Resumming Non-global Logs

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QCD Evolution at JLab 2015

arXiv: 1501.04596

May 27, 2015

Motivation

- Collider observables often do not constrain all of phase-space.
- Such observables have so-called **non-global logarithms (NGLs)**, corresponding to correlated splittings in different phase-space regions [Dasgupta, Salam].
- Resummation has resisted exclusive factorization theorem techniques, relying on evolution equations [Banfi, Marchesini, Smye; Weigert; Caron-Huot] or Monte Carlos [Dasgupta, Salam].
- The pattern of these NGLs at fixed order do not exhibit any straightforward exponentiation, but still have rich structure [Schwartz, Zhu].

Outline

- What is an NGL?
- Isolating phase-space regions that dominate NGLs.
- Exhibit factorization theorems for such phase-space regions.
- From the RG structure of such factorization theorems, resum NGLs.
- Realizing Jets as the quasi-particles of pQCD: integrating jets not gluons to resum more inclusive quantities.

Global Logarithms

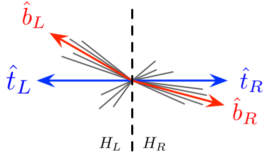
Global Logarithms directly tied to Sudakov resummation from hard production scale down to emission scales.

- $\alpha_s \ln^2 \tau$: thrust in e^+e^- annihilation.
- $\alpha_s \ln^2 \frac{k_t}{Q}$ for transverse momentum distributions.
- Directly tied to soft and collinear IR divergences of QCD.
- Resummation: $\sim \mathrm{Exp}\Big[-\alpha_s \ln^2 \tau\Big]$

Global Logarithms

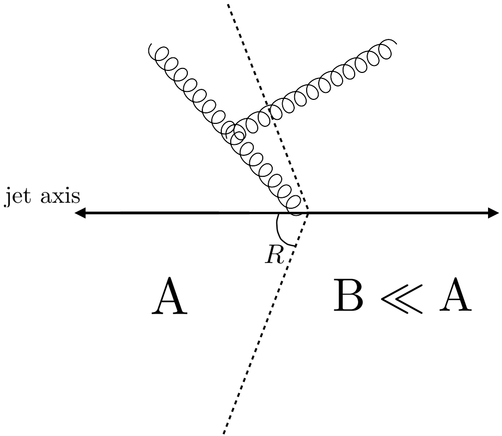
- Resummation controlled by RG evolution equations.
- Consequence of factorization theorems.

$$\frac{d\sigma}{d\tau} = \sigma_0 H \int \delta(\tau - \tau_n - \tau_{\bar{n}} - \tau_s) J_n(\tau_n) J_{\bar{n}}(\tau_{\bar{n}}) S(\tau_s)$$



NGL: Correlated soft emmission.

- Measure different quantities in phase space regions.
- At fixed order, soft configuration giving rise to NGLs:



Apply Global Factorization.

The hard, collinear, and soft factorization of the cross-section:

$$\frac{d\sigma}{dAdB} = H_{n\bar{n}} J_n(A) \otimes J_{\bar{n}}(B) \otimes S_{n\bar{n}}(A,B)$$

- Simply integrates over these soft splittings in $S_{n\bar{n}}$.
- Details of the dynamics leading to the NGL lost.
- No RG equation results to resum $\alpha_s \ln\frac{A}{B}$.
- Large N_c Leading Log NGL expansion:
  $1-\frac{\pi^2}{24}L^2+\frac{\zeta(3)}{6}L^3+\ldots,\ L=\frac{\alpha_s C_A}{\pi}\ln\frac{A}{B}$.

Isolating Soft Jets

To study the NGLs,

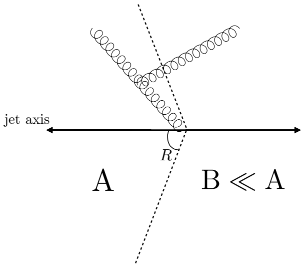
- Need a detailed understanding of the history of soft radiation
- Move from the inclusive cross-section to a more exclusive cross-section.

$$\frac{d\sigma}{dAdB} \to \frac{d\sigma}{de_2^{(\alpha)}de_2^{(\beta)}de_3^{(\beta)}dB}$$

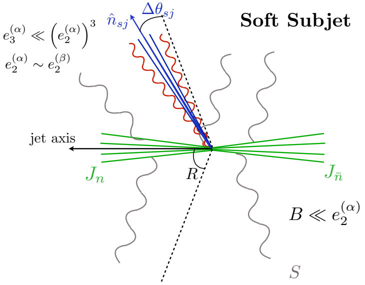
Study its factorization properties.

