Twist-3 Spin Asymmetries in $\ell N \rightarrow h X$
Studied in Collinear Factorization

(A. Metz, Temple University, Philadelphia)

- Introduction and Motivation
- A related observable: double-spin asymmetry $A_{LT}$ for $\vec{\ell} N^\uparrow \rightarrow \ell X$
- Single-spin asymmetry $A_{UT}$ ($A_N$) for $\ell N^\uparrow \rightarrow h X$
- Single-spin asymmetry $A_{UT}$ ($A_N$) for $\ell N \rightarrow \Lambda^\uparrow X$
- Double-spin asymmetry $A_{LT}$ for $\vec{\ell} N^\uparrow \rightarrow h X$
- Summary

talk mainly based on
arXiv:1411.6459, Kanazawa, A.M., Pitonyak, Schlegel
Introduction and Motivation

1. Data exist for twist-3 spin asymmetries in $\ell N \rightarrow h X$
   - example: $A_N$ for $\ell N^\uparrow \rightarrow h X$ (HERMES, 2013 / JLab Hall A, 2013)

   $$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

   $$x_F = \frac{2P_{hL}}{\sqrt{s}}$$

   (HERMES, 2013)

   - more data: $A_N$ for $\ell N \rightarrow \Lambda^\uparrow X$ (HERMES, 2014)
   - $A_{LT}$ for $\ell N^\uparrow \rightarrow h X$ (JLab Hall A, 2015)

   - can one understand those data, and what can one learn from them?
2. Related asymmetries in processes like $p\,p \rightarrow h\,X$

- example: $A_N$ for $p\,p^\uparrow \rightarrow \pi\,X$ (→ talk by Pitonyak)

(= plot from Aidala, Bass, Hasch, Mallot, 2012)

- data exist for $A_N$ in $p\,p \rightarrow \Lambda^\uparrow\,X$ (Bunce et al, 1976 / ...)

- calculation available for $A_{LT}$ for $p\,p^\uparrow \rightarrow (h, \gamma, \text{jet})\,X$

- it is challenging to reveal the underlying physics of the available data

- maybe the asymmetries for $\,\ell\,N$ help to understand the asymmetries for $p\,p$
3. **Playground to solidify and streamline theory tools**
   (due to small number of Feynman graphs)
   - gauge-invariance of calculation
   - frame-independence of results
   - higher order corrections

4. **Explore potential of those observables for future measurements**
   - EIC should be ideal for future studies
     → measurements possible for large transverse momenta $P_{h,\perp}$
Reminder: double-spin asymmetry $A_{LT}$ for $\vec{\ell}N^\uparrow \rightarrow \ell X$
(see also talk by Meziani)

- Re-scattering of struck quark matters at twist-3 (gluon with physical polarization)

- Contributing correlators after factorization
- collinear quark-quark correlator at twist-3 → \( g_T(x) \)
- \( k_\perp \)-dependent quark-quark correlator → \( \tilde{g}(x) = \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{2M^2} g_{1T}(x, \vec{k}_\perp^2) \)
- (collinear) quark-gluon-quark correlator → \( F_{FT}(x, x_1), G_{FT}(x, x_1) \)

- Exploit relations between functions
  - relation due to QCD equation of motion

\[
x g_T(x) = \int dx_1 \left[ G_{DT}(x, x_1) - F_{DT}(x, x_1) \right]
\]

- Final result

\[
\frac{l''^0 d\sigma_{LT}}{d^3\vec{l}'} = -8 \alpha_{em}^2 x_B^2 \sqrt{1 - y} \frac{M}{Q^5} \lambda_\ell |\vec{S}_\perp| \cos \phi_S \sum_q e_q^2 g_{qT}^q(x_B)
\]

- twist-3 effect
- final result looks rather simple
- comparable twist-3 observables may have more complicated structure
Single-spin asymmetry $A_N$ for $\ell \, N^\uparrow \rightarrow h \, X$

(Gamberg, Kang, A.M., Pitonyak, Prokudin, 2014)

