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# $A_N$ in single-inclusive particle production from proton-proton collisions

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# Outline

## ➤ Motivation

- What are transverse single-spin asymmetries (TSSAs)?
- Collinear twist-3 vs. Generalized Parton Model (GPM) formalisms

## ➤ TSSAs in single-inclusive processes

$$\underline{p^\uparrow p \rightarrow \pi X}$$

- The “sign mismatch” issue between the Qiu-Sterman (QS) and Sivers functions
- Towards an explanation using collinear twist-3 fragmentation
- Further tests using  $e N^\uparrow \rightarrow \pi X$  measurements

$$\underline{p^\uparrow p \rightarrow \gamma X}$$

- “Clean” access to the QS function
- Could test the process dependence of the Sivers function (on same footing as  $A_N$  in DY)
- Could distinguish between collinear twist-3 and GPM frameworks

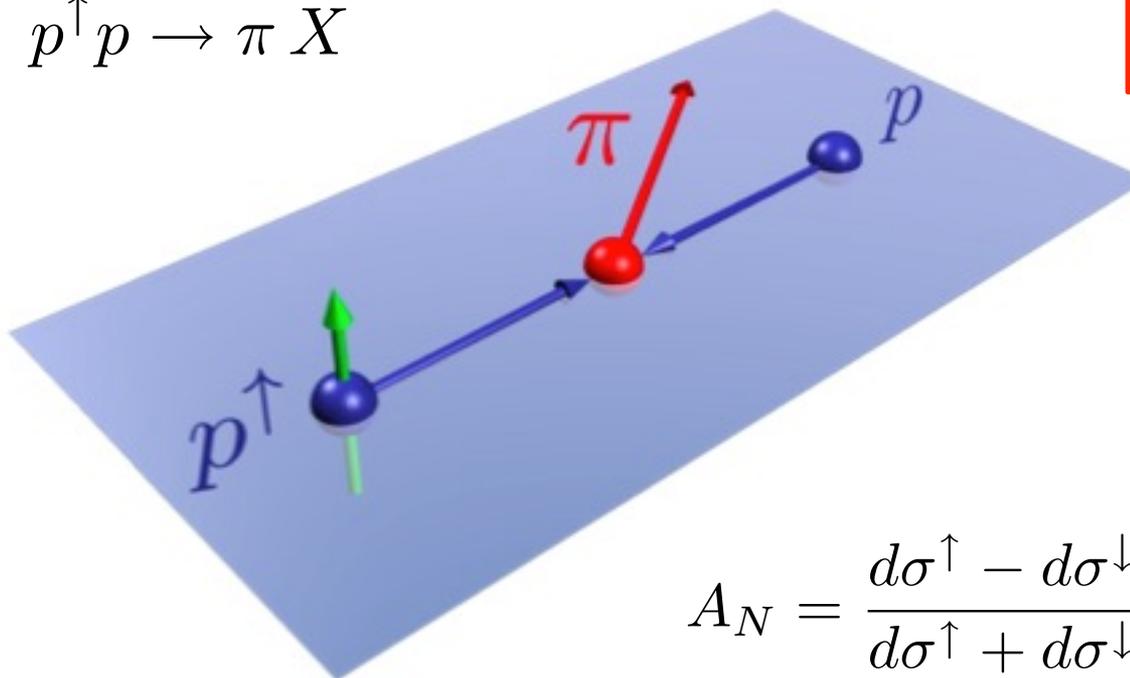
## ➤ Summary and outlook



# Motivation

➤ What are TSSAs?

$$p^\uparrow p \rightarrow \pi X$$



- Naïve T-odd effect
- $P_{h\perp}$  is the only scale

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$

Data available from RHIC (BRAHMS, PHENIX, STAR),  
FNAL (E704, E581), AGS, and ANL

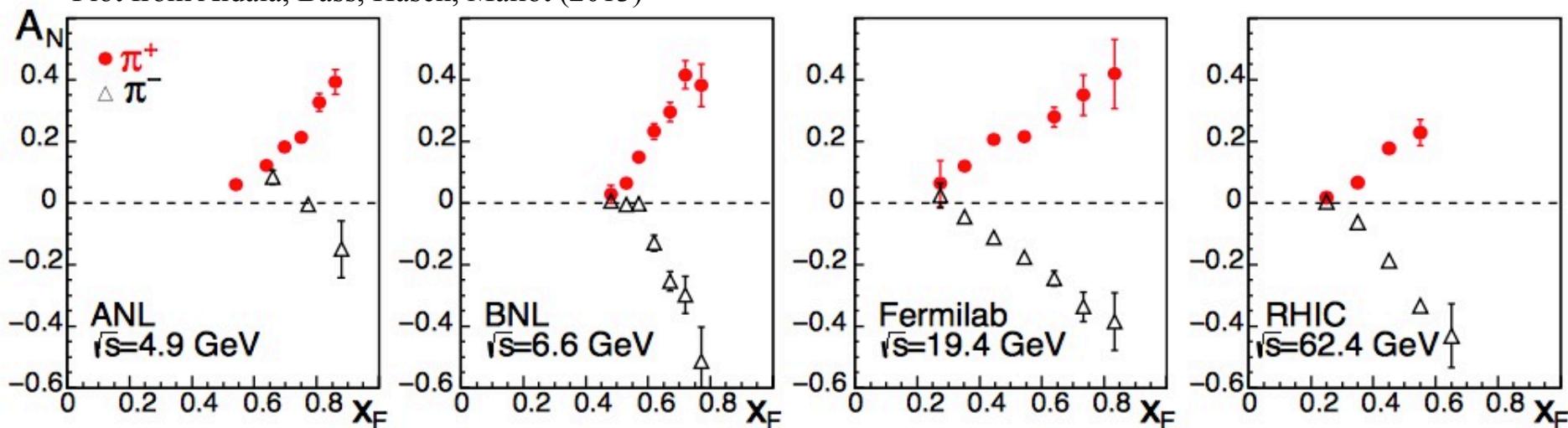
(Figure thanks to K. Kanazawa)



# Motivation

## ➤ What are TSSAs?

Plot from Aidala, Bass, Hasch, Mallot (2013)



$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$

Data available from RHIC (BRAHMS, PHENIX, STAR),  
FNAL (E704, E581), AGS, and ANL



- Large TSSAs observed in the mid-1970s in the detection of hyperons from proton-beryllium collisions (Bunce, et al. (1976)) and charged pions from proton-proton collisions (Klem, et al. (1976))
- Initially thought to contradict pQCD (Kane, Pumplin, Repko (1978)) – within the naïve collinear parton model:

$$A_N \sim \alpha_s m_q / P_{h\perp}$$

“If  $P(A)$  is significantly different from zero, then either it is not valid to apply QCD in this region...or QCD cannot be applied perturbatively...or, conceivably, something is wrong with the present formulation of QCD itself.”

- Higher-twist approach to calculating TSSAs in  $pp$  collisions introduced in the 1980s – large  $A_N$  possible (Efremov and Teryaev (1982, 1985))
- Benchmark calculations performed starting in the early 1990s (Qiu and Sterman (1992, 1999); Kouvaris, et al. (2006); Koike and Tomita (2009), etc.)
- GPM approach first used starting in the mid-1990s (Anselmino, Boglione, Murgia (1995); Anselmino and Murgia (1998); Anselmino, et al. (2006, 2012, 2013), etc.)



➤ Collinear twist-3 vs. Generalized Parton Model (GPM)

Collinear twist-3

Uses collinear functions ( $P_{h\perp} \gg \Lambda_{QCD}$ )

$$\begin{aligned} d\sigma &= H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\ &+ H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\ &+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)} \end{aligned}$$

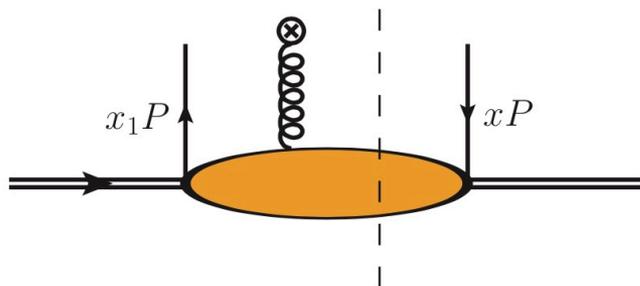


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 &+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
 \end{aligned}$$



$$\begin{array}{ccc}
 \text{SGP} & & \text{SFP} \\
 F_{FT}(x, x) & & F_{FT}(0, x), G_{FT}(0, x) \\
 \downarrow & & \\
 \text{QS (Sivers-type) function} & & 
 \end{array}$$

Note: Can also have tri-gluon correlators at SGPs (Beppu, et al. (2013))



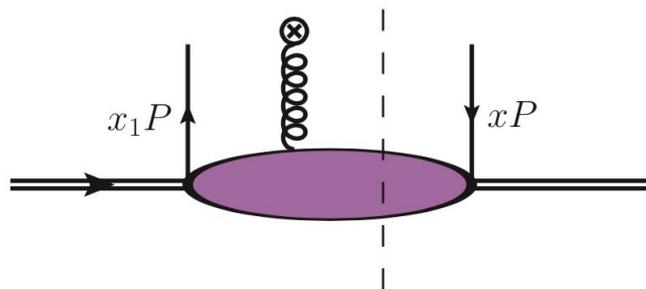
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 &+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
 \end{aligned}$$

<u>SGP</u>	<u>SFP</u>
$F_{FT}(x, x)$	$F_{FT}(0, x), G_{FT}(0, x)$
$H_{FU}(x, x)$	$H_{FU}(0, x)$
↓	
Boer-Mulders-type function	



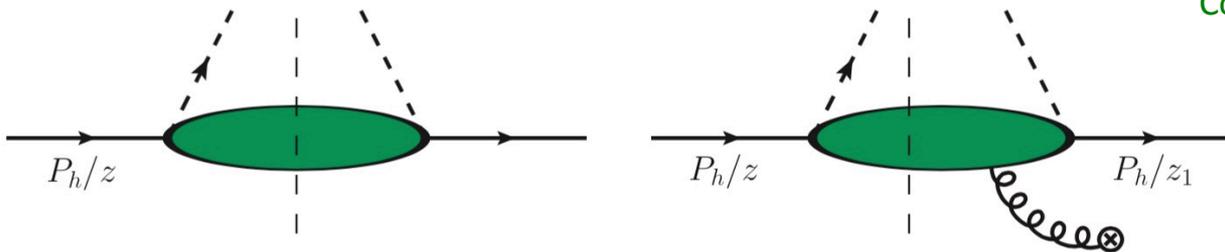
➤ Collinear twist-3 vs. Generalized Parton Model (GPM)

Collinear twist-3

Uses collinear functions ( $P_{h\perp} \gg \Lambda_{QCD}$ )

$$\begin{aligned}
 d\sigma = & H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} & F_{FT}(x, x) & F_{FT}(0, x), G_{FT}(0, x) \\
 & + H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} & H_{FU}(x, x) & H_{FU}(0, x) \\
 & + H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)} & \hat{H}(z), H(z), & \hat{H}_{FU}(z, z_1)
 \end{aligned}$$

Collins-type function





➤ Collinear twist-3 vs. Generalized Parton Model (GPM)

Collinear twist-3

Uses collinear functions ( $P_{h\perp} \gg \Lambda_{QCD}$ )

$$\begin{aligned}
 d\sigma = & H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} && F_{FT}(x, x) && F_{FT}(0, x), G_{FT}(0, x) \\
 & + H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} && H_{FU}(x, x) && H_{FU}(0, x) \\
 & + H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)} && \hat{H}(z), H(z), && \hat{H}_{FU}(z, z_1)
 \end{aligned}$$

GPM

Uses TMD functions ( $P_{h\perp} \gg ?? \sim \Lambda_{QCD}$ )

$$\begin{aligned}
 d\sigma = & H \otimes f_{1T}^\perp \otimes f_1 \otimes D_1 \\
 & + H' \otimes h_1 \otimes h_1^\perp \otimes D_1 \\
 & + H'' \otimes h_1 \otimes f_1 \otimes H_1^\perp
 \end{aligned}$$



➤ Collinear twist-3 vs. Generalized Parton Model (GPM)