From Gluons to Jets

Inclusive to exclusive cross-section: Study soft jets, not gluons.



Versus



Isolating Soft Jets

To control the soft jet spectrum: energy-energy correlators:

$$e_2^{(\alpha)} = \frac{1}{E_J^2}\sum_{i<j} E_i E_j \left(\frac{p_i\cdot p_j}{E_i E_j}\right)^{\alpha/2}$$

$$e_3^{(\beta)} = \frac{1}{E_J^3}\sum_{i<j<k} E_i E_j E_k \left(\frac{p_i\cdot p_j}{E_i E_j}\frac{p_j\cdot p_k}{E_j E_k}\frac{p_k\cdot p_i}{E_k E_i}\right)^{\beta/2}$$

[Larkoski, Thaler, Salam]

$d\sigma$... what?? Why so many observables?

We simultaneously impose constraints from $e_2^{(\alpha)}, e_2^{(\beta)}$, and $e_3^{(\beta)}$:

- Inclusive and IRC safe definition of the splitting angle and energy of the subjet.
- $e_2^{(\alpha)}, e_2^{(\beta)} \to z, \theta$.
- $e_3^{(\beta)}$ demarcates resolved partons (jets) from unresolved partons (also called partons).
- Relative scalings of $e_2^{(\alpha)}, e_2^{(\beta)}$ and $e_3^{(\beta)}$ map out 1v2-prong structure

1 v 2 Boundary from Comparing ECFs

Assume 2 prong structure dominates observables:

$$e_2^{(\alpha)} = z_{sj}\Big(n \cdot n_{sj}\Big)^{\alpha/2} + \ldots$$

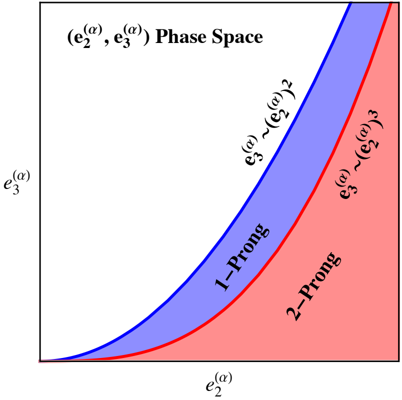
$$e_3^{(\alpha)} = z_{sj}\Big(n \cdot n_{sj}\Big)^{\alpha/2} \sum_k z_k \Big(n \cdot n_k \, n_{sj} \cdot n_k\Big)^{\alpha/2} + \ldots$$

For extra soft partons k:

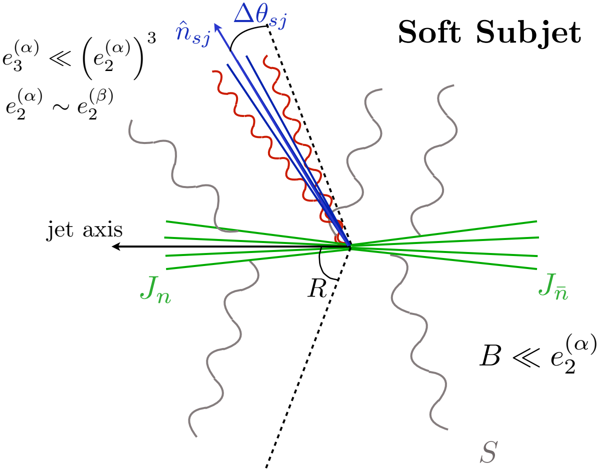
- If $z_{sj} \sim z_k$, $e_3^{(\alpha)} \sim (e_2^{(\alpha)})^2$
- If $z_{sj}^2 \sim z_k$, $e_3^{(\alpha)} \sim (e_2^{(\alpha)})^3$

Jet phase-space

Division of jets in e_2, e_3 phase-space:



