effects?

$$e^+e^- \to \pi\pi X$$

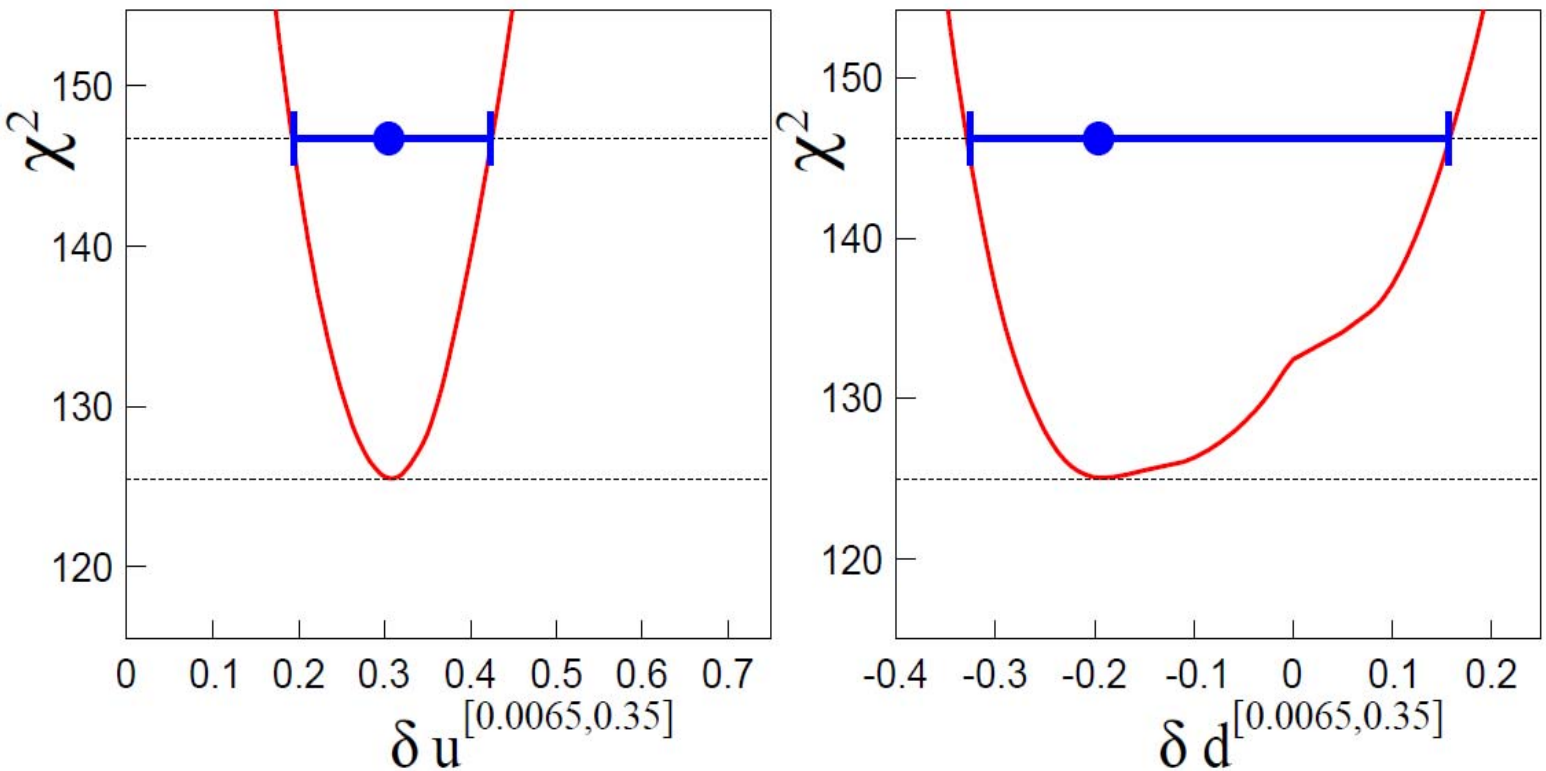
No evolution:

$Q^2 = 2.4\ \mathrm{GeV}^2$

No A_2



Scan δq for transversity

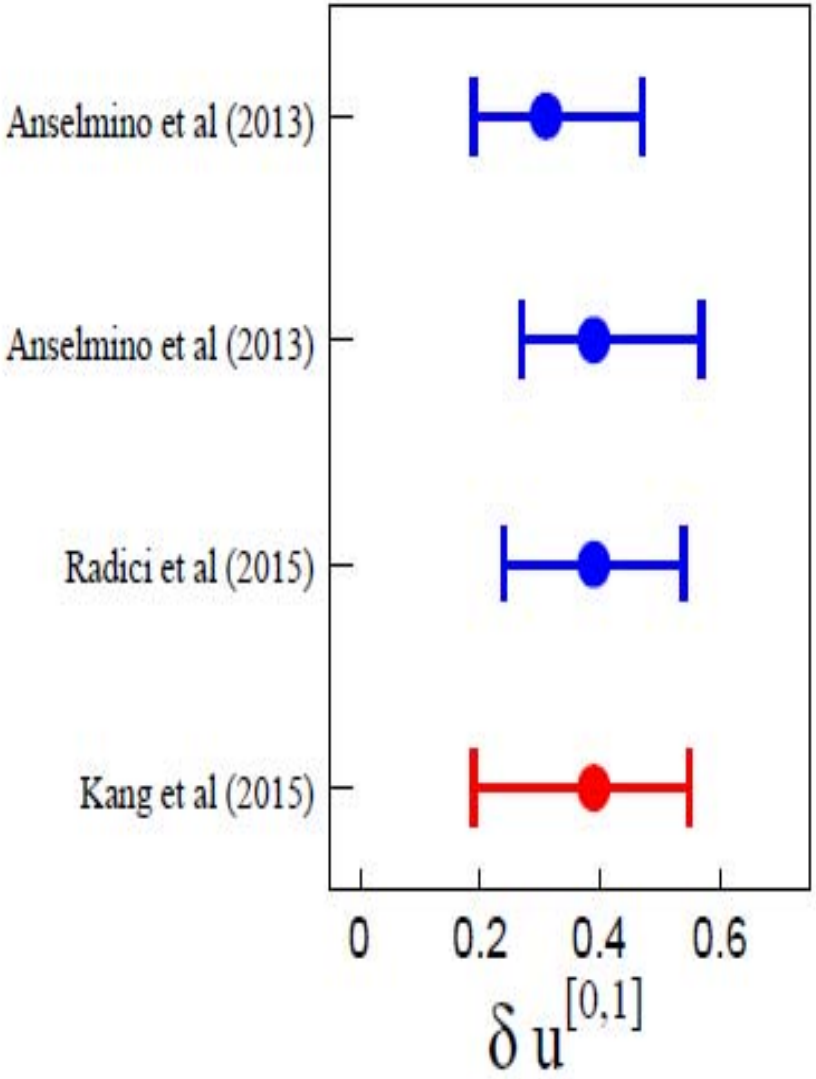


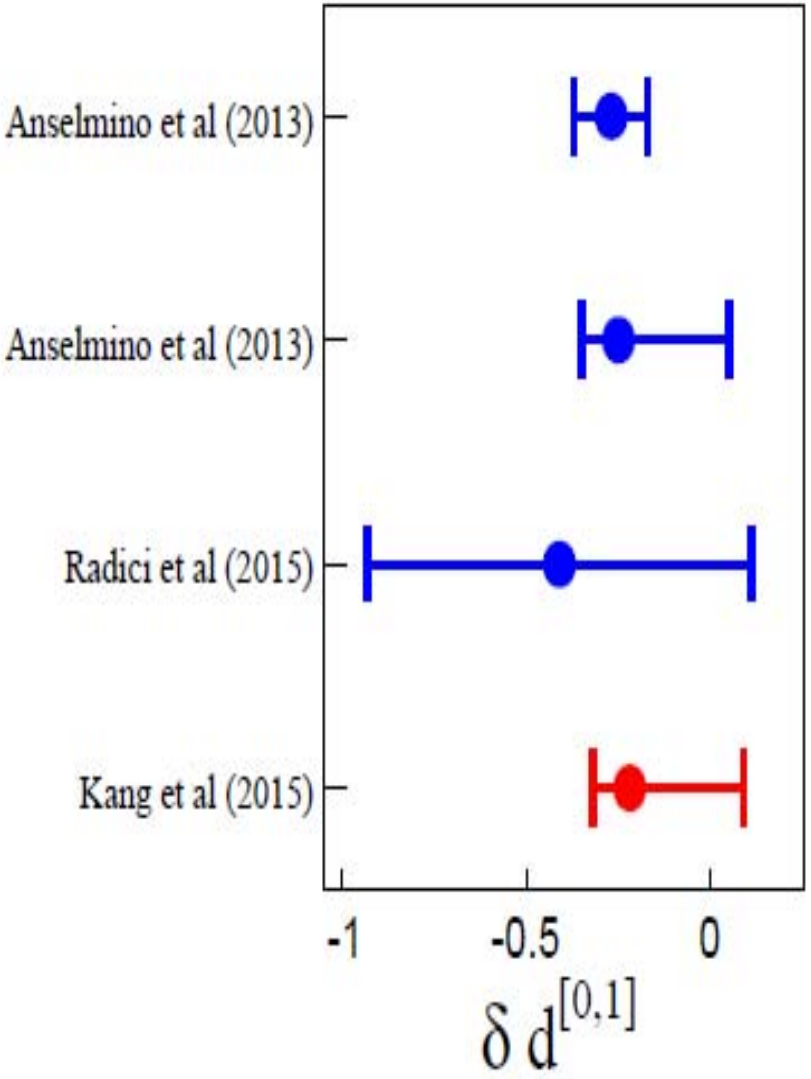
$$\delta u^{[0.0065,0.35]} = +0.30^{+0.12}_{-0.11}$$