**Collinear twist-3**

Uses collinear functions ( $P_{h\perp} \gg \Lambda_{QCD}$ )

$$\begin{aligned}
 d\sigma = & H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} && F_{FT}(x, x) && F_{FT}(0, x), G_{FT}(0, x) \\
 & + H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} && H_{FU}(x, x) && H_{FU}(0, x) \\
 & + H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)} && \hat{H}(z), H(z), \hat{H}_{FU}(z, z_1)
 \end{aligned}$$

**GPM**

Uses TMD functions ( $P_{h\perp} \gg ?? \sim \Lambda_{QCD}$ )

$$\begin{aligned}
 d\sigma = & H \otimes f_{1T}^\perp \otimes f_1 \otimes D_1 && \text{Sivers} \\
 & + H' \otimes h_1 \otimes h_1^\perp \otimes D_1 && \text{Boer-Mulders} \\
 & + H'' \otimes h_1 \otimes f_1 \otimes H_1^\perp && \text{Collins}
 \end{aligned}
 \left. \vphantom{\begin{aligned} d\sigma = \\ + H' \\ + H'' \end{aligned}} \right\} \text{Enter in azimuthal asymmetries in SIDIS} \\
 \left. \vphantom{\begin{aligned} d\sigma = \\ + H' \\ + H'' \end{aligned}} \right\} (Q \gg P_{h\perp} \sim \Lambda_{QCD})$$



➤ Collinear twist-3 vs. Generalized Parton Model (GPM)

**Collinear twist-3**

Uses collinear functions ( $P_{h\perp} \gg \Lambda_{QCD}$ )

$$\begin{aligned}
 d\sigma = & H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} && F_{FT}(x, x) && F_{FT}(0, x), G_{FT}(0, x) \\
 & + H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} && H_{FU}(x, x) && H_{FU}(0, x) \\
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 \end{aligned}$$

**GPM**

Uses TMD functions ( $P_{h\perp} \gg \text{??} \sim \Lambda_{QCD}$ )

$$\begin{aligned}
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 & + H' \otimes h_1 \otimes h_1^\perp \otimes D_1 && \text{Boer-Mulders} \\
 & + H'' \otimes h_1 \otimes f_1 \otimes H_1^\perp && \text{Collins}
 \end{aligned}$$

There is no soft scale



➤ Collinear twist-3 vs. Generalized Parton Model (GPM)

**Collinear twist-3**

Uses collinear functions ( $P_{h\perp} \gg \Lambda_{QCD}$ )

$$\begin{aligned}
 d\sigma = & H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} & F_{FT}(x, x) & F_{FT}(0, x), G_{FT}(0, x) \\
 & + H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} & H_{FU}(x, x) & H_{FU}(0, x) \\
 & + H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)} & \hat{H}(z), H(z), \hat{H}_{FU}(z, z_1) &
 \end{aligned}$$

**GPM**

Uses TMD functions ( $P_{h\perp} \gg ?? \sim \Lambda_{QCD}$ )

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 d\sigma = & H \otimes f_{1T}^\perp \otimes f_1 \otimes D_1 & \text{Sivers} \\
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 & + H'' \otimes h_1 \otimes f_1 \otimes H_1^\perp & \text{Collins}
 \end{aligned}$$

$$\begin{aligned}
 \pi F_{FT}(x, x) &= f_{1T}^{\perp(1)}(x) \Big|_{SIDIS} & \hat{H}(z) &= H_1^{\perp(1)}(z) \\
 \pi H_{FU}(x, x) &= h_1^{\perp(1)}(x) \Big|_{SIDIS} & &
 \end{aligned}$$

## ➤ Collinear twist-3 vs. Generalized Parton Model (GPM)

**Collinear twist-3**

 Uses collinear functions ( $P_{h\perp} \gg \Lambda_{QCD}$ )

NO (twist-2) TMD analogues

$$\begin{aligned}
 d\sigma &= H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\
 &+ H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\
 &+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
 \end{aligned}$$

$$\begin{aligned}
 &F_{FT}(x, x) && F_{FT}(0, x), G_{FT}(0, x) \\
 &H_{FU}(x, x) && H_{FU}(0, x) \\
 &\hat{H}(z), \hat{H}(z) && \hat{H}_{FU}(z, z_1)
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**GPM**

 Uses TMD functions ( $P_{h\perp} \gg ?? \sim \Lambda_{QCD}$ )

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 d\sigma &= H \otimes f_{1T}^\perp \otimes f_1 \otimes D_1 && \text{Sivers} \\
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 \end{aligned}$$

$$\hat{H}(z) = H_1^{\perp(1)}(z)$$



# TSSAs in Single-Inclusive Processes

Collinear twist-3

Uses collinear functions ( $P_{h\perp} \gg \Lambda_{QCD}$ )

$$\begin{aligned} d\sigma &= H \otimes f_{a/A\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\ &+ H' \otimes f_{a/A\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\ &+ H'' \otimes f_{a/A\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)} \end{aligned}$$

➔ For many years the SGP term involving the QS/Sivers-type function  $F_{FT}$  was thought to be the dominant contribution to TSSAs in  $p^\uparrow p \rightarrow \pi X$

$$\begin{aligned} E_\ell \frac{d^3 \Delta\sigma(\vec{s}_T)}{d^3 \ell} &= \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\min}}^1 \frac{dz}{z^2} D_{c \rightarrow h}(z) \int_{x'_{\min}}^1 \frac{dx'}{x'} \frac{1}{x'S + T/z} \phi_{b/B}(x') \\ &\times \sqrt{4\pi\alpha_s} \left( \frac{\epsilon^{\ell s_T n \bar{n}}}{z \hat{u}} \right) \frac{1}{x} \left[ T_{a,F}(x, x) - x \left( \frac{d}{dx} T_{a,F}(x, x) \right) \right] H_{ab \rightarrow c}(\hat{s}, \hat{t}, \hat{u}) \end{aligned}$$

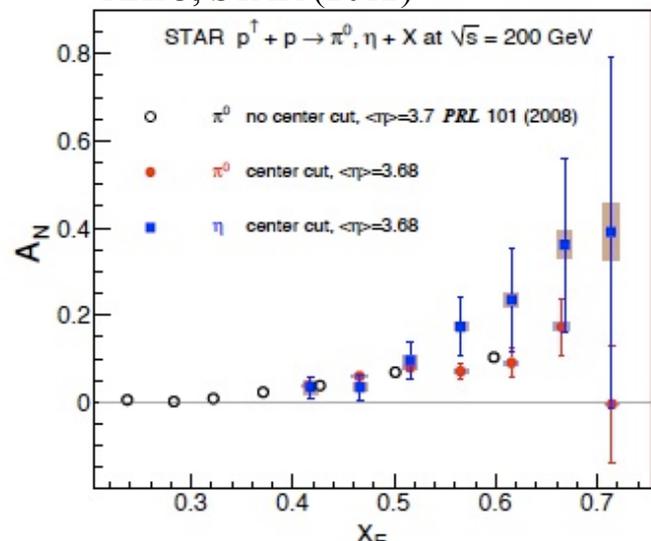
$$F_{FT} \sim T_F$$

(Qiu and Sterman (1999), Kouvaris, et al. (2006))

➤ The “sign mismatch” issue

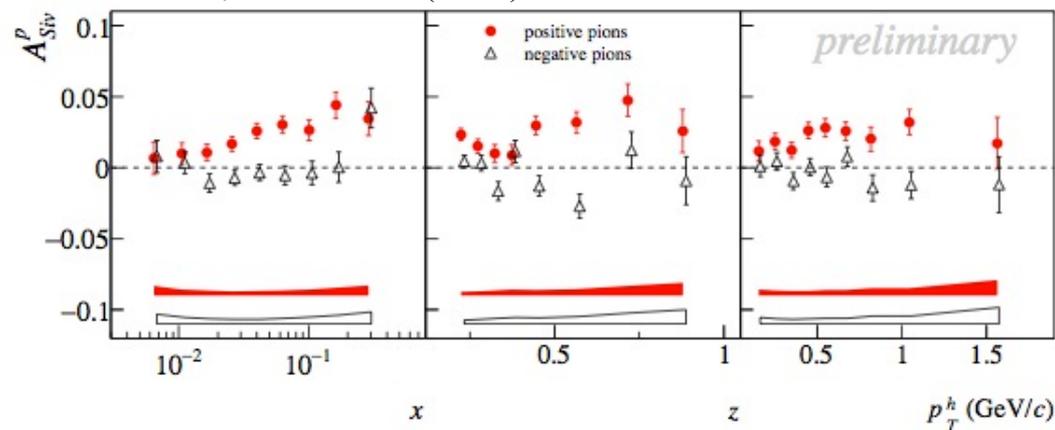
$$p^\uparrow p \rightarrow h X$$

RHIC, STAR (2012)



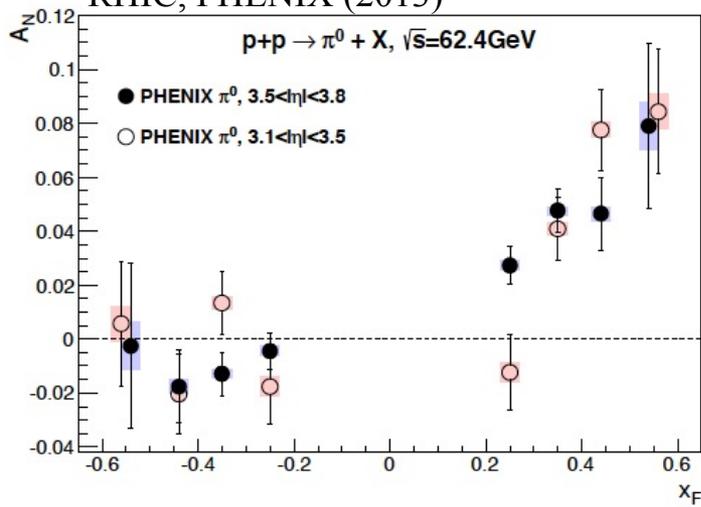
$$\ell N^\uparrow \rightarrow \ell' h X$$

CERN, COMPASS (2013)



$$\pi F_{FT}(x, x) = f_{1T}^{\perp(1)}(x)$$

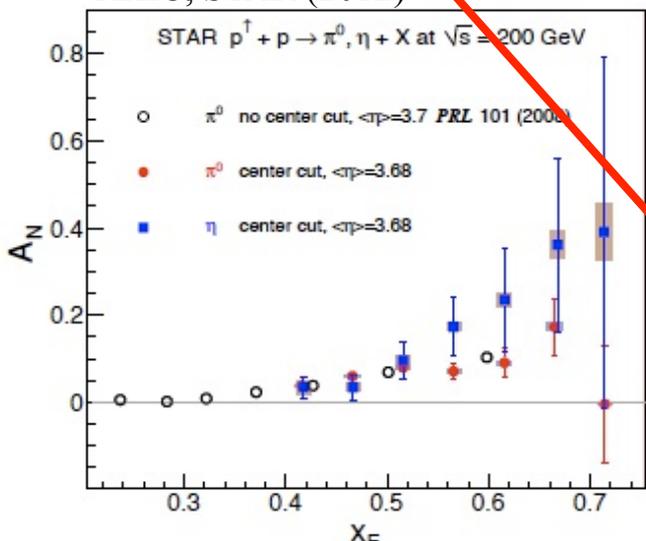
RHIC, PHENIX (2013)



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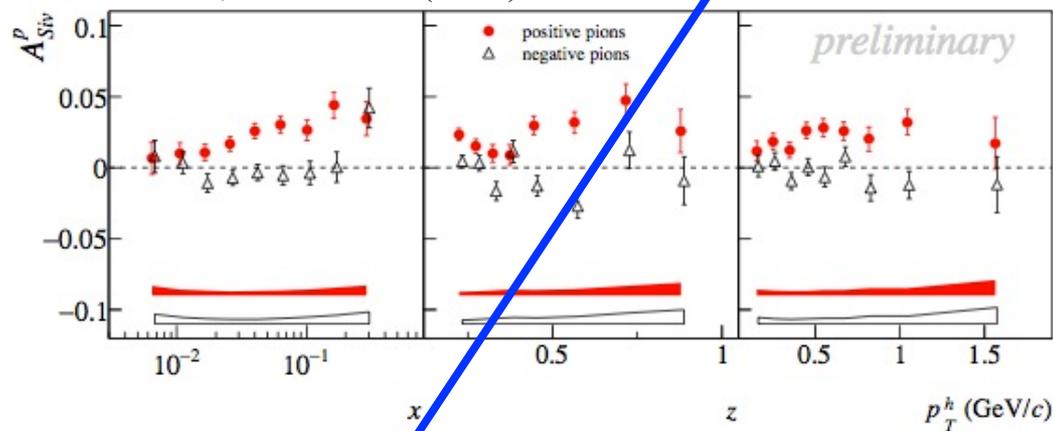
$$p^\uparrow p \rightarrow h X$$