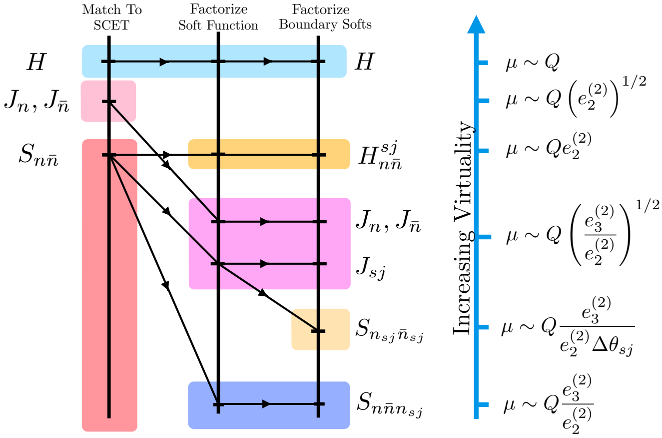
Where Are We?



$\Delta\theta_{sj}$
$\hat{n}_{sj}$
Soft Subjet
$e_3^{(\alpha)} \ll \left(e_2^{(\alpha)}\right)^3$
$e_2^{(\alpha)} \sim e_2^{(\beta)}$
jet axis
J_n
R
$J_{\bar{n}}$
$B \ll e_2^{(\alpha)}$
S

Soft Subjet Factorization in SCET

$e^+e^- \to 2_j + 1_{sj}$:



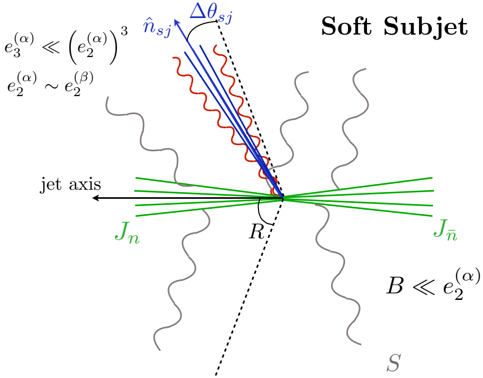
Soft Subjet Factorization

$e^+e^- \to 2_j + 1_{sj}$:

$$\frac{d\sigma}{de_2^{(\alpha)} de_2^{(\beta)} de_3^{(\beta)} dB} = H_{n\bar{n}} H_{n\bar{n}}^{sj}(e_2^{(\alpha)}, e_2^{(\beta)}) J_{n_{sj}}(e_3^{(\beta)}) \otimes S_{n_{sj}\bar{n}_{sj}}(e_3^{(\beta)}) \otimes S_{n\bar{n}n_{sj}}(e_3^{(\beta)}; B) \otimes J_n(e_3^{(\beta)}) \otimes J_{\bar{n}}(B)$$

- $H_{n\bar{n}}$ creation of initial dijets.
- $H_{n\bar{n}}^{sj}$ creation of soft subjet off $n\bar{n}$ dipole.
- $J_{n_{sj}}$ soft jet collinear modes.
- $S_{n_{sj}\bar{n}_{sj}}$ *boundary soft* modes.
- $S_{n\bar{n}n_{sj}}$ soft modes.

The Role of Boundary Softs



Boundary soft modes sensitive to the soft jet's angular distance to fat-jet boundary $\Delta\theta_{sj}$.

The Role of Boundary Softs

The boundary softs are subtracted from the $S_{n\bar{n}n_{sj}}$ function.

- Following the zero-bin procedure of [Manohar, Stewart].
- Removes $\Delta\theta_{sj}$ from the "in-jet" region of $S_{n\bar{n}n_{sj}}$.
- Does *not* remove $\Delta\theta_{sj}$ from the "out-jet" region (scaleless).
- Anom. Dim. dependence on $\Delta\theta_{sj}$ cancels between "out-jet" region and boundary soft modes.
- Thus resums a hierarchy of *soft* energy scales, with support in differing angular regions of phase-space.

Connecting to Inclusive Cross-Section

Having resummed an NGL in exclusive cross-section.