$$\delta d^{[0.0065,0.35]} = -0.20^{+0.35}_{-0.13}$$

$$\delta q^{[x_{\min},x_{\max}]}(Q^2) \equiv \int_{x_{\min}}^{x_{\max}} dx\, h_1^q(x,Q^2)$$

at $Q^2 = 10\ \mathrm{GeV}^2$


Anselmino et al (2013)
Anselmino et al (2013)
Radici et al (2015)
Kang et al (2015)
0
0.2
0.4
0.6
δ u[0,1]


Anselmino et al (2013)
Anselmino et al (2013)
Radici et al (2015)
Kang et al (2015)
-1
-0.5
0
δ d[0,1]

Summary

- TMD evolution is studied for the Collins effects in e^+e^- annihilation and SIDIS with CSS resummation scheme

- The QCD evolution effect plays an important role in our fitting.

Soft Gluon Resummations in Dijet Azimuthal Angular Correlations at the Collider

Peng Sun,[1] C.-P. Yuan,[2] and Feng Yuan[1]

Abstract

We derive the all order soft gluon resummation in dijet azimuthal angular correlation in pp collisions at the next-to-leading logarithmic level. The relevant coefficients for the resummation Sudakov factor, and the soft and hard factors are calculated. The theory predictions agree well with the experimental data from D0 collaboration at the Tevatron.

Ref: Peng Sun, C.-P. Yuan, Feng Yuan, PRL 113, 232001 (2014);

Motivations:

reummation of large logs in dijet production

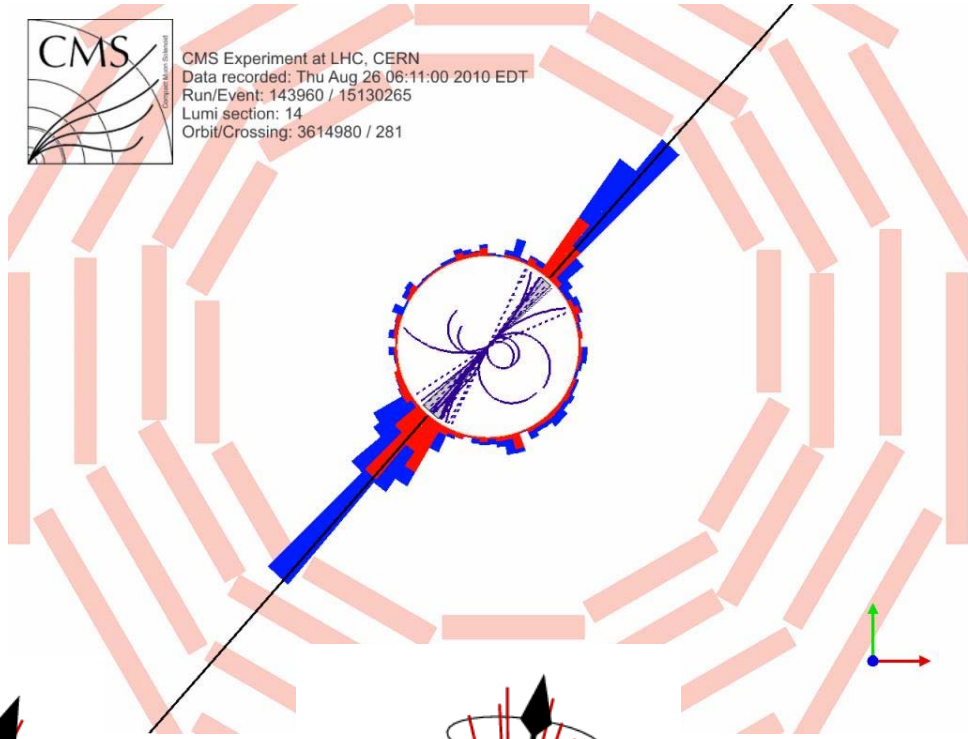
factorization breaking effects

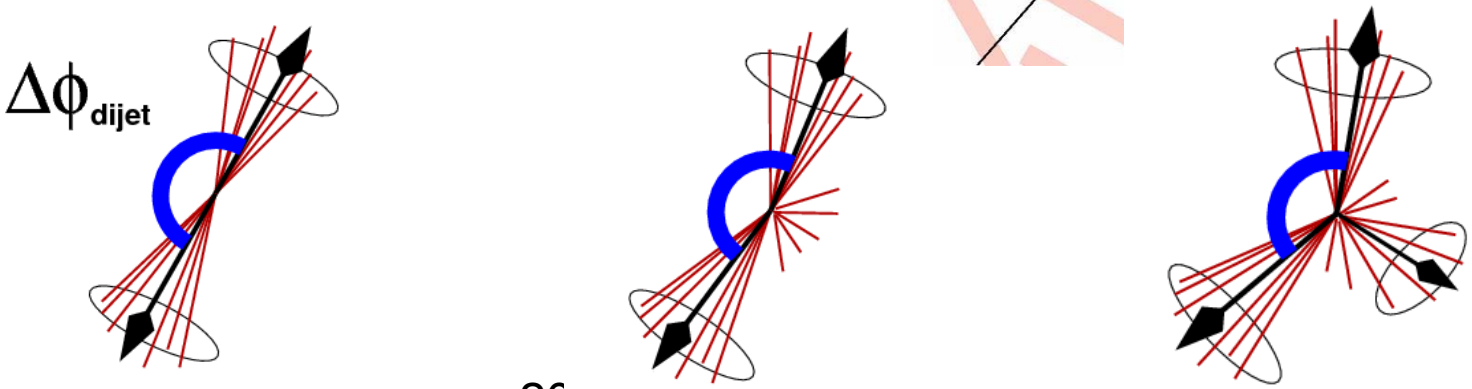
Collins-Qiu, 2007; Vogelsang-Yuan, 2007;

Rogers-Mulders 2010

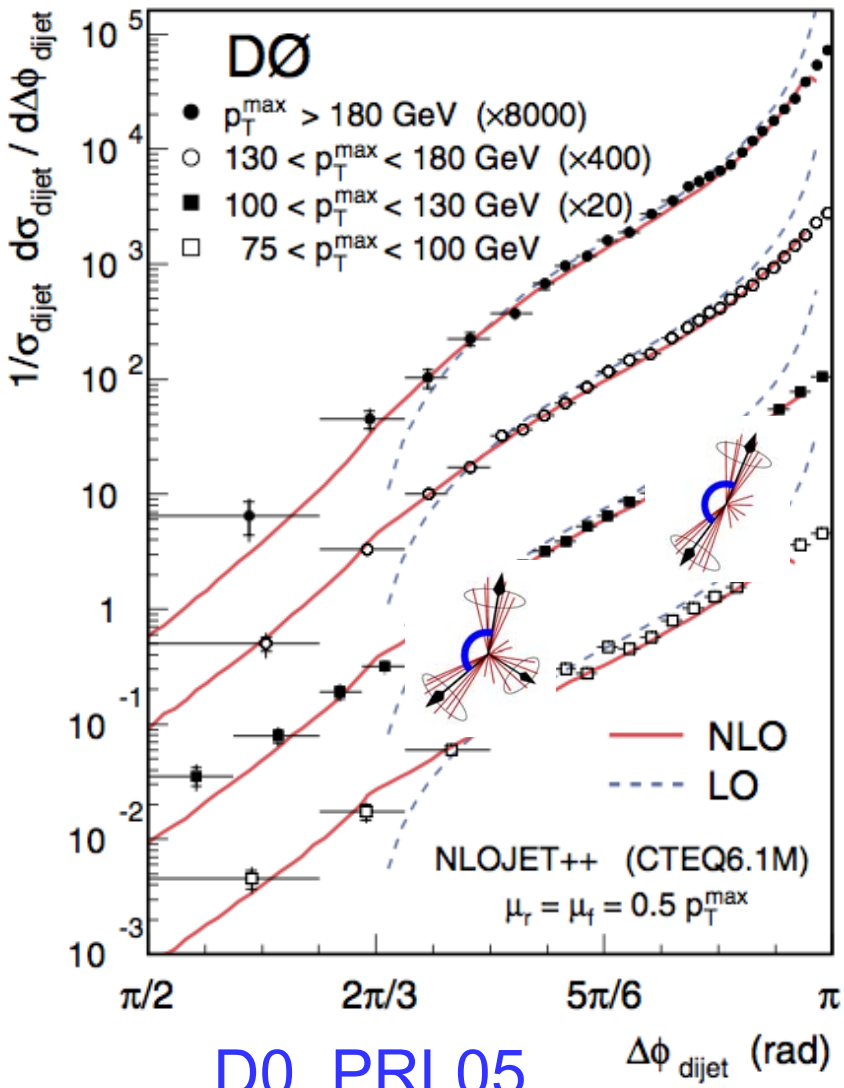
Dijet production at the hadron colliders