RHIC, STAR (2012)



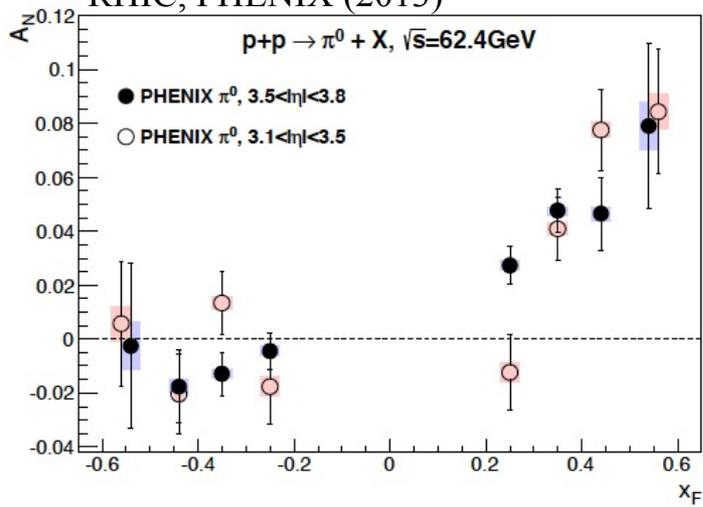
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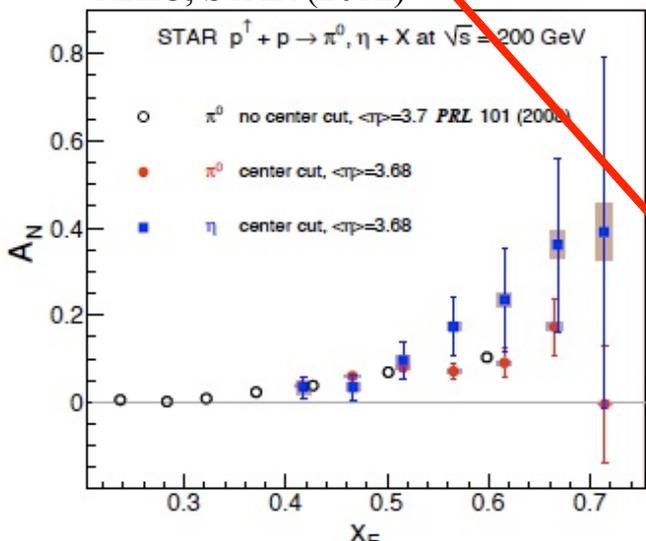
RHIC, PHENIX (2013)



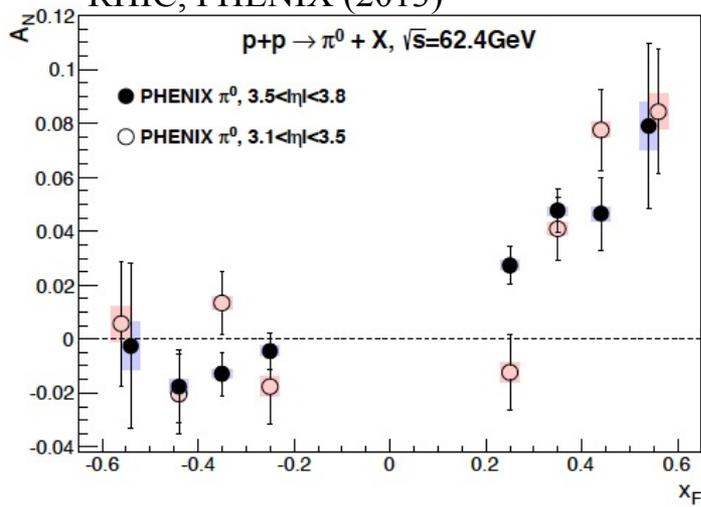
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$$p^\uparrow p \rightarrow h X$$

RHIC, STAR (2012)

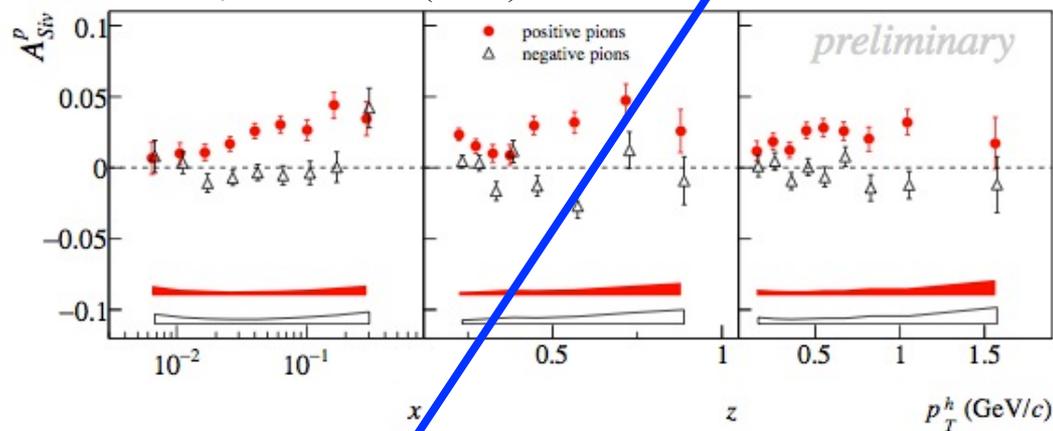


RHIC, PHENIX (2013)



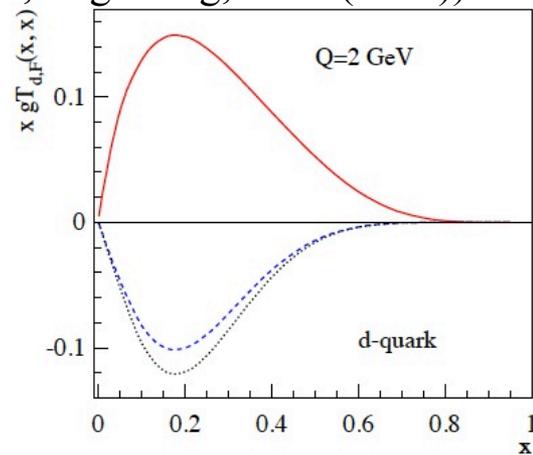
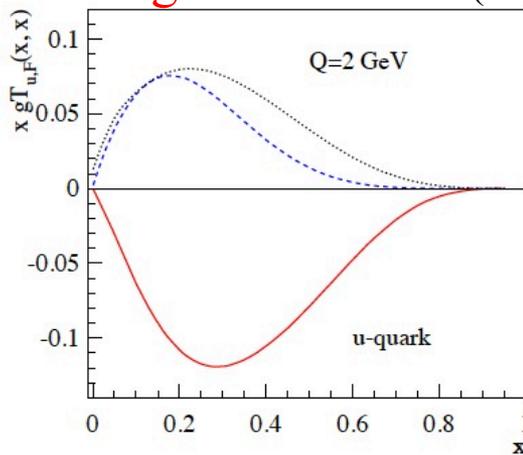
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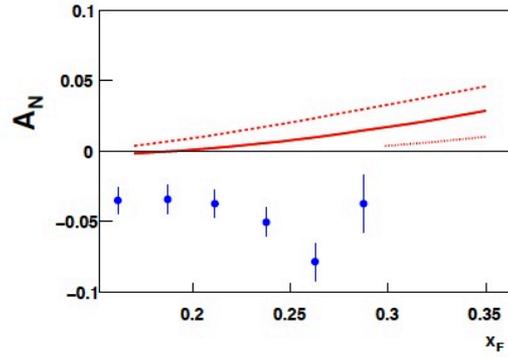
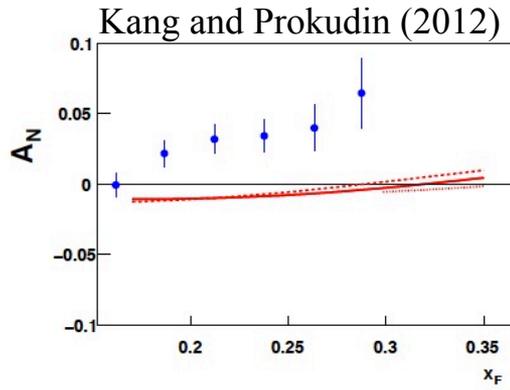
CERN, COMPASS (2013)



$$\pi F_{FT}(x, x) = f_{1T}^{\perp(1)}(x)$$

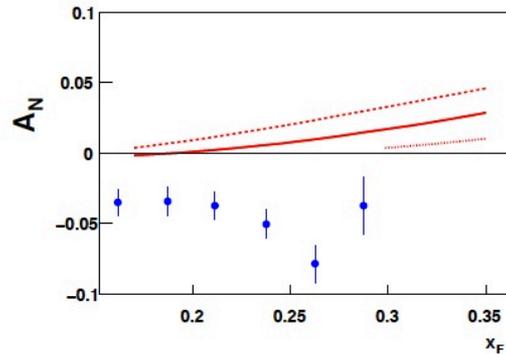
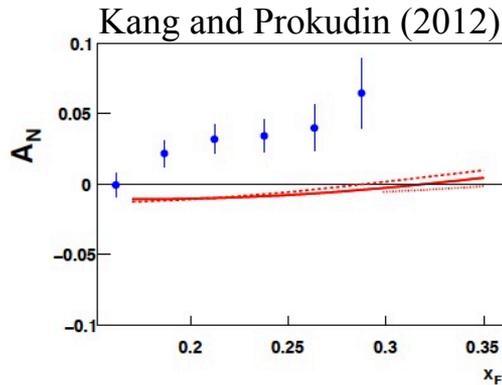
“sign mismatch” (Kang, Qiu, Vogelsang, Yuan (2011))





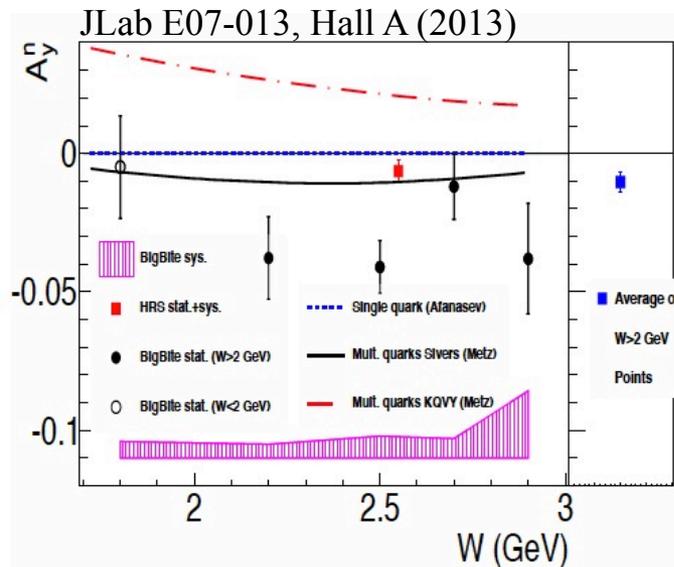
Proton-proton data from  
BRAHMS for  $\pi^+$  (left) and  $\pi^-$   
(right)

Nodes in Sivers cannot  
resolve issue



Proton-proton data from BRAHMS for  $\pi^+$  (left) and  $\pi^-$  (right)

Nodes in Sivers cannot resolve issue



Neutron TSSA in inclusive DIS

Metz, DP, Schäfer, Schlegel, Vogelsang, Zhou - PRD 86 (2012)

Sivers input agrees reasonably well with the JLab data  $\Rightarrow$  **FIRST INDICATION** on the **PROCESS DEPENDENCE** of the Sivers function (see also Gamberg, Kang, Prokudin (2013))

**KQVY** input gives the wrong sign  $\Rightarrow$  **QS function cannot be the main cause of the large TSSAs seen in pion production**



$$\begin{aligned}d\sigma &= H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\ &+ H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\ &+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}\end{aligned}$$