- Feynman diagrams for LO calculation

- twist-3 effects associated with nucleon, and with fragmentation process
- large scale for pQCD calculations provided by $P_{h\perp}$
- in LO formalism, hadron recoils against (undetected) final state lepton $\rightarrow Q^2$ large
Analytical result

\[
P_h^0 \frac{d\sigma_{UT}}{d^3 \vec{P}_h} = -\frac{8\alpha_{em}^2}{S} \varepsilon_{\perp \mu \nu} S_{P \perp}^\mu P_{h \perp}^\nu \sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3} \frac{1}{S + T/z} \frac{1}{x} \frac{\pi M}{\hat{u}} D_1^{h/q}(z) \left( F_{FT}^q(x, x) - x \frac{dF_{FT}^q(x, x)}{dx} \right) \left[ \hat{s}(\hat{s}^2 + \hat{u}^2) \right] \frac{2\hat{t}^3}{}\n
+ \frac{M_h}{-x\hat{u} - \hat{t}} h_1^q(x) \left\{ \left( \hat{H}_{h/q}(z) - z \frac{d\hat{H}_{h/q}(z)}{dz} \right) \left[ (1 - x)\hat{s}\hat{u} \right] \frac{\hat{t}^2}{\right \}

+ \frac{1}{z} H_{h/q}(z) \left[ \frac{\hat{s}(\hat{s}^2 + (x - 1)\hat{u}^2)}{\hat{t}^3} \right] + 2z^2 \int_{z_1}^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{h/q, \perp}(z, z_1) \left[ \frac{x\hat{s}^2\hat{u}}{\xi_z \hat{t}^3} \right] \right\}
\]

Ingredients for numerics

- twist-2 FF $D_1$ (de Florian, Sassot, Stratmann, 2007)
- Qiu-Sterman function $F_{FT}$, from Sivers function $f_{1T}^{\perp}$ (Anselmino et al, 2008)
- transversity $h_1$ (Anselmino et al, 2013)
- twist-3 FF $\hat{H}$, from Collins function $H_{1}^{\perp}$ (Anselmino et al, 2013)
- twist-3 FF $H$ and $\hat{H}^S_{FU}$ enter $A_N$ for $p p^\uparrow \rightarrow h X$ (A.M., Pitonyak, 2012)
use model-independent relation

$$H^{h/q}(z) = -2z\hat{H}^{h/q}(z) + 2z^3 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{z-1} \hat{H}^{h/q, S}_{FU}(z, z_1)$$

and fit to RHIC data for $A_N$ (Kanazawa, Koike, A.M., Pitonyak, 2014)

* good fit can be obtained ($\chi^2/d.o.f = 1.03$)
* $A_N$ dominated by twist-3 FFs (beyond moment of Collins function)
* data on $A_N$ for $\ell N^\uparrow \rightarrow h X$ may allow cross check
Numerical results and discussion

- comparison with HERMES data (JLab data are at very low $P_{h\perp}$)

- error band based on uncertainties of $f_{1T}^\perp$, $h_1$, $H_1^\perp$ only
- relatively poor comparison with data, especially for $\pi^+$ production
- potential reasons for discrepancy:
  1. no error band for twist-3 FF $\hat{H}_{FU}^3$ and hence for FF $H$
  2. (significant) other source(s) for $A_N$ in $p p_{\uparrow} \rightarrow h X$
  3. leading order formalism not appropriate for rather low $P_{h\perp}$ of available data;
    HERMES: even data at highest $P_{h\perp}$ dominated by quasi-real photo-production
    $\rightarrow$ calculation of NLO correction needed

(Hinderer, Schlegel, Vogelsang, arXiv:1505.06415 / $\rightarrow$ talk by Schlegel)
– better comparison with "DIS" sub-set of HERMES data \((Q^2 \geq 1 \text{ GeV}^2)\)

\[
P_{\text{NL}} \text{(GeV)}
\]

\[
A_N
\]