- Most abundant events
- Almost back-to-back
- De-correlation comes
  - Hard gluon jet
  - Soft gluon radiation



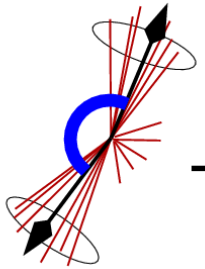


QCD calculations



D0, PRL05

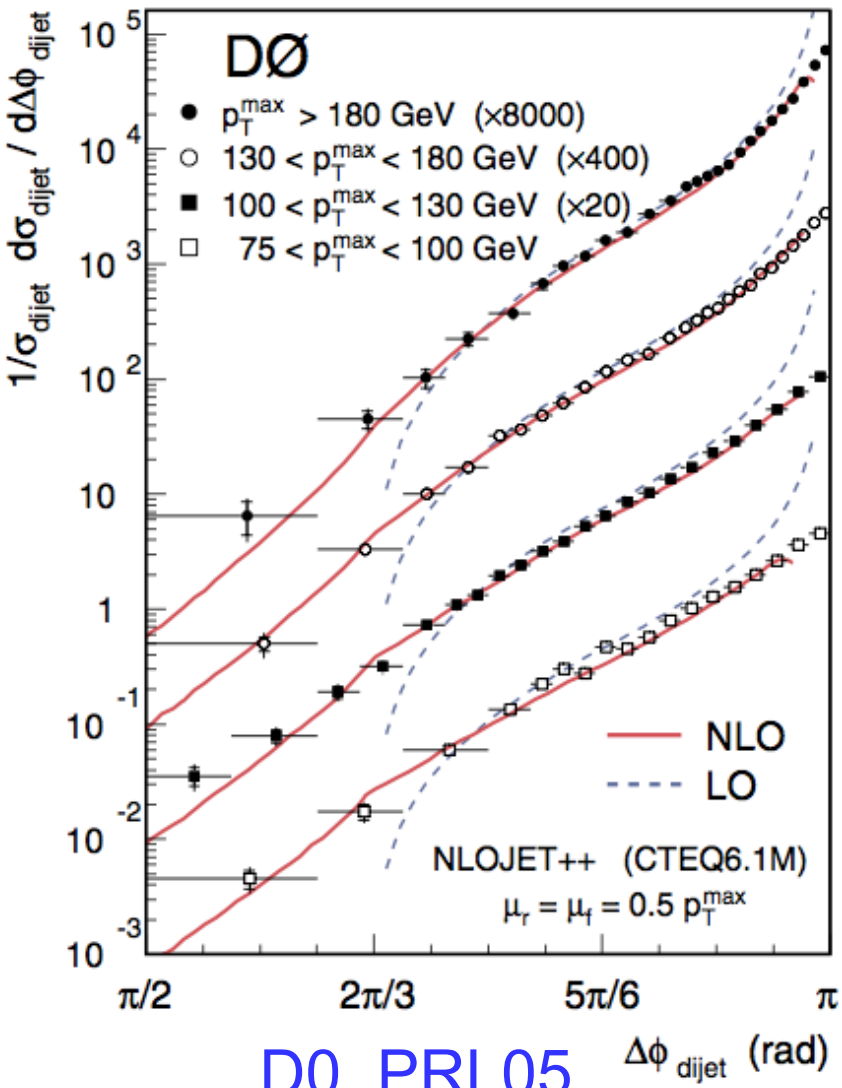
- Fixed order calculations divergent around π, where soft gluon radiation dominate
- All order resummation is needed to understand the physics around here
  - Two separate scales $P_T >> q_T$