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Negligible  
(Kanazawa and  
Koike (2000))



$$d\sigma = H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)}$$

$$+ H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)}$$

$$+ H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}$$

Negligible  
(Kanazawa and  
Koike (2000))



➤ Collinear twist-3 fragmentation term:  $H'' \otimes h_1 \otimes f_1 \otimes (\hat{H}, H, \hat{H}_{FU}^{\mathfrak{S}})$

$$\hat{H}(z) = H_1^{\perp(1)}(z) \quad \text{Collins-type function}$$

$$2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{\mathfrak{S}}(z, z_1) = H(z) + 2z\hat{H}(z) \quad \text{3-parton correlator}$$

$$\begin{aligned} \frac{P_h^0 d\sigma_{pol}}{d^3\vec{P}_h} = & -\frac{2\alpha_s^2 M_h}{S} \epsilon_{\perp\mu\nu} S_{\perp}^{\mu} P_{h\perp}^{\nu} \sum_i \sum_{a,b,c} \int_{z_{min}}^1 \frac{dz}{z^3} \int_{x'_{min}}^1 \frac{dx'}{x'} \frac{1}{x'S + T/z} \frac{1}{-x\hat{u} - x'\hat{t}} \\ & \times \frac{1}{x} h_1^a(x) f_1^b(x') \left\{ \left( \hat{H}^{C/c}(z) - z \frac{d\hat{H}^{C/c}(z)}{dz} \right) S_{\hat{H}}^i + \frac{1}{z} H^{C/c}(z) S_H^i \right. \\ & \left. + 2z^2 \int \frac{dz_1}{z_1^2} PV \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{C/c,\mathfrak{S}}(z, z_1) \frac{1}{\xi} S_{\hat{H}_{FU}}^i \right\} \end{aligned}$$

(Metz and DP - PLB 723 (2013))



➤ Towards an explanation using twist-3 fragmentation  
(Kanazawa, Koike, Metz, DP - PRD 89(RC) (2014))

- Numerical study (Note: we only use  $\sqrt{S} = 200$  GeV data → higher  $P_{h\perp}$  values)

Distribution term

- ➔ SGP:  $\pi F_{FT}(x, x) = f_{1T}^{\perp(1)}(x)$ , Sivvers function taken from Torino group (2009/2013)
- ➔ SFP/Tri-gluon: neglect for now

Fragmentation term

- ➔ Transversity: taken from Torino group (2013), but allow  $\beta$  parameters to be free
- ➔  $\hat{H}^{h/q}(z)$ : use Collins function extracted by the Torino group (2013)

$$\hat{H}^{h/q}(z) = z^2 \int d^2\vec{k}_{\perp} \frac{\vec{k}_{\perp}^2}{2M_h^2} H_1^{\perp h/q}(z, z^2\vec{k}_{\perp}^2)$$

- ➔  $\hat{H}_{FU}^{h/q, \mathfrak{S}}(z, z_1)$  → use the following ansatz:

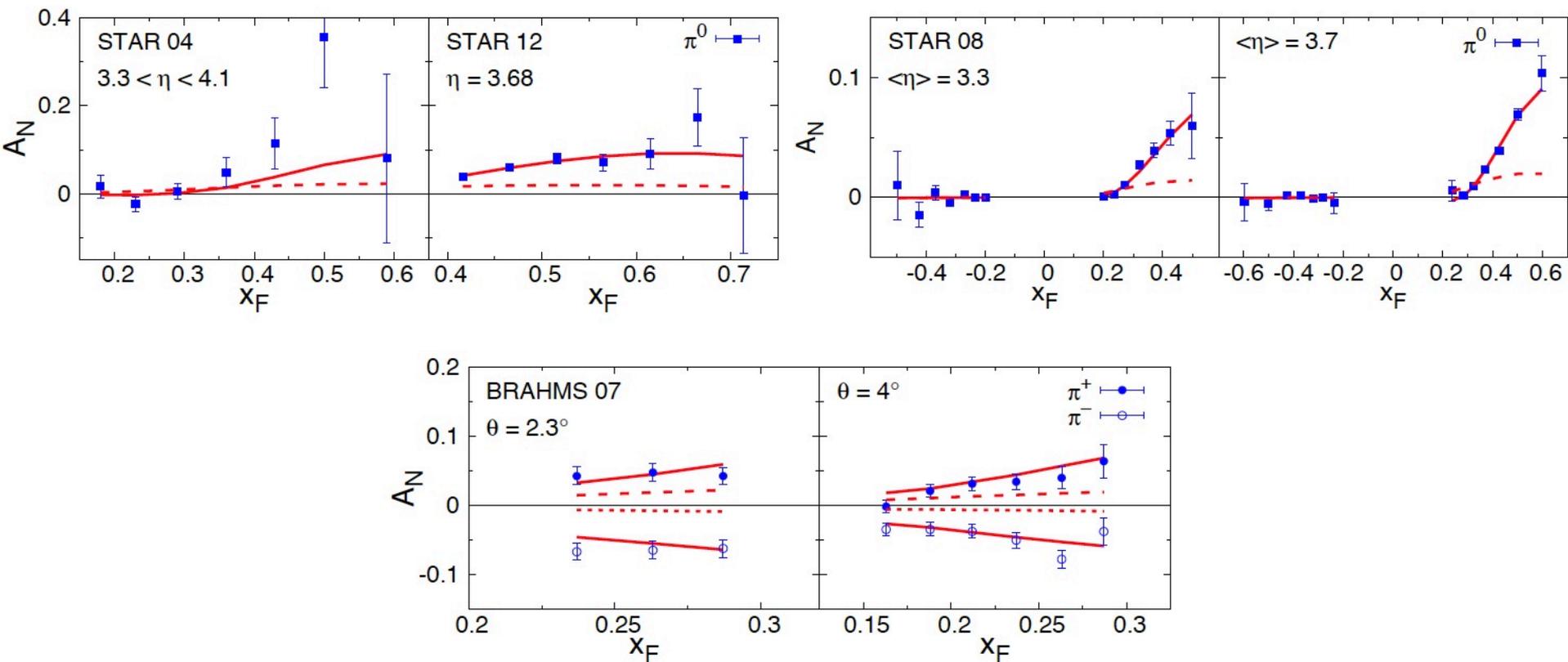
$$\frac{\hat{H}_{FU}^{\pi^+/(u, \bar{d}), \mathfrak{S}}(z, z_1)}{D^{\pi^+/(u, \bar{d})}(z) D^{\pi^+/(u, \bar{d})}(z/z_1)} = \frac{N_{\text{fav}}}{2I_{\text{fav}} J_{\text{fav}}} z^{\alpha_{\text{fav}}} (z/z_1)^{\alpha'_{\text{fav}}} (1-z)^{\beta_{\text{fav}}} (1-z/z_1)^{\beta'_{\text{fav}}}$$

(similar for disfavored,  $\pi^-$  defined through c.c.,  $\pi^0$  defined as average of  $\pi^+$  and  $\pi^-$ )

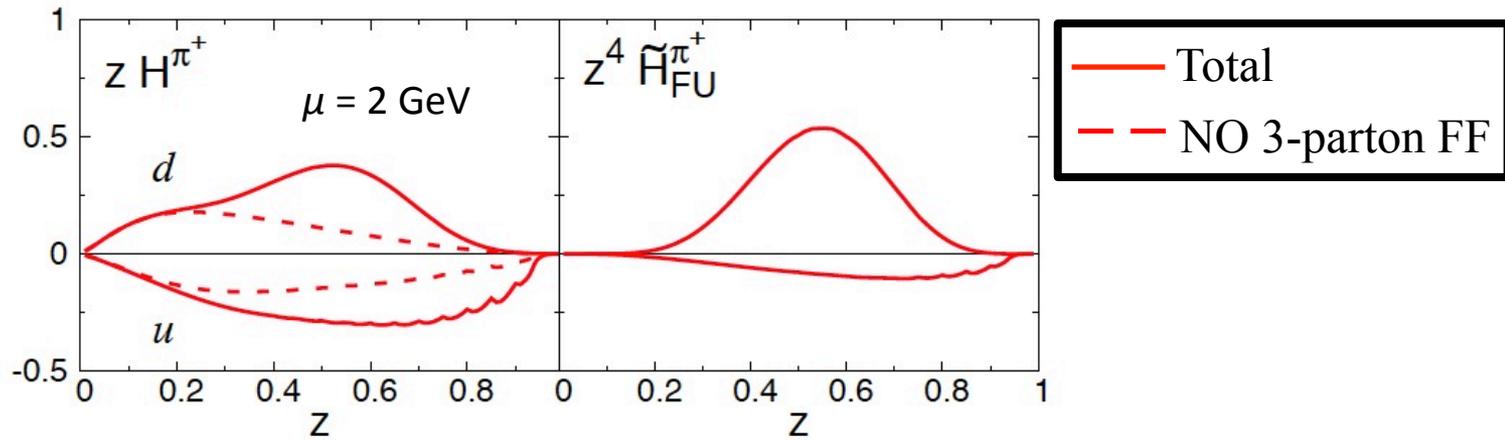
8 free parameters



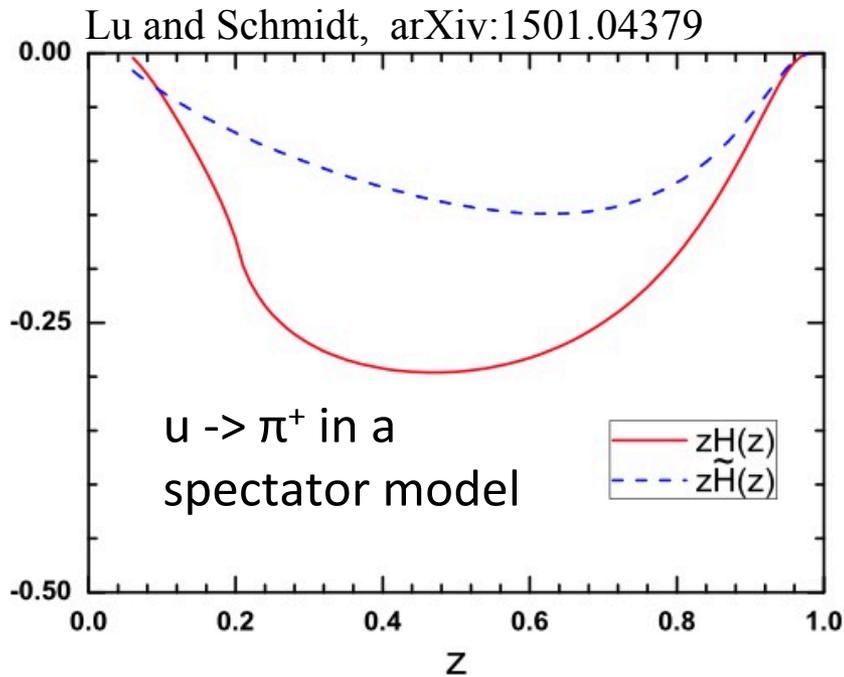
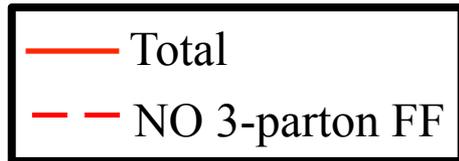
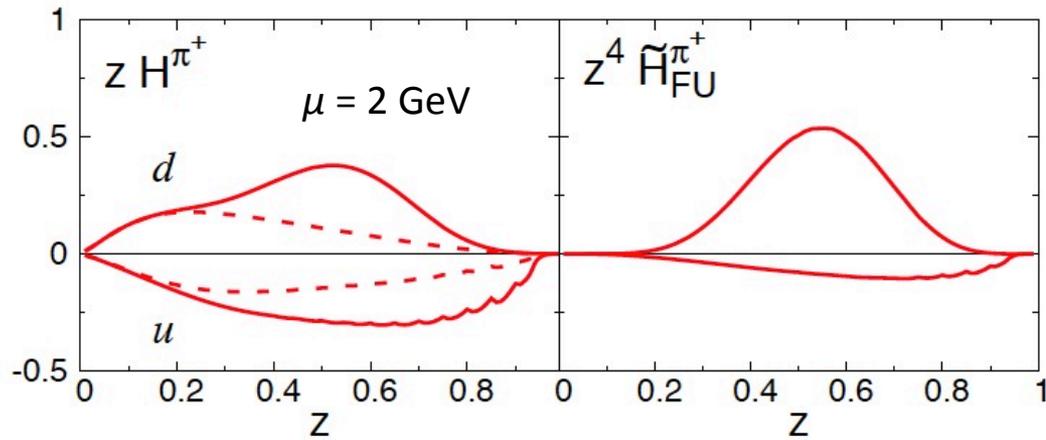
$\chi^2/\text{d.o.f.} = 1.03$