- Return to inclusive cross-section by laplace transforms:

$$\frac{d\sigma}{dz d\theta d\tilde{e}_3^{(\beta)} dB} = \int_0^\infty de_3^{(\beta)}\, e^{-\tilde{e}_3^{(\beta)} e_3^{(\beta)}} \frac{d\sigma}{dz d\theta de_3^{(\beta)} dB}$$

- And marginalizing:

$$\frac{d\sigma}{dAdB} = \int dz d\theta\, \delta\Big(A - F(z,\theta)\Big) \lim_{\tilde{e}_3^{(\beta)} \to 0} \frac{d\sigma}{dz d\theta d\tilde{e}_3^{(\beta)} dB}$$

This is the one soft sub-jet contribution to $\frac{d\sigma}{dAdB}$.

Dressing the Gluon as a Soft Jet.

Re-associate elements of the factorization theorem using their RG equations [Hornig, Lee, Walsh, Zuberi]:

$$W_{n\bar{n}}(z,\theta) = \lim_{\tilde{e}_3^{(\beta)}\to 0} H_{n\bar{n}}^{sj}(z,\theta) J_{n_{sj}}(\tilde{e}_3^{(\beta)}) S_{n_{sj}\bar{n}_{sj}}(\tilde{e}_3^{(\beta)}) \frac{S_{n\bar{n}n_{sj}}(\tilde{e}_3^{(\beta)})}{S_{n\bar{n}}(\tilde{e}_3^{(\beta)})}\Big|_{in}$$

$$G_{n\bar{n}n_{sj}}(B) = \lim_{\tilde{e}_3^{(\beta)}\to 0} \frac{S_{n\bar{n}n_{sj}}(\tilde{e}_3^{(\beta)};B)}{S_{n\bar{n}}(\tilde{e}_3^{(\beta)};B)}\Big|_{out+NG}$$

So:

$$\lim_{\tilde{e}_3^{(\beta)}\to 0} \frac{d\sigma}{dz\,d\theta\,d\tilde{e}_3^{(\beta)}\,dB} = \lim_{\tilde{e}_3^{(\beta)}\to 0} H_{n\bar{n}} W_{n\bar{n}}(z,\theta) G_{n\bar{n}n_{sj}}(B)$$
$$J_n(\tilde{e}_3^{(\beta)}) J_{\bar{n}}(B) S_{n\bar{n}}(\tilde{e}_3^{(\beta)};B)$$

$W_{n\bar{n}}(z,\theta)G_{n\bar{n}n_{sj}}(B)$ *is an RG invariant!*

Resumming the NGLs

Evolving $G_{n\bar{n}n_{sj}}(B)$ resums the NGLs from out-of-jet emmissions off of the soft sub-jet:

$$W_{n\bar{n}}(z,\theta)G_{n\bar{n}n_{sj}}(B) = \frac{\alpha_s C_F}{4\pi^2}\frac{1}{z\sin^2\theta}\left(1-\tan^2\frac{\theta}{2}\right)^{\frac{\alpha_s C_A}{\pi}\ln\left(\frac{\mu}{B}\right)}$$

Connecting to Inclusive Cross-Section: Thrust

$$S^{(1)}(t_H,t_L)=\int [d^dp]_+\frac{n\cdot\bar{n}}{n\cdot p\,p\cdot\bar{n}}\theta(t_H-n\cdot p)\theta(\bar{n}\cdot p-n\cdot p)$$
$$\left\{\theta(t_L-n\cdot p)+\theta(n\cdot p-t_L)\left(1-\frac{n\cdot p}{\bar{n}\cdot p}\right)^{\frac{\alpha_s C_A}{\pi}\ln\left(\frac{n\cdot p}{t_L}\right)}\right\}$$

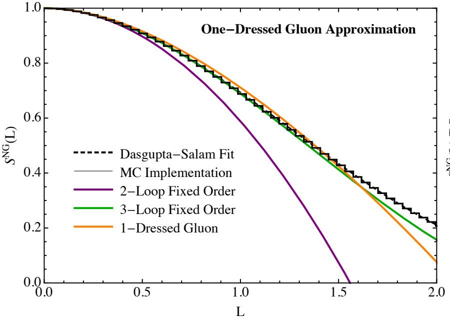
- $A=t_H$ and $B=t_L$.
- Use soft jet description when the factorization is valid: $z\sim t_H>t_L$, and $0\ll\theta<\frac{\pi}{2}$.