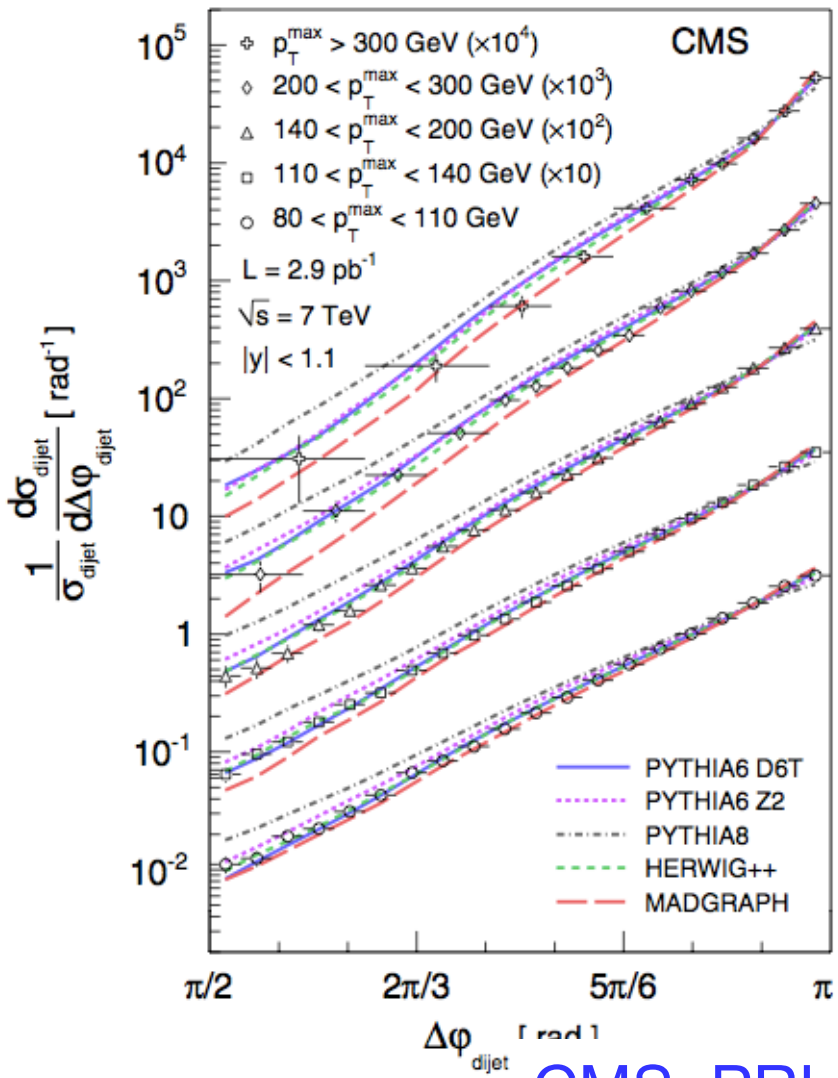
Leading P_T

Total $q_T \approx P_T \mathrm{Sin}(\pi - \Delta\phi)$

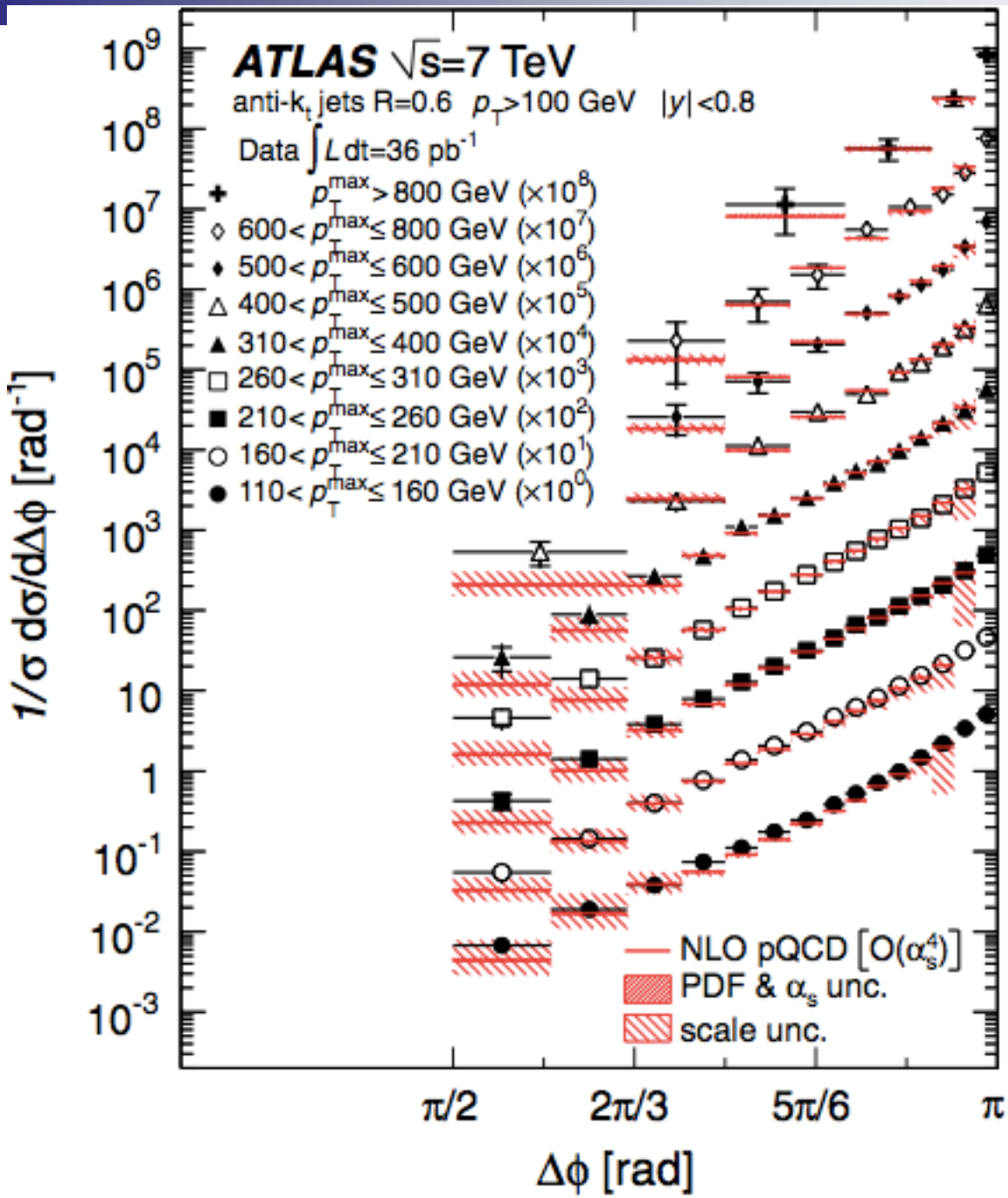
Beautiful data from Tevatron/LHC



D0, PRL05



CMS, PRL11


ATLAS √s=7 TeV
anti-k$_t$ jets R=0.6 p_T>100 GeV |y|<0.8
Data ∫L dt=36 pb^{-1}
p_T^{max}>800 GeV (×10^8)
600<p_T^{max}≤800 GeV (×10^7)
500<p_T^{max}≤600 GeV (×10^6)
400<p_T^{max}≤500 GeV (×10^5)
310<p_T^{max}≤400 GeV (×10^4)
260<p_T^{max}≤310 GeV (×10^3)
210<p_T^{max}≤260 GeV (×10^2)
160<p_T^{max}≤210 GeV (×10^1)
110<p_T^{max}≤160 GeV (×10^0)
NLO pQCD [O(α_s^4)]
PDF & α_s unc.
scale unc.
1/σ dσ/dΔφ [rad^{-1}]
Δφ [rad]
π/2
2π/3
5π/6
π

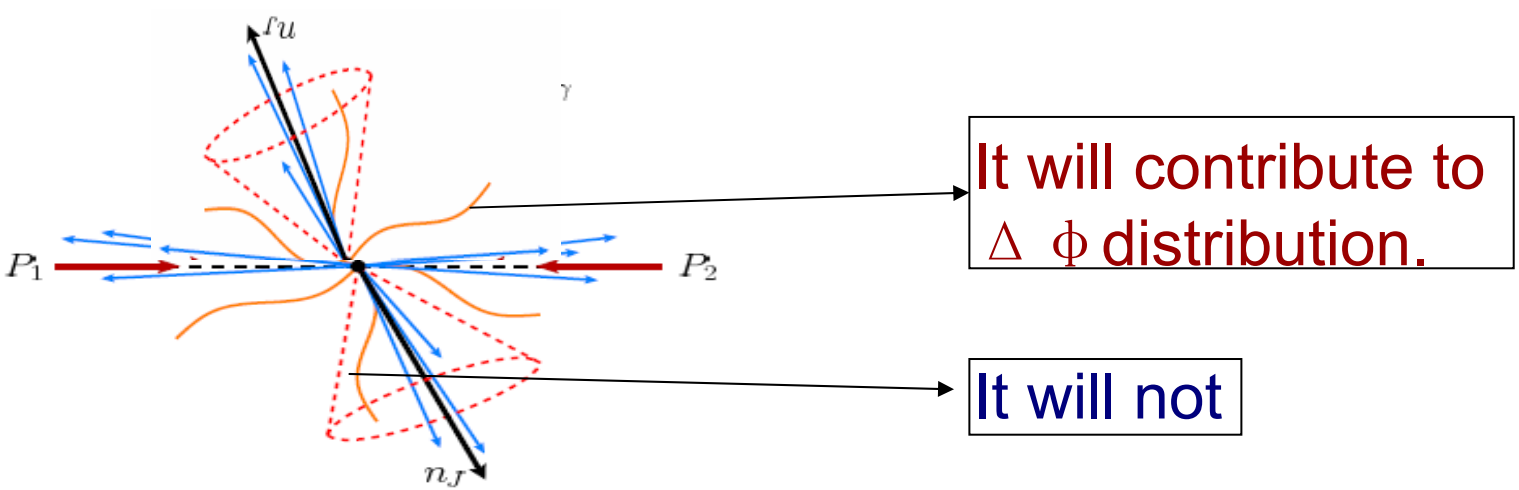
ATLAS, PRL11

There three kinds of large logarithms in the processes: $(\mathrm{Log}(q_\perp/P_J))^2$, $\mathrm{Log}(q_\perp/P_J)$ and $\mathrm{Log}(R)\mathrm{Log}(q_\perp/P_J)$

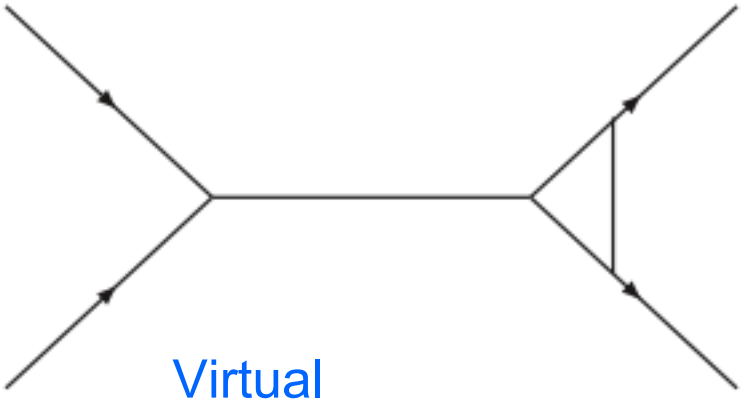
$$\frac{d^4\sigma}{dy_1 dy_2 dP_J^2 d^2q_\perp} = \sum_{ab}\sigma_0\left[\int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-iq_\perp\cdot b_\perp} W_{ab\to cd}(x_1,x_2,b_\perp) + Y_{ab\to cd}\right]$$

where

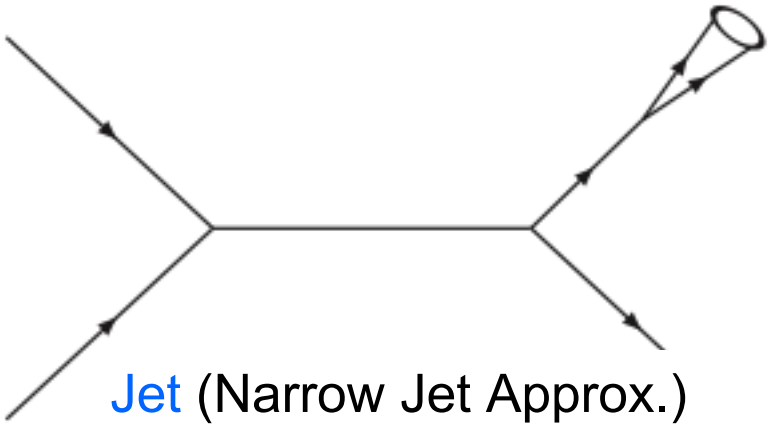
$$W_{ab\to cd}(x_i,b) = x_1 f_a(x_1,b,\zeta^2,\mu^2,\rho) x_2 f_b(x_2,b,\bar{\zeta}^2,\mu^2,\rho)\mathrm{Tr}\left[\mathbf{H}_{ab\to cd}(Q^2,\mu^2,\rho)\mathbf{S}_{ab\to cd}(b,\mu^2,\rho)\right]$$