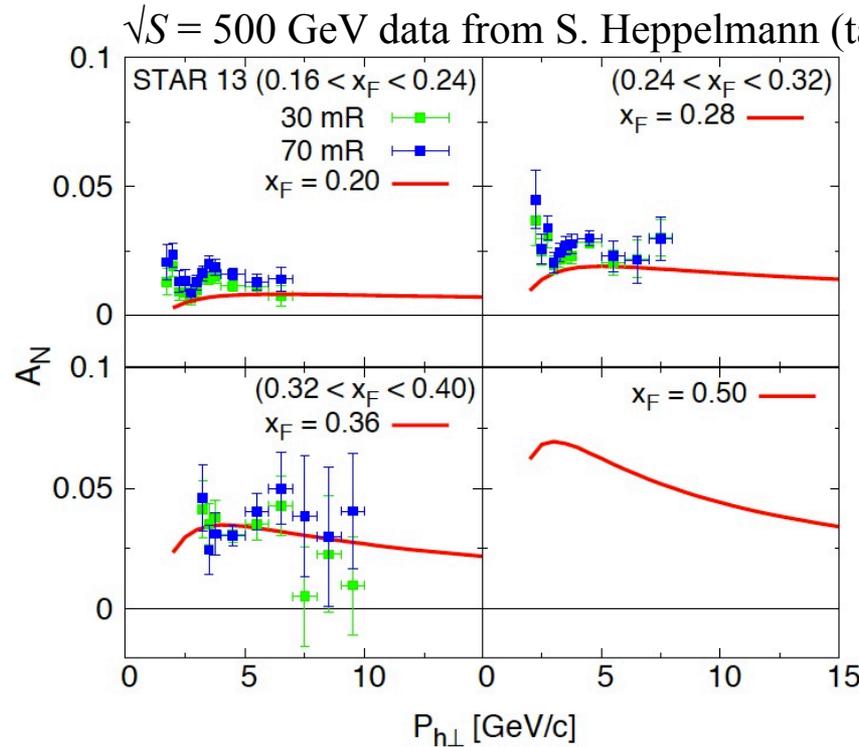
- ➡ Including the (total) fragmentation term leads to very good agreement with the RHIC data, especially with its characteristic rise towards large  $x_F$
- ➡ Without the 3-parton FF, one has difficulty describing the RHIC data
- ➡  $H$  term dominates the asymmetry



➔ Favored and disfavored collinear twist-3 FFs are roughly equal in magnitude but opposite in sign



Note:  $\tilde{H}(z)$  is not defined with  $1/(1-z/z_1)$  factor in the integral



— Theory

Note: 500 GeV data  
was NOT included  
in the fit

➔ Our analysis shows a flat  $P_{h\perp}$  dependence for  $A_N$  seen so far at RHIC ➔ remains flat even to larger  $P_{h\perp}$  values



- TSSA in  $e N^\uparrow \rightarrow \pi X$   
 (Gamberg, Kang, Metz, DP, Prokudin - PRD 90 (2014)) (see talk by A. Metz)

$$\begin{aligned}
 P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = & -\frac{8\alpha_{\text{em}}^2}{S} \varepsilon_{\perp\mu\nu} S_{P_\perp}^\mu P_{h\perp}^\nu \sum_q e_q^2 \int_{z_{\text{min}}}^1 \frac{dz}{z^3} \frac{1}{S+T/z} \frac{1}{x} \\
 & \times \left\{ -\frac{\pi M}{\hat{u}} D_1^{h/q}(z) \left( F_{FT}^q(x, x) - x \frac{dF_{FT}^q(x, x)}{dx} \right) \left[ \frac{\hat{s}(\hat{s}^2 + \hat{u}^2)}{2\hat{t}^3} \right] \right. \\
 & \quad \left. + \frac{M_h}{-x\hat{u} - \hat{t}} h_1^q(x) \left\{ \left( \hat{H}^{h/q}(z) - z \frac{d\hat{H}^{h/q}(z)}{dz} \right) \left[ \frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2} \right] \right. \right. \\
 & \quad \left. \left. + \frac{1}{z} H^{h/q}(z) \left[ \frac{\hat{s}(\hat{s}^2 + (x-1)\hat{u}^2)}{\hat{t}^3} \right] + 2z^2 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{h/q, \mathfrak{S}}(z, z_1) \left[ \frac{x\hat{s}^2\hat{u}}{\xi_z \hat{t}^3} \right] \right\} \right\}
 \end{aligned}$$



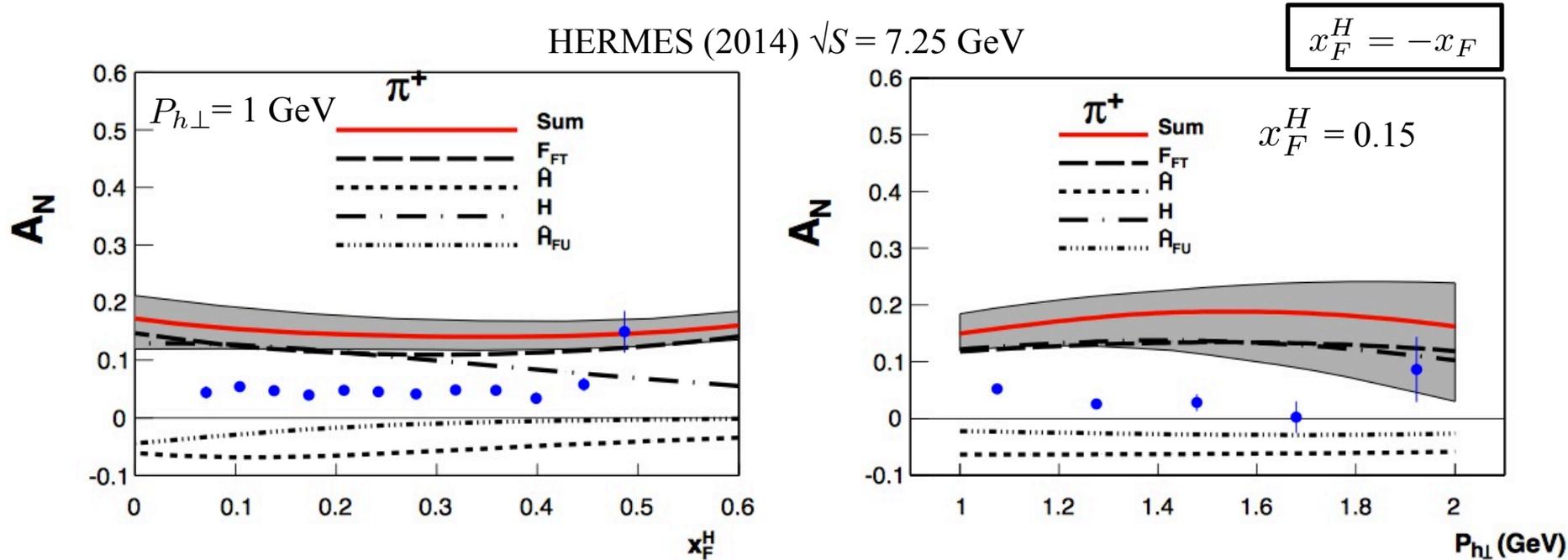
➤ TSSA in  $e N^\uparrow \rightarrow \pi X$   
 (Gamberg, Kang, Metz, DP, Prokudin - PRD 90 (2014))

$$\begin{aligned}
 P_h^0 \frac{d\sigma_{UT}}{d^3\vec{P}_h} = & -\frac{8\alpha_{em}^2}{S} \varepsilon_{\perp\mu\nu} S_{P\perp}^\mu P_{h\perp}^\nu \sum_q e_q^2 \int_{z_{min}}^1 \frac{dz}{z^3} \frac{1}{S+T/z} \frac{1}{x} \\
 & \times \left\{ -\frac{\pi M}{\hat{u}} D_1^{h/q}(z) \left( F_{FT}^q(x, x) - x \frac{dF_{FT}^q(x, x)}{dx} \right) \left[ \frac{\hat{s}(\hat{s}^2 + \hat{u}^2)}{2\hat{t}^3} \right] \right. \\
 & + \frac{M_h}{-x\hat{u} - \hat{t}} h_1^q(x) \left\{ \hat{H}^{h/q}(z) \frac{d\hat{H}^{h/q}(z)}{dz} \left[ \frac{(1-x)\hat{s}\hat{u}}{\hat{t}^2} \right] \right. \\
 & \left. \left. + \frac{1}{z} H^{h/q}(z) \left[ \frac{\hat{s}(\hat{s}^2 + (x-1)\hat{u}^2)}{\hat{t}^3} \right] + 2z^2 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{h/q, \mathcal{S}}(z, z_1) \left[ \frac{x\hat{s}^2\hat{u}}{\xi_z \hat{t}^3} \right] \right\} \right\}
 \end{aligned}$$

Use Siverts function from SIDIS (Anselmino, et al. (2009))

Use Collins function from SIDIS/ $e^+e^-$  (Anselmino, et al. (2013))

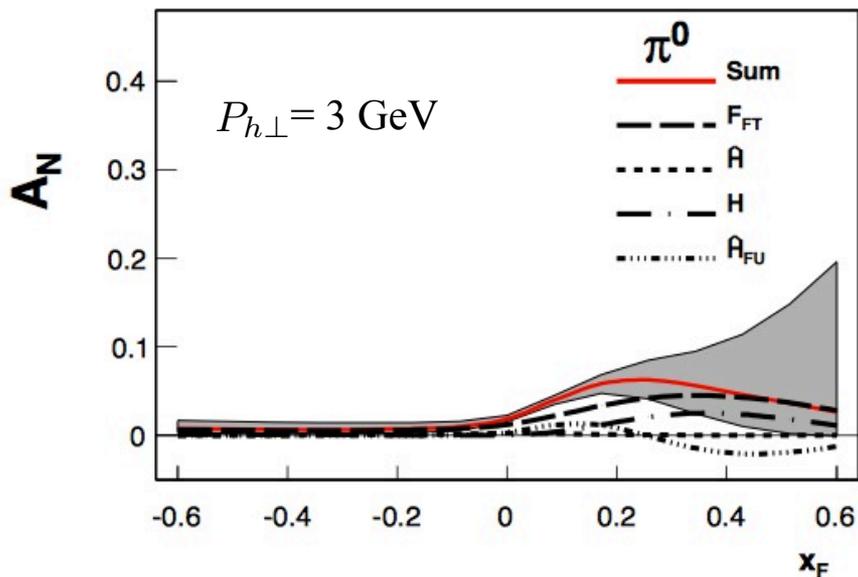
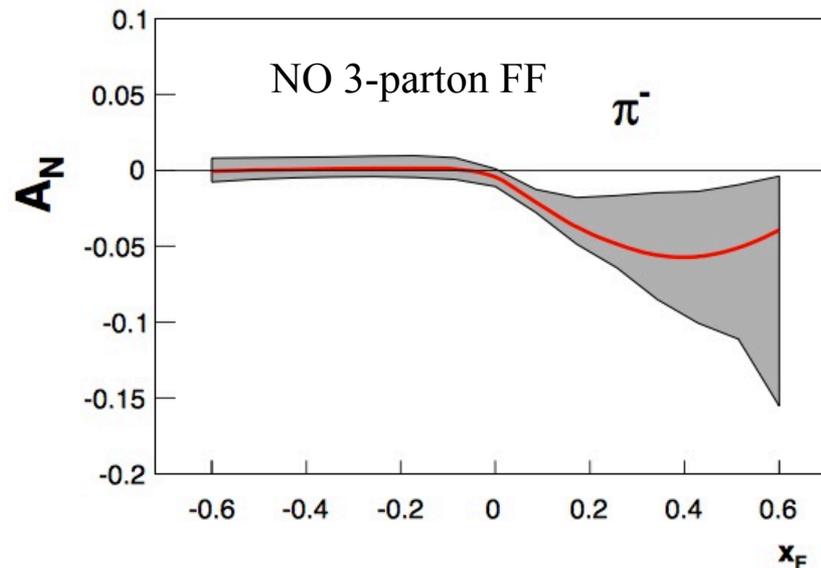
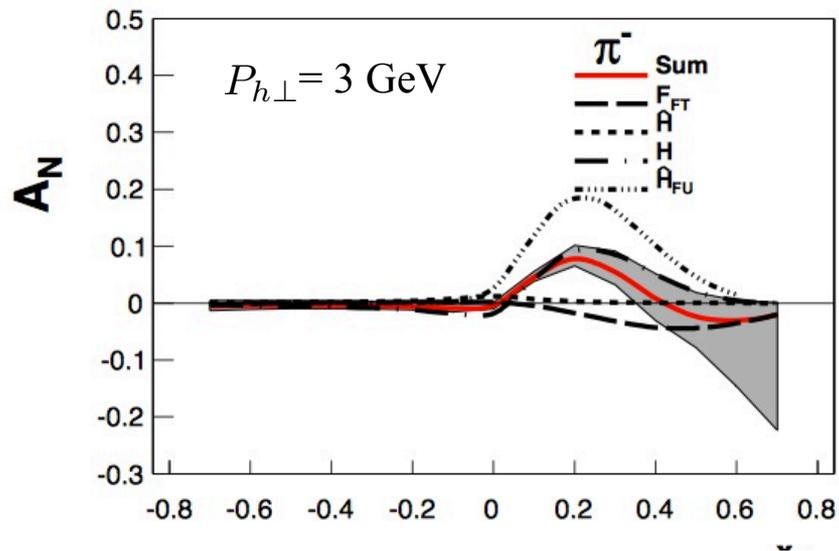
Take from  $pp$  fit KKMP