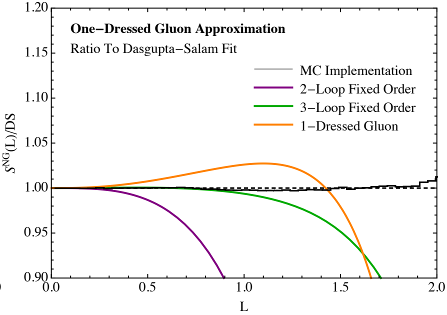
Connecting to Inclusive Cross-Section: Thrust

$$S^{(1)}(t_H, t_L) = \int [d^d p]_+ \frac{n\cdot\bar{n}}{n\cdot p\, p\cdot\bar{n}} \theta(t_H - n\cdot p)\theta(\bar{n}\cdot p - n\cdot p)$$
$$\left\{1 + \theta(n\cdot p - t_L)\left(\left(1 - \frac{n\cdot p}{\bar{n}\cdot p}\right)^{\frac{\alpha_s C_A}{\pi}\ln\left(\frac{n\cdot p}{t_L}\right)} - 1\right)\right\}$$

- "1" is global contribution.
- Remove region where soft subjet is not a soft subjet but an unresolved parton.
- When unresolved, linked to the virtual corrections via KLN.

Single Soft Jet Contribution To Full LL NGLs





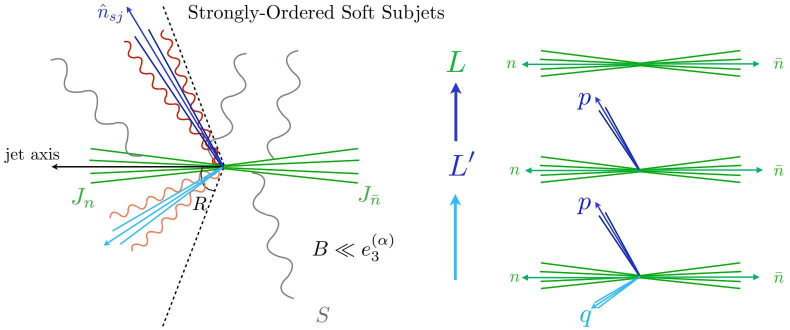
$$L = \frac{\alpha_s C_A}{\pi} \ln \frac{t_H}{t_L}$$

Single Soft Jet NGL contribution

Begins to break down at $L \sim 1$.

- A single soft jet factorization resums only the NGLs originating off of the jet.
- Does not resum NGLs of $\frac{e_3^{(\beta)}}{B}$.
- The inclusive hemi-sphere thrust distribution includes contributions from potentially arbitrary number soft subjets.

Iterating Soft jet factorization.



Sequence of EFTs [Bauer,Schwartz; Baumgart, Marcantonini, Stewart] to improve NGL resummation:

$$0_{sj} \to 1_{sj} \to 2_{sj}$$

Including arbitrary subjets.

Need a general factorization formula for $N \to N + m_{sj}$ process. At two soft subjets, we have:

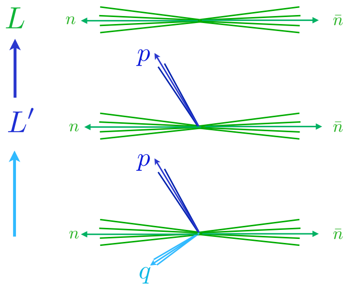
$$\frac{d\sigma(B_N)}{dz_2 d\Omega_1\, dz_2 d\Omega_2\, de_{res}} = \mathrm{tr}\left[\mathbf{H}_{n12\bar{n}}\mathbf{S}_{n12\bar{n}}\mathbf{H}^{\dagger}_{n12\bar{n}}\right]\tilde{J}_1\tilde{J}_2\, J_n\, J_{\bar{n}}$$

- $\mathbf{H}_{n12\bar{n}}$ can be expanded in strongly ordered limit (iteration).
- Diagonalize color trace by going to large-N_c limit.
- Remove global contributions from original di-jet factorization ($S_{n\bar{n}}$).