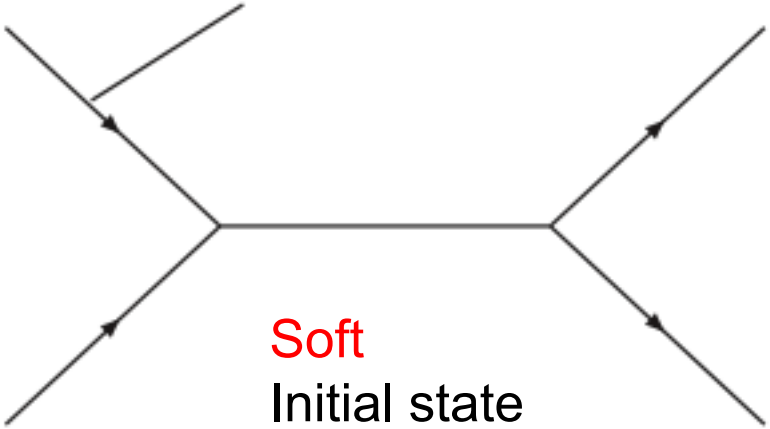
Soft and collinear gluon at one-loop



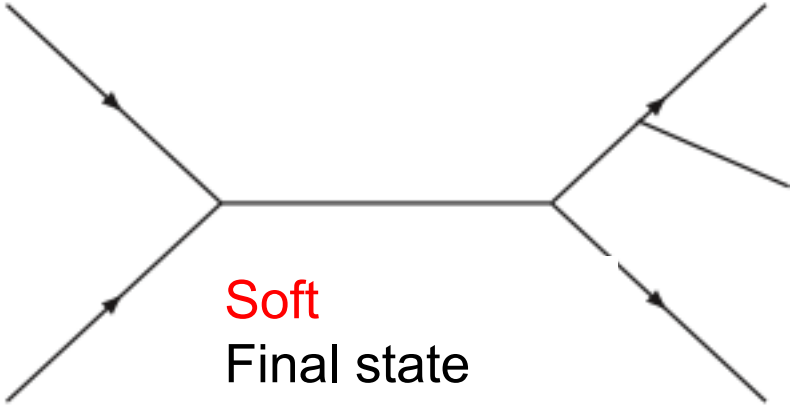
Virtual
Ellis-Sexton 86



Jet (Narrow Jet Approx.)
Jager-Stratmann-Vogelsang
2004



Soft
Initial state



Soft
Final state
(out of jet cone)

$$S_{IJ} = \int_0^\pi \frac{d\phi_0}{\pi} C^{bb'}_{Iii'} \boxed{C^{aa'}_{Jll'}} \langle 0|\mathcal{L}^\dagger_{vcb'}(b_\perp)\mathcal{L}_{vbc}(b_\perp)\mathcal{L}^\dagger_{\bar{v}ca'}(0)\mathcal{L}_{\bar{v}ac}(0)\mathcal{L}^\dagger_{nji}(b_\perp)\mathcal{L}_{\bar{n}i'k}(b_\perp)\mathcal{L}^\dagger_{\bar{n}kl}(0)\mathcal{L}_{nl'j}(0)|0\rangle$$

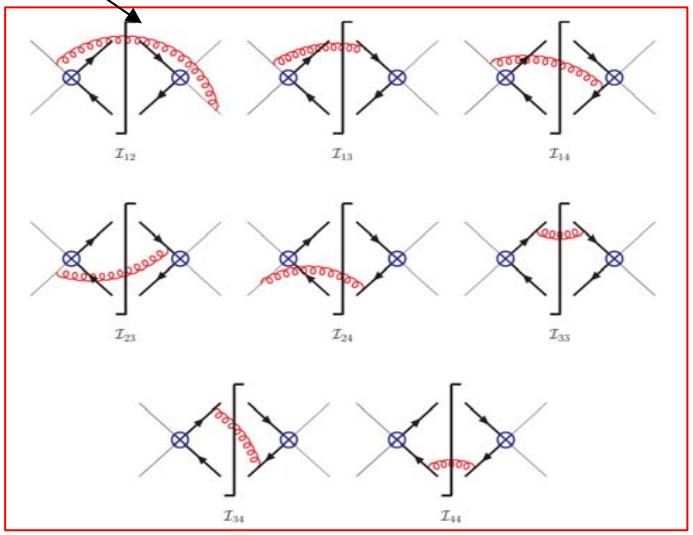
The soft factor satisfies

$$\frac{d}{d\ln\mu} S_{IJ}(\mu) = -\boxed{\Gamma^{s\dagger}_{IJ'}} S_{J'J}(\mu) - S_{IJ'}(\mu)\Gamma^s_{J'J}$$

$$c_1 = f^{a_1a_2c_1} f_{a_3a_4c_1}, \quad c_2 = f^{a_1a_3c_1} f_{a_2a_4c_1} + f^{a_1a_4c_1} f_{a_2a_3c_1}, \quad c_3 = d^{a_1a_2c_1} f_{a_3a_4c_1},$$

$$c_4 = f^{a_1a_2c_1} d_{a_3a_4c_1}, \quad c_5 = d^{a_1a_4c_1} f_{a_2a_3c_1}, \quad c_6 = \delta^{a_1a_2}\delta^{a_3a_4}, \quad c_7 = \delta^{a_1a_3}\delta^{a_2a_4}, \quad c_8 = \delta^{a_1a_4}\delta^{a_2a_3}$$

$$c_1 = \delta^{a_1a_2}\delta_{a_3a_4}, \quad c_2 = i f^{a_1a_2c} t^c_{a_3a_4}, \quad c_3 = d^{a_1a_2c} t^c_{a_3a_4}$$



Cross checks

- Divergences cancelled out between virtual, jet, sot contributions (dimension regulation applied)
- Final results :double logs, single logs, ..

$$W^{(1)}(b_\perp)|_{logs.} = \frac{\alpha_s}{2\pi}\left\{ h^{(0)}_{q_iq_j\to q_iq_j}\left[-\ln\left(\frac{\mu^2 b_\perp^2}{b_0^2}\right)\left(\mathcal{P}_{qq}(\xi)\delta(1-\xi') + \mathcal{P}_{qq}(\xi')\delta(1-\xi)\right) - \delta(1-\xi) \right.\right.$$
$$\left. \times\delta(1-\xi')\left(C_F \ln^2\left(\frac{Q^2 b_\perp^2}{b_0^2}\right) + \ln\left(\frac{Q^2 b_\perp^2}{b_0^2}\right)\left(-3C_F + C_F \ln\frac{1}{R_1^2} + C_F \ln\frac{1}{R_2^2}\right)\right)\right]$$
$$\left. -\delta(1-\xi)\delta(1-\xi')\ln\left(\frac{Q^2 b_\perp^2}{b_0^2}\right)\Gamma^{(qq')}_{sn}\right\}, \qquad (71)$$

Quark channel: $q_iq_j \rightarrow q_iq_j$

After solving the evolution equations

$$W_{ab\to cd}(x_1,x_2,b) = x_1 f_a(x_1,\mu=c_0/b_\perp) x_2 f_b(x_2,\mu=c_0/b_\perp) e^{-S_{Sud}(Q^2,b_\perp)}$$
$$\times \ \mathrm{Tr}\left[\mathbf{H}_{ab\to cd}\exp[-\int_{c_0/b_\perp}^{Q}\frac{d\mu}{\mu}\gamma^{s\dagger}]\mathbf{S}_{ab\to cd}\exp[-\int_{c_0/b_\perp}^{Q}\frac{d\mu}{\mu}\gamma^{s}]\right]$$

where

$$S_{Sud}(Q^2,b_\perp,C_1,C_2) = \int_{C_1^2/b_\perp^2}^{C_2^2Q^2}\frac{d\mu^2}{\mu^2}\left[\ln\left(\frac{Q^2}{\mu^2}\right)A + B + D_1\ln\boxed{\frac{2}{R_1}} + D_2\ln\frac{2}{R_2}\right]$$