- Theoretical results are above the data, but NLO calculation most likely needed given that the data are dominated by quasi-real photoproduction
- Jefferson Lab Hall A also has data for a neutron target, but  $P_{h\perp}$  is too low  
 → 12 GeV update will give valuable data at higher  $P_{h\perp}$
- This process can help better constrain the 3-parton FF that has been fitted in  $pp$   
 → crucial to measure at at EIC



EIC  $\sqrt{S} = 63$  GeV



- EIC is a unique position to measure  $A_N$  in the forward region like in  $pp$  collisions
- Clearly nonzero signal ( $\sim 10\%$ ) predicted for  $\pi^0$  for  $x_F > 0$ , like in  $pp$
- Can provide further constraints/tests of the mechanism used to describe  $A_N$  in  $pp$



➤ TSSA in  $p^\uparrow p \rightarrow \gamma X$   
(Kanazawa, Koike, Metz, DP – PRD 91 (2015))

$$E_\gamma \frac{d^3 \Delta \sigma^{\chi-o}}{d^3 \vec{q}} = \frac{\alpha_{em} \alpha_s \pi M_N}{S} \epsilon^{pmqS_\perp} \int \frac{dx'}{x'} \int \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u})$$

$$\times \sum_a e_a^2 \left[ \left( E_F^a(x, x) - x \frac{dE_F^a(x, x)}{dx} \right) h^{\bar{a}}(x') \frac{\hat{\sigma}_1^{\text{SGP}}}{-\hat{t}} + E_F^a(x, x) h^{\bar{a}}(x') \hat{\sigma}_2^{\text{SGP}} \right]$$

$$E_F \sim H_{FU}$$

$$E_\gamma \frac{d^3 \Delta \sigma^{\chi-e, \text{SGP}}}{d^3 \vec{q}} = -\frac{\alpha_{em} \alpha_s \pi M_N}{S N C_F} \epsilon^{pmqS_\perp} \int \frac{dx'}{x'} \int \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u})$$

$$\times \sum_a e_a^2 \frac{1}{-\hat{u}} \left[ \frac{1}{2N} f^{\bar{a}}(x) \hat{\sigma}_{\bar{a}a} - \frac{N}{2} f^g(x) \hat{\sigma}_{ga} \right] \left[ x' \frac{dG_F^a(x', x')}{dx'} - G_F^a(x', x') \right]$$

$$E_\gamma \frac{d^3 \Delta \sigma^{\chi-e, \text{SFP}}}{d^3 \vec{q}} = -\frac{\alpha_{em} \alpha_s \pi M_N}{S 2N} \epsilon^{pmqS_\perp} \int \frac{dx'}{x'} \int \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u})$$

$$\times \sum_a \left[ \sum_b e_a e_b \hat{\sigma}_{ab}^{\text{SFP}} \left\{ G_F^a(0, x') + \tilde{G}_F^a(0, x') \right\} f^b(x) \right.$$

$$+ \sum_b e_a e_b \hat{\sigma}_{ab}^{\text{SFP}} \left\{ G_F^a(0, x') + \tilde{G}_F^a(0, x') \right\} f^{\bar{b}}(x)$$

$$\left. + e_a^2 \hat{\sigma}_{ag}^{\text{SFP}} \left\{ G_F^a(0, x') + \tilde{G}_F^a(0, x') \right\} f^g(x) \right]$$

$$T_F \sim G_F \sim F_{FT}$$

$$\tilde{T}_F \sim \tilde{G}_F \sim G_{FT}$$



➤ TSSA in  $p^\uparrow p \rightarrow \gamma X$   
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$$\times \sum_a e_a^2 \left[ \left( E_F^a(x, x) - x \frac{dE_F^a(x, x)}{dx} \right) h^{\bar{a}}(x') \frac{\hat{\sigma}_1^{\text{SGP}}}{-\hat{t}} + E_F^a(x, x) h^{\bar{a}}(x') \hat{\sigma}_2^{\text{SGP}} \right]$$

New result from this work

$$E_F \sim H_{FU}$$

$$E_\gamma \frac{d^3 \Delta \sigma^{\chi-e, \text{SGP}}}{d^3 \vec{q}} = -\frac{\alpha_{em} \alpha_s \pi M_N}{S N C_F} \epsilon^{pmqS_\perp} \int \frac{dx'}{x'} \int \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u})$$

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Qiu and Sterman (1992);  
Kouvaris, et al. (2006);

Gamberg and Kang (2012)

Gamberg, et al. (2013)

Include fragmentation photons

$$E_\gamma \frac{d^3 \Delta \sigma^{\chi-e, \text{SFP}}}{d^3 \vec{q}} = -\frac{\alpha_{em} \alpha_s \pi M_N}{S 2N} \epsilon^{pmqS_\perp} \int \frac{dx'}{x'} \int \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u})$$

$$\times \sum_a \left[ \sum_b e_a e_b \hat{\sigma}_{ab}^{\text{SFP}} \left\{ G_F^a(0, x') + \tilde{G}_F^a(0, x') \right\} f^b(x) \right.$$

$$+ \sum_b e_a e_b \hat{\sigma}_{ab}^{\text{SFP}} \left\{ G_F^a(0, x') + \tilde{G}_F^a(0, x') \right\} f^{\bar{b}}(x)$$

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Ji, et al. (2006);

Kanazawa and Koike (2011, 2013)

$$T_F \sim G_F \sim F_{FT}$$

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➤ TSSA in  $p^\uparrow p \rightarrow \gamma X$   
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$$E_\gamma \frac{d^3 \Delta \sigma^{\chi-o}}{d^3 \vec{q}} = \frac{\alpha_{em} \alpha_s \pi M_N}{S} \epsilon^{pmqS_\perp} \int \frac{dx'}{x'} \int \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u})$$

$$\times \sum_a e_a^2 \left[ \left( E_F^a(x, x) - x \frac{dE_F^a(x, x)}{dx} \right) h^{\bar{a}}(x') \frac{\hat{\sigma}_1^{\text{SGP}}}{-\hat{t}} + E_F^a(x, x) h^{\bar{a}}(x') \hat{\sigma}_2^{\text{SGP}} \right]$$

New result from this work

$$E_F \sim H_{FU}$$

$$E_\gamma \frac{d^3 \Delta \sigma^{\chi-e, \text{SGP}}}{d^3 \vec{q}} = -\frac{\alpha_{em} \alpha_s \pi M_N}{S N C_F} \epsilon^{pmqS_\perp} \int \frac{dx'}{x'} \int \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u})$$

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Qiu and Sterman (1992);  
Kouvaris, et al. (2006);  
Gamberg and Kang (2012);  
Gamberg, et al. (2013)

$$E_\gamma \frac{d^3 \Delta \sigma^{\chi-e, \text{SFP}}}{d^3 \vec{q}} = -\frac{\alpha_{em} \alpha_s \pi M_N}{S 2N} \epsilon^{pmqS_\perp} \int \frac{dx'}{x'} \int \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u})$$

$$\times \sum_a \left[ \sum_b e_a e_b \hat{\sigma}_{ab}^{\text{SFP}} \left\{ G_F^a(0, x') + \tilde{G}_F^a(0, x') \right\} f^b(x) \right.$$

$$+ \sum_b e_a e_b \hat{\sigma}_{ab}^{\text{SFP}} \left\{ G_F^a(0, x') + \tilde{G}_F^a(0, x') \right\} f^{\bar{b}}(x)$$

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Ji, et al. (2006);  
Kanazawa and Koike (2011, 2013)

$$T_F \sim G_F \sim F_{FT}$$

$$\tilde{T}_F \sim \tilde{G}_F \sim G_{FT}$$

Note: Contribution from tri-gluon correlators calculated by Koike and Yoshida (2012)



➤ TSSA in  $p^\uparrow p \rightarrow \gamma X$   
(Kanazawa, Koike, Metz, DP – PRD 91 (2015))

Use Boer-Mulders function from DY (Barone, et al. (2010))

$$E_\gamma \frac{d^3 \Delta \sigma^{\chi-o}}{d^3 \vec{q}} = \frac{\alpha_{em} \alpha_s \pi M_N}{S} e^{pnqS_\perp} \int \frac{dx'}{x'} \int \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u})$$

$$\times \sum_a e_a^2 \left[ \left( E_F^a(x, x) - x \frac{dE_F^a(x, x)}{dx} \right) h^{\bar{a}}(x') \frac{\hat{\sigma}_1^{\text{SGP}}}{-\hat{t}} + E_F^a(x, x) h^{\bar{a}}(x') \hat{\sigma}_2^{\text{SGP}} \right]$$

Use Siverson function from SIDIS (Anselmino, et al. (2009))

$$E_\gamma \frac{d^3 \Delta \sigma^{\chi-e, \text{SGP}}}{d^3 \vec{q}} = -\frac{\alpha_{em} \alpha_s \pi M_N}{S N C_F} e^{pnqS_\perp} \int \frac{dx'}{x'} \int \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u})$$

$$\times \sum_a e_a^2 \frac{1}{-\hat{u}} \left[ \frac{1}{2N} f^{\bar{a}}(x) \hat{\sigma}_{\bar{a}a} - \frac{N}{2} f^g(x) \hat{\sigma}_{ga} \right] \left[ x' \frac{dG_F^a(x', x')}{dx'} - G_F^a(x', x') \right]$$