– prospects for measurement at an EIC \(\sqrt{S} = 63 \text{ GeV}, \ P_{h\perp} = 3 \text{ GeV}\)

with \(\hat{H}_{FU}^3\) without \(\hat{H}_{FU}^3\)

\[
A_N
\]

\[
P_{\text{NL}} \text{(GeV)}
\]

\[
A_N
\]

\[
x_F
\]

\[
A_N
\]

\[
x_F
\]

* description of \(A_N\) in \(p p^\uparrow \rightarrow h X\) through twist-3 fragmentation may be checked
Single-spin asymmetry $A_N$ for $\ell N \rightarrow \Lambda^\uparrow X$

(Kanazawa, A.M., Pitonyak, Schlegel, 2015)

- Same Feynman graphs as for $A_N$ in $\ell N^\uparrow \rightarrow h X$

- Analytical results
  - twist-3 distribution contribution

\[
\frac{P_h d\sigma_{\text{Dist}}^{LC}}{d^3 P_h} = \frac{8\pi M \alpha_{\text{em}}^2}{S} \epsilon_{\perp} P_{h\perp} S_{HT} \sum_q e_q^2 \int_{z_{\text{min}}}^{1} \frac{dz}{xz^3} \frac{1}{S + T/z} \frac{1}{\hat{u}} H_1^q(z) \left( x \frac{dH_{FU}^q(x,x)}{dx} \right) \left[ -\frac{s^2 \hat{u}}{\ell^3} \right]
\]

* no non-derivative term (first observed by Zhou, Yuan, Liang, 2008)

- twist-3 fragmentation contribution

\[
\frac{P_h d\sigma_{\text{Frag}}^{LC}}{d^3 P_h} = \frac{2M_h \alpha_{\text{em}}^2}{S} \epsilon_{\perp} P_{h\perp} S_{HT} \sum_q e_q^2 \int_{z_{\text{min}}}^{1} \frac{dz}{xz^3} \frac{1}{S + T/z} \frac{1}{-\hat{t} - x\hat{u}} f_1^q(x)
\]

\[
\times \left[ z \frac{d\hat{D}_T^q(z)}{dz} \hat{\sigma}_D + \hat{D}_T^q(z) \hat{\sigma}_N + \frac{1}{z} \hat{D}_T^q(z) \hat{\sigma}_2 + \int_{z}^{\infty} \frac{dz_1}{z_1^2} \frac{1}{1/z_1 - 1/z} \frac{1}{\xi} \hat{D}_{FT}^q(z_1, z_1) \hat{\sigma}_3 \right]
\]

* (again) contributions from three sources
Discussion
  – identified all relevant parton correlation functions and their relations
  – first complete result in twist-3 collinear factorization
  – results obtained in both light-cone gauge and Feynman gauge
    (for the first time for twist-3 fragmentation contribution to transverse SSA)
  – checked fragmentation contribution to $A_N$ for $\ell N \to h X$ in Feynman gauge
  – numerical estimate needed
  – ingredients available for calculation of $A_N$ for process like $pp \to \Lambda X$
Double-spin asymmetry $A_{LT}$ for $\vec{\ell} N^{↑} \rightarrow h X$

(Kanazawa, A.M., Pitonyak, Schlegel, 2014)

- Same Feynman graphs as before, but somewhat different treatment of kinematics
- Analytical results (calculation in lepton-nucleon cm frame)
  - results obtained in both light-cone gauge and Feynman gauge
  - twist-3 distribution contribution

\[
\frac{P_h^0 d\sigma_{LT}^{\text{Dist}}(\lambda_\ell, \vec{S}_\perp)}{d^3 \vec{P}_h} = -\frac{8\alpha^2_{\text{em}}}{S} M \vec{P}_h \cdot \vec{S}_\perp \lambda_\ell \sum_q e_q^2 \int_{z_{\text{min}}}^1 \frac{dz}{z^3} \frac{1}{S + T/z} \frac{1}{x\hat{u}} D^h_{1/q}(z) \\
\times \left\{ \left( \tilde{g}^q(x) - x \frac{d\tilde{g}^q(x)}{dx} \right) \left[ \frac{\hat{s}(\hat{s} - \hat{u})}{2\hat{t}^2} \right] + x g_T^q(x) \left[ \frac{\hat{u}}{2\hat{t}} \right] + \int dx_1 G_{DT}^q(x, x_1) \left[ \frac{\hat{u}(\hat{s} - \hat{u})}{\xi\hat{t}^2} \right] \right\}
\]