For g g → jj $A_{gg}=C_A\, a_s/\pi$ $B_{gg}=-2C_A\, \beta_0\, a_s/\pi$

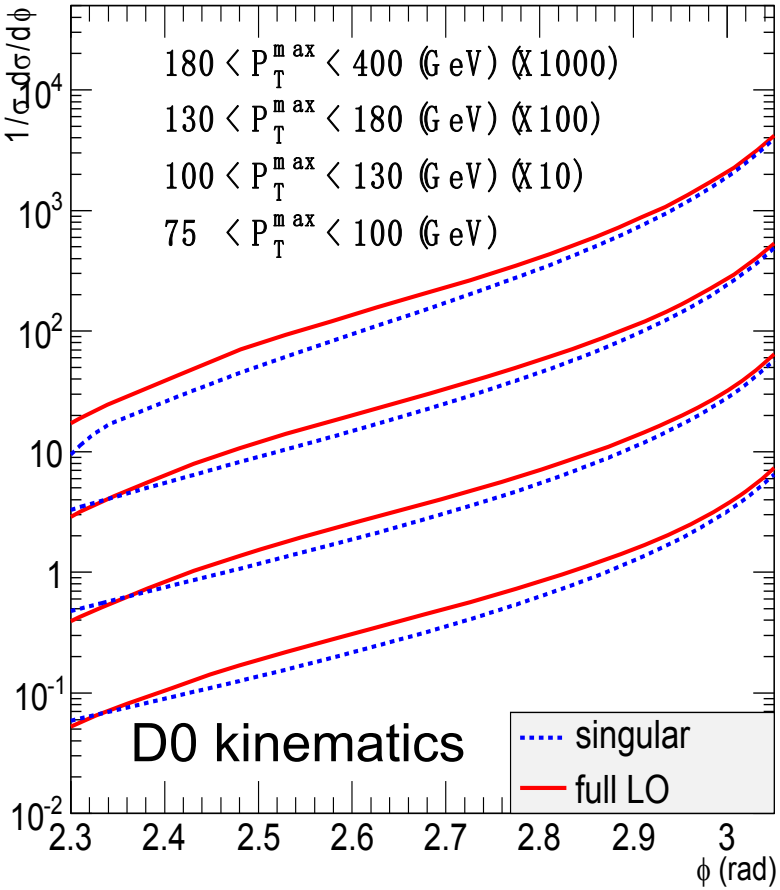
For q q → jj $A_{qq}=C_F\, a_s/\pi$ $B_{qq}=-2C_F/3\, a_s/\pi$

For q g → jj $A_{qg}=(A_{gg}+A_{qq})/2$ $B_{qg}= =(B_{gg}+B_{qq})/2$

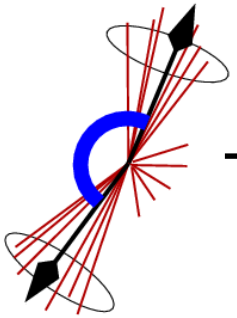
for quark jet $D_i=C_F\, a_s/\pi$

for gluon jet $D_i=C_A\, a_s/\pi$

Compared to full calculations



$$\frac{\alpha_s}{2\pi^2}\frac{1}{q_\perp^2}\sum_{ab,a'b'}\sigma_0\int\frac{dx_1'}{x_1'}\frac{dx_2'}{x_2'}x_1' f_a(x_1',\mu)x_2' f_b(x_2',\mu)$$
$$\times\Big\{h^{(0)}_{a'b'\to cd}\Big[\xi_1\mathcal{P}_{a'/a}(\xi_1)\delta(1-\xi_2)+\xi_2\mathcal{P}_{b'/b}(\xi_2)\delta(1-\xi_1)$$
$$+\delta(1-\xi_1)\delta(1-\xi_2)\delta_{aa'}\delta_{bb'}\left((C_a+C_b)\ln\frac{Q^2}{q_\perp^2}+C_c\ln\frac{1}{R_1^2}+C_d\ln\frac{1}{R_2^2}\right)\Big]$$
$$+\delta(1-\xi_1)\delta(1-\xi_2)\delta_{aa'}\delta_{bb'}\Gamma^{ab\to cd}_{sn}\Big\}\ , \qquad (10$$