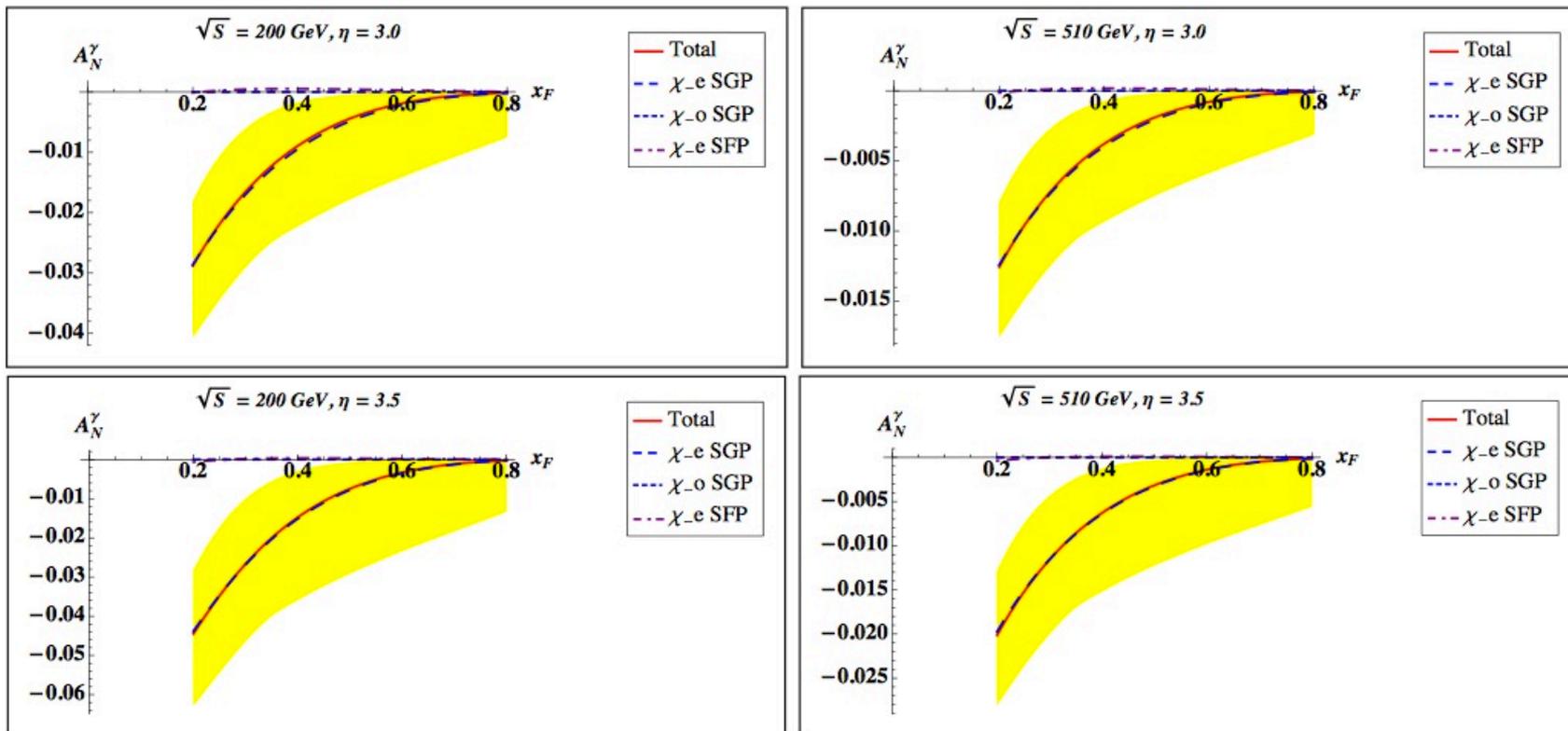
$$E_\gamma \frac{d^3 \Delta \sigma^{\chi-e, \text{SFP}}}{d^3 \vec{q}} = -\frac{\alpha_{em} \alpha_s \pi M_N}{S 2N} e^{pnqS_\perp} \int \frac{dx'}{x'} \int \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u})$$

$$\times \sum_a \left[ \sum_b e_a e_b \hat{\sigma}_{ab}^{\text{SFP}} \left\{ G_F^a(0, x') + \tilde{G}_F^a(0, x') \right\} f^b(x) \right.$$

$$+ \sum_b e_a e_b \hat{\sigma}_{ab}^{\text{SFP}} \left\{ G_F^a(0, x') + \tilde{G}_F^a(0, x') \right\} f^{\bar{b}}(x)$$

$$\left. + e_a^2 \hat{\sigma}_{ag}^{\text{SFP}} \left\{ G_F^a(0, x') + \tilde{G}_F^a(0, x') \right\} f^g(x) \right]$$

Assume  $G_F(0, x') + \tilde{G}_F(0, x') = G_F(x', x')$



- Measurements ongoing by PHENIX and STAR at RHIC
- Sivers-type contribution is dominant, others are negligible
  - ➡ Can “cleanly” extract QS function to help resolve “sign mismatch” issue
  - ➡ Clear measurement of a negative  $A_N$  would be a strong indication on the process dependence of the Sivers function (see also TSSA in inclusive DIS – Metz, et al. (2012), and in jet production from  $A_N \text{DY}$  – Gamberg, Kang, Prokudin (2013))



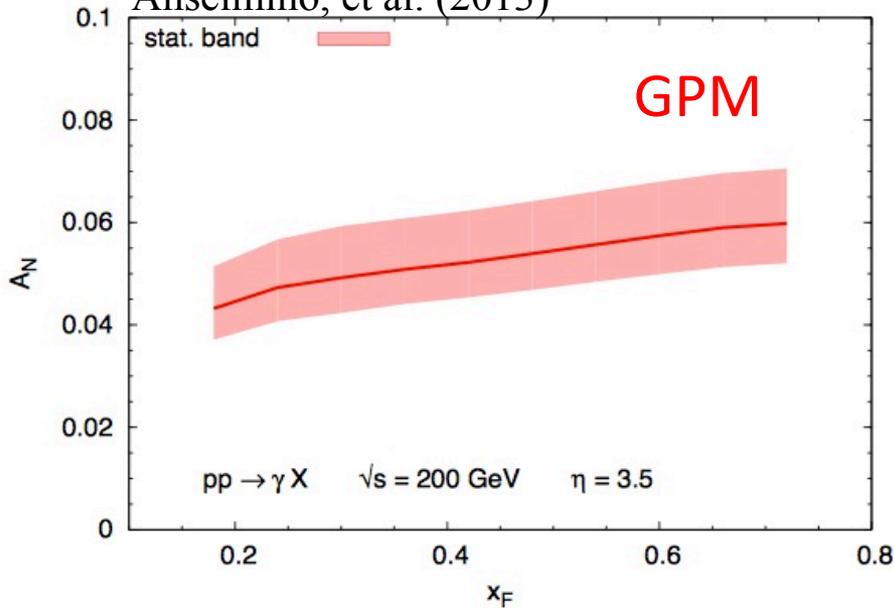
- GPM has been used to calculate  $A_N$  in all of the processes discussed
- How can we distinguish between GPM and twist-3? Which one is “right”?



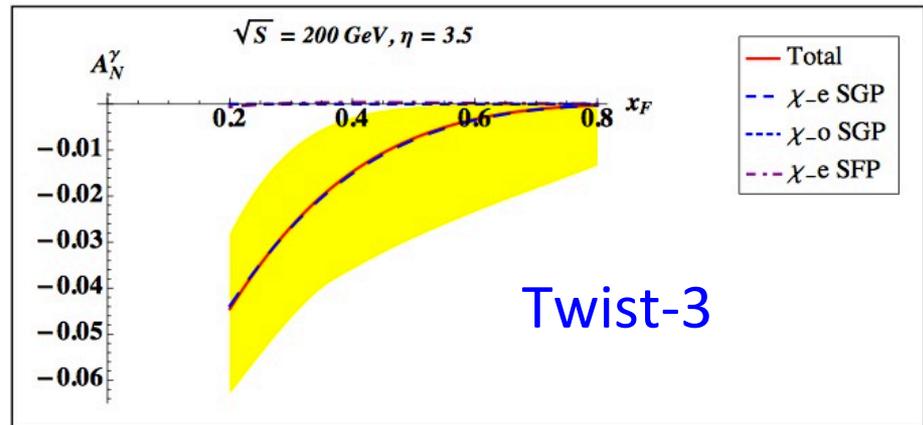
- GPM has been used to calculate  $A_N$  in all of the processes discussed
- How can we distinguish between GPM and twist-3? Which one is “right”?

Answer could be found through  $A_N$  in direct photon production

Anselmino, et al. (2013)



Kanazawa, et al. (2015)



GPM predicts **positive** asymmetry while **twist-3** predicts **negative**



# Summary and outlook

- Collinear twist-3 and GPM both provide frameworks to analyze TSSAs, but the underlying mechanism causing  $A_N$  remained unclear for close to 40 years
- Twist-3 fragmentation could finally give us an explanation
  - Describes RHIC pion data very well
  - Our analysis provides a consistency between spin/azimuthal asymmetries in  $pp$  (collinear) and SIDIS,  $e^+e^-$  (TMD); In particular, the “sign mismatch” is NOT an issue (DO NOT need QS function to be dominant mechanism causing  $A_N$ )
  - Future work: include SFPs (can help with charged hadrons), proper evolution of the 3-parton FF; analyze kaons (BRAHMS), etas (PHENIX), and jets ( $A_N$ DY, STAR)

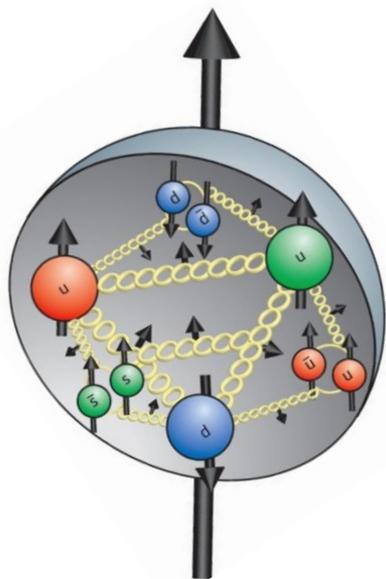


- $e N^\uparrow \rightarrow \pi X$  measurements (both current and future) at HERMES, JLab, COMPASS, and an EIC can provide further tests/constraints
- $p^\uparrow p \rightarrow \gamma X$  (measurements ongoing by PHENIX and STAR) can provide a clean extraction of the QS function, test the process dependence of the Sivers function, and distinguish between the twist-3 and GPM formalisms
- Sivers and Collins asymmetries at large  $P_{h\perp}$  measured in SIDIS at COMPASS, JLab12, and an EIC also can give valuable information
- Proposed fixed target experiment (AFTER) at the LHC plans to look into TSSAs (see Kanazawa, Koike, Metz, DP, arXiv:1502.04021, to appear in a Special Issue of *Advances in High Energy Physics*)



Further measurements of TSSAs in  $pp$  and  $lN$  collisions along with continued theoretical work is crucial in order to understand this fundamental hadronic spin physics phenomenon

Backup slides



$$A_N = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$

- Data tells us (if fragmentation mechanism dominates) that the pions care about the transverse spin of the fragmenting quark → fragment in a particular direction (left or right)
- Small and negative  $x_F$  → probe sea quarks and gluons in  $p^\uparrow$ 
  - ➔  $gg \rightarrow gg$  channel gives large contribution to unpolarized cross section, but NO gluon “transversity” → no such channel in spin-dependent cross section
  - ➔ Little information on sea quark “transversity” → might speculate sea quarks, on average, are less likely to emerge from  $p^\uparrow$  with a transverse spin in a certain direction
- Large  $x_F$  → probe valence quarks in  $p^\uparrow$ 
  - ➔ From SIDIS we know  $u$  quarks ( $d$  quarks) are more likely emerge from  $p^\uparrow$  with their transverse spin aligned (anti-aligned) with  $p^\uparrow$  → pions more likely to fragment in a particular direction (left or right)
  - ➔  $gg \rightarrow gg$  channel dies out in this region → unpolarized cross section becomes smaller



**Distribution term (SGP)**

$$E_\ell \frac{d^3 \Delta \sigma(\vec{s}_T)}{d^3 \ell} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\min}}^1 \frac{dz}{z^2} \underbrace{D_{c \rightarrow h}(z)} \int_{x'_{\min}}^1 \frac{dx'}{x'} \frac{1}{x' S + T/z} \underbrace{\phi_{b/B}(x')}$$

$$\times \sqrt{4\pi\alpha_s} \left( \frac{\epsilon^{\ell s_T n \bar{n}}}{z \hat{u}} \right) \frac{1}{x} \underbrace{T_{a,F}(x, x)} - x \left( \frac{d}{dx} T_{a,F}(x, x) \right) \Big] H_{ab \rightarrow c}(\hat{s}, \hat{t}, \hat{u})$$

**Unpolarized FF (DSS)**
**Unpolarized PDF (GRV98)**

**Fragmentation term**

$$\frac{P_h^0 d\sigma_{pol}}{d^3 \vec{P}_h} = -\frac{2\alpha_s^2 M_h}{S} \epsilon_{\perp \mu\nu} S_{\perp}^{\mu} P_{h\perp}^{\nu} \sum_i \sum_{a,b,c} \int_{z_{\min}}^1 \frac{dz}{z^3} \int_{x'_{\min}}^1 \frac{dx'}{x'} \frac{1}{x' S + T/z} \frac{1}{-x\hat{u} - x'\hat{t}}$$

$$\times \frac{1}{x} \underbrace{h_1^a(x)} f_1^b(x') \left\{ \underbrace{\hat{H}^{C/c}(z)} - z \frac{d\hat{H}^{C/c}(z)}{dz} \right\} S_{\hat{H}}^i + \frac{1}{z} \underbrace{H^{C/c}(z)} S_H^i$$

$$+ 2z^2 \int \frac{dz_1}{z_1^2} PV \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \underbrace{\hat{H}_{FU}^{C/c, \mathfrak{S}}(z, z_1)} \frac{1}{\xi} S_{\hat{H}_{FU}}^i \Big\}$$