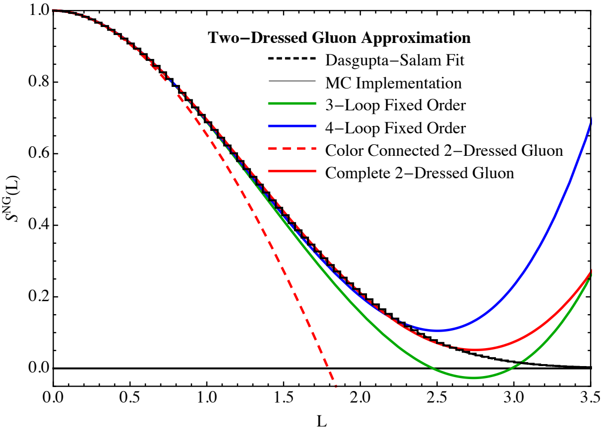
Jets as the quasi-particles of pQCD

In order to resum the NGLs of an inclusive cross-section:

- Reorganize perturbative series as an expansion in **jets (quasi-particles)**, not partons.
- Define jets through physical observables: z, θ, e_{res}
- Factorizations in corners of phase-space resum NGLs.
- Keep quasi-particle description to regions where it is valid.
- Sum over the possible jet histories.



Two Soft Jet Contribution



$$L = \frac{\alpha_s C_A}{\pi} \ln \frac{t_H}{t_L}$$

Soft Jet Expansion Versus Evolution equations.

The soft jet expansion can be related to evolution equation resummation of NGLs [Banfi, Marchesini, Smye; Weigert; Caron-Huot]:

$$\partial_L g_{n\bar{n}}(L) = \int_{in} \frac{d\Omega_j}{4\pi} W_{n\bar{n}}(j) \Big\{ U_{n\bar{n}j}(L) g_{nj}(L) g_{j\bar{n}}(L) - g_{n\bar{n}}(L) \Big\}$$

$$U_{n\bar{n}j}(L) = \mathrm{Exp}\left[L \int_{out} \frac{d\Omega_\ell}{4\pi} W_{n\bar{n}}(\ell) - W_{nj}(\ell) - W_{j\bar{n}}(\ell) \right]$$

$$g_{n\bar{n}} = 1 + \sum_{k=1}^{\infty} g_{n\bar{n}}^{(k)}$$

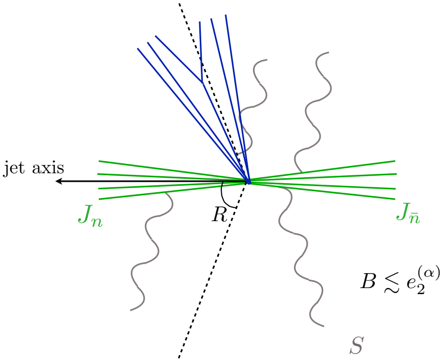
Each $g_{n\bar{n}}^{(k)}$ corresponds to a marginalization over a resummed factorization theorem with k-soft jets with all collinear limits subtracted.

Conclusions

- Resummation of NGLs realize jets as the quasi-particles of perturbative QCD.
- Expansion for NGLs can be mapped to physically observable processes.
- Full NGL resummation must sum over all possible physically realizable histories.
- The diluteness of weak coupling jets and collinear subtractions implies only few such physical processes needed to provide an accurate description for phenomological NGLs.

Directions

- Collinear Splittings at the Jet Boundary.
- Conformal relationship between small-x evolution and NGL [Marchesini, Mueller (2003); Hatta et. al. (2008, 2009)].
- Relationship of exclusive jet factorization theorems to Balitsky's High Energy Effective Action.
- Size of $L \leftrightarrow$ average (sub)jet multiplicity.
- What does truncation of soft jet expansion truncate in?
- Buffer region of k soft jets $\to$ asymptotic behavoir of $g_{n\bar{n}}^{(k)}$ as $L \to \infty$.



At fixed two-loop order, collinear splitting phase space region determines subleading NGLs.
Fixed Order: [Hornig, Lee, Stewart, Walsh, Zuberi (2011); Kelley, Schwartz, Zhu (2011)]