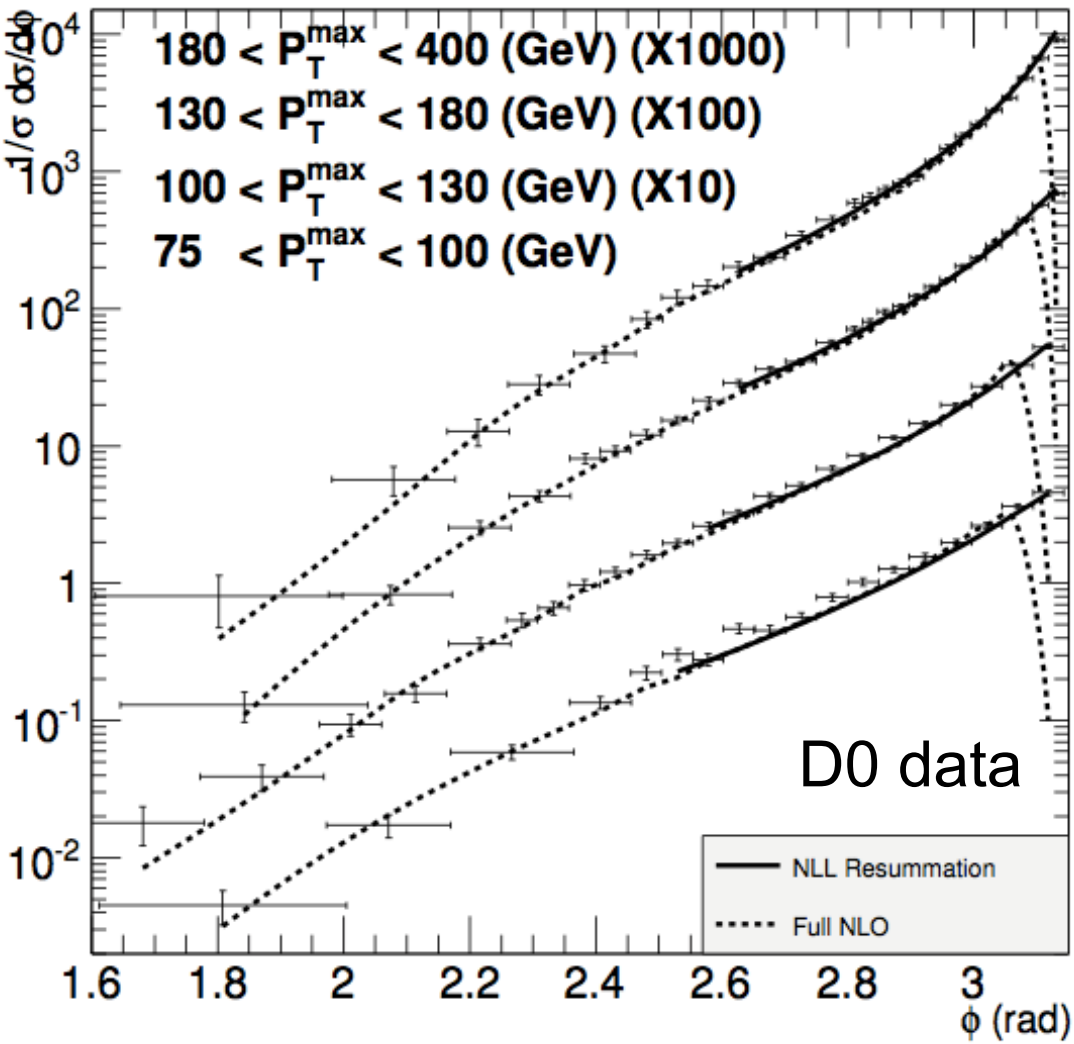


Leading P_T

Total $q_T \approx P_T \mathrm{Sin}(\pi - \phi)$

full LO: Nagy 2002, NLOJET++

Compared to the data



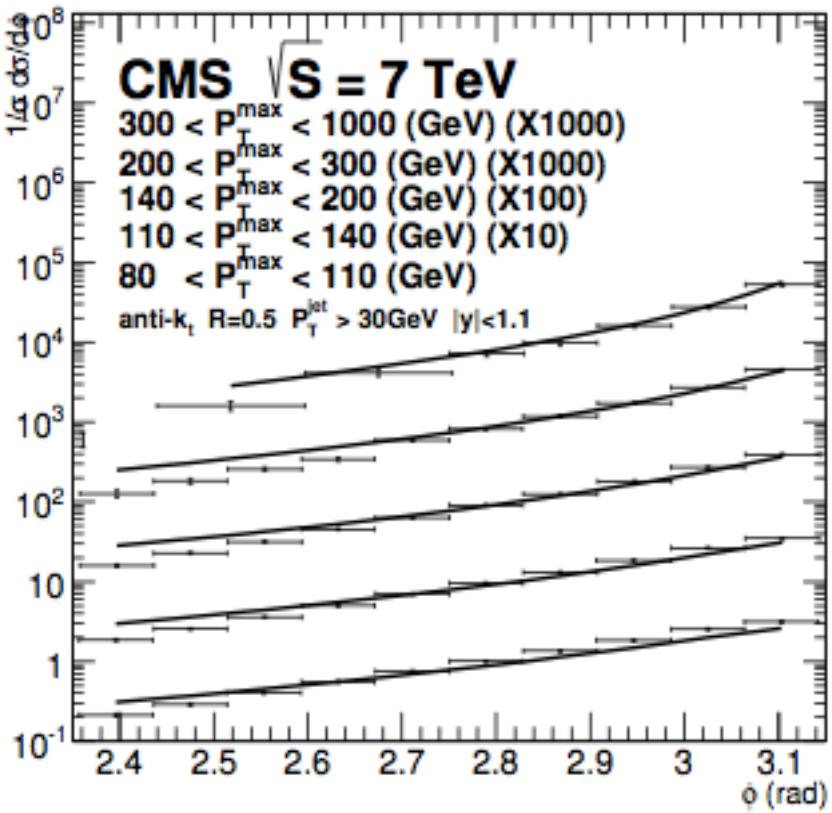
NLL Resummation:
Sun,C.P.Yuan, F.Yuan, PRL2014

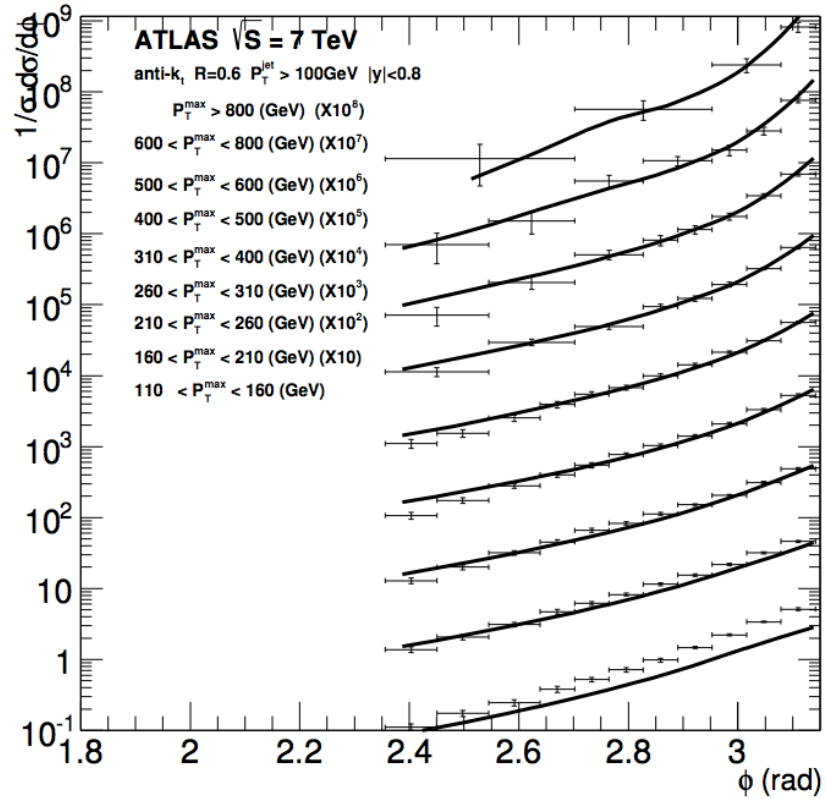
$$x_1 f_a(x_1, \mu = b_0/b_\perp) x_2 f_b(x_2, \mu = b_0/b_\perp) e^{-S_{\rm Sud}(Q^2, b_\perp)}$$

$$\mathrm{Tr}\left[\mathbf{H}_{ab\to cd} \exp\left[-\int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu}\gamma^{s\dagger}\right] \mathbf{S}_{ab\to cd} \exp\left[-\int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu}\gamma^{s}\right]\right]$$

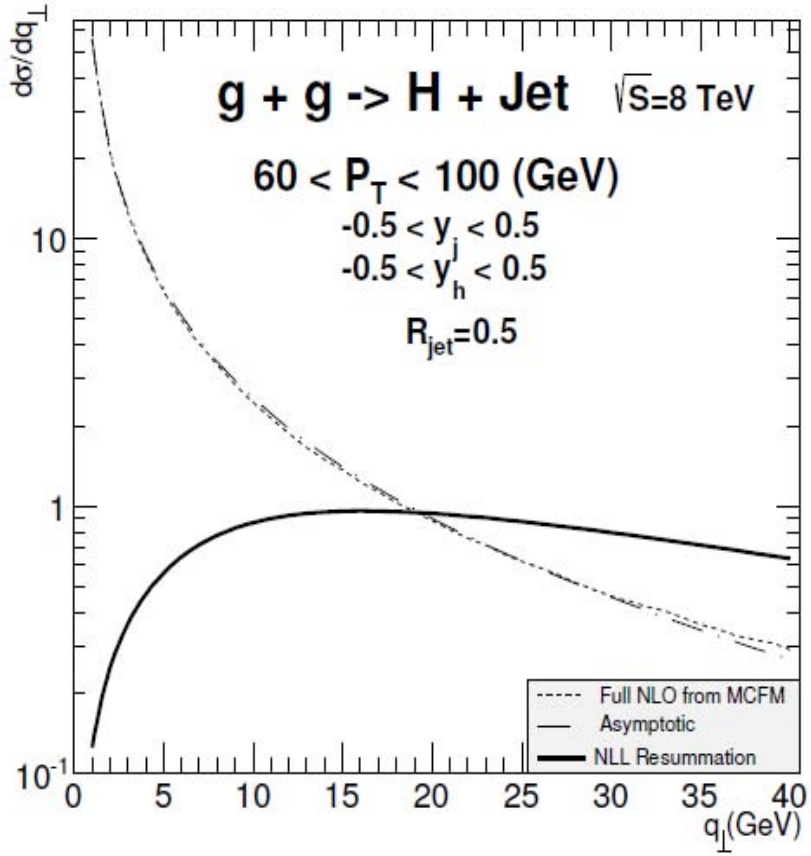
Full NLO: Nagy 2002, NLOJET++

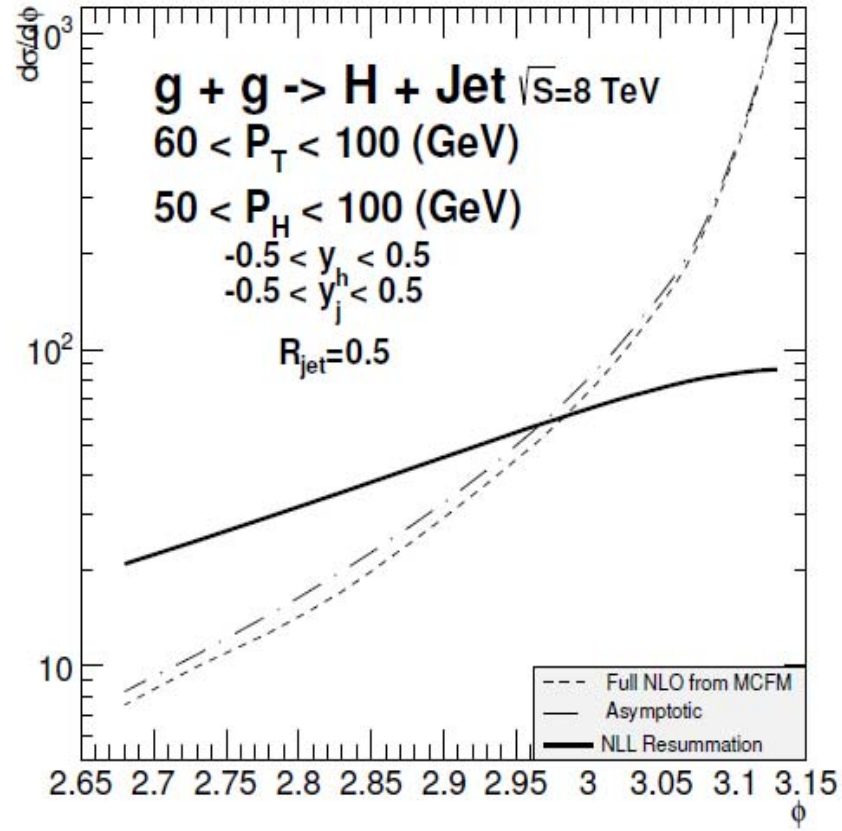
At the LHC


CMS √S = 7 TeV
300 < P_T^max < 1000 (GeV) (X1000)
200 < P_T^max < 300 (GeV) (X1000)
140 < P_T^max < 200 (GeV) (X100)
110 < P_T^max < 140 (GeV) (X10)
80 < P_T^max < 110 (GeV)
anti-k_t R=0.5 P_T^jet > 30GeV |y|<1.1
1/σ dσ/dφ
φ (rad)