**Transversity PDF (Torino13)**

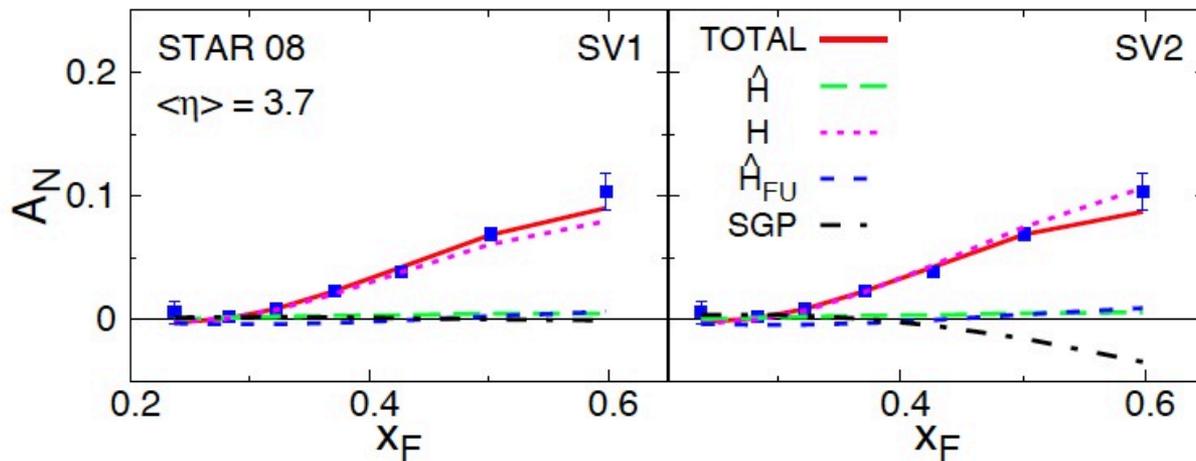
**Recall:**  $H^{h/q}(z) = -2z \hat{H}^{h/q}(z) + 2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{h/q, \mathfrak{S}}(z, z_1)$



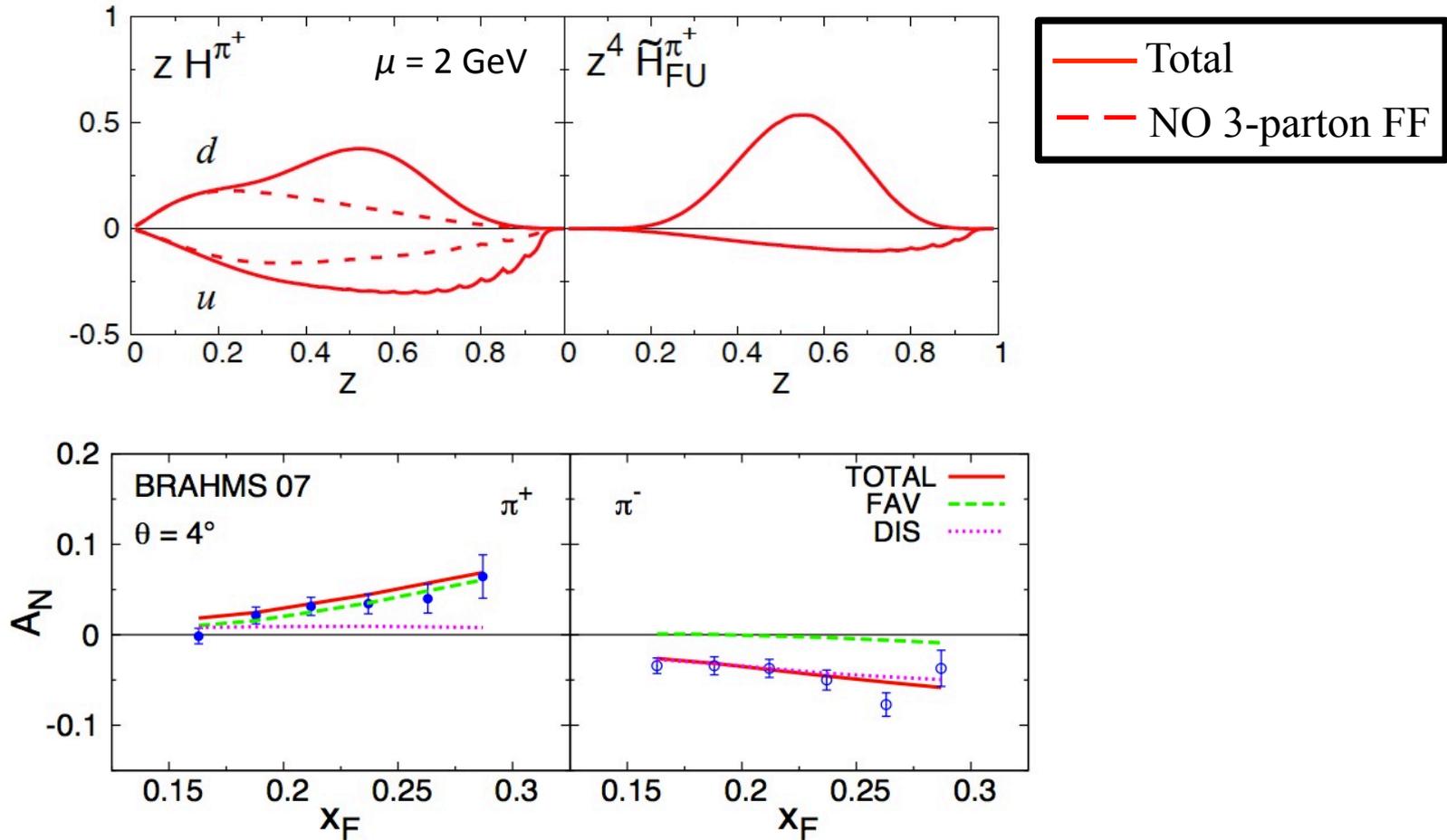
8 free parameters:  $N_{fav}, \alpha_{fav} = \alpha'_{fav}, \beta_{fav}, \beta'_{fav} = \beta'_{dis}$   
 $N_{dis}, \alpha_{dis} = \alpha'_{dis}, \beta_{dis}, \beta_u^T = \beta_d^T$

$\chi^2/d.o.f. = 1.03$	
$N_{fav} = -0.0338$	$N_{dis} = 0.216$
$\alpha_{fav} = \alpha'_{fav} = -0.198$	$\beta_{fav} = 0.0$
$\beta'_{fav} = \beta'_{dis} = -0.180$	$\alpha_{dis} = \alpha'_{dis} = 3.99$
$\beta_{dis} = 3.34$	$\beta_u^T = \beta_d^T = 1.10$

➔ Above parameters are from using 2009 Siverts function (SV1). Using 2013 Siverts function (SV2) gives similar values and  $\chi^2/d.o.f. = 1.10$



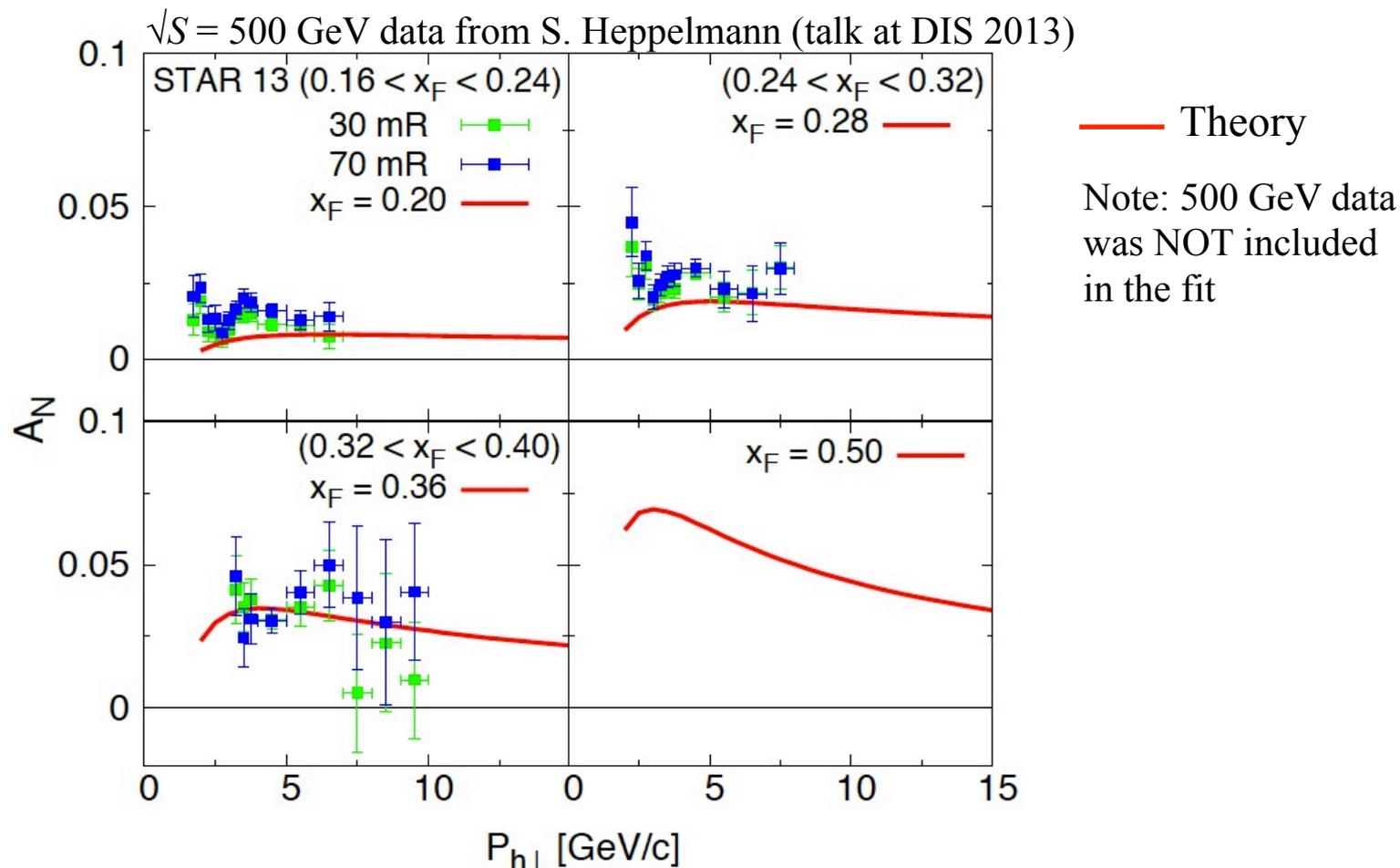
- ➔  $H$  term is dominant; Siverts-type, Collins-type, and  $\hat{H}_{FU}$  terms are negligible
- ➔ SV1 – 2009 Siverts function from Torino group ➔ flavor-*independent* large- $x$  behavior
- ➔ SV2 – 2013 Siverts function from Torino group ➔ flavor-*dependent* large- $x$  behavior and slower decrease at large- $x$  than SV1
  - Including 3-parton FF, one can accommodate such a Siverts function through the  $H$  term
  - Without the 3-parton FF, one would have serious issues handling such a (negative) SGP contribution to obtain a (large) positive  $A_N$



➡ Favored and disfavored (chiral-odd) collinear twist-3 FFs are roughly equal in magnitude but opposite in sign ➔ similar to Collins FF

➡  $A_N$  for  $\pi^+$  ( $\pi^-$ ) dominated by favored (disfavored) fragmentation

- ➔ Flat  $P_T$  dependence thought to be an issue for collinear twist-3 approach ➔  $A_N \sim 1/P_T$
- ➔ First argued by Qiu and Sterman (1998) and later shown by Kanazawa and Koike (2011) that this does not have to be the case



- ➔ Our analysis also shows a flat  $P_T$  dependence for  $A_N$  seen so far at RHIC ➔ remains flat even to larger  $P_T$  values