* (again) contributions from three sources
- twist-3 fragmentation contribution

\[
\frac{P_h^0 d\sigma_{LT}^{\text{Frag}}(\lambda_\ell, \vec{S}_\perp)}{d^3 \vec{P}_h} = -\frac{8\alpha^2_{\text{em}}}{S} M_h \vec{P}_h \cdot \vec{S}_\perp \lambda_\ell \sum_q e_q^2 \int_{z_{\text{min}}}^1 \frac{dz}{z^3} \frac{1}{S + T/z} \frac{1}{z\hat{t} h_1^q(x) E^{h/q}(z) \left[ -\frac{\hat{s}}{\hat{t}} \right]}
\]

* final result becomes rather simple
- Comparison between lepton-nucleon cm-frame and nucleon-hadron cm-frame
  - agreement after taking into account the following relation (LIR):
    (Bukhvostov, Kuraev, Lipatov, 1984 / ...)
    \[ g_T(x) = g_1(x) - \frac{2}{x} \int dx_1 \frac{1}{\xi} G_{DT}(x, x_1) \]
  - matters here only for twist-3 distribution contribution
  - LIR also allows one to simplify the final result

\[ \frac{P_h^0 d\sigma_{LT}^{Dist}(\lambda, \bar{S})}{d^3 \vec{P}_h} = -\frac{8 \alpha_{em}^2}{S} M \vec{P}_h \cdot \bar{S} \lambda \sum_q e_q^2 \int_{z_{min}}^{1} \frac{dz}{z^3} \frac{1}{S + T/z} \frac{1}{x \hat{u}} D_{1}^{h/q}(z) \]

\[ \times \left\{ \left( \bar{g}_q(x) - x \frac{d\bar{g}_q(x)}{dx} \right) \left[ \frac{\hat{s}(\hat{s} - \hat{u})}{2\hat{t}^2} \right] + x g_1^q(x) \left[ -\frac{\hat{u}}{\hat{t}^2} \right] + x g_1^q(x) \left[ \frac{\hat{u}(\hat{s} - \hat{u})}{2\hat{t}^2} \right] \right\} \]

- in particular, numerics becomes much easier \((g_1\) instead of \(G_{DT}\))
• Numerical results and discussion (twist-3 distribution term only)
  – use Wandzura-Wilceck approximation for $g_T$
  – two models for $\tilde{g}$
    (1) WW-type approximation: $\tilde{g}(x) = g_{1T}^{(1)}(x) \approx x \int_x^1 \frac{dy}{y} g_1(y)$
    (2) large $N_c$ analysis suggests: $\tilde{g}(x) \approx -f_{1T}^{(1)}(x)$ (Pobylitsa, 2003)
  – comparison with data from JLab Hall A

* (mostly) agreement in sign
* quantitative description only for $\pi^+$ for WW-type approximation
* but, keep in mind that $P_{h\perp}$ of data certainly too low, errors in input, higher order terms
* in general, $A_{LT}$ may allow one to study the TMD $g_{1T}$
- prediction for COMPASS ($\sqrt{S} = 17.3$ GeV, $P_{h\perp} = 2$ GeV)

- prediction for an EIC ($\sqrt{S} = 63$ GeV)

- $A_{LT}$ becomes very small at higher energies
Summary

- Twist-3 spin asymmetries for $\ell N \rightarrow h X$ are interesting "new" observables
- They may give new insight into non-perturbative parton correlation functions
- They may give new insight into corresponding asymmetries for hadronic collisions
- First data available for three such asymmetries
- LO calculations available for those three asymmetries
  - helped to solidify theory tools (gauge invariance, frame independence)
  - phenomenology is at exploratory stage
- Calculation of higher order terms needed