ATLAS √S = 7 TeV
anti-k_t R=0.6 P_T^jet > 100GeV |y|<0.8
P_T^max > 800 (GeV) (X10^8)
600 < P_T^max < 800 (GeV) (X10^7)
500 < P_T^max < 600 (GeV) (X10^6)
400 < P_T^max < 500 (GeV) (X10^5)
310 < P_T^max < 400 (GeV) (X10^4)
260 < P_T^max < 310 (GeV) (X10^3)
210 < P_T^max < 260 (GeV) (X10^2)
160 < P_T^max < 210 (GeV) (X10)
110 < P_T^max < 160 (GeV)
1/σ dσ/dφ
φ (rad)

Higgs + jet production in pp collision





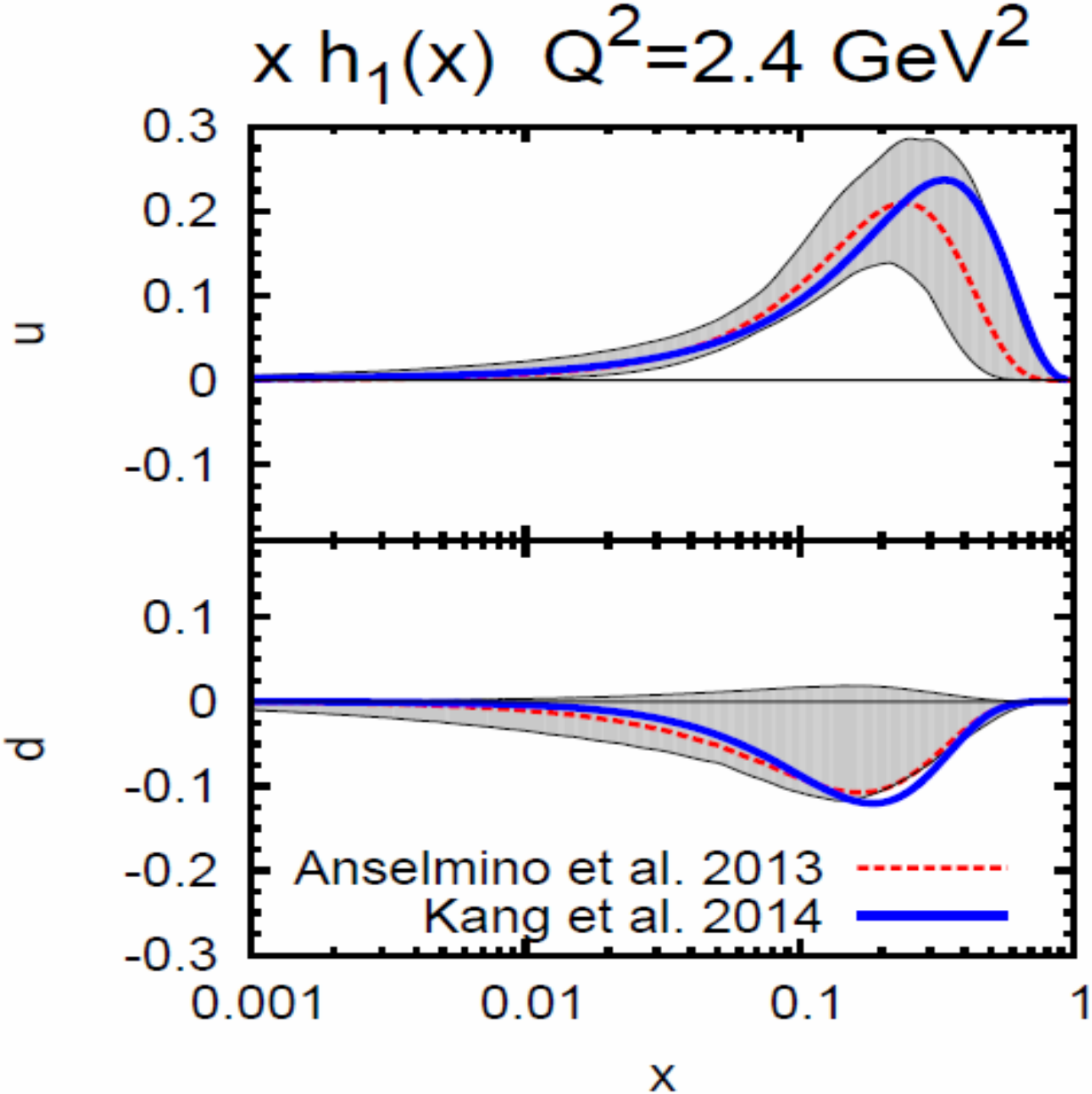
- **Higgs+Jet, Sun, C.-P. Yuan, F. Yuan, phys.Rev.Lett.(2015)**

Summary

- Soft gluon resummation for dijet correlation at the next-to-leading logarithmic order agrees well with the experimental data.

- Extending to EW boson plus jet production will be interesting to follow

Thank you very much!


x h1(x) Q2=2.4 GeV2
0.3
0.2
0.1
0
-0.1
u
0.1
0
-0.1
-0.2
-0.3
d
Anselmino et al. 2013
Kang et al. 2014
0.001
0.01
0.1
1
x

Approaches to TMD evolution

Collins-Soper-Sterman (CSS) resummation framework

Collins-Soper-Sterman 1985
ResBos: C.P. Yuan, P. Nadolsky
Qiu-Zhang 1999, Vogelsang tetc...
Kang-Xiao-Yuan 2011, Sun-Yuan 2013
Prokudin-Kang-Sun-Yuan 2014

“New” Collins approach

Collins 2011
Aybat-Rogers 2011,
Aybat-Collins-Rogers-Qiu, 2012
Aybat-Prokudin-Rogers 2012
Anselmino-Boglione-Melis 2012
Prokudin-Bacchetta 2013
Echevarria-Idilbi-Kang-Vitev 2014

Soft Collinear Effective Theory (SCET)

Echevarria-Idilbi-Schafer-Scimemi 2012
D'Alesio-Echevarria-Melis-Scimemi 2014

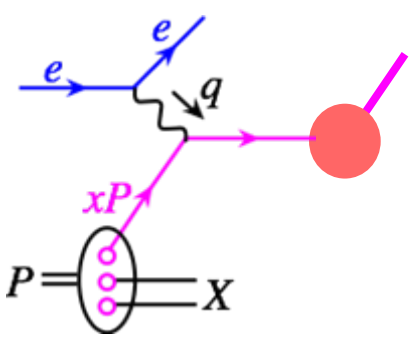
Why do we need QCD evolution?

TMD factorization is applicable in case two different scales are observed in processes such as SIDIS, Drell-Yan, W/Z production in hadron-hadron collisions. Kinematical regime: $Q_T \ll Q$

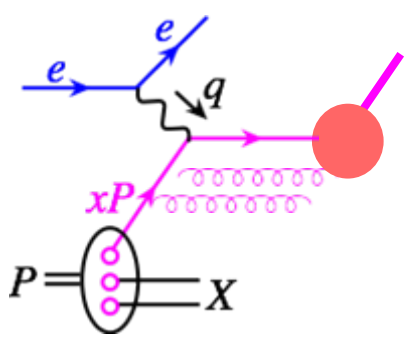
For SIDIS Q_T is transverse momentum of final parton

And Q is the invariant mass of virtual photon

Double and Single logarithms will appear order by order in perturbative calculations



$f(x)$



$f(x, k_\perp; Q)$

$$\left(\alpha_s \ln^2 \frac{Q^2}{Q_T^2}\right)^n$$

$$\left(\alpha_s \ln \frac{Q^2}{\mu^2}\right)^n